Física do spin

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(buscar: física do spin)

Plano do curso

13/03 aula 1: Partículas elementares e idéias da física quântica

20/03 aula 2: Átomo de Bohr, quantização do momento angular

27/03 aula 3: Momento de dipolo magnético, Stern - Gerlach

10/04 aula 4: Efeito Zeeman anômalo

17/04 1ª Prova

24/04 aula 5: Equações de autovalores, matrizes de Pauli

08/05 aula 6: Comutadores, medidas SG sequenciais

15/05 aula 7: Medidas e valores médios

22/05 aula 8: Precessão e adição de spins

29/05 aula 9: Adição de spins

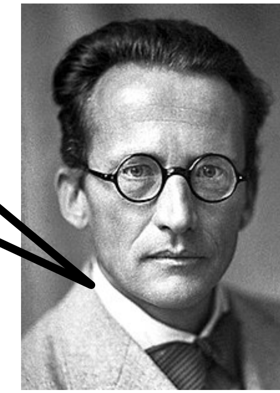
O momento angular é quantizado !

Bohr

Eu chutei que era assim :

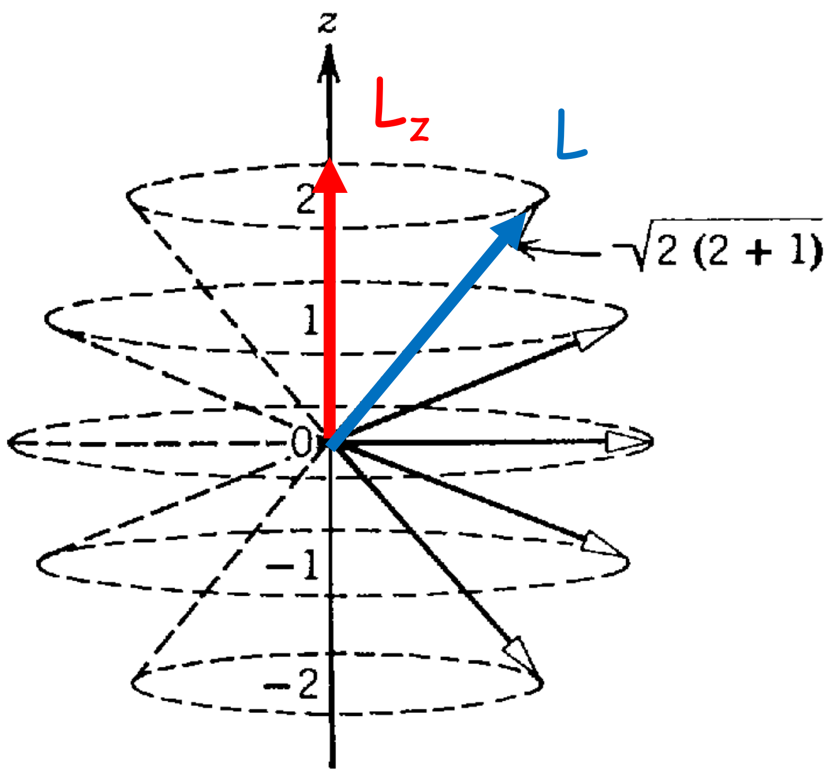
$$L = n\,\hbar$$

Mas tava errado ! Certo é assim:



Schrödinger

"Rotação quântica"



$$L^2 = l\,(\,l\, + \,1\,)\,\hbar^2$$

$$l = 0\,,\;1\,,\;2\,,\;...$$

$$L_z = \,m_l\,\hbar$$

$$-l\;\leq m_l \leq l$$

Aula 3

Momento de dipolo magnético

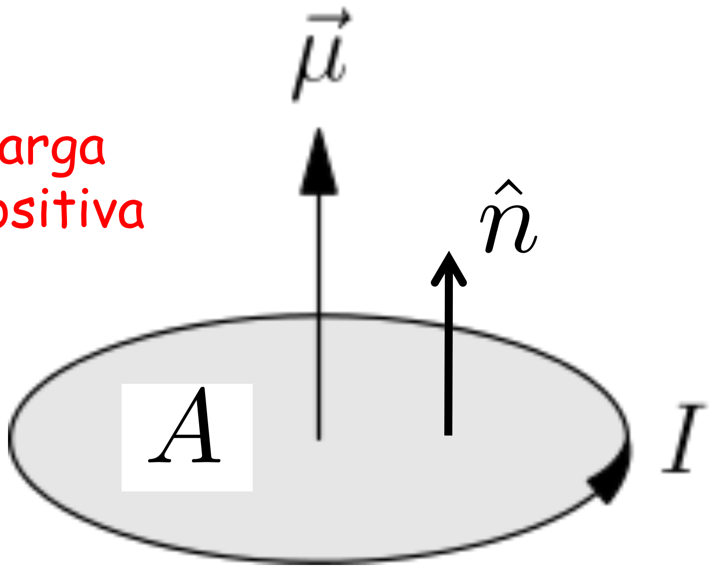
Espira num campo magnético

Experiência de Stern-Gerlach

Momento de dipolo magnético

Espira de corrente

carga positiva

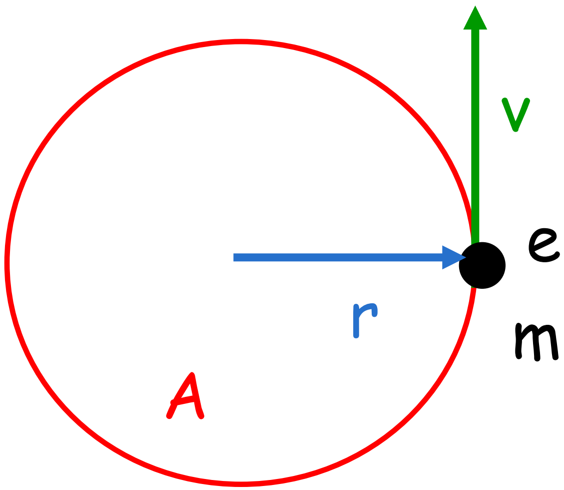


regra da mão direita

$$\vec{A} = A\,\hat{n}$$

$$\vec{\mu} = I\,\vec{A}$$

Momento magnético e momento angular



corrente = carga / tempo

$$i = \frac{e}{T}$$

$$v = \frac{2\pi r}{T} \longrightarrow \frac{1}{T} = \frac{v}{2\pi r}$$

$$A = \pi r^2$$

$$i = \frac{ev}{2\pi r}$$

$$\mu = i A = \frac{e v}{2 \pi r} \pi r^2 = \frac{e v r}{2}$$

$$L = m v r$$

$$\frac{\mu}{L} = \frac{e}{2 m}$$

$$\frac{\mu}{L} = \frac{e}{2\,m}\,\frac{\hbar}{\hbar}$$

$$\mu_b = \frac{e\,\hbar}{2\,m}$$ magneton de Bohr

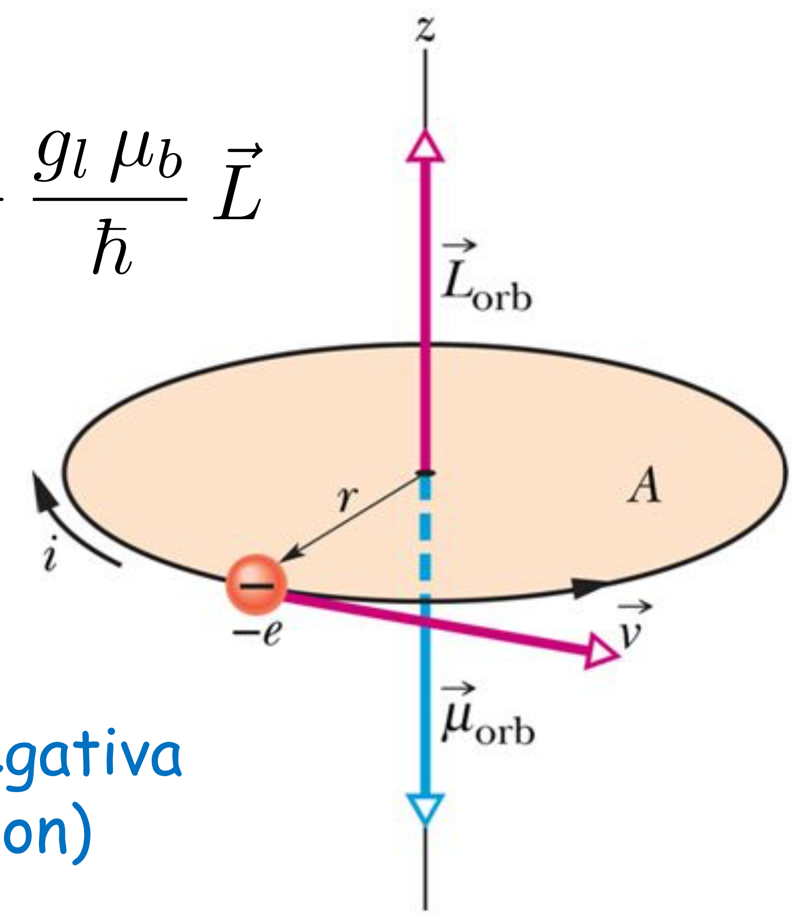
$$g_l = 1$$ fator g orbital

$$\frac{\mu}{L} = \frac{g_l\,\mu_b}{\hbar}$$

$$\vec{\mu} = \frac{g_l\,\mu_b}{\hbar}\,\vec{L}$$

carga positiva

$$\vec{\mu} = -\,\frac{g_l\,\mu_b}{\hbar}\,\vec{L}$$



carga negativa
(eletron)

Momento magnético do elétron é quantizado !

$$L = \sqrt{l\,(l+1)}\,\hbar$$

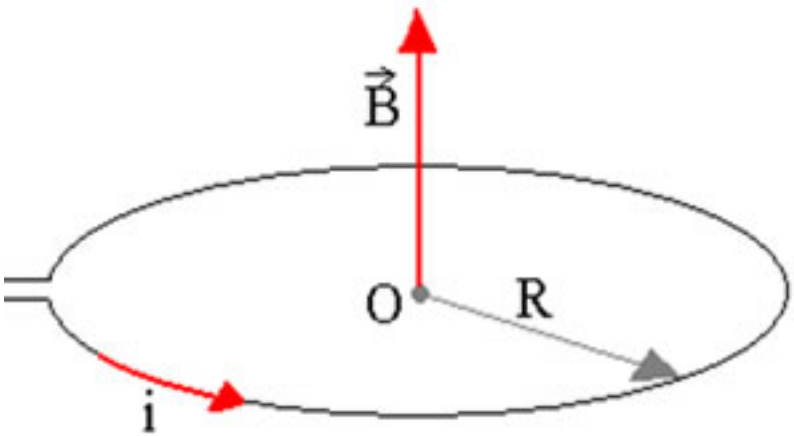
$$\mu = \frac{g_l\,\mu_b}{\hbar}\,L = \frac{g_l\,\mu_b}{\hbar}\sqrt{l\,(l+1)}\,\hbar = g_l\,\mu_b\sqrt{l\,(l+1)}$$

$$\mu_z = \frac{g_l\mu_b}{\hbar}L_z = \frac{g_l\mu_b}{\hbar}m_l\hbar = g_l\mu_b m_l$$

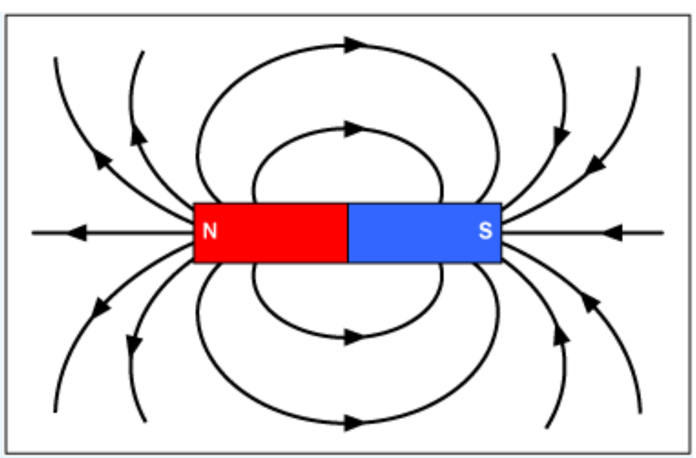
$l = 0\,,\,1\,,\,2\,,\,\ldots$ inteiro

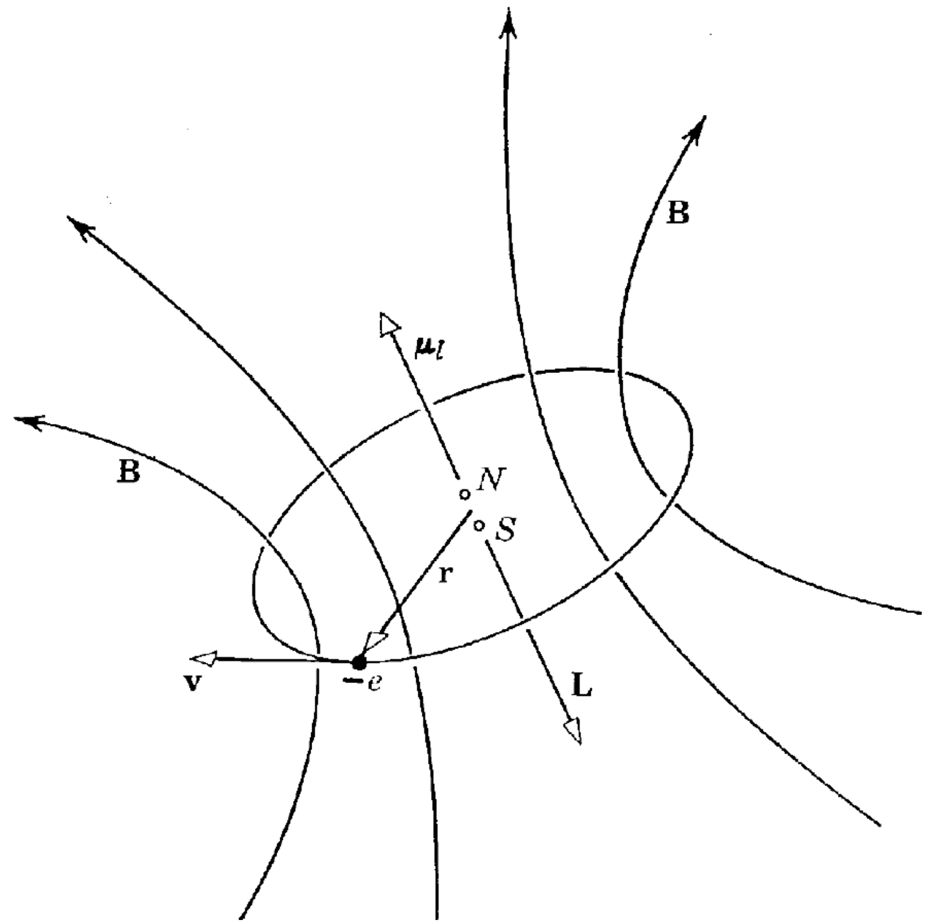
$-l \leq m_l \leq l$ inteiro

Espira é um dipolo magnético



Espira gera campo magnético
semelhante ao de um imã :



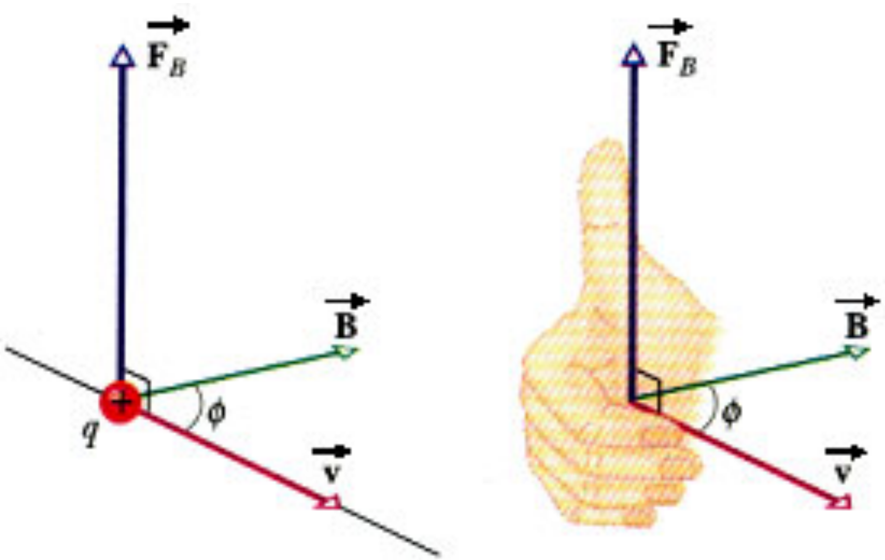


Dipolo magnético !

Espira num campo magnético uniforme

Força magnética
sobre uma carga
em movimento

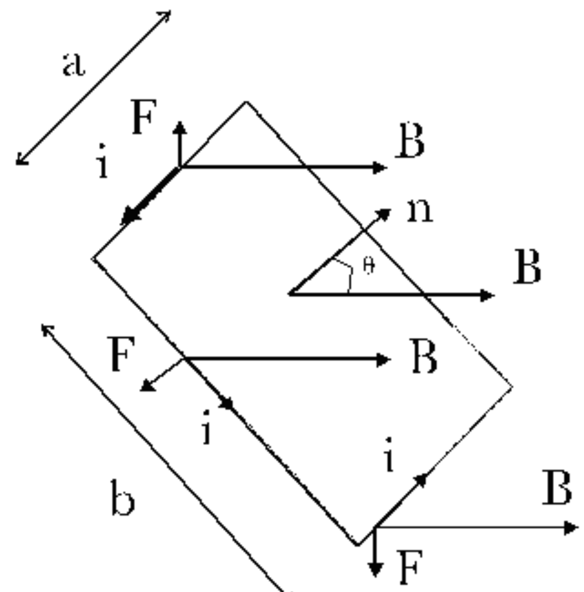
$$\vec{F} = q\,\vec{v} \times \vec{B}$$



Força magnética
sobre uma corrente

$$\vec{F} = I\,\vec{L} \times \vec{B}$$

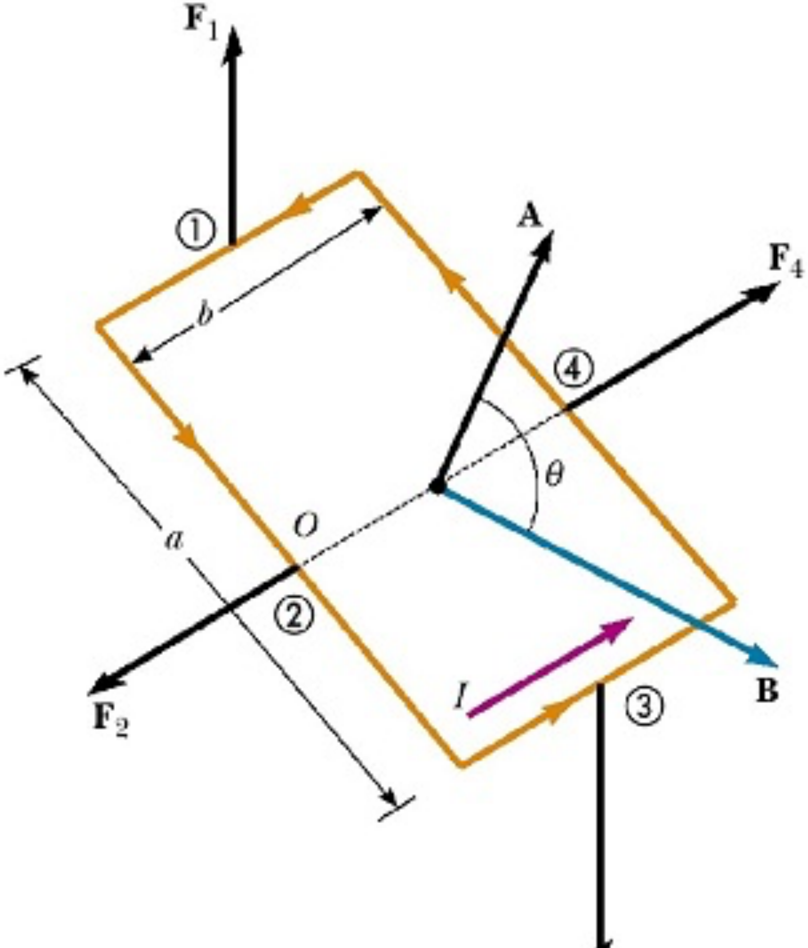
($\vec{L}$ no sentido da corrente)



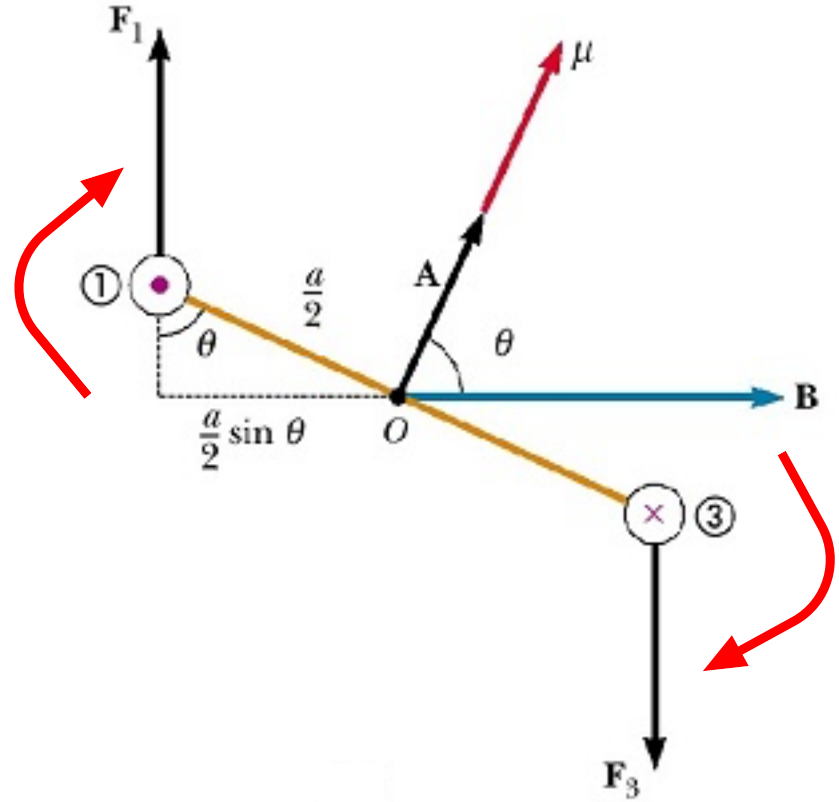
Espira sofre um torque !

$$\vec{\mu} = I\,\vec{A}$$

$$\mu = I\,a\,b$$

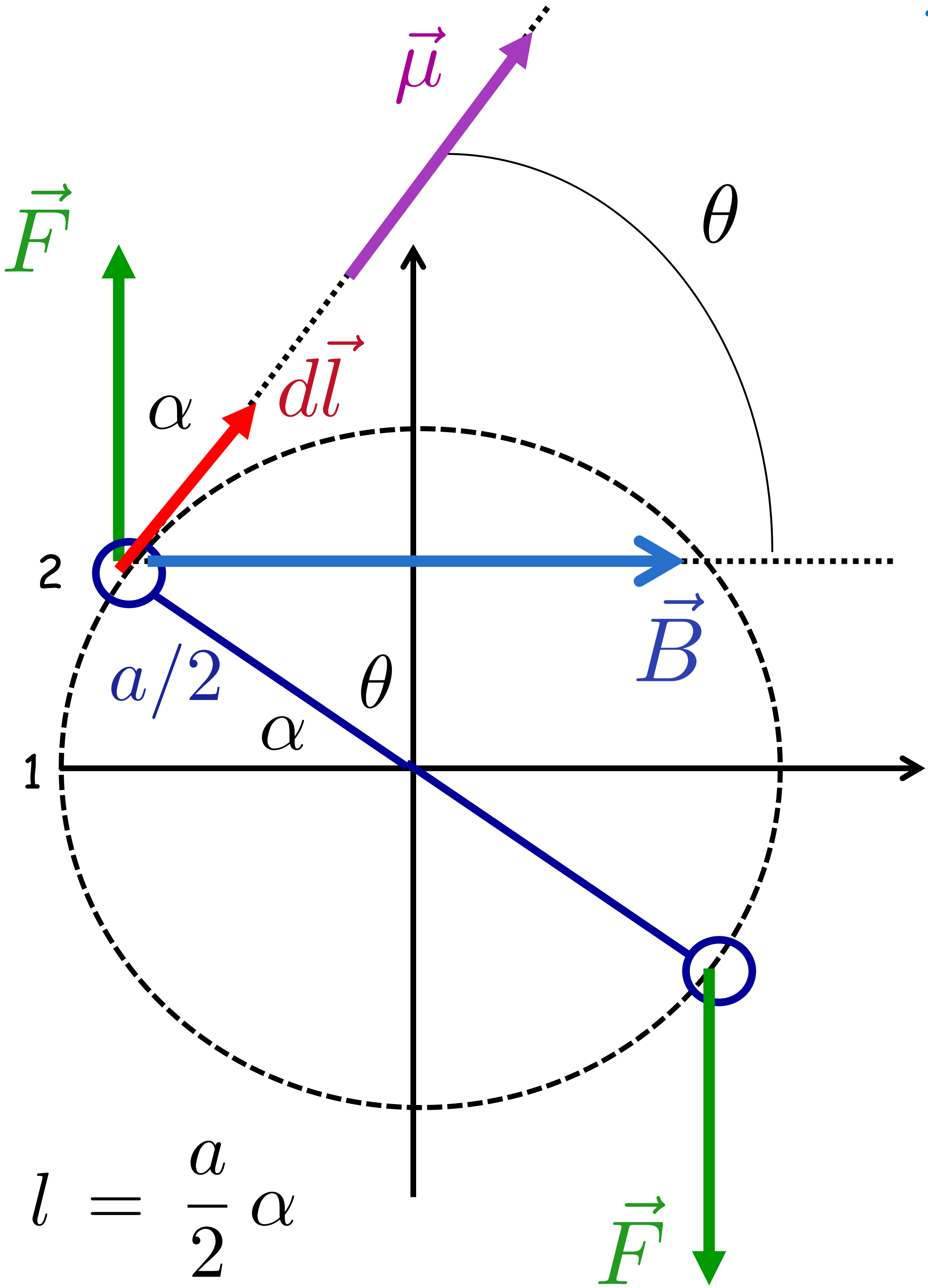


(a)



$$\vec{F} = I\,\vec{L} \times \vec{B}$$

$$F = I\,b\,B$$



Trabalho da força F quando a espira vai de 1 a 2. :

$$dW = \vec{F} \, . \, d\vec{l}$$

$$= F \, dl \, cos\alpha$$

$$= F \, \frac{a}{2} \, d\alpha \, cos\alpha$$

$$W = \int_0^{\alpha} F \, \frac{a}{2} \, cos\,\alpha \; d\alpha$$

$$W = F \, \frac{a}{2} \, sen\,\alpha$$

$$W = F \, \frac{a}{2} \, cos\,\theta$$

$$W = F\,\frac{a}{2}\,cos\,\theta$$

Somando a contribuição da barra inferior (3) :

$$W = F\,a\,cos\,\theta$$

$$F = I\,b\,B$$

$$W = I\,a\,b\,B\,cos\,\theta$$

$$\mu = I\,a\,b$$

$$W = \mu\,B\,cos\,\theta$$

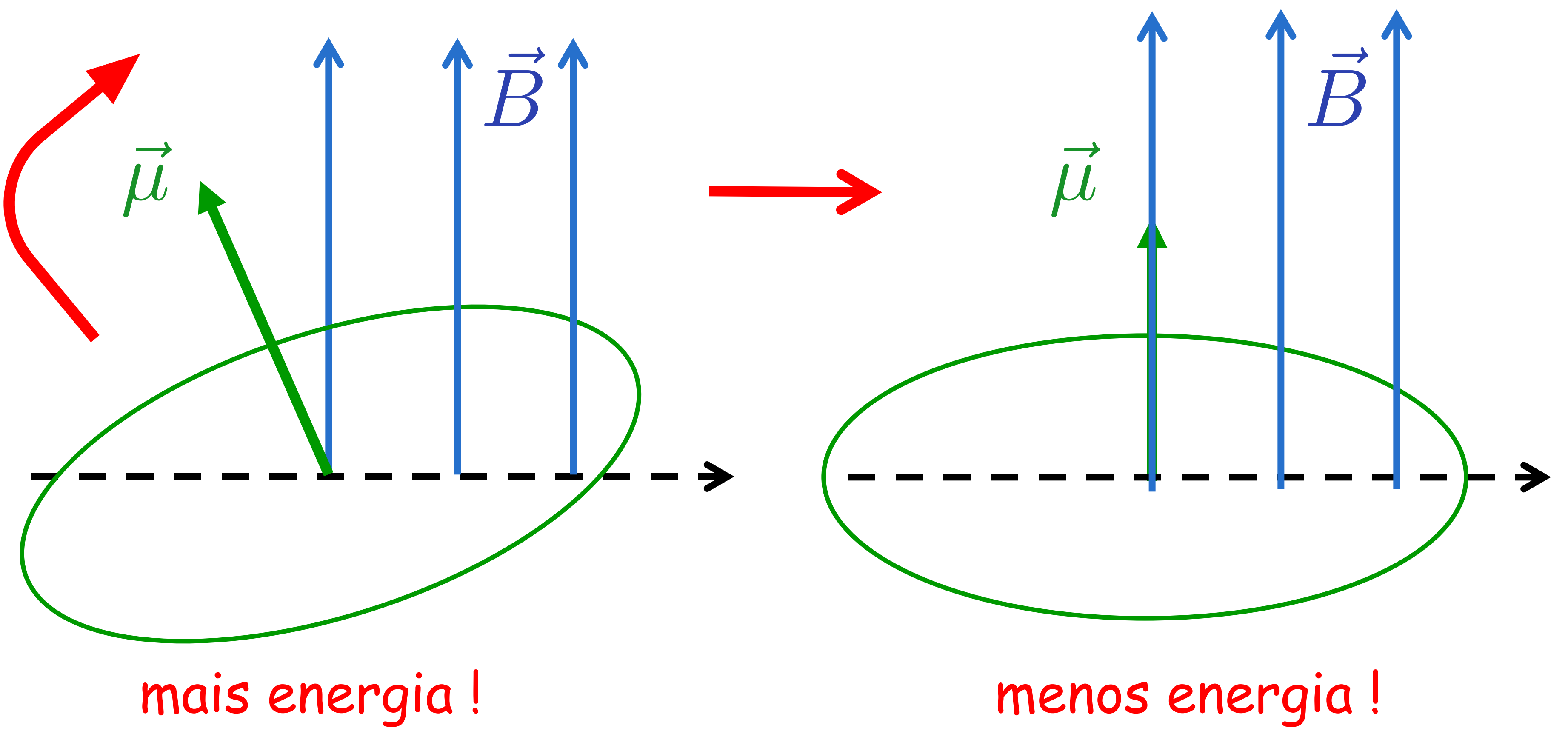
$$W = \vec{\mu}\,.\,\vec{B}$$

$$W = -U$$

$$U = -\vec{\mu}\,.\,\vec{B}$$

$$U = -\mu\,B\,cos\,\theta$$

Espira tenta se alinhar como uma bússola !

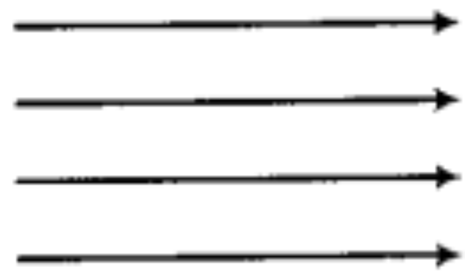


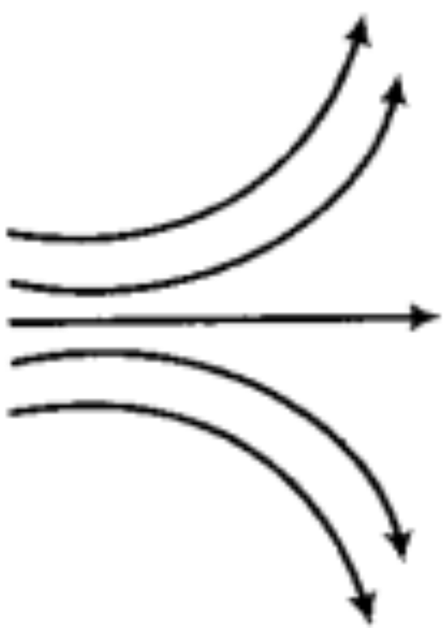
Energia potencial de orientação :

$$U = -\vec{\mu} \, . \, \vec{B}$$

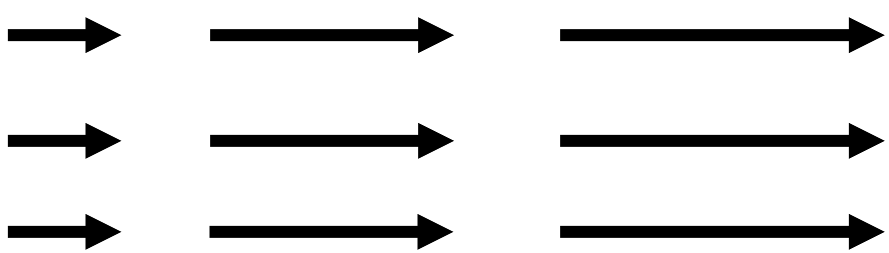
$$U = -\mu \, B \, cos \, \theta$$

Espira num campo magnético não uniforme



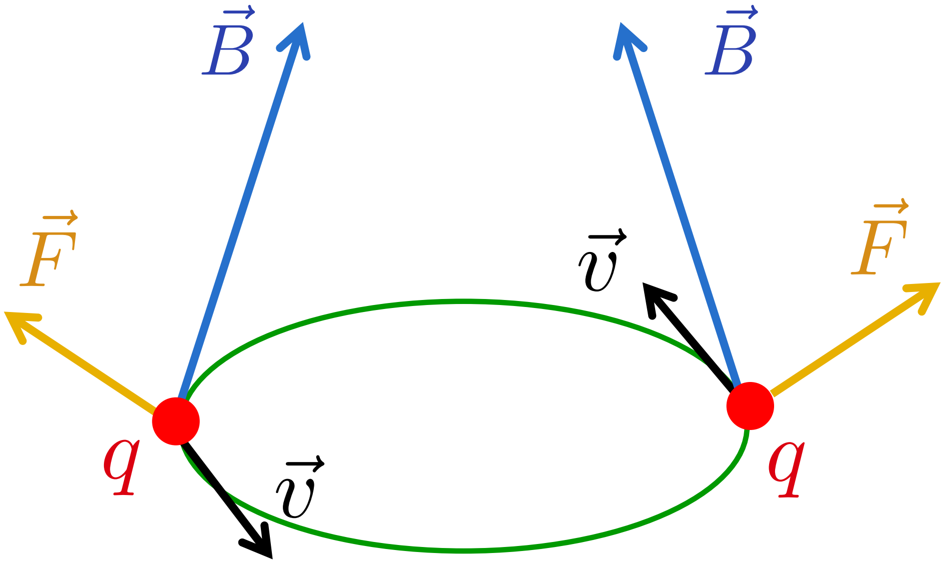


não uniforme

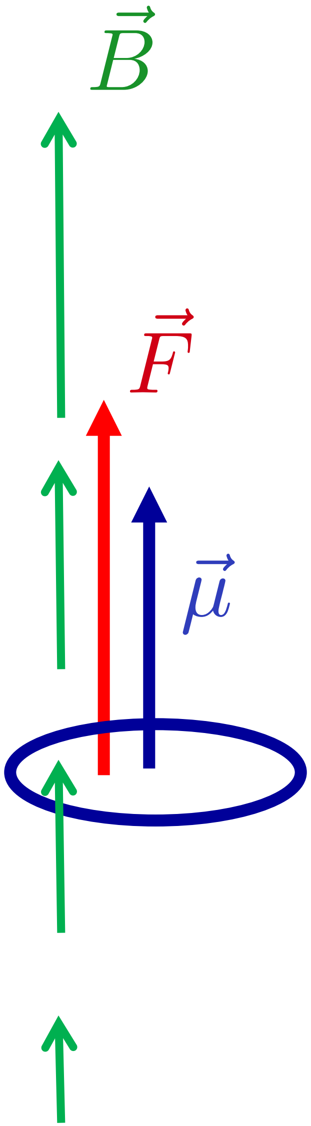


não uniforme

Espira num campo magnético não uniforme



Campo magnético não uniforme



$$\vec{B} = B_z\,\hat{k} \qquad \vec{\mu} = \mu_z\,\hat{k}$$

$$U = -\vec{\mu}\,.\,\vec{B} = -\mu_z\,B_z$$

energia potencial mínima

$$\vec{F} = -\vec{\nabla}\,U = -\left[\hat{i}\,\frac{\partial}{\partial x} + \hat{j}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}\right] U$$

$$\vec{F} = \mu_z\,\frac{\partial\,B_z}{\partial z}\,\hat{k} \qquad \mu_z = \frac{g_l\,\mu_b}{\hbar}\,L_z$$

$$F = \frac{g_l\,\mu_b\,L_z}{\hbar}\,\frac{\partial\,B_z}{\partial z}$$

Mecânica clássica :

L_z assume valores contínuos !

$$F = \frac{g_l\,\mu_b\,L_z}{\hbar}\,\frac{\partial B_z}{\partial z}$$

F assume valores contínuos

Mecânica quântica :

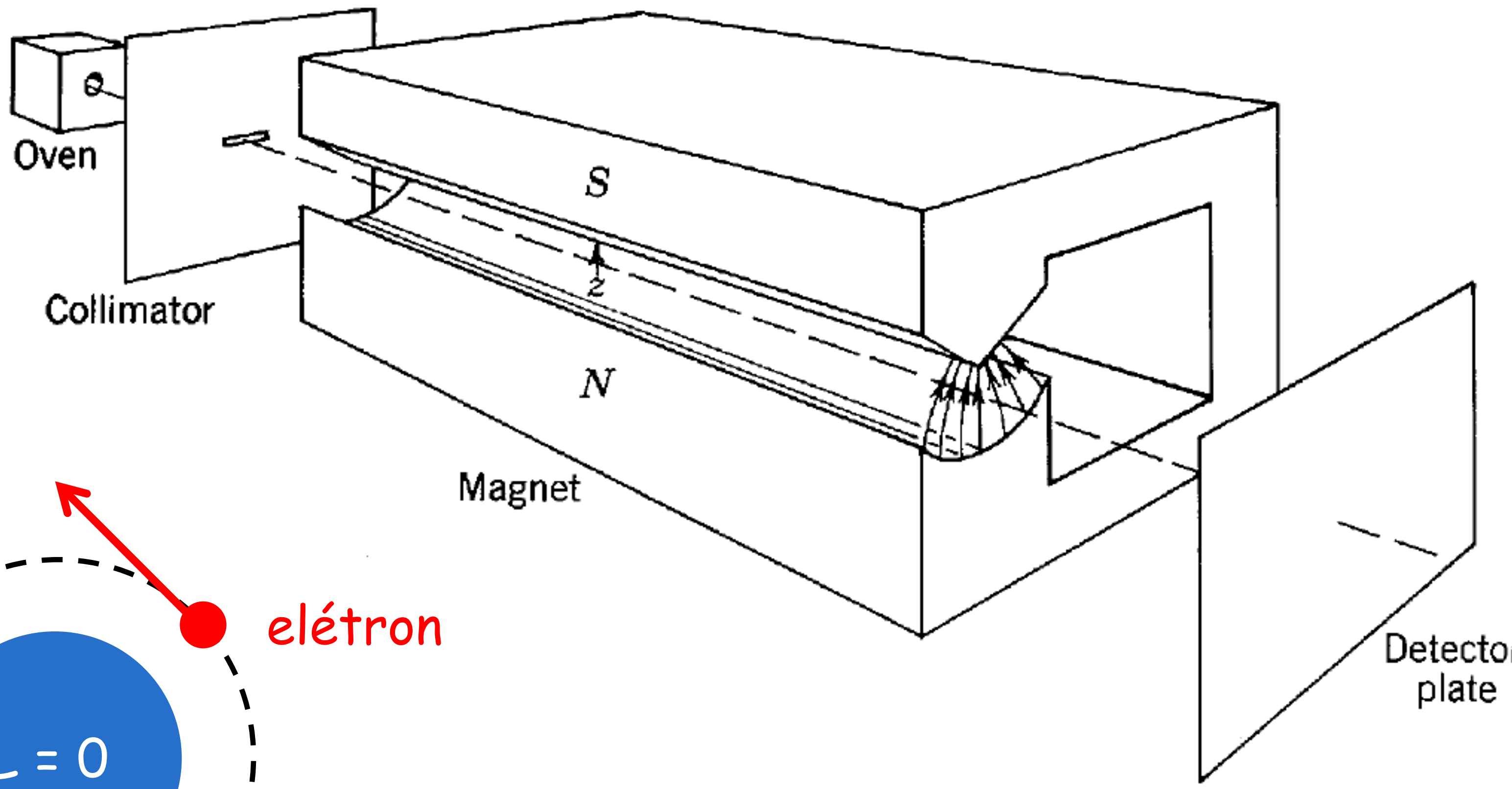
$$L_z = m_l\,\hbar \quad \begin{cases} l = 0\,,\,1\,,\,2\,,\,\dots & \text{inteiro} \\ -l \leq m_l \leq l & \text{inteiro} \end{cases}$$

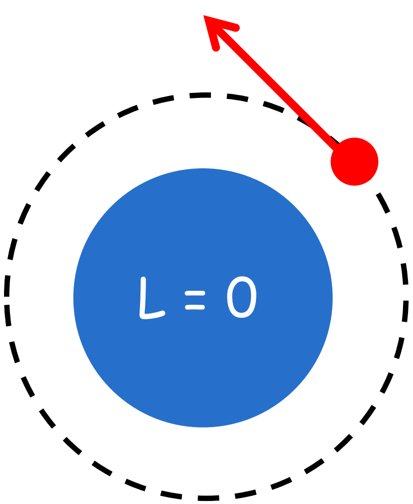
$$\boxed{F = g_l\,\mu_b\,m_l\,\frac{\partial B_z}{\partial z}}$$

F assume valores discretos

Experiência de Stern-Gerlach (1922)

Medida do momento magnético de átomos de prata

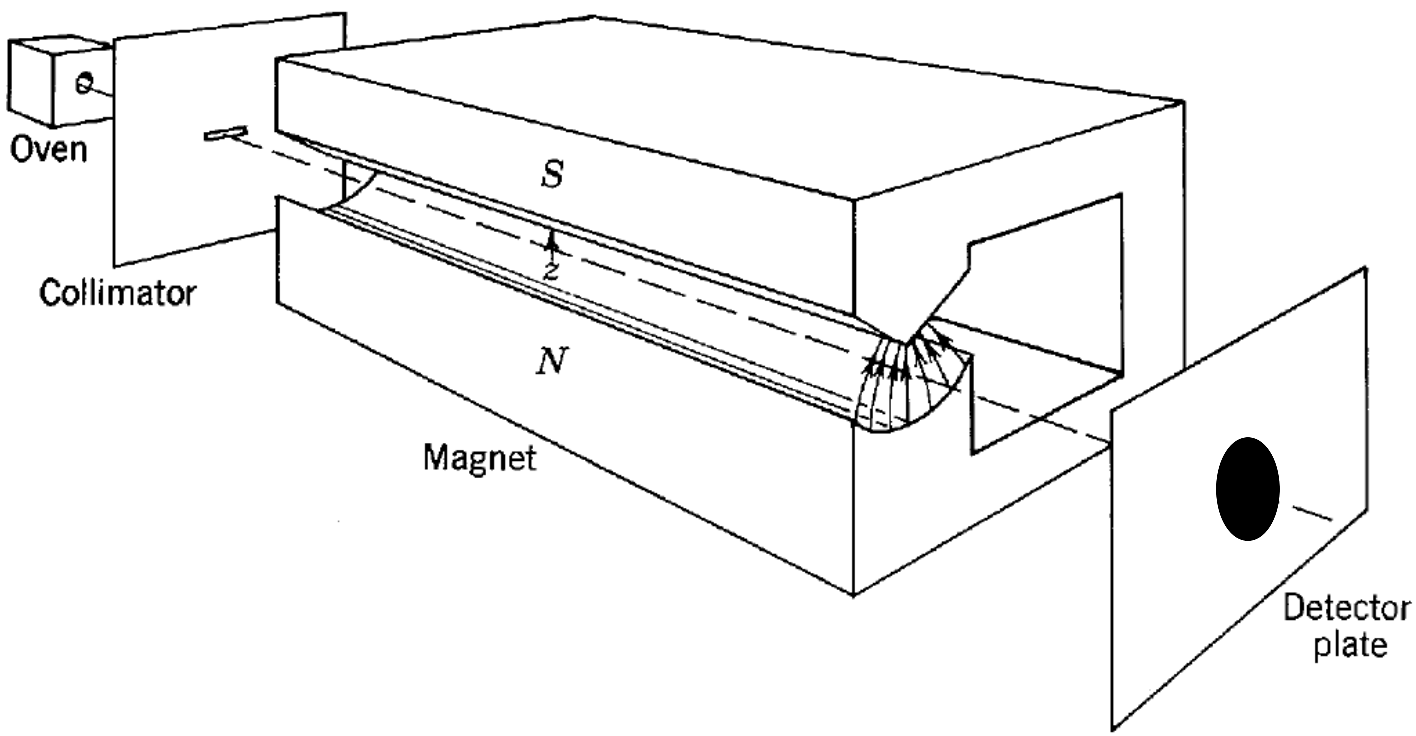




átomo de prata

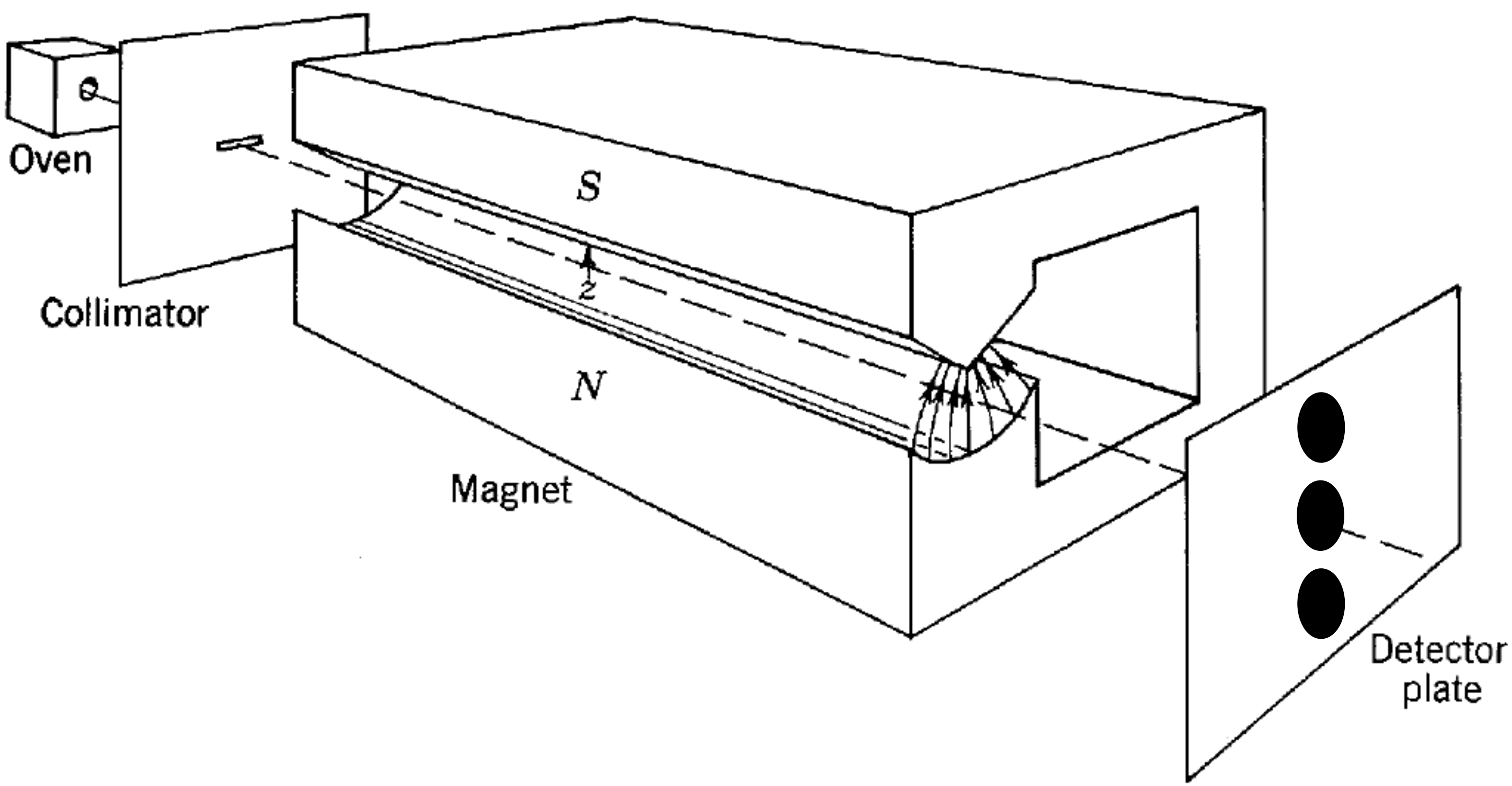
Se o elétron tiver momento angular zero:

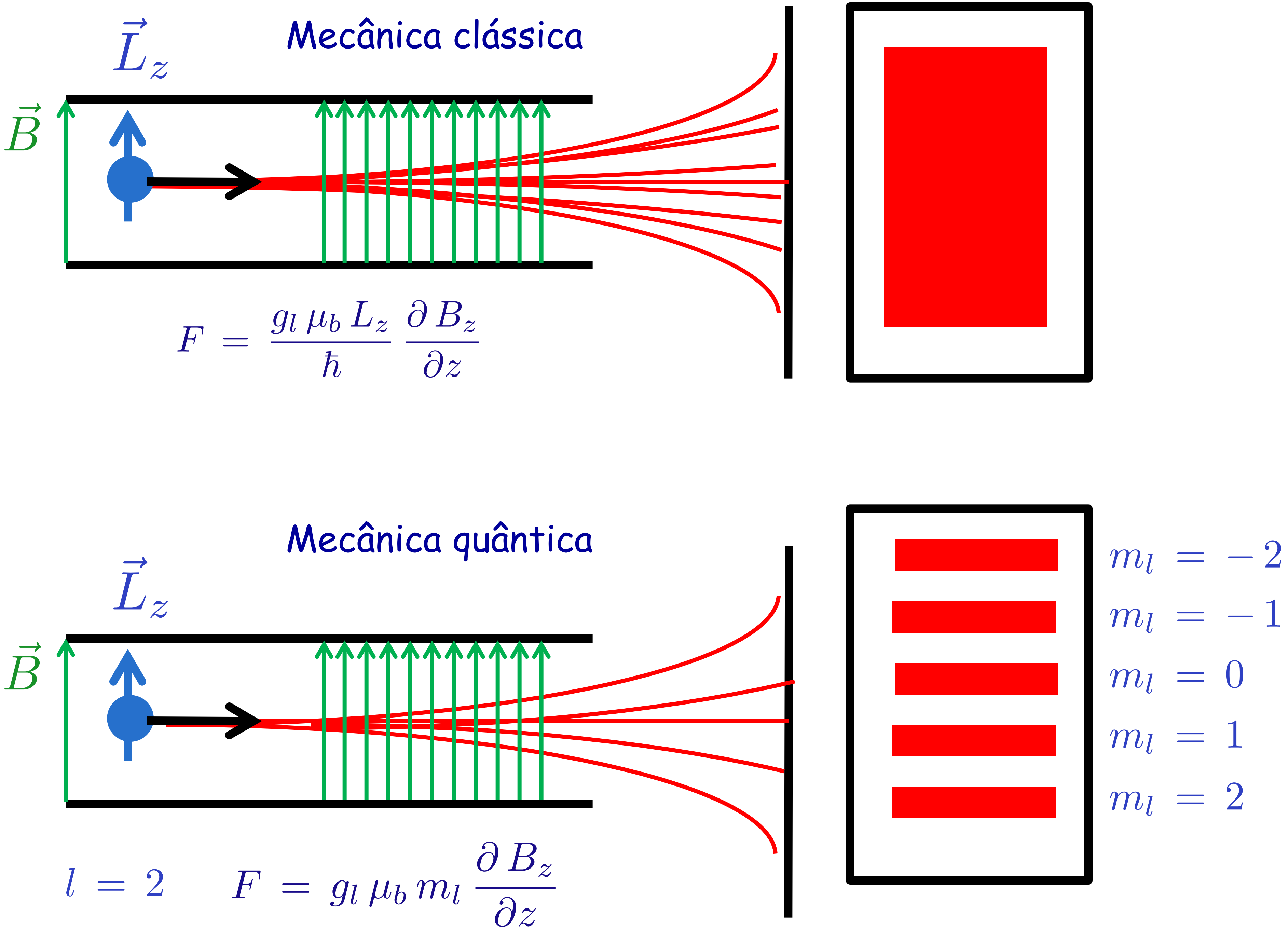
$$l = 0 \qquad m_l = 0 \qquad F = g_l \, \mu_b \, m_l \, \frac{\partial B_z}{\partial z} \qquad F = 0$$



Se o elétron tiver momento angular "um" :

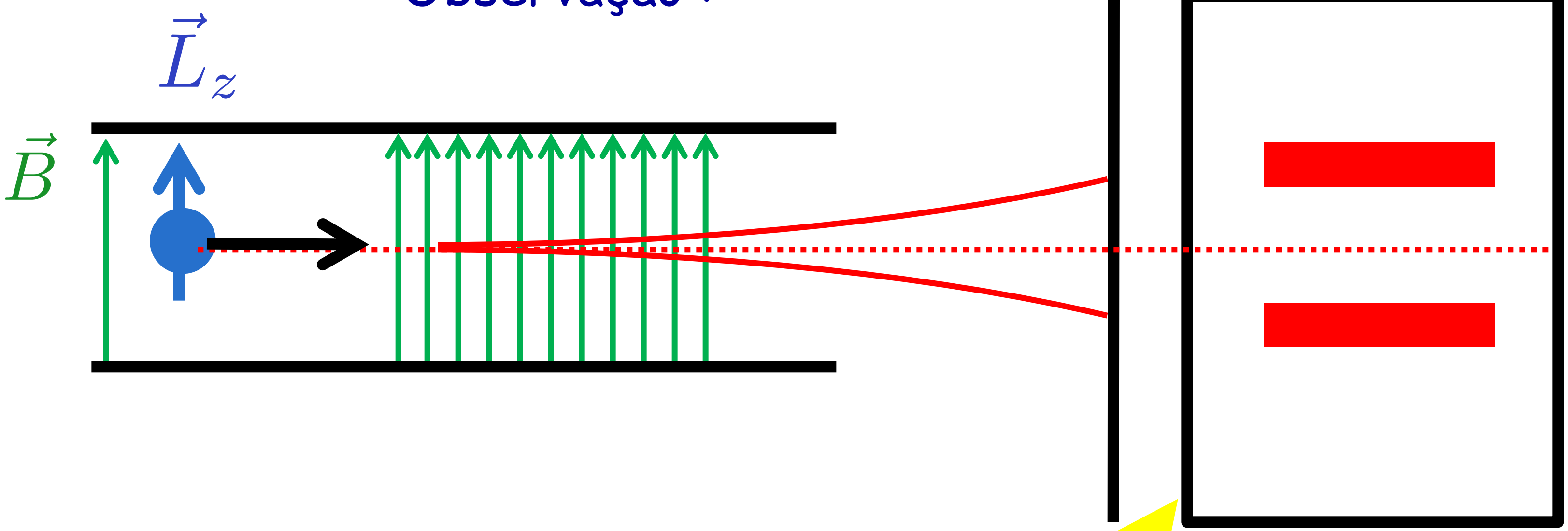
$$l = 1 \begin{cases} m_l = -1 \\ m_l = 0 \\ m_l = 1 \end{cases} \qquad F = g_l \, \mu_b \, m_l \, \frac{\partial B_z}{\partial z} \begin{cases} F = -g_l \mu_b \frac{\partial B_z}{\partial z} \\ F = 0 \\ F = +g_l \mu_b \frac{\partial B_z}{\partial z} \end{cases}$$





Mecânica clássica
$\vec{L}_z$
$\vec{B}$
$F = \frac{g_l\,\mu_b\,L_z}{\hbar}\frac{\partial B_z}{\partial z}$
Mecânica quântica
$\vec{L}_z$
$\vec{B}$
$m_l = -2$
$m_l = -1$
$m_l = 0$
$m_l = 1$
$m_l = 2$
$l = 2$
$F = g_l\,\mu_b\,m_l\,\frac{\partial B_z}{\partial z}$

Observação :



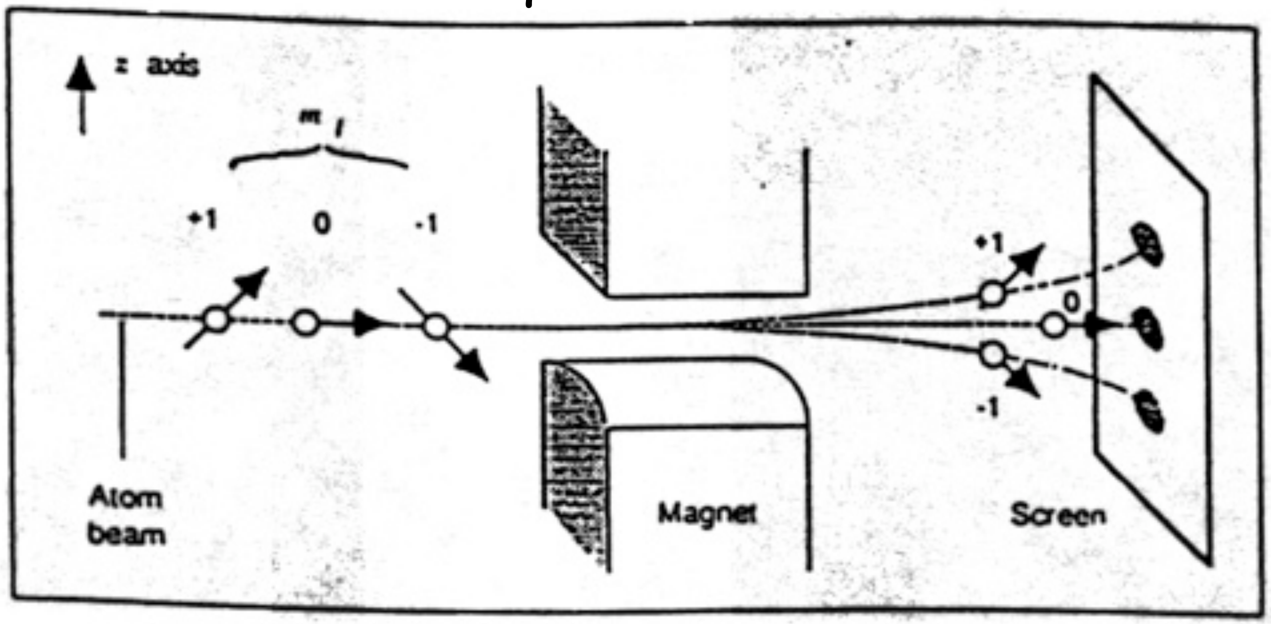
Só duas franjas sempre ! Dois valores de m : + m e - m

Não existe m = 0 !

clássico

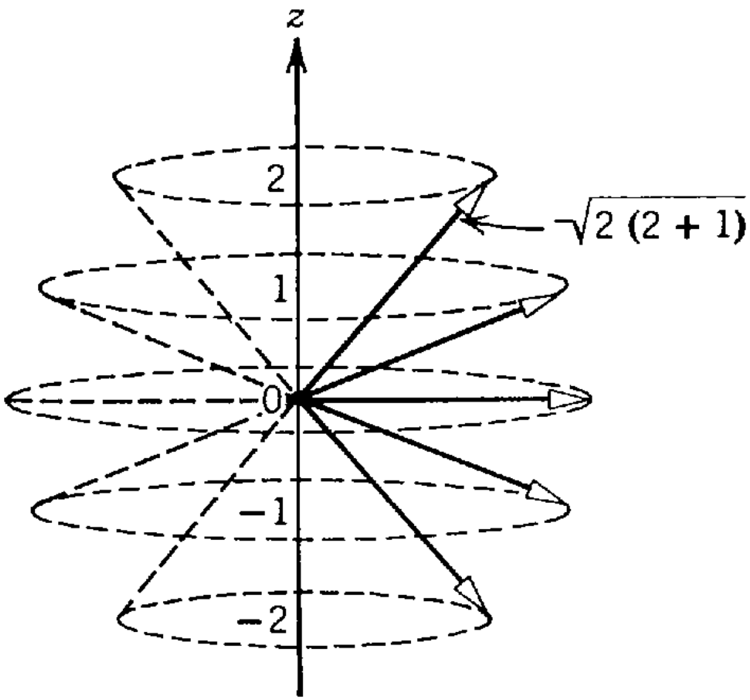
quântico

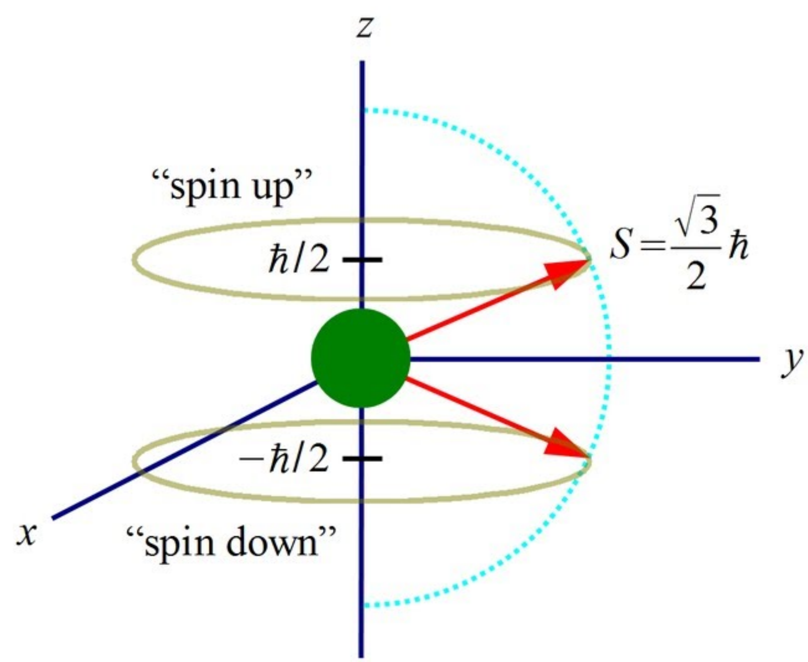
observado



Elétron tem momento angular intrínseco : spin !

Copiamos a teoria do momento angular





$$L = \sqrt{l\,(l+1)}\,\hbar$$

$$L_z = m_l\,\hbar$$

$$-l \le m_l \le l$$

$$l = 0\,,\,1\,,\,2\,,\,\ldots$$

$$S = \sqrt{s\,(s+1)}\,\hbar$$

$$S_z = m_s\,\hbar$$

$$m_s = -s\,,\,-s+1\,,\ldots s$$

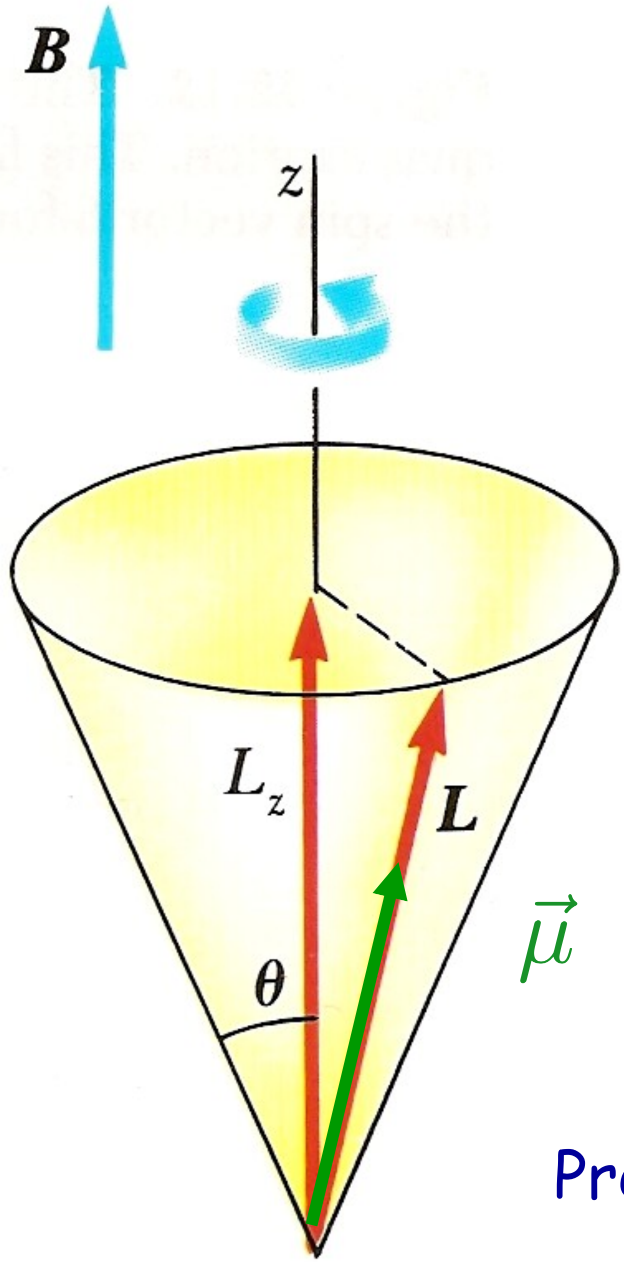
$$s = \frac{1}{2} \qquad m_s = -\frac{1}{2}\,,\,+\frac{1}{2}$$

FIM

A Little History of Spin in Quantum Mechanics

- 1922 – Otto Stern & Walter Gerlach – The existence of spin angular momentum is inferred from their experiment, in which particles (Ag atoms) are observed to possess angular momentum that cannot be accounted for by orbital angular momentum alone.
- 1924 – Wolfgang Pauli – proposed a new quantum degree of freedom (or quantum number) with two possible values and formulated the Pauli exclusion principle.
- 1925 – Ralph Kronig, George Uhlenbeck & Samuel Goudsmit – identified Pauli's new degree of freedom as electron spin and suggested a physical interpretation of particles spinning around their own axis.
- 1926 – Enrico Fermi & Paul Dirac – formulated (independently) the Fermi-Dirac statistics, which describes distribution of many identical particles obeying the Pauli exclusion principle (fermions with half-integer spins – contrary to bosons satisfying the Bose-Einstein statistics)
- 1926 – Erwin Schrödinger – formulated his non-relativistic Schrödinger equation, but it incorrectly predicted the magnetic moment of H to be zero in its ground state.
- 1927 – T.E. Phipps & J.B. Taylor – reproduced the effect using H atoms in the ground state, thereby eliminating any doubts that may have been caused by the use of Ag atoms.
- 1927 – Wolfgang Pauli – worked out on mathematical formulation of spin (2×2 matrices).
- 1928 – Paul Dirac – showed that spin comes naturally from his relativistic Dirac equation.

Mas nunca se alinha ! Precessiona !

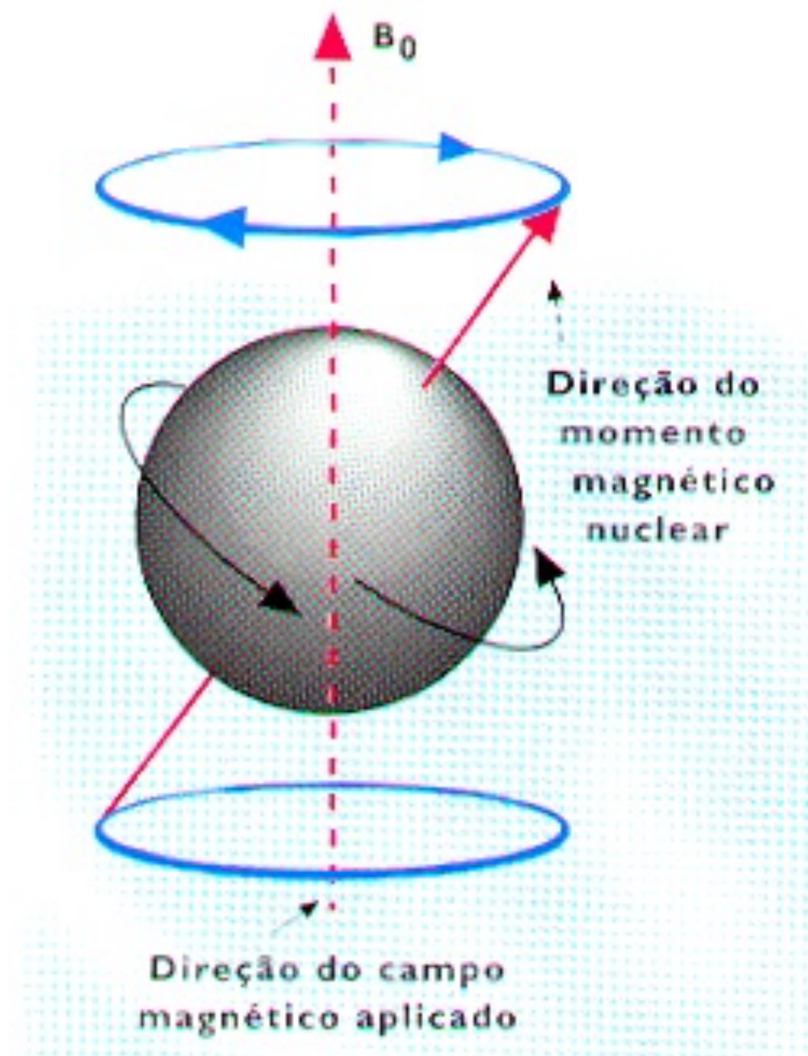


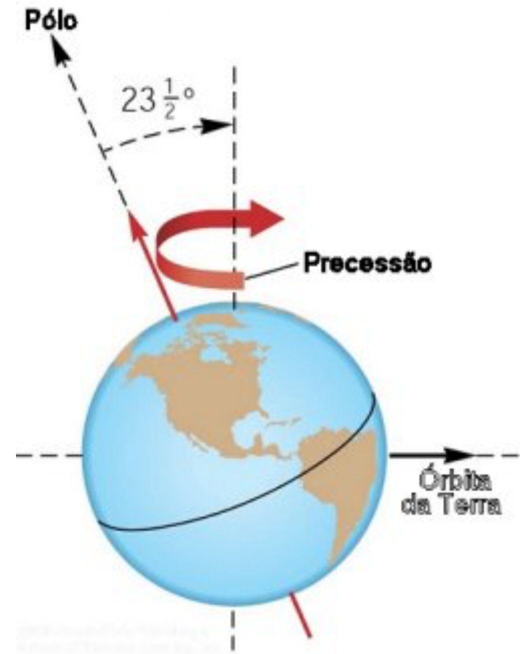
$$\vec{\mu} = \frac{g_l \, \mu_b}{\hbar} \vec{L}$$

Precessão de Larmor

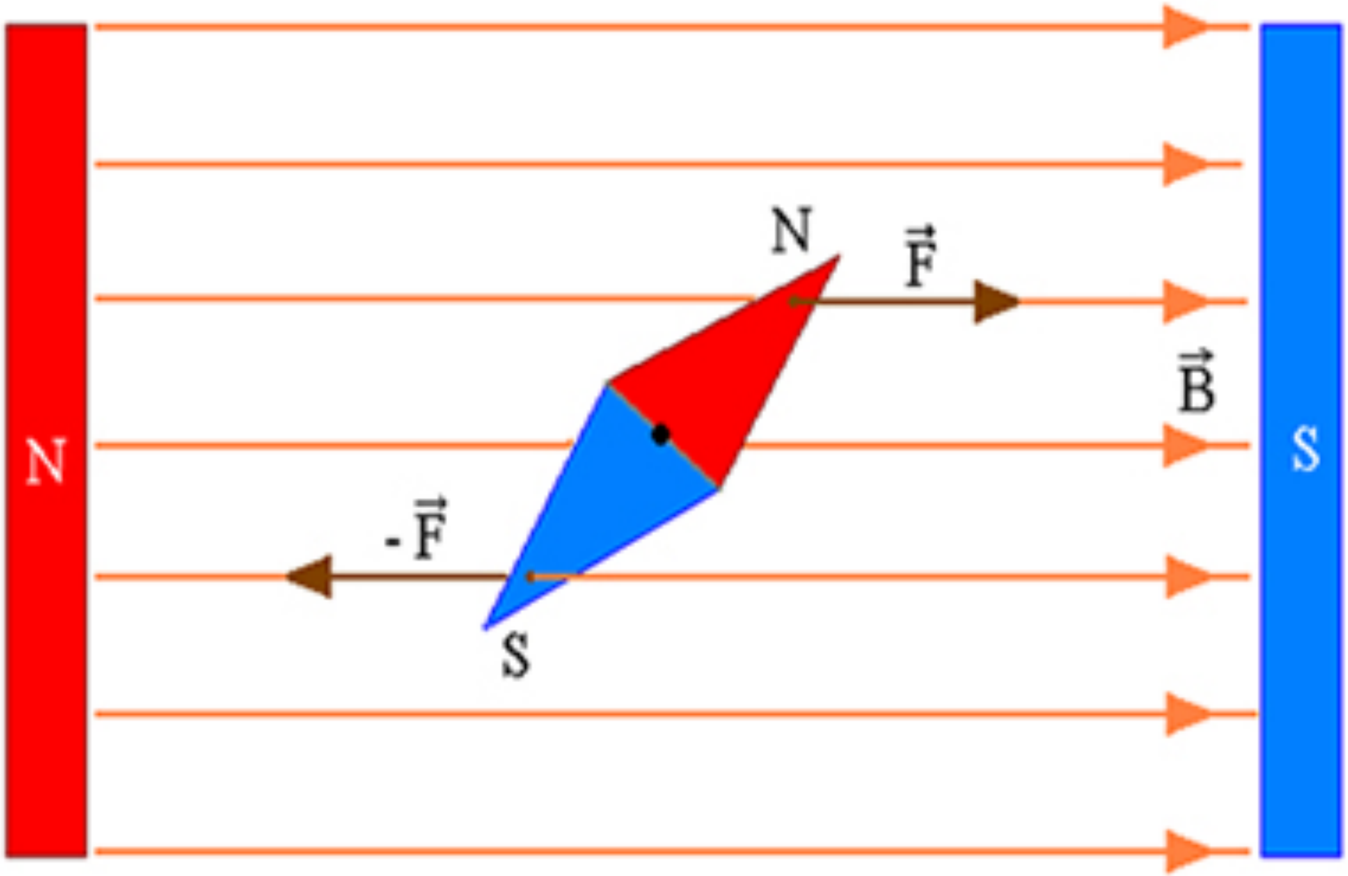
Mas nunca se alinha ! Precessiona !

Precessão de Larmor



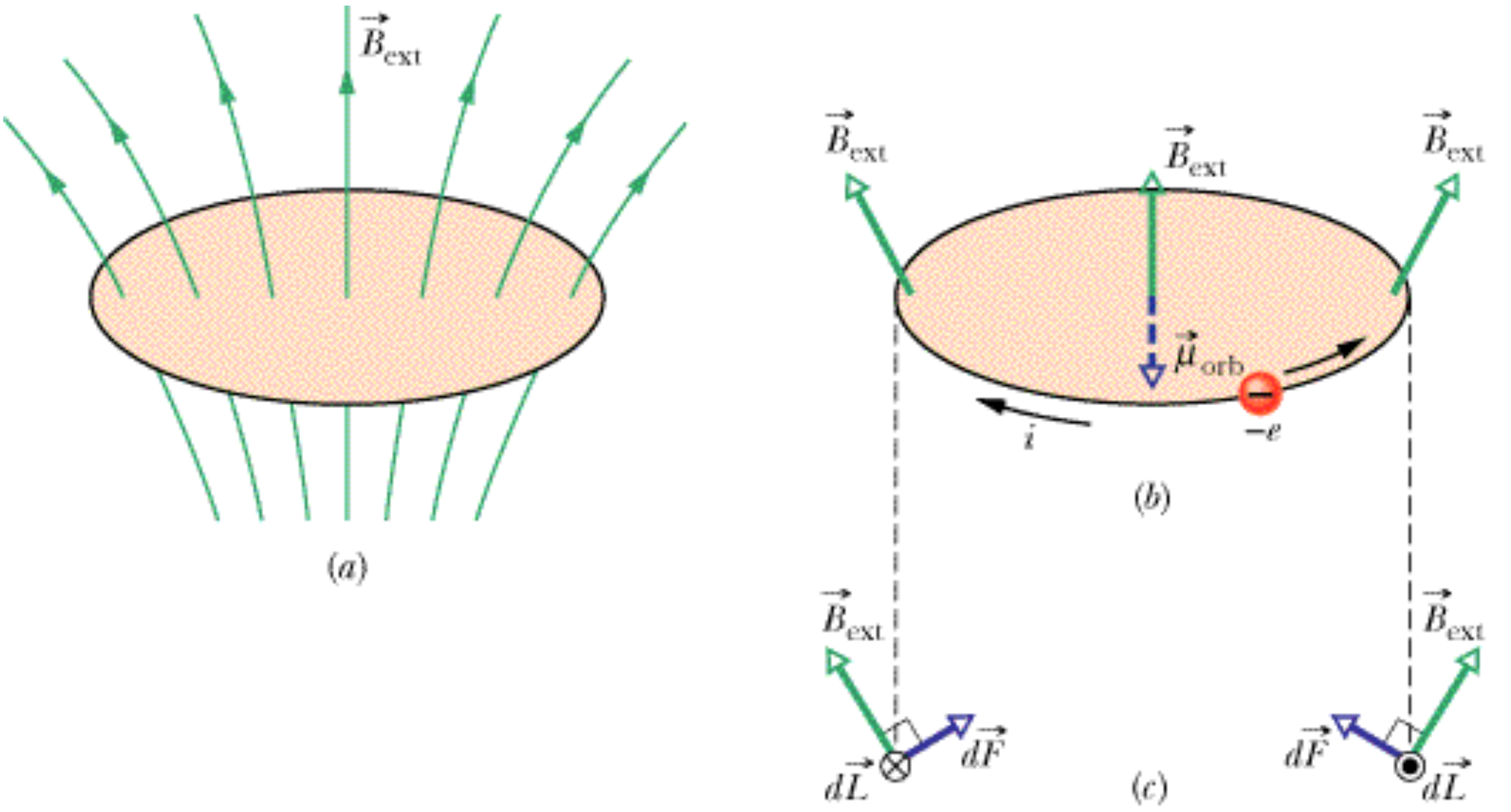


Espira se alinha como uma bússola !



Um imã é uma espira ?

Espira num campo magnético não uniforme



Levitação magnética!