$$\lim_{x\to 9}\frac{\overset{a}{\sqrt{x}}-\overset{b}{3}}{x-9}\cdot\frac{\overset{(a+b)}{(\sqrt{x}+3)}}{(\sqrt{x}+3)}=\lim_{x\to 9}\frac{(\sqrt{x})^2-3^2}{x-9}\cdot\frac{1}{(\sqrt{x}+3)}$$

$\to 0$, $\to 0$, $=1$, a^2-b^2

$$a^2-b^2=(a-b)(a+b)$$

$$=\lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{x-9}}\cdot\frac{1}{(\sqrt{x}+3)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{6}$$

$\to 6$

$$(\sqrt{x^2+1})^2 = x^2+1$$

$$\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{x} \cdot \frac{(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}+1)}$$

$$= \lim_{x\to 0} \frac{(x^2+\cancel{1})-\cancel{1}}{x\cdot(\sqrt{x^2+1}+1)} = \lim_{x\to 0} \frac{\cancel{x}\cdot x}{\cancel{x}\cdot(\sqrt{x^2+1}+1)}$$

$$= \lim_{x\to 0} \frac{x}{(\sqrt{x^2+1}+1)} = \frac{0}{2} = 0$$

$$\lim_{x\to 2}\frac{\overbrace{\sqrt[3]{x^2+4}}^{a}-\overbrace{2}^{b}}{x-2}\to 0\cdot\frac{\left(\left(\sqrt[3]{x^2+4}\right)^2+\left(\sqrt[3]{x^2+4}\cdot 2\right)+2^2\right)}{\left(\left(\sqrt[3]{x^2+4}\right)^2+\left(\sqrt[3]{x^2+4}\cdot 2\right)+2^2\right)}$$

$a^2 + ab + b^2$

$x-2 \to 0$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$=\lim_{x\to 2}\frac{\left(\sqrt[3]{x^2+4}\right)^3-2^3}{x-2}\cdot\frac{1}{\left(\left(\sqrt[3]{x^2+4}\right)^2+\left(\sqrt[3]{x^2+4}\cdot 2\right)+2^2\right)}$$

$$=\lim_{x\to 2}\frac{x^2+4-8}{(x-2)\left(\left(\sqrt[3]{x^2+4}\right)^2+\sqrt[3]{x^2+4}\cdot 2+2^2\right)}=\lim_{x\to 2}\frac{\overbrace{(x+2)}^{\to 4}\cancel{(x-2)}}{\cancel{(x-2)}\underbrace{\left(\left(\sqrt[3]{x^2+4}\right)^2+\left(\sqrt[3]{x^2+4}\cdot 2\right)+4\right)}_{\to 12}}=\frac{4}{12}=\frac{1}{3}$$

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{\sqrt[3]{x+26}-3} \cdot \frac{(\sqrt{x+3}+2)}{(\sqrt{x+3}+2)} \cdot \frac{g(x)}{g(x)}$$

$\to 0$

$\to 0$

$$g(x) = (\sqrt[3]{x+26})^2 + \sqrt[3]{x+26} \cdot 3 + 3^2 \xrightarrow[x\to 1]{} 9+9+9=27$$

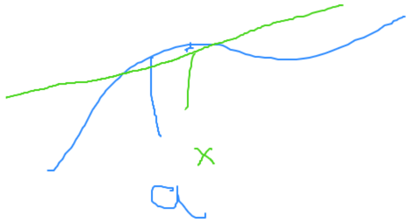
$$= \lim_{x\to 1} \frac{\cancel{(x-1)} \cdot g(x)}{\cancel{(x-1)}(\sqrt{x+3}+2)} = \frac{27}{4}$$

$\to 27$

$\to 4$

$$(\sqrt[3]{x+26})^3 - 27 = x+26-27 = x-1$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$



$$\lim_{x \to a} \frac{x^3 - a^3}{x - a} \qquad f(x) = x^3$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}\left(\overset{a^2}{x^2} + \overset{a^2}{ax} + a^2\right)}{\cancel{(x-a)}} = 3a^2$$

$$g(x) = ax^2 + bx + c \qquad a \neq 0$$

$$g'(x_0) = \lim_{x \to x_0} \frac{g(x) - g(x_0)}{x - x_0}$$

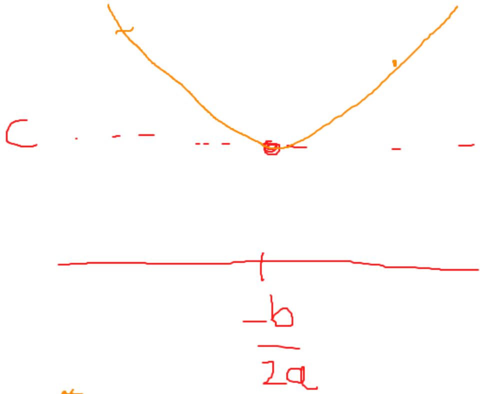
$$= \lim_{x \to x_0} \frac{ax^2 + bx + \cancel{c} - (ax_0^2 - bx_0 - \cancel{c})}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{a(x^2 - x_0) + b(x - x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\cancel{(x - x_0)}[a(x + x_0) + b]}{\cancel{(x - x_0)}}$$

$$= 2ax_0 + b$$

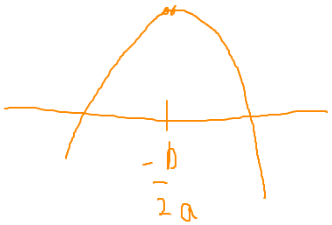
$g'(x_0)=0 \Leftrightarrow x_0 = -\frac{b}{2a}$

$(a>0)$



$a>0$

x		$-\frac{b}{2a}$	
$g'(x)$	$-$	0	$+$



$a<0$

x		$-\frac{b}{2a}$	
$g'(x)$	$+$	0	$-$

$$\lim_{x\to x_0} f(x) = \lim_{h\to 0} f(x_0+h)$$

$\forall \varepsilon > 0 \ \exists \delta > 0$

$0 < |x - x_0| < \delta$

$\Rightarrow |f(x) - f(x_0)| < \varepsilon$

$\forall \varepsilon > 0 \ \exists \delta > 0$

$0 < |h| < \delta$

$\Rightarrow |f(x_0+h) - f(x_0)| < \varepsilon$

$h = x - x_0$

$x = x_0 + h$

$$(x^3)' = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

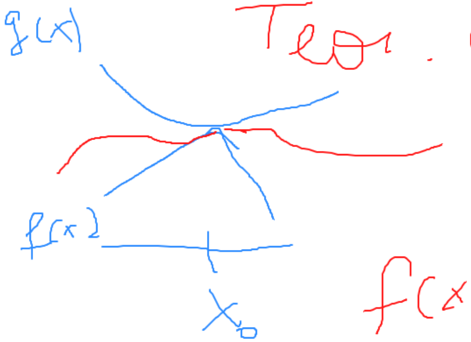
$x > 0$ $$(\sqrt{x})' = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\underbrace{\sqrt{x+h}}_{\to\sqrt{x}} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x \to a} f(x)+g(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{si} \quad \lim_{x \to a} g(x) \neq 0$$



Teor. do confronto

(a.k.a sanduíche).

$f(x) \leq h(x) \leq g(x) \quad \forall x \neq x_0$ próximo a x_0

e $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = L$

então $\lim_{x \to x_0} h(x) = L$

$f(x) \leq h(x) \leq g(x)$

$\lim_{x \to x_0} f(x) = L = \lim_{x \to x_0} g(x)$

Fixa $\varepsilon > 0$

$\exists \delta_1 > 0 \quad 0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$\Rightarrow \underline{L - \varepsilon < f(x)} < L + \varepsilon$

$\exists \delta_2 > 0 \quad 0 < |x - x_0| < \delta_1 \Rightarrow |g(x) - L| < \varepsilon$

$L - \varepsilon < \underline{g(x) < L + \varepsilon}$

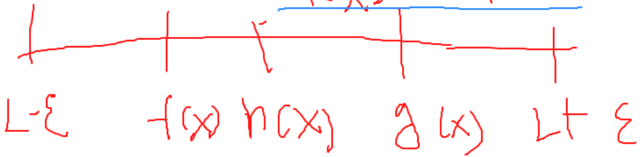
$\delta = \min\{\delta_1, \delta_2\}$

$0 < |x - x_0| < \delta \Rightarrow$ $0 < |x - x_0| < \delta_1 \Rightarrow L - \varepsilon < f(x) \leq h(x)$

$0 < |x - x_0| < \delta_2 \Rightarrow h(x) \leq g(x) < L + \varepsilon$

$\Rightarrow L - \varepsilon < h(x) < L + \varepsilon$

$\Rightarrow |h(x) - L| < \varepsilon$



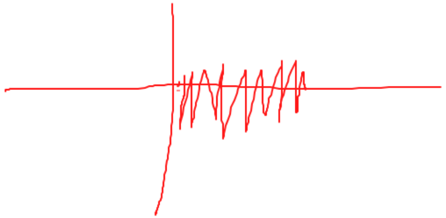
$\therefore \lim_{x \to x_0} h(x) = L$

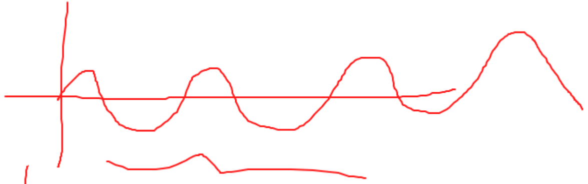
Def: f é limitada em torno de x_0 se $\exists\, M$ e $\delta > 0$ tal que

$$0<|x-x_0|<\delta \Rightarrow |f(x)| \le M$$

e

$$\left\{\operatorname{sen}\frac{1}{x} : x>0\right\}$$





$\operatorname{sen}\frac{1}{x}$ é limitada mas não tem limite

Teo: Se f é limitada em torno de x_0 e $\lim_{x \to x_0} g(x) = 0$

então $\lim_{x \to x_0} f(x) \cdot g(x) = 0$

limitada

limitada

Ex $\lim_{x \to 0} \left(\operatorname{sen}\left(\frac{1}{x}\right) \right) \cdot x = 0$

$$\lim_{x\to 0} \operatorname{sen}\frac{1}{x}\,\overset{g(x)}{(x+1)}$$ se tivesse limite

$$\lim_{x\to 0} g(x) \cdot \frac{1}{x+1}$$ existe, pois $\lim_{x\to 0}\frac{1}{x+1}$ existe

$\lim_{x\to 0}$ sen $\frac{1}{x}$ não existe