Exercises and Solutions of the book

Light-Emitting Diodes

2nd edition

E. F. Schubert

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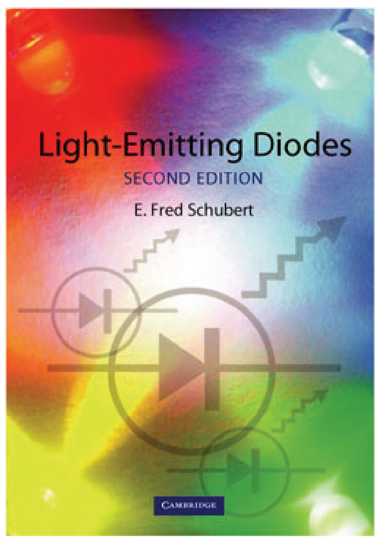

Chapter 2

Exercise: ***Minority carrier lifetimes***. Calculate the minority carrier lifetime in p-type GaAs at doping concentrations of 10^{15} and 10^{18} cm^{-3} using a bimolecular recombination coefficient of $B = 10^{-10}$ cm^3/s. Assume that one could fabricate GaAs without any impurities. What is the carrier lifetime in intrinsic GaAs with an intrinsic carrier concentration of 2×10^6 cm^{-3} ?

Solution:

$\tau_n = 10\ \mu\text{s}$	for $N_A = 10^{15}\ \text{cm}^{-3}$
$\tau_n = 10\ \text{ns}$	for $N_A = 10^{18}\ \text{cm}^{-3}$
$\tau = 2500\ \text{s}$	for undoped GaAs.

Discuss how the modulation speed of communication LEDs is affected by the radiative lifetime and the doping concentration.

Solution: When the injection current of an LED is switched off instantaneously at $t = 0$ and the radiative lifetime is τ, then light emission from the LED essentially ceases after the time $t = \tau$. Thus an LED cannot be switched "on" or "off" faster than the time τ. Assuming that both the switch-on and switch-off time is τ, then the maximum modulation frequency is given by $f \approx 1 / (2\tau)$.

We have shown that the radiative lifetime τ depends strongly on the doping concentration. Thus by highly doping the active region of a device, the radiative lifetime τ is shortened and the maximum modulation speed is increased.

Exercise: ***Concentration of point defects***. Assume that the energy required to move a substitutional lattice atom into an interstitial position is $E_a = 1.1$ eV. What is the equilibrium concentration of interstitial defects of a simple cubic lattice with lattice constant $a_0 = 2.5$ Å?

Solution: The concentration of lattice atoms of a simple cubic lattice is given by $N = a_0^{-3} = 6.4 \times 10^{22}\ \text{cm}^{-3}$. The concentration of interstitial defects under equilibrium conditions at room temperature is then given by

$$N_{\text{defect}} \;=\; N \exp(-E_a / kT) \;=\; 2.7 \times 10^4\ \text{cm}^{-3}\ .$$

Note that the calculated concentration of defects is small when compared to the typical concentrations of electrons and holes. If the defect discussed here forms a level in the gap, non-radiative recombination through the defect level can occur.

Chapter 3

Exercise: ***Radiative efficiency***. Analyze the temperature dependence of the radiative lifetime based on the van Roosbroeck–Shockley model and the non-radiative lifetime based on the Shockley–Read model and predict the temperature dependence of the radiative efficiency in semiconductors.

Solution: The *radiative* recombination rate (van Roosbroeck–Shockley rate) has a weak temperature dependence, and it depends on temperature according to $R = B\,n\,p$, where $B \propto T^{-3/2}$, as concluded from Eqs. (3.24) and (3.25). The concentrations n and p mostly depend on the excitation strength (injection current) and can be assumed to be temperature independent. Thus, it is $R \propto \tau_{\text{radiative}}{}^{-1} \propto T^{-3/2}$.

The *non-radiative* recombination rate (Shockley–Read) has a strong temperature dependence and it increases very rapidly with increasing temperature. The Shockley–Read recombination rate includes the term $\cosh\,[(E_\text{T} - E_\text{Fi})\,/\,(kT)]$. Because $\cosh x = ½\,(e^x + e^{-x})$ and $(E_\text{T} - E_\text{Fi})$ is either < 0 or > 0 (depending on the location of the trap with respect to the intrinsic Fermi level, E_Fi), one of the exponential functions (e^x or e^{-x}) dominates, so that an approximately exponential temperature dependence is obtained. Thus, it is $R \propto \tau_{\text{non-radiative}}{}^{-1} \propto \exp\,[-1\,/\,(kT)]$.

The exponential dependence (Shockley–Read: $R \propto \exp\,[-1\,/\,(kT)]$) is much stronger than the power-law dependence (Shockley–Read: $R \propto T^{-3/2}$), so that the radiative efficiency of semiconductors strongly decreases with increasing temperature.

Chapter 4

Exercise: ***Critical points of diode current–voltage characteristics***. The *I–V* characteristics of diodes are frequently characterized in terms of ***four critical points***, namely *forward voltage one*, V_{f1}, *forward voltage two*, V_{f2}, *forward voltage three*, V_{f3}, and *reverse saturation current*, I_s, specified at the operating current (e.g. 100 mA), a small forward current (e.g. 10 μA), a very small forward current (e.g. 1 μA), and at negative bias (e.g. – 5 V), respectively. The critical points are shown in Fig. 4.6.

(a) Explain the relevance of the critical points.

(b) Two GaInN diodes have the following data: (1) V_{f1} = 3.2 V, V_{f2} = 2.5 V, V_{f3} = 2.3 V, I_s = 0.8 μA; (2) V_{f1} = 3.4 V, V_{f2} = 2.0 V, V_{f3} = 1.8 V, I_s = 0.8 μA. Which device has the more favorable characteristics?

Solution: (a) For devices emitting at the same peak wavelength, V_{f1} should be as *low* as possible, as high values indicate a high series resistance. The forward voltage two, V_{f2}, should be as *high* as possible (as close to V_{f1} as possible), as low values of V_{f2} indicate excessive sub-threshold leakage. The same argument applies to V_{f3}. The reverse saturation current should be as low as possible as high values of I_s indicate excessive leakage paths (e.g. surface leakage or bulk leakage mediated by surface states, bulk point defects and dislocations). Low values of V_{f1}, high values of V_{f2} and V_{f3}, and low values of I_s are consistently correlated with high device reliability. (b) Device (1) has more favorable characteristics due to lower series resistance and lower sub-threshold leakage.

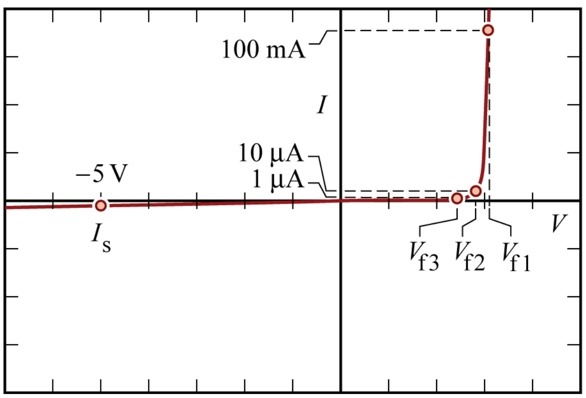

Fig. 4.6. Critical points of diode *I–V* characteristic, namely "forward voltage one", V_{f1} (measured at operating current, e.g. 100 mA), "forward voltage two", V_{f2} (measured at low current, e.g. 10 μA), "forward voltage three", V_{f3} (measured at very low current, e.g. 1 μA), and reverse saturation current (measured at e.g. – 5.0 V).

Exercise: ***Grading of heterostructures***. Assume that the conduction band discontinuity of an AlGaAs/GaAs heterostructure is given by ΔE_C = 300 meV and that the structure is uniformly doped with donors of concentration $N_D = 5 \times 10^{17}$ cm^{-3}. Over what distance should the interface be graded in order to minimize the resistance occurring in abrupt heterostructures?

Solution: Calculating the depletion layer thickness from Eq. (4.22) yields W_D = 30 nm. Thus the heterostructure should be graded over 30 nm to minimize the heterostructure resistance. The graded region should have *two* parabolic regions as shown in Fig. 4.9 (b).

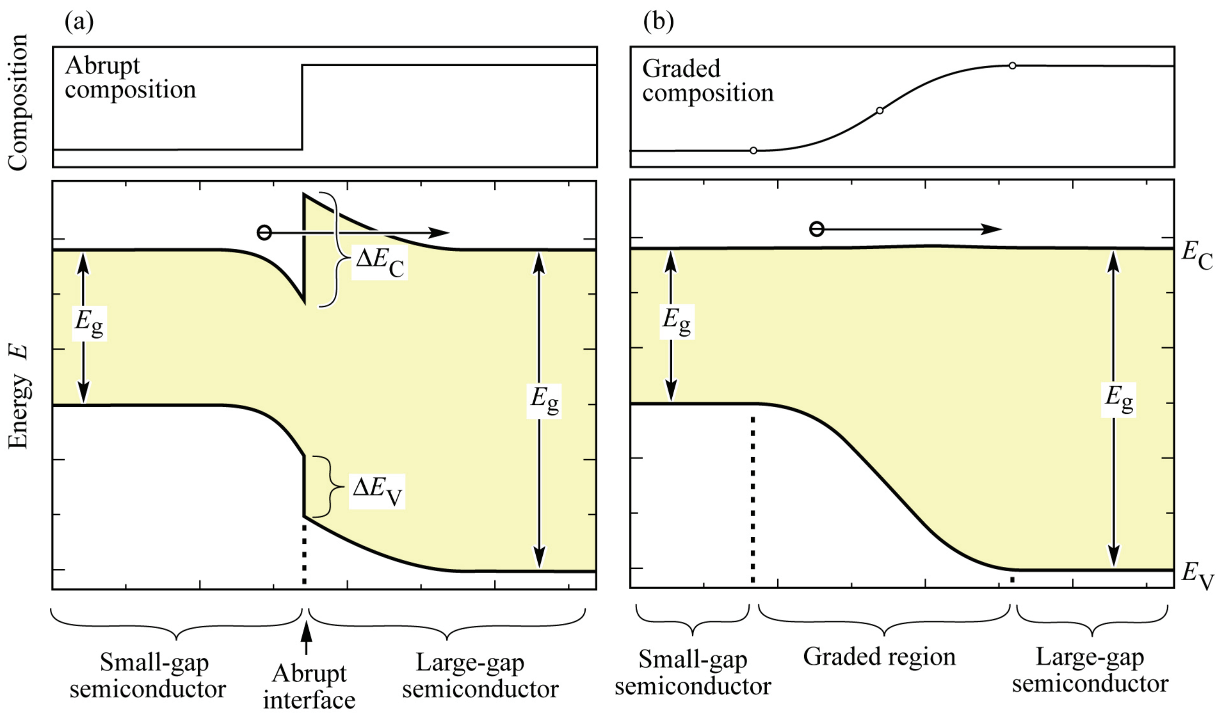

Fig. 4.9. Band diagram of (a) an abrupt n-type–n-type heterojunction and (b) a graded heterojunction of two semiconductors with different bandgap energy. The abrupt junction is more resistive than the graded junction due to the electron barrier forming at the abrupt junctions (after Schubert *et al*., 1992).

Exercise: ***Carrier leakage over a barrier***. Electrons in the active region of a GaAs structure have a concentration of 2×10^{18} cm^{-3}. Calculate the current density of the carrier loss over the barrier for barrier heights of 200 and 300 meV, assuming an electron mobility of 2000 cm^2/(V s) and a minority carrier lifetime of 5 ns. Compare the calculated leakage current to LED injection currents of 0.1–1.0 kA/cm^2.

Solution: The Fermi level in GaAs with electron density of 2×10^{18} cm^{-3} is 77 meV above the conduction band edge. Assuming that the effective density of states in the barrier is the same as in the GaAs active region, the carrier concentrations at the edge of the barrier are 3.9×10^{15} cm^{-3} for a 200 meV barrier and 8.3×10^{13} cm^{-3} for a 300 meV barrier. The diffusion constant, as inferred from the Einstein relation, is $D_n = 51.7$ cm^2/s. The diffusion length is then given by $L_n = (D_n \tau_n)^{1/2} = 5.1$ µm. The leakage current is calculated using Eq. (4.27), and one obtains 63 A/cm^2 for the 200 meV barrier and 1.3 A/cm^2 for the 300 meV barrier. Comparison with diode current densities of 0.1–1.0 kA/cm^2 suggests that leakage currents can be a significant loss mechanism, particularly for small barrier heights.

Exercise: ***Carrier overflow in a double heterostructure***. Consider electrons in a GaAs double heterostructure with a barrier height of $\Delta E_C = 200$ meV and an active region thickness of $W_{DH} = 500$ Å. Calculate the current level at which the electron well overflows.

Solution: Using $N_c = 4.4 \times 10^{17}$ cm^{-3} and $B = 10^{-10}$ cm^3/s, one obtains from Eq. (4.33) a current level of $J_{max} = 3990$ A/cm^2.

Exercise: ***Drive voltages of LEDs***. Calculate the approximate forward diode voltage of LEDs emitting in the blue, green, and red parts of the visible spectrum. Also calculate the forward diode voltage of LEDs emitting at 870 nm and 1.55 µm.

Solution:

Emission color	*Wavelength*	*Photon energy*	*Drive voltage*
Blue	470 nm	2.6 eV	2.6 V
Green	550 nm	2.2 eV	2.2 V
Red	650 nm	1.9 eV	1.9 V
IR	870 nm	1.4 eV	1.4 V
IR	1550 nm	0.8 eV	0.8 V

Chapter 5

Exercise: ***LED efficiency***. Consider an LED with a threshold voltage of $V_{th} = E_g / e = 2.0$ V with a differential resistance of $R_s = 20\ \Omega$, so that the I–V characteristic in the forward direction is given by $V = V_{th} + I R_s$. When the device is operated at 20 mA it emits a light power of 4.0 mW of energy $h\nu = E_g$. Determine the (a) external quantum efficiency, (b) internal quantum efficiency, and (c) power efficiency, assuming that the light-extraction efficiency is 50%.

Solution: (a) External quantum efficiency: Number of emitted photons per second is given by 4.0 mW / 2.0 eV = 4.0×10^{-3} CV s^{-1} / ($2.0 \times 1.602 \times 10^{-19}$ CV) = 1.25×10^{16} s^{-1}. Number of injected electrons per second is given by 20 mA / e = 20×10^{-3} C s^{-1} / (1.602×10^{-19} C) = 1.25×10^{17} s^{-1}. Thus the external quantum efficiency is 1.25×10^{16} s^{-1} / 1.25×10^{17} s^{-1} = 10%. (b) Using $\eta_{external} = \eta_{internal} \times \eta_{extraction}$ and using the light-extraction efficiency value of 50%, the internal quantum efficiency is 20%. (c) The power efficiency is given by $P_{optical}$ / (I V) = 4 mW / [20 mA × (2.0V + 20 mA × 20 Ω)] = 4 mW / 48 mW = 8.33%

Exercise: ***Light escape from planar GaAs, GaN, and polymer LED structures***. The refractive indices of GaAs, GaN, and light-emitting polymers are 3.4, 2.5, and 1.5, respectively. Calculate the critical angle of total internal reflection for GaAs, GaN, and for polymers. Also calculate the fraction of light power that can escape from a planar GaAs and GaN semiconductor structures and a polymer LED structure.

What improvement can be attained if a planar GaAs LED is encapsulated in a transparent polymer of refractive index 1.5, if the reflection at the polymer–air interface is neglected?

Solution:

Critical angle for total internal reflection:

GaAs $\phi_c = 17.1°$ GaN $\phi_c = 23.6°$ Polymer $\phi_c = 41.8°$.

Fraction of light that can escape:

GaAs 2.21% GaN 4.18% Polymer 12.7%.

Improvement of the GaAs planar LED due to polymer encapsulation: 232%.

Exercise: ***LED-to-fiber coupling efficiency***. Consider a GaAs LED with a point-like light-emitting region located in close proximity to the planar GaAs LED surface. An optical fiber has an acceptance angle of 12° in air. What fraction of the light emitted by the active region can be coupled into the fiber? Assume a GaAs refractive index of 3.4. Neglect Fresnel reflection losses at the semiconductor–air and air–fiber interfaces.

Solution: The acceptance angle in the semiconductor is obtained from Snell's law and is 3.5°. Thus 0.093% of the power emitted by the active region can be coupled into the fiber.

Chapter 6

Exercise: ***Temperature dependence of diode forward voltage***. Experimentally determined linear temperature coefficients (dV_f / dT) for GaAs diodes range from 1.2 to 1.4 mV/K. Calculate the linear temperature coefficient of the forward voltage of a GaAs diode with $N_A = N_D = 2 \times 10^{17}$ cm^{-3} at room temperature. What is the decrease in forward voltage if the ambient temperature is increased from 20 to 40 °C and the internal heating in the diode can be neglected?

Solution: For GaAs with $\alpha = 5.41 \times 10^{-4}$ eV/K and $\beta = 204$ K, one obtains at room temperature $dV_f / dT = -1.09$ mV/K. The decrease in diode voltage for the 20 °C temperature increase is ΔV_f = 21.9 mV.

Exercise: ***Compensation of the temperature dependence of an LED with a drive circuit***. Consider an LED with a characteristic temperature T_1 = 100 K, a turn-on voltage of 1.4 V at 20 °C, a temperature coefficient of the turn-on voltage of –2.1 mV/K, and a linear I–V characteristic with a differential resistance of 5 Ω for forward voltages larger than the turn-on voltage. Assume that the temperature dependence of the emission intensity is given by $I = I|_{300\text{ K}} \exp[-(T - 300\text{ K}) / T_1]$.

Design a drive circuit consisting of a constant-voltage source and a resistor, which compensates for the temperature dependence of the emission intensity of the LED so that the LED emission intensity is the same at the water freezing-point temperature (0 °C) and 60 °C. The LED should draw 20 mA at the freezing-point temperature.

Solution: At 60 °C, the current needs to be 36.4 mA in order to keep the emission intensity independent of temperature. Constructing a load line that intersects the 0 °C and 60 °C diode I–V characteristic at 20 mA and 36.4 mA, respectively, yields the following values for the drive circuit: Constant-voltage source with V = 1.6 V and series resistance of 2.7 Ω.

Chapter 8

Exercise: ***Current crowding occurring at very high current levels in devices with current-spreading layer***. In device structures with vertical current flow (current flowing from top to bottom of chip), the current-spreading layer ensures that the current spreads out over the entire p-n junction area. However, as the current increases to very high levels, the current tends to crowd under the top contact. This is illustrated in Fig. 8.7 (a) and (b). Explain the phenomenon of current crowding occurring at very high current densities.

Solution 1: The equation for the current-spreading length has the dependence $L_s \propto J_0^{-1/2}$. Thus, as the current density increases, L_s decreases, and the current "bunches" under the top contact.

Solution 2: An intuitive explanation for current crowding can be obtained from the equivalent circuit shown in Fig. 8.7 (c). At very high current densities, the resistors that represent the p-n junction decrease (whereas the resistors representing the current-spreading layer remain constant), thereby causing the current to flow directly downward from the top contact.

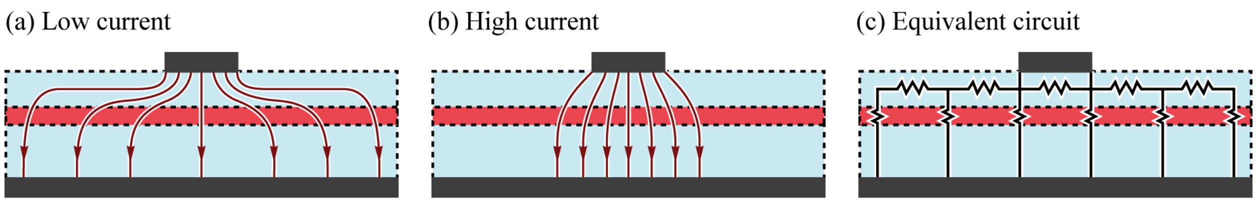

Fig. 8.7. Schematic current flow in device with current-spreading layer at (a) low and (b) high current. Current spreading decreases at very high current densities which results in current "bunching" under the top contact as shown in (b). (c) Equivalent circuit.

Chapter 10

Exercise: ***Lambertian reflectors in LEDs***. Assume a lambertian reflector with reflectivity 1.0 that is incorporated in a lossless GaAs LED structure with refractive index of 3.5. Assume that the outside medium is air. Calculate the critical angle of the escape cone, the probability that a reflected light ray falls within the escape cone and the average number of reflection events before a photon escapes from the high-index GaAs layer.

Solution: Critical angle $\Theta_c = 16.6°$; Probability of escape $p = 8.2\%$; Average number of reflection events before a photon escapes $N = 11.7$.

Would a hypothetical planar reflector that reflected light coming from any incoming direction towards the surface normal be useful? Is there a physical principle that prevents a reflector from reflecting light in such a way?

Solution: Although such a reflector would be very useful, such a reflector would unfortunately violate the ***conservation of radiance theorem*** (previously called the conservation of brightness theorem), which states that it is impossible to increase the radiance of light by a passive optical system beyond a value of L/n^2 where L is the radiance in vacuum and n is the refractive index of the medium in which the light propagates.

Chapter 13

Exercise: ***Activation of Mg acceptors in GaN***. Mg acceptors in GaN have an activation energy of $E_a = 200$ meV. (a) Calculate the fraction of acceptors that are ionized at 300 K for an acceptor concentration of $N_{Mg} = 10^{18}$ cm^{-3} using the formula $p = (g^{-1} N_{Mg} N_v)^{1/2} \exp(-E_a / 2kT)$ where g is the acceptor ground state degeneracy ($g = 4$) and N_v is the effective density of states at the valence band edge of GaN. (b) What would be the activation of acceptors if a hydrogen atom were bonded to each acceptor?

Solution: (a) Using the formula given above, one obtains that only about 6% of the acceptors are ionized. (b) If acceptors are passivated, p-type conductivity cannot be established.

Exercise: ***Cracking***. Why does cracking occur in epitaxial layers that are under biaxial tensile strain but not in epilayers that are under biaxial compressive strain?

Solution: Wafer bowing and ultimately cracking of an epitaxial film that is under biaxial tensile strain releases the strain energy stored in the film. For epitaxial layers that are under compressive strain, the strain energy can be released by wafer bowing, film buckling, and film delamination. Due to the compressive strain, there is "no room" for fissures or cracks, so that cracks generally do not form in compressively strained films.

The strain energy stored in a homo-epitaxial film, that is lattice mismatched to the substrate, is proportional to the thickness of the film. As the thickness of a strained film increases, it will at some point become energetically more favorable to reduce the strain energy by creating misfit dislocations and cracks. Thus, at a certain thickness, the film will form misfit dislocations to release the strain energy. The critical thickness, at which a homo-epitaxial film starts forming misfit dislocations, is given by the ***Matthews–Blakeslee law*** (Matthews and Blakeslee, 1976). As the film thickness increases further, misfit dislocations do not suffice to release the strain energy, so that at some point the film will start to crack. A formula for the critical thickness at which a film under biaxial tensile strain starts to crack was given by Hearne *et al*. (2000).

Chapter 14

Exercise: ***Transmission through a Fabry–Perot cavity***. Derive Eq. (14.1) by calculating the transmitted wave intensity in terms of a geometric series as illustrated in Fig. 14.1 (a).

Solution: As illustrated in Fig. 14.1 (a), the amplitude of the electric field of the transmitted wave, $\mathcal{E}_T$, is obtained by the following sum

$$\begin{aligned}\mathcal{E}_T &= \mathcal{E}_0 t_1 t_2 + \mathcal{E}_0 t_1 t_2 r_1 r_2 e^{i2\phi} + \mathcal{E}_0 t_1 t_2 r_1^2 r_2^2 e^{i4\phi} + \mathcal{E}_0 t_1 t_2 r_1^3 r_2^3 e^{i6\phi} + \ldots \\ &= \mathcal{E}_0 t_1 t_2 \left(1 + r_1 r_2 e^{i2\phi} + r_1^2 r_2^2 e^{i4\phi} + r_1^3 r_2^3 e^{i6\phi} + \ldots\right)\end{aligned}$$

where t and r are the electric field Fresnel transmittance and reflectance coefficients, respectively, and $\phi = 2\pi\,(\bar{n}\,d/\lambda)$ is the phase change incurred by the wave when traveling the distance between the two reflectors. Using the formula for the geometric series, i.e. $1 + x + x^2 + x^3 + \ldots = 1/(1-x)$, we obtain

$$t = \frac{\mathcal{E}_T}{\mathcal{E}_0} = \frac{t_1 t_2}{1 - r_1 r_2 e^{i2\phi}} = \frac{t_1 t_2}{1 - r_1 r_2 \cos(2\phi) - i r_1 r_2 \sin(2\phi)}$$

Making the transition from electric-field amplitude to electric-field intensity yields

$$T = |t|^2 = \left|\frac{A}{B - iC}\right|^2 = \frac{A^2}{B^2 + C^2} = \frac{t_1^2 t_2^2}{1 - 2 r_1 r_2 \cos(2\phi) + r_1^2 r_2^2 (\cos^2 2\phi + \sin^2 2\phi)}$$

$$= \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\,\cos(2\phi)}$$

what was to be shown.

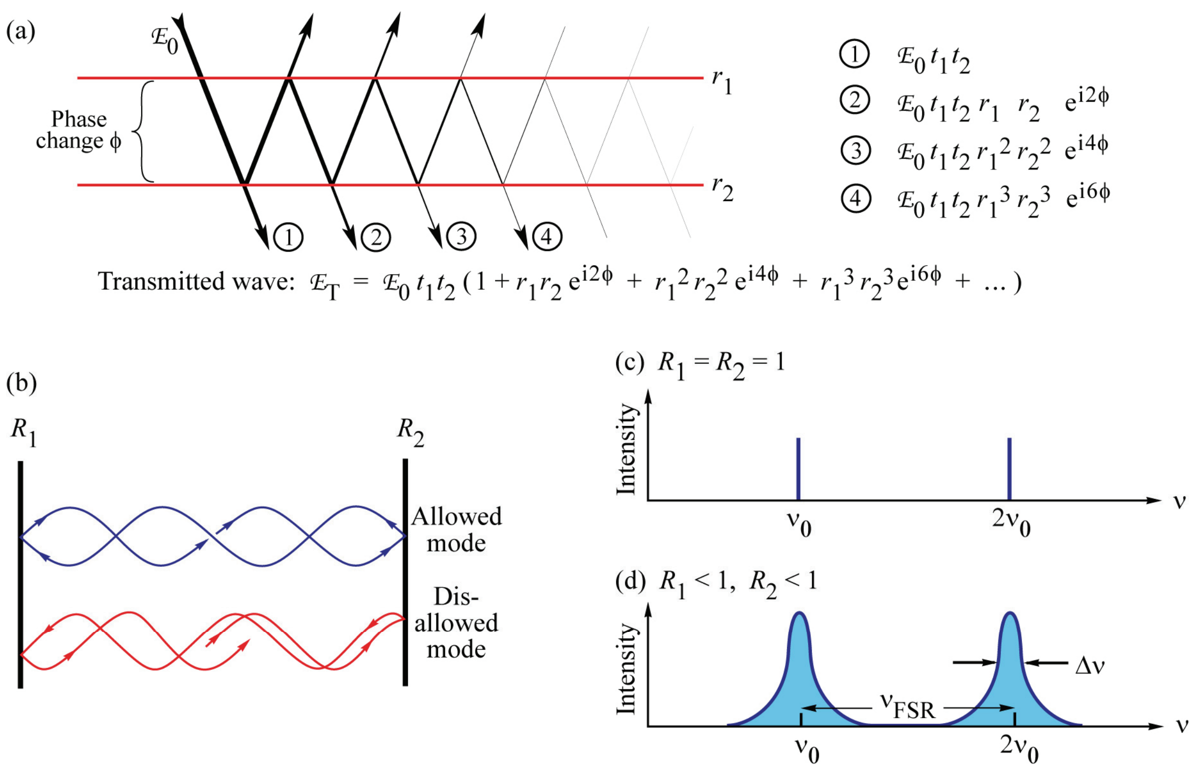

Fig. 14.1. (a) Transmission of a light wave with electric field ampltitude $\mathcal{E}_0$ through a Fabry–Perot resonator. (b) Schematic illustration of allowed and disallowed optical modes in a Fabry–Perot cavity consisting of two coplanar reflectors. Optical mode density for a resonator with (c) no mirror losses ($R_1 = R_2 = 100\%$) and (d) mirror losses.

Exercise: ***Optical mode density***. Derive Eq. (14.8), i.e. optical mode density in a 1D space. Also derive the 3D and 2D optical mode density.

Solution: To derive the *3D optical mode density*, we consider a cubic volume with length L, and volume $V = L^3$, as shown in Fig. 14.4. The possible k vectors of an optical wave inside the volume are given by

$$k = \frac{2\pi}{\lambda_0} m = \frac{2\pi}{2L} m = \frac{\pi}{L} m \qquad \text{for } m = 1, 2, 3 \ldots$$

where m is the mode index and λ_0 is the wavelength of the fundamental optical mode. Note that the modes are equidistant in k space. Thus the "volume element" of one mode in k space is given by

$$\mathrm{d}k_x \, \mathrm{d}k_y \, \mathrm{d}k_z = (\pi / L)^3 \ .$$

Furthermore, the spherical volume in k-space defined by the wave vector k is given by $V = (4/3)\ \pi k^3$. Since k is restricted to *positive* values, the volume is reduced to only the positive quadrant and we use only one eighth of the space, i.e.

$$V = \frac{1}{8} \frac{4}{3} \pi k^3 \ .$$

Thus the number of modes is obtained by dividing the volume in k space used by one optical mode through the volume element in k space, i.e.

$$N_k = \frac{\frac{1}{8}\frac{4}{3}\pi k^3}{(\pi / L)^3} \times 2 = \frac{1}{3} \frac{1}{\pi^2} k^3 V \ .$$

where the factor of 2 ("× 2") is due to the two possible polarizations of the optical mode. Using the free-space dispersion relationship $k = \omega (\varepsilon \mu)^{1/2} = 2\pi\nu\ \bar{n} / c$, one obtains

$$N_\nu = \frac{1}{3} \frac{1}{\pi^2} \left(\frac{2\pi\nu\bar{n}}{c} \right)^3 V \ .$$

Thus the density of optical modes per unit volume per unit frequency is given by

$$\boxed{\rho(\nu) = \frac{1}{V} \frac{\mathrm{d}N_\nu}{\mathrm{d}\nu} = 8\pi\nu^2 \frac{\bar{n}^3}{c^3}} \qquad \textit{\textbf{3D optical mode density}}$$

For two degrees of freedom (2D case), we derive the *2D optical mode density* as follows: The area in k space of one optical mode is given by

$$\mathrm{d}k_x \, \mathrm{d}k_y = (\pi / L)^2 \ .$$

The area in k-space defined by the wave vector k is given by $(1/4)\ \pi k^2$. Thus

$$N_k = \frac{(1/4)\,\pi k^2}{(\pi / L)^2} \times 2 = \frac{1}{2\pi} k^2 A \qquad \text{and} \qquad N_\nu = \frac{1}{2\pi} \left(\frac{2\pi\nu\bar{n}}{c} \right)^2 A = 2\pi \left(\frac{\nu\bar{n}}{c} \right)^2 A$$

where the area $A = L^2$. We thus obtain the density of optical modes per unit area per unit frequency

$$\boxed{\rho(\nu) = \frac{1}{A} \frac{\mathrm{d}N_\nu}{\mathrm{d}\nu} = 4\pi\nu \frac{\bar{n}^2}{c^2}} \qquad \textit{\textbf{2D optical mode density}}$$

For one degree of freedom (1D case), we derive the *1D optical mode density* as follows: The length in k space of one optical mode is given by

$$\mathrm{d}k_x = \pi / L \ .$$

The length in k-space defined by the wave vector k is given by $(½)\,k$. Thus

$$N_k = \frac{(1/2)\,k}{\pi / L} \times 2 = \frac{1}{\pi}\,k\,L \quad \text{and} \quad N_\nu = \frac{1}{\pi}\,\frac{2\pi\nu\bar{n}}{c}\,L = \frac{2\nu\bar{n}}{c}\,L\ .$$

We thus obtain the density of optical modes per unit length per unit frequency

$$\boxed{\rho(\nu) = \frac{1}{L}\,\frac{dN_\nu}{d\nu} = \frac{2\bar{n}}{c}} \qquad \textbf{\textit{1D optical mode density}}$$

what was to be shown.

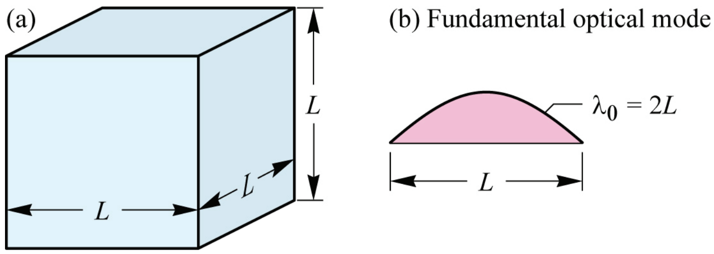

Fig. 14.4. (a) Volume element used to derive the optical mode density. (b) Fundamental optical mode.

Chapter 16

Exercise: ***Photometric units***. A 60 W incandescent light bulb has a luminous flux of 1000 lm. Assume that light is emitted isotropically from the bulb.

(a) What is the luminous efficiency (i.e. the number of lumens emitted per watt of electrical input power) of the light bulb?
(b) What number of standardized candles emit the same luminous intensity?
(c) What is the illuminance, E_{lum}, in units of lux, on a desk located 1.5 m below the bulb?
(d) Is the illuminance level obtained under (c) sufficiently high for reading?
(e) What is the luminous intensity, I_{lum}, in units of candela, of the light bulb?
(f) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous intensity, measured in *candela*.
(g) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous flux, measured in *lumen*.
(h) The definition of the cd involves the optical power of (1/683) W. What, do you suppose, is the origin of this particular power level?

Solution: (a) 16.7 lm/W. (b) 80 candles. (c) $E_{\text{lum}} = 35.4 \text{ lm/m}^2 = 35.4$ lux. (d) Yes.
(e) 79.6 lm/sr = 79.6 cd. (f) $E_{\text{lum}}\, r^2 = I_{\text{lum}}$. (g) $E_{\text{lum}}\, 4\pi r^2 = \Phi_{\text{lum}}$.
(h) Originally, the unit of luminous intensity had been defined as the intensity emitted by a real candle. Subsequently the unit was defined as the intensity of a light source with specified wavelength and optical power. When the power of that light source is (1/683) W, it has the same intensity as the candle. Thus this particular power level has a historical origin and results from the effort to maintain continuity.

Exercise: ***Luminous efficacy of radiation and luminous efficiency of LEDs***. Consider a red and an amber LED emitting at 625 and 590 nm, respectively. For simplicity, assume that the emission spectra are monochromatic ($\Delta\lambda \rightarrow 0$). What is the luminous efficacy of radiation of the two light sources? Calculate the luminous efficiency of the LEDs, assuming that the red and amber LEDs have an external quantum efficiency of 50%. Assume that the LED voltage is given by $V = E_g / e = h\nu / e$.

Assume next that the LED spectra are thermally broadened and have a gaussian lineshape with a linewidth of $1.8\,kT$. Again calculate the luminous efficacy of radiation and luminous efficiency of the two light sources. How accurate are the results obtained with the approximation of monochromaticity?

Solution: The LED emitting at 625 nm has a luminous efficacy of radiation of 219.2 lm/W.
The LED emitting at 590 nm has a luminous efficacy of radiation of 517.0 lm/W.
The LED emitting at 625 nm has a power efficiency of 50% and a luminous efficiency of 109.6 lm/W.
The LED emitting at 590 nm has a power efficiency of 50% and a luminous efficiency of 258.5 lm/W.
We use a gaussian lineshape with a full-width at half-maximum (FWHM) of $1.8\,kT = 1.8 \times 25$ meV = 45 meV. The FWHM and standard deviation of a gaussian function, σ, are related by: FWHM = $2\,(2\ln 2)^{1/2}\,\sigma = 2.355\,\sigma$. Thus the gaussian function's standard deviation is σ = 19.11 meV or 6.02 nm (for the 625 nm LED) and 5.37 nm (for the 590 nm LED). A numerical calculation reveals the following:
625 nm LED: Luminous efficacy of radiation = 221.4 lm/W Luminous efficiency = 110.7 lm/W
590 nm LED: Luminous efficacy of radiation = 515.7 lm/W Luminous efficiency = 257.8 lm/W
Comparison of the results obtained with the approximation of monochromaticity with the results obtained by the accurate numerical calculation reveals a difference of only a few percent.

Chapter 19

Exercise: ***Color rendering***. The color of a physical object, as seen by a human being, is not just a function of the object but also a function of the light source illuminating the object! In fact, the color of an object can depend very strongly on the light source illuminating the object. Some light sources do render the natural colors of an object (true color rendering) while some light sources do not (false color rendering).

(a) What is the color of a yellow banana when illuminated with a red LED?
(b) What is the color of a green banana when illuminated with a yellow LED?
(c) Could it be advantageous for a grocer to illuminate meat with red LEDs, bananas with yellow LEDs, and oranges with orange LEDs?
(d) Is it possible for two physical objects of different colors to appear to have the same color under certain illumination conditions?
(e) Why are low-pressure Na vapor lights used despite their low color-rendering index?
(f) What would be the advantage and disadvantage of using green LEDs for illumination?

Solution:

(a) Red. (b) Yellow. (c) Yes – but his truthfulness in displaying fruit could be questioned. (d) Yes. (e) Because of their high luminous efficiency (and thus low electricity consumption). (f) High luminous efficacy would be an advantage but low color-rendering properties would be a disadvantage.

Chapter 22

Exercise: ***Modal dispersion in waveguides***. Calculate the time delay between the slowest and the fastest modes, and the maximum possible bit rate for a 1 km long multimode fiber waveguide with core refractive index $\bar{n}_1 = 1.45$ and cladding refractive index $\bar{n}_2 = 1.4$.

Solution: Using Snell's law (Eq. 22.2), one obtains $\theta_c \approx 15^o$. The time delay calculated from Eq. (22.3) for a 1 km long fiber amounts to $\Delta\tau = 170$ ns. The minimum time required to transmit one bit of information is given by $\Delta\tau$. This yields an approximate maximum bit rate of $f_{max} = 1/170$ ns = 5.8 Mbit/s. The calculation shows that modal dispersion can be a significant limitation in optical communication. Graded-index multimode fibers or single-mode fibers are therefore required for high-speed communication systems.

Exercise: ***Material dispersion in waveguides***. Derive Eqs. (22.6) and (22.7). Why does material dispersion have a much smaller significance for semiconductor lasers than for LEDs?

Solution:

Derivation of Eq. (22.6): The group refractive index is defined as

$$\bar{n}_{gr} = \frac{c}{v_{gr}} = c\frac{\mathrm{d}k}{\mathrm{d}\omega} = c\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{\omega\,\bar{n}}{c} .$$

In this equation we have used

$$k = \frac{2\pi}{\lambda} = \bar{n}\,k_0 = \bar{n}\frac{2\pi}{\lambda_0} = \bar{n}\frac{2\pi}{c/\nu} = \bar{n}\frac{\omega}{c} .$$

Performing the derivative yields

$$\bar{n}_{gr} = c\left(\frac{\bar{n}}{c} + \frac{\omega}{c}\frac{\mathrm{d}\bar{n}}{\mathrm{d}\omega}\right) = \bar{n} + \omega\frac{\mathrm{d}\bar{n}}{\mathrm{d}\omega} .$$

Using $\omega = 2\pi\nu = 2\pi\, c/\lambda_0$, and using $\mathrm{d}\,(1/\lambda_0) = -\lambda_0^{2}\,\mathrm{d}\lambda_0$, one obtains

$$\bar{n}_{gr} = \bar{n} + 2\pi\frac{c}{\lambda_0}\frac{\mathrm{d}\bar{n}}{2\pi c\,\mathrm{d}\frac{1}{\lambda_0}} = \bar{n} - \lambda_0\frac{\mathrm{d}\bar{n}}{\mathrm{d}\lambda_0} ,$$

what was to be shown.

Derivation of Eq. (22.7): It is $v_{gr} = c/\bar{n}_{gr}$. Forming the derivative with respect to λ yields $\mathrm{d}v_{gr}/\mathrm{d}\lambda = c\,(\mathrm{d}/\mathrm{d}\lambda)\,\bar{n}_{gr}^{-1} = c\,(-1/\bar{n}_{gr}^{2})\,(\mathrm{d}\bar{n}_{gr}/\mathrm{d}\lambda)$. Thus $\Delta v_{gr} = c\,(-1/\bar{n}_{gr}^{2})\,(\mathrm{d}\bar{n}_{gr}/\mathrm{d}\lambda)\,\Delta\lambda$. Taking the absolute value of both sides of the equation yields the equation that was to be shown.

Why is material dispersion much less relevant for lasers than for LEDs? According to Eq. (22.8), material dispersion is proportional to the spectral linewidth of the source, $\Delta\lambda_0$. Because lasers (longitudinal multi-mode as well as single-mode lasers) have a much narrower linewidth than LEDs, material dispersion is generally much less relevant for laser-based communication systems than it is for LED-based communication systems.

Exercise: ***Comparison of material and modal dispersion***. Consider a 62.5 µm core diameter multimode step-index fiber of 3 km length with a core index of $\bar{n}_1 = 1.45$ and a cladding index of $\bar{n}_2 = 1.4$. Assume that the fiber inputs come from either an LED or a laser emitting at 850 nm. Assume that the LED and the laser have a linewidth of 50 and 5 nm, respectively. Calculate the modal and the material dispersion for each case and explain the result.

Solution: ***Modal dispersion***: The calculation of the modal dispersion depends only on the fiber and is independent of the source. Using Snell's law, one obtains $\theta_c \approx 15°$. The time delay due to modal dispersion calculated from Eq. (22.3) for a 3 km long optical fiber amounts to $\Delta\tau_{\text{modal dispersion}} = 510$ ns. ***Material dispersion***: Figure 22.5 shows that at 850 nm, the material dispersion in silica is $\Delta\tau_{\text{material dispersion}} / (\Delta\lambda_0\, L) \approx -65$ ps / (nm km). For a 3 km long optical fiber that is fed by a source with a 50 nm line width (LED) and 5 nm line width (laser), the dispersion amounts to 9.75 ns and 0.975 ns, respectively. This exercise shows that modal dispersion typically dominates over material dispersion when multimode fibers are used.

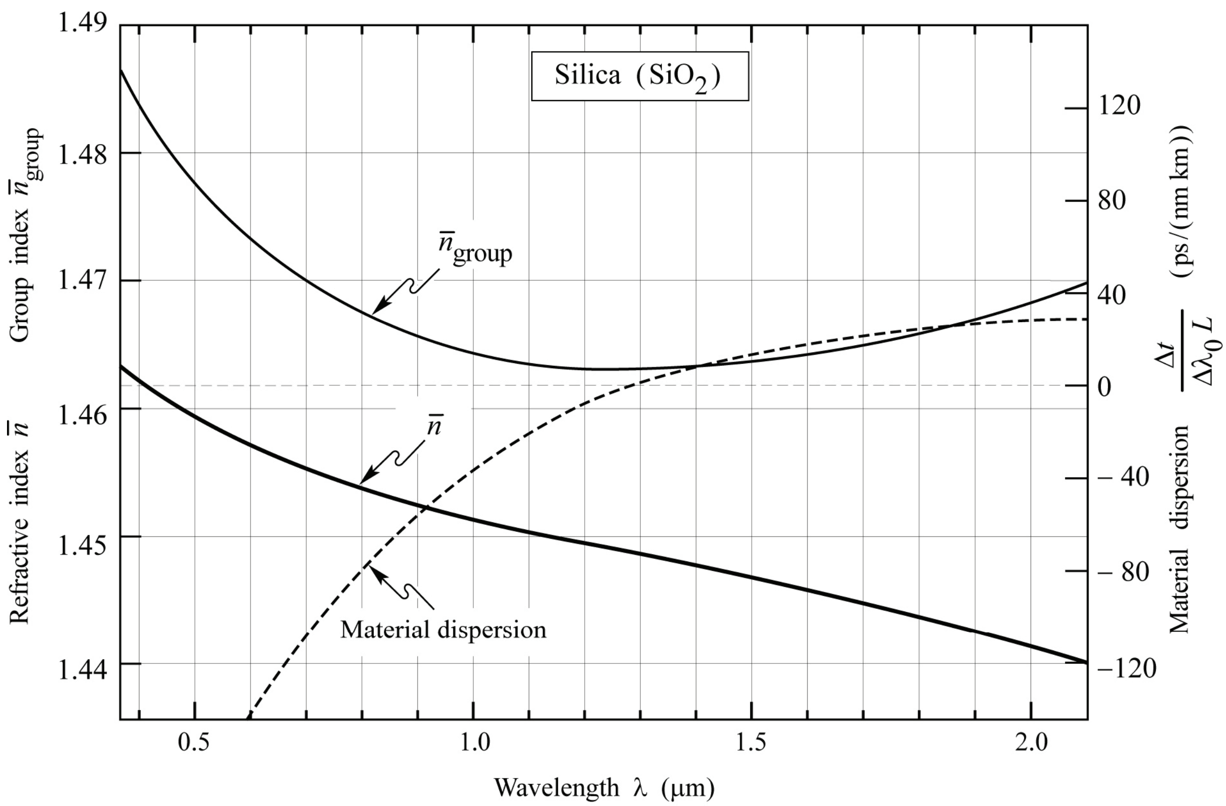

Fig. 22.5. Refractive index, group index, and material dispersion of a silica fibers for an optical signal spectral width $\Delta\lambda_0$ in vacuum. The material dispersion of regular silica fibers is zero at $\lambda = 1.3$ µm.

Exercise: ***Coupling efficiency of a fiber butt-coupled to an LED***. Consider an LED with a point-like emission region that emits an optical power of 1 mW into the hemisphere. For simplicity, assume that the intensity emitted by the LED is independent of the emission angle. What is the maximum acceptance angle of a single-mode fiber with $NA = 0.1$ and multimode fiber with $NA = 0.25$? What is the power that can be coupled into the two fibers?

Solution: The maximum acceptance angles of the single-mode and multimode fibers in air are $\theta_{air} = 5.7°$ and 14.5°, respectively. The solid angle defined by an acceptance angle θ_{air} is given by $\Omega = 0.031$ and 0.20 for the single-mode and multimode fiber, respectively. Since the entire hemisphere has a solid angle of 2π, the power coupled into the single-mode and multimode fibers is given by 0.0049 mW and 0.032 mW, respectively.

Exercise: ***Coupling efficiency of a fiber coupled to an LED with a lens***. Consider an LED circular emission region with diameter 20 μm coupled to a silica multimode fiber with NA = 0.2 and a core diameter of 62.5 μm. The LED emits a power of 1 mW into the hemisphere lying above the planar LED surface. For simplicity, assume that the LED emission intensity is independent of the emission angle. What is the maximum power that can be coupled into the multimode fiber?

Solution: Improved coupling can be obtained by imaging the LED emission region on to the core of the optical fiber. For maximum coupled power, a convex lens with magnification M = 62.5 μm / 20 μm = 3.125 can be used. Using the lens, the acceptance angle of the fiber is increased from $\theta_{air} = 11.5°$ to $\theta_{LED} = 35.9°$. The solid angle defined by the LED acceptance angle θ_{LED} is given by $\Omega = 1.19$. Since the LED emits 1 mW into the entire hemisphere (with solid angle $\Omega = 2\pi$), the power coupled into the fiber is given by 0.189 mW.

Chapter 24

Exercise: ***Derivation of equations***. Derive Eqs. (24.3), (24.4), (24.5) and (24.8).

Solution:

Derivation of Eq. (24.3): Consider an exponential decay with time constant τ, i.e. exp $(-t/\tau)$. Assume that the time at which the amplitude has decreased to 90% and 10% of its maximum value are $t_{90\%}$ and $t_{10\%}$, respectively. Accordingly

$$e^{-t_{90\%}/\tau} = 90\% \qquad \text{and} \qquad e^{-t_{10\%}/\tau} = 10\% \ .$$

Then

$$\frac{t_{90\%}}{\tau} - \frac{t_{10\%}}{\tau} = -\ln 0.1 + \ln 0.9 = \ln 9 \qquad \text{or} \qquad t_{90\%} - t_{10\%} = \tau \ln 9$$

what was to be shown. An analogous consideration applies to the rise time.

Derivation of Eq. (24.4): The voltage transfer function of an RC voltage-divider circuit is given by

$$H(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I\,(\mathrm{i}\,\omega C)^{-1}}{I\,R + I\,(\mathrm{i}\,\omega C)^{-1}} = \frac{1}{1 + \mathrm{i}\,\omega RC} = \frac{1}{1 + \mathrm{i}\,\omega\tau}$$

where $\tau = RC$; this is what was to be shown.

Derivation of Eq. (24.5): Using $|H(\omega_{3\text{dB}})|^2 = |(1 + \mathrm{i}\,\omega\tau)^{-1}|^2 = ½$, and

$$|H(\omega)|^2 = \left|\frac{A}{A + \mathrm{i}B}\right|^2 = \frac{|A|^2}{|A + \mathrm{i}B|^2} = \frac{A^2}{A^2 + B^2} = \frac{1}{1 + \omega^2\tau^2}$$

we obtain $\omega_{3\text{dB}} = \tau^{-1} = (RC)^{-1}$ or $f_{3\text{dB}} = (2\pi\,\tau)^{-1}$, what is what was to be shown.

Derivation of Eq. (24.8): Using $|H(\omega_{3\text{dB}})|^2 = |(1 + \mathrm{i}\,\omega_{3\text{dB}}\,\tau)^{-1}| = ½$, and

$$|H(\omega)|^2 = \left|\frac{A}{A + \mathrm{i}B}\right| = \frac{A}{\sqrt{A^2 + B^2}} = \frac{1}{\sqrt{1 + \omega^2\tau^2}}$$

we obtain

$$\frac{1}{2} = \frac{1}{\sqrt{1 + \omega_{3\text{dB}}^2\,\tau^2}} \ .$$

Solving for $\omega_{3\text{dB}}$ yields $\omega_{3\text{dB}} = 3^{1/2}/\tau$ or $f_{3\text{dB}} = 3^{1/2}/(2\pi\tau)$, what is what was to be shown.

Exercise: ***Rise and fall time and 3 dB frequency***. Consider an LED with a rise time of 1.75 ns. Assume that the fall time of the LED is identical to the rise time. What is the 3 dB frequency of the device? Give the physical reasons as to why Eq. (24.8) gives only an *approximate* value of the 3 dB frequency.

Solution: A 3 dB frequency of 343 MHz is expected on the basis of Eq. (24.8). In practice the 3 dB frequency can be lower or higher than the calculated value since the rise and fall are frequently not exponential. As a practical rule, the numerical factor 1.2 in the numerator of Eq. (24.8) can vary between 1.0 and 1.5.

Exercise: ***Calculation of carrier sweep-out time***. Calculate the carrier sweep-out time for typical values of the electric field in the p-n junction depletion region, typical carrier velocity, and an active region thickness of 0.1–1 μm.

Solution: The carrier sweep-out time can be very short. For typical diode parameters, the carrier sweep-out time is about 1–100 ps, i.e. much shorter than the spontaneous recombination time. As an example, let us assume that a carrier drifts with the drift-saturation velocity, which is about 10^7 cm/s, across a reverse-biased active region. The time needed to drift across a 1.0 μm thick active region is given by 1.0 μm / 10^7 cm/s = 10 ps.

Exercise: ***Photometric units***. A 60 W incandescent light bulb has a luminous flux of 1000 lm. Assume that light is emitted isotropically from the bulb.

(a) What is the luminous efficiency (i.e. the number of lumens emitted per watt of electrical input power) of the light bulb?
(b) What number of standardized candles emit the same luminous intensity?
(c) What is the illuminance, E_{lum}, in units of lux, on a desk located 1.5 m below the bulb?
(d) Is the illuminance level obtained under (c) sufficiently high for reading?
(e) What is the luminous intensity, I_{lum}, in units of candela, of the light bulb?
(f) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous intensity, measured in *candela*.
(g) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous flux, measured in *lumen*.
(h) The definition of the cd involves the optical power of (1/683) W. What, do you suppose, is the origin of this particular power level?

Solution: (a) 16.7 lm/W. (b) 80 candles. (c) E_{lum} = 35.4 lm/m^2 = 35.4 lux. (d) Yes.
(e) 79.6 lm/sr = 79.6 cd. (f) $E_{\text{lum}}\, r^2 = I_{\text{lum}}$. (g) $E_{\text{lum}}\, 4\pi r^2 = \Phi_{\text{lum}}$.
(h) Originally, the unit of luminous intensity had been defined as the intensity emitted by a real candle. Subsequently the unit was defined as the intensity of a light source with specified wavelength and optical power. When the power of that light source is (1/683) W, it has the same intensity as the candle. Thus this particular power level has a historical origin and results from the effort to maintain continuity.