# Utility Analysis in the Decision Trees

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In this paper it is presented an integrated decision analysis procedure for the decision trees. While utility theory (Allais, 1979; Fishburn, 1970; French, 1994; Neumann, 1947), remains the most valuable prescriptive approach for the inclusion of risk in decision analysis, it is difficult to obtain accurate and consistent utility function values. It may be worth developing an integrated decision analysis procedure that includes the calculation of the expected values, simulation of the risk profiles, and as a final step the sensitivity analysis using utility functions with different levels of risk aversion.

## *1. PHASE I. Expected monetary value analysis*

*Step I.1.* Analyze the decision tree in term of the expected monetary value. Very often this provides an adequate basis for making the decision.

*Step I.2.* Examine the value of additional information about the states of nature in order to revise or update the probability estimates and develop an optimal decision strategy for the problem. The concepts of expected of sample information, expected value of perfect information, and efficiency of information can be used to evaluate the contribution of the sample information.

The expected monetary value approach examines the average outcome of a strategy. However, because of this it fails to capture what is the essence of risk, by ignoring variability of outcome. Go to Phase II, in order to take risk into account in making decision.

## 2. PHASE II. Risk profiles analysis

*Step II.1.* Define the possible strategies available to the decision maker in the decision tree. For each strategy identified, determine the cumulative probability distribution of the outcomes.

*Step II.2.* Use Monte Carlo simulation (Bonini, 1997; Eppen, 2001; Luban, 2000) for developing the risk profile for each strategy.

*Step II.3*. Use the stochastic dominance criterion to choose among strategies.

However, if the outcomes appear unreasonably large or unreasonable small, and if the decision maker feels monetary values do not adequately reflect his true preferences for the outcomes, a utility analysis of the problem should be considered.

## 3. PHASE III. Utility analysis

The analysis based on the utility functions for a risk - averse decision maker can be carried out as follows.

*Step III.1.* Determine the best and the worst possible payoffs in the decision problem. Assign utility values to the best and worst payoffs. Any values will work as long as the utility assigned to the best payoff is greater than the utility assigned to the worst payoff. Suppose that $U$(worst payoff) = 0 and $U$(best payoff) = 1;

*Step III.2.* Assessing a utility function. We propose an exponential function in order to avoid the difficulty regarding the construction of a utility function that often causes decision makers to feel uncomfortable. It has the form:

$$U(\phi) = \alpha - \beta \exp(-\phi/R^L)$$

where:

$\phi$ is a monetary payoff that must be converted to utility,

$\alpha = \beta * \exp(-(\text{worst payoff})/R^L)$

$\beta = 1/(\exp(-(\text{worst payoff})/R^L) - \exp(-(\text{best payoff})/R^L))$.

The risk aversion parameter $R^L$, can be obtained by trying different values of $R$ such that

$$U(L) = 0.5*U(R^L)+0.5*U((R^L)/2) = U(0)$$

*Step III.3.* Do the sensitivity analysis to determine how risk averse the decision maker would have to be toward the selected strategy.

Determine the certainty equivalent $CE$ for the lottery $L$ involving a 50/50 chance of winning $R^L$ monetary units and losing $(R^L)/2$ monetary units, i.e.

$$CE(L) = - R^L*\ln((\alpha - U(L))/\beta).$$

If the decision maker values the lottery $L$ at more than $CE(L)$, he is less risk averse than $R^L$ utility function.

Continue the sensitivity analysis to determine how large the risk aversion parameter R would have to be for the decision maker to be indifferent between different alternatives. For $R^L$, determine the expected utility for each of the decision alternatives:

$$EU(v_i) = \sum_{j=1}^{n} p_j U(\phi_{ij}), \qquad \text{for } i = 1,\dots, m$$

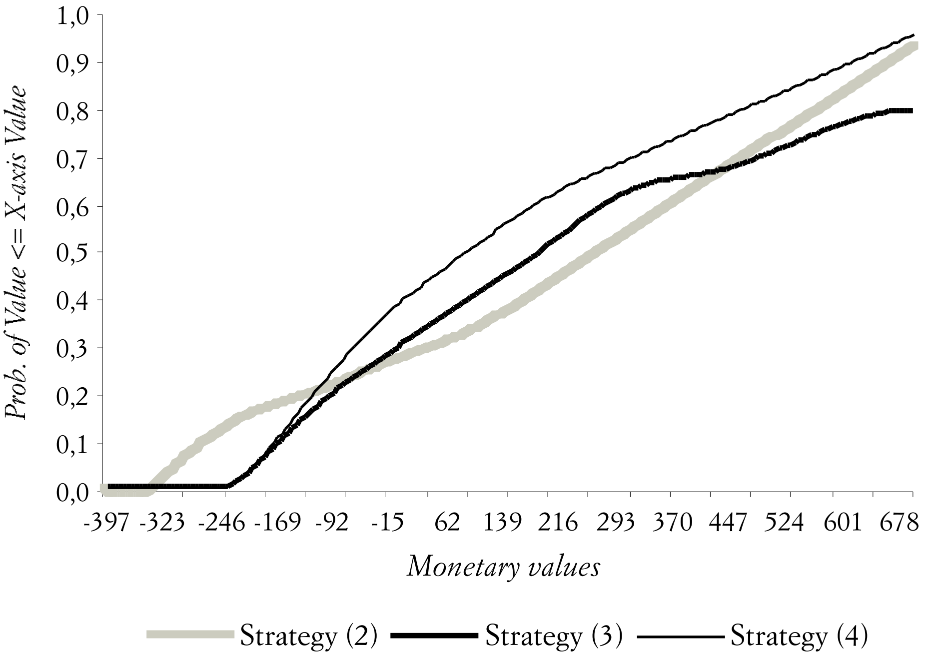
and select the alternative yielding the best expected utility.

This procedure can be implemented on spreadsheets like EXCEL.

In the paper, to illustrate the proposed procedure *numerical results* are presented. There are four possible strategies available to the decision-maker:

- Strategy (1): Do not introduce the product.
- Strategy (2): Introduce the product;
  if a competitive product is introduced, set a high price;
  if no competitive product is introduced, set a high price.
- Strategy (3): Introduce the product;
  if a competitive product is introduced, set a medium price;
  if no competitive product is introduced, set a high price.
- Strategy (4): Introduce the product;
  if a competitive product is introduced, set a low price;
  if no competitive is introduced, set a high price.

*Fig. 1 Risk profiles obtained by simulation*




In Figure 1 are shown the cumulative distributions for profit for all strategies provided by Monte Carlo simulation.

The strategy (2) dominates by the second-degree stochastic dominance the other strategies because its cumulative curve is below the cumulative curves of the other for the outcome between -43 m.u. and 458 m.u.

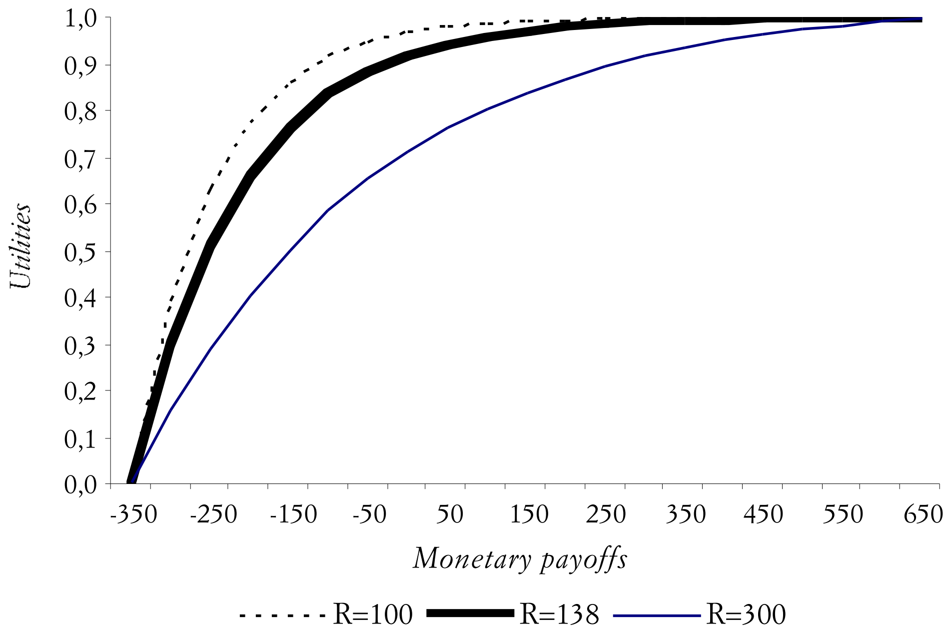
In contrast, the expected monetary optimal strategy is strategy (3) and the expected monetary value is 265 m.u.

To determine the utility associated with every other payoff we use the exponential utility function:

$$U(\phi) = \alpha - \beta \exp(-\phi/R^{L})$$

Firstly, the value of the risk aversion parameter $R$ must to be calculated such that the decision maker to be indifferent between a bet where

*Fig. 2 Utility function for indifference values of R*




he wins $R$ m.u. or loses $R/2$ m.u. with equal probability and not betting at all. By trying different values of $R$, it was found

$$R^L = 138 \text{ m.u.}$$
$$\alpha = 1.001$$
$$\beta = 0.079$$

such that

$$U(\phi) = 1.001 - 0.079 \exp(-\phi/138).$$

The curves with different value of $R$ are plotted in Figure 2.

For $R^L$ = 138 m.u. there is indifference between “Introduce the product” and “Do not introduce the product”.

With this curve, the decision maker has only to decide in which range his risk aversion lies.

## 4. Selected references

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