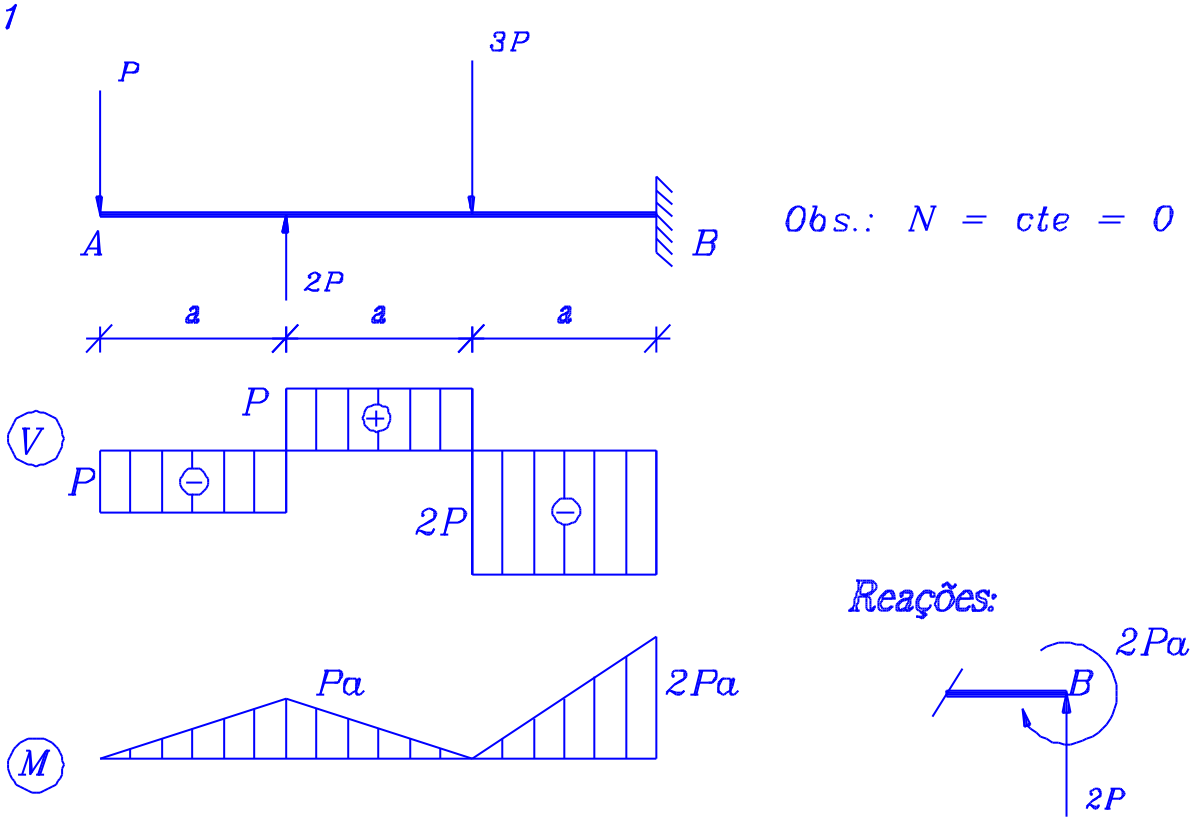
Lista de exercícios nº 3
Diagramas de esforços solicitantes

1 - Traçar os diagramas de esforços solicitantes e indicar as reações de apoio:

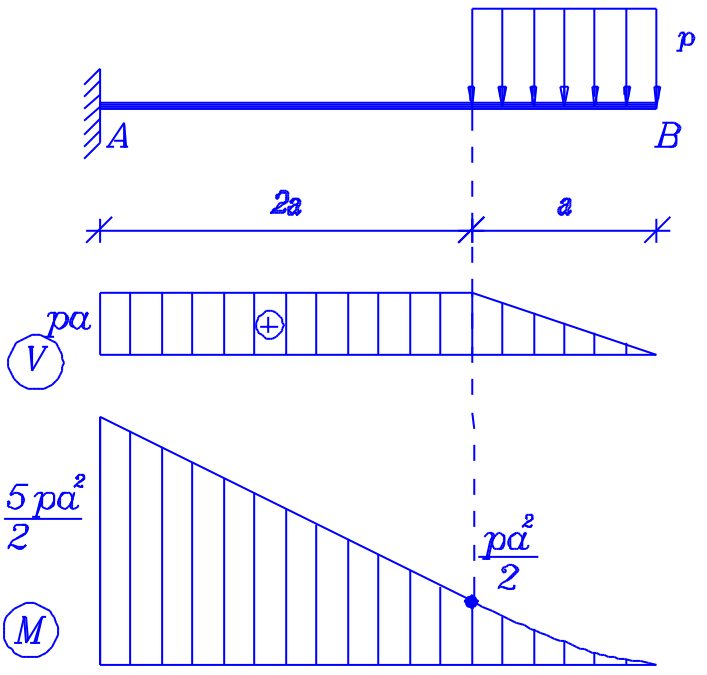
1.1



Obs.: N = cte = 0

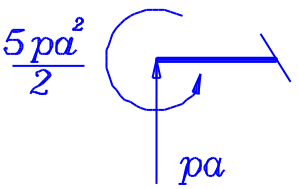
Reações:

1.2

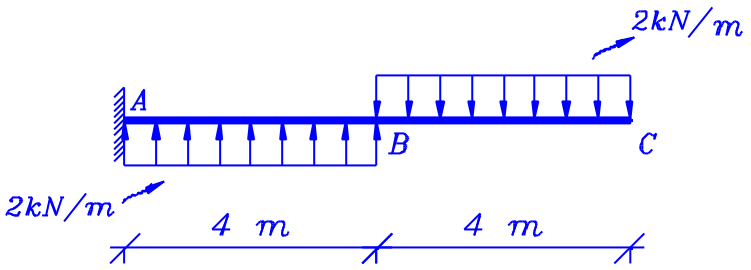


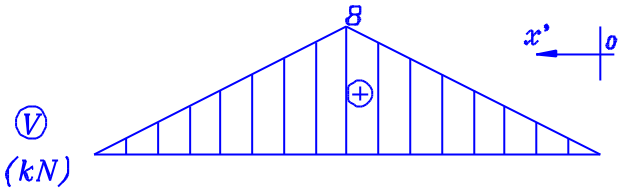
Obs.: N = cte = 0

Reações:



1.3



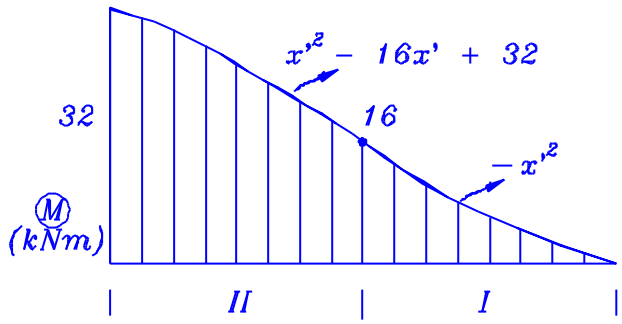


Trecho I : $0 \leqslant x' \leqslant 4$

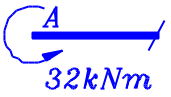
$M = -x'^2$

Trecho II: $4 \leqslant x' \leqslant 8$

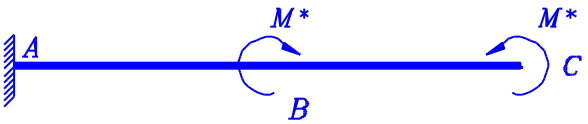
$M = x'^2 - 16x' + 32$



Reações:

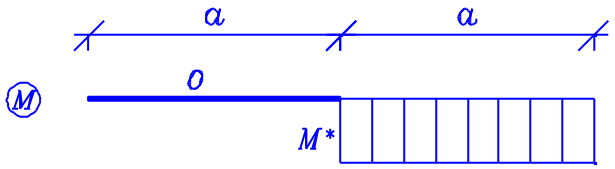


1.4



Obs.: $N = cte = 0$

$V = cte = 0$



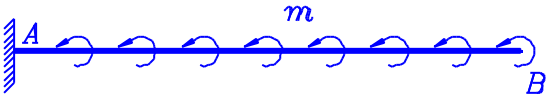
Reações:

$Y_A = 0$

$X_A = 0$

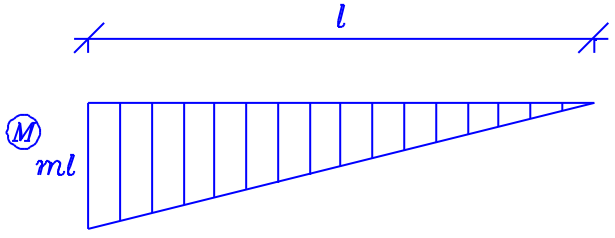
$M_A = 0$

1.5

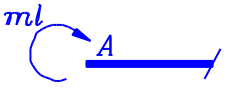


Obs.: $N = cte = 0$

$V = cte = 0$

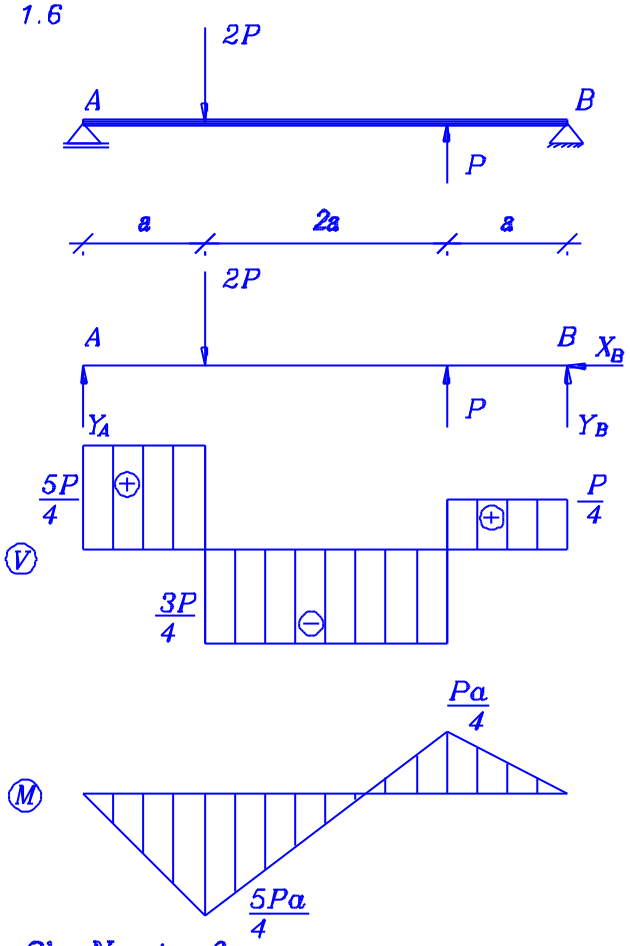


Reações:



Obs.: A viga é carregada por um momento uniformemente distribuído ao longo de seu comprimento.

1.6



Obs.: N = cte = 0

Cálculo das reações de apoio:

$\sum X = 0 : \boxed{X_B = 0}$

$\sum Y = 0 : Y_A - 2P + P + Y_B = 0 \Rightarrow$

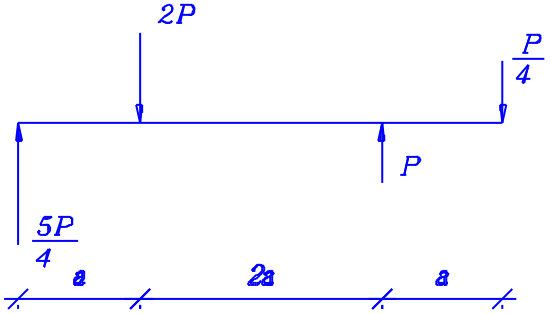
$Y_A + Y_B = P \ (1)$

$\sum M_A = 0$

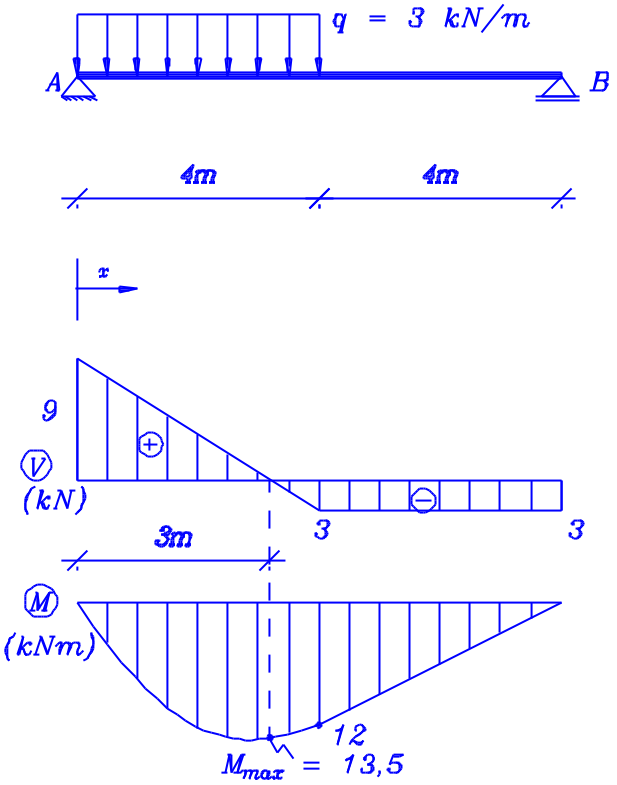
$-2P \cdot a + P \cdot 3a + V_B \cdot 4a = 0$

$\boxed{Y_B = -P/4} \ (2)$

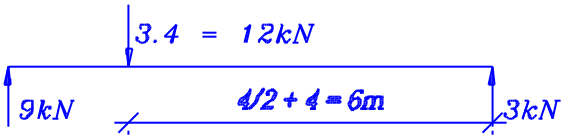
(2) em (1): $\boxed{Y_A = 5P/4}$



1.7



Reações de apoio:



$Y_A = \frac{(3 \ . \ 4) \ . \ (4/2 \ + \ 4)}{4 \ + \ 4} = 9 \ kN$

$Y_B = 3 \ . \ 4 \ - \ Y_A = 3 \ kN$

$X_A = 0 \ kN$

Diagramas:

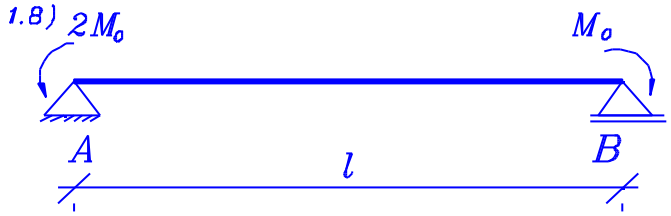
$0 \leq x \leq 4$

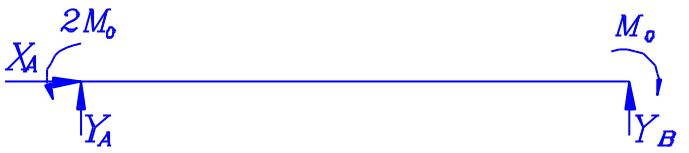
$M = 9x - \frac{3x^2}{2}$

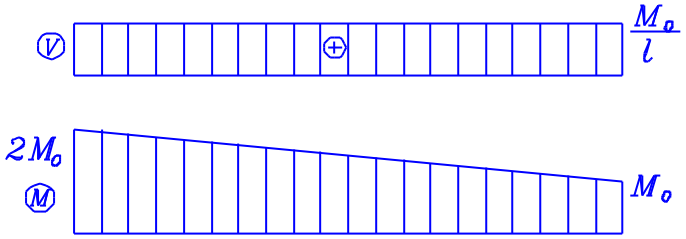
$V = 9 - 3x$

$V = 0 \Rightarrow M = M_{max} \ M(x=3)$

1.8)







Reações de apoio:

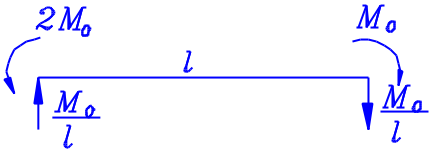
$$\Sigma M_A = 0$$

$$\Rightarrow 2M_0 - M_0 + Y_B \cdot l = 0 \Rightarrow Y_B = -\frac{M_0}{l}$$

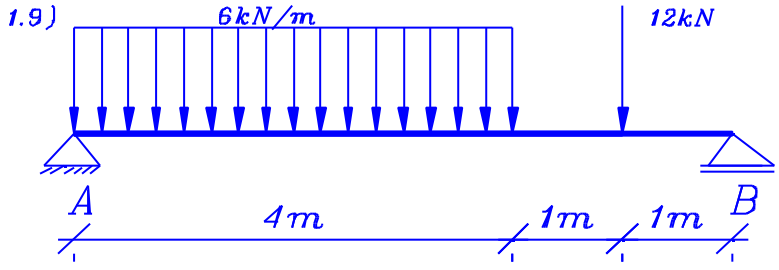
$$\Sigma Y = 0 \Rightarrow Y_A + Y_B = 0$$

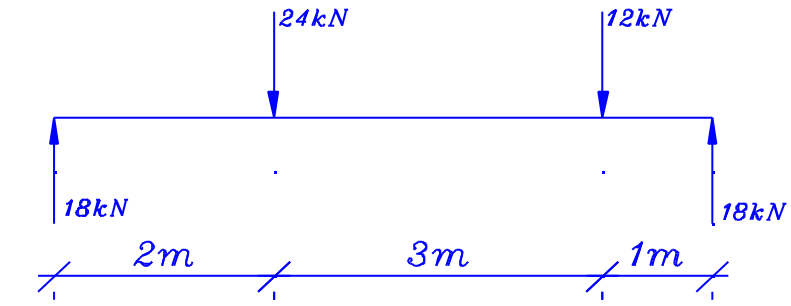
$$\Rightarrow Y_A = +\frac{M_0}{l}$$

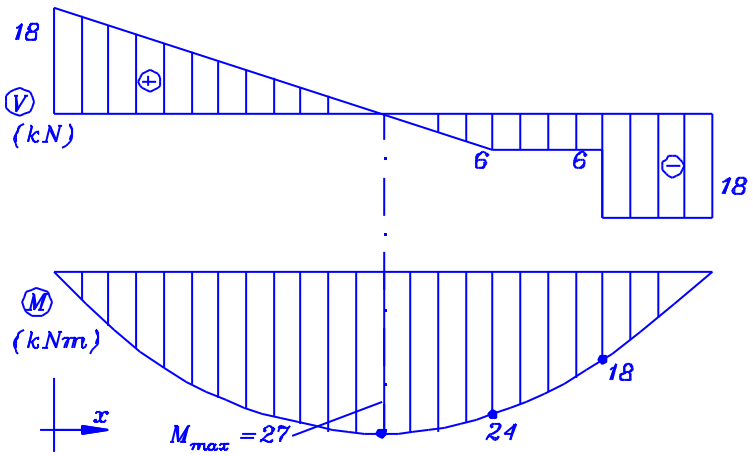
$$\Sigma X = 0 \Rightarrow X_A = 0$$



1.9)







Reações de apoio:

$$Y_A = \frac{(6 \cdot 4) \cdot (2+2) + 12 \cdot 1}{6} = 18kN$$

$$Y_B = 6 \cdot 4 + 12 - 18 = 18kN$$

$$X_A = 0$$

Diagramas: $0 \le x \le 4$

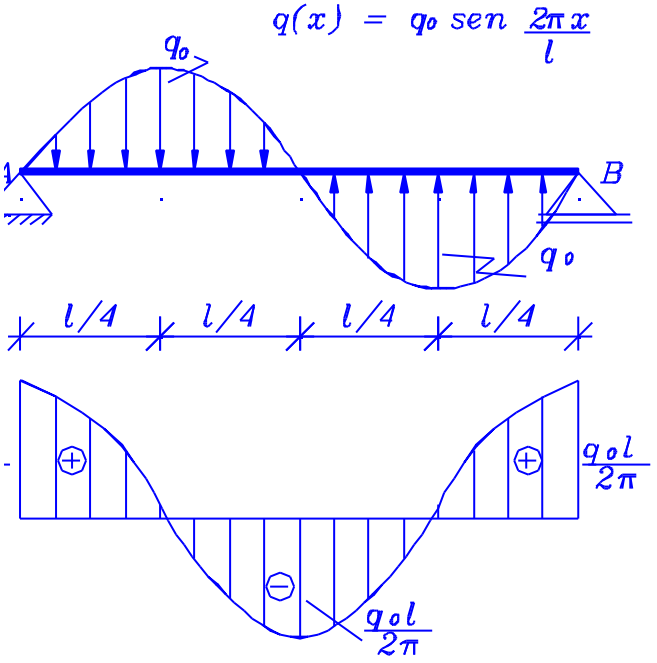
$$M(x) = 18x - 3x^2$$

$$V(x) = 18 - 6x$$

$$V = 0 \Rightarrow x = 3m$$

$$M_{\max} = M(x=3) = 27kNm$$

Determinar as reações de apoio e traçar os diagramas de esforços solicitantes da estrutura abaixo:



$$\frac{dM^2}{dx^2} = -p(x) = -q_0 \operatorname{sen}\frac{2\pi x}{l}$$

$$\frac{dM}{dx} = V(x) = -q_0 \operatorname{sen}\frac{2\pi x}{l}\,dx = \frac{q_0 l}{2\pi}\cos\frac{2\pi x}{l} + C_1$$

$$M = \frac{q_0 l^2}{2^2}\cos\frac{2\pi x}{l}\,dx + C_1 x = \frac{q_0 l}{4\pi^2}\operatorname{sen}\frac{2\pi x}{l} + C_1 x + C_2$$

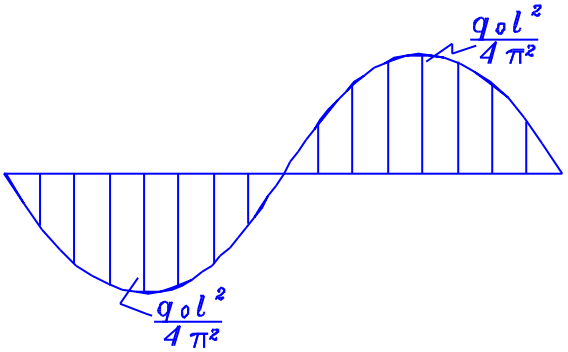
$$M(0) = 0 \Rightarrow C_2 = 0$$

$$M(l) = 0 \Rightarrow C_1 = 0$$

Tem-se então:

$$V(x) = \frac{q_0 l}{2\pi}\cos\frac{2\pi x}{l}$$

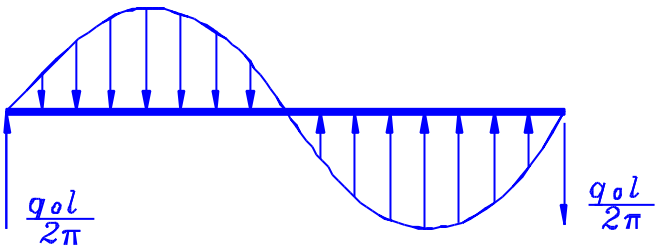
$$M(x) = \frac{q_0 l^2}{4\pi^2}\operatorname{sen}\frac{2\pi x}{l} = \frac{l^2}{4\pi^2}q(x)$$



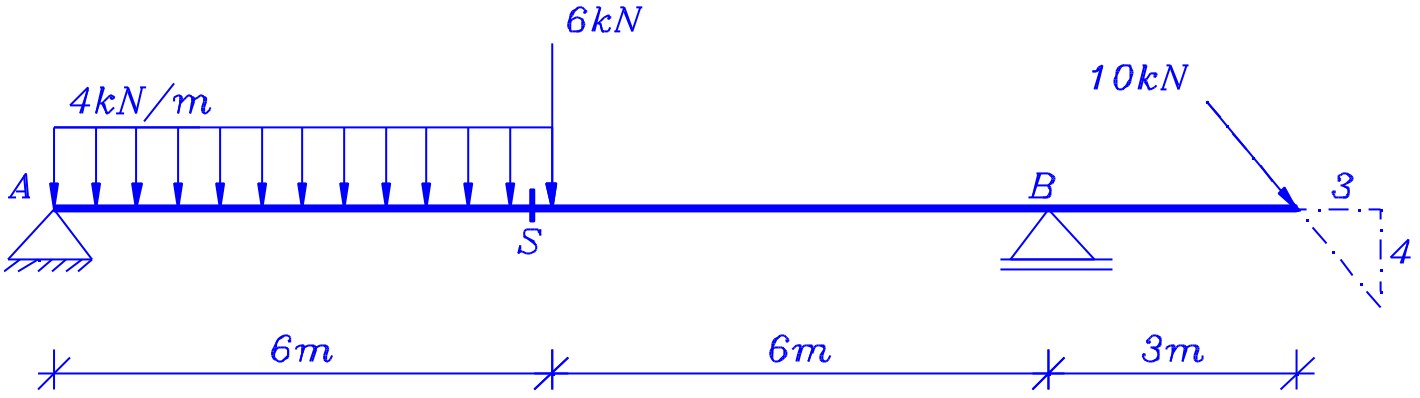
Reações:

$$Y_A = V(0) = \frac{q_0 l}{2\pi}$$

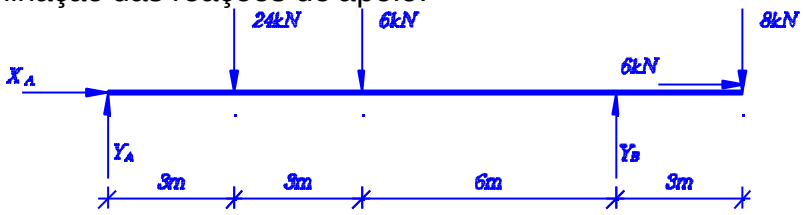
$$Y_B = -V(l) = -\frac{q_0 l}{2\pi}$$



3 - Determinar os esforços solicitantes na seção **S**:



Determinação das reações de apoio:

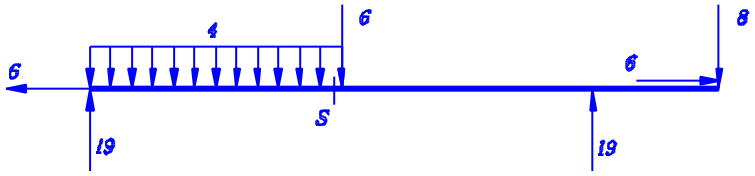


$\sum X = 0 \Rightarrow X_A + 6 = 0 \therefore \boldsymbol{X_A = -6kN}$

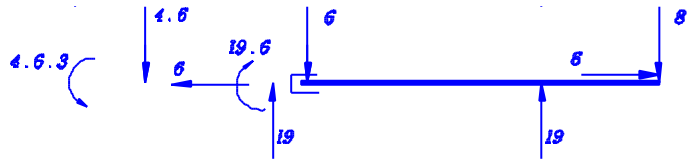
$\sum Y = 0 \Rightarrow Y_A + Y_B = 24 + 6 + 8 = 38$

$\sum M_A = 0 \Rightarrow -24 \cdot 3 - 6 \cdot 6 + 12Y_B - 8 \cdot 15 = 0 \therefore \boldsymbol{Y_B = 19kN}$

$Y_A = 38 - Y_B \therefore \boldsymbol{Y_A = 19kN}$



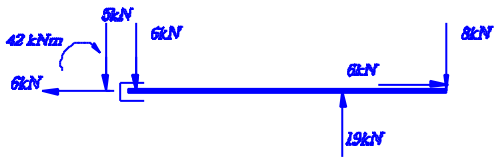
Determinação dos esforços solicitantes:



$N_S = 6kN$

$V_S = 19 - 24 \therefore V_S = -5kN$

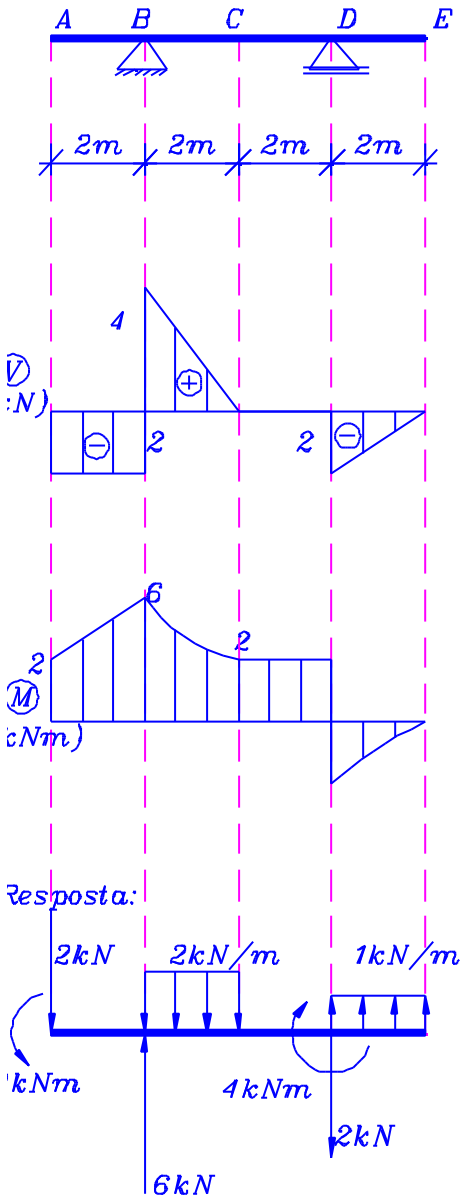
$M_S = 19 \cdot 6 - 4 \cdot 6 \cdot 3 \therefore M_S = +42kNm$



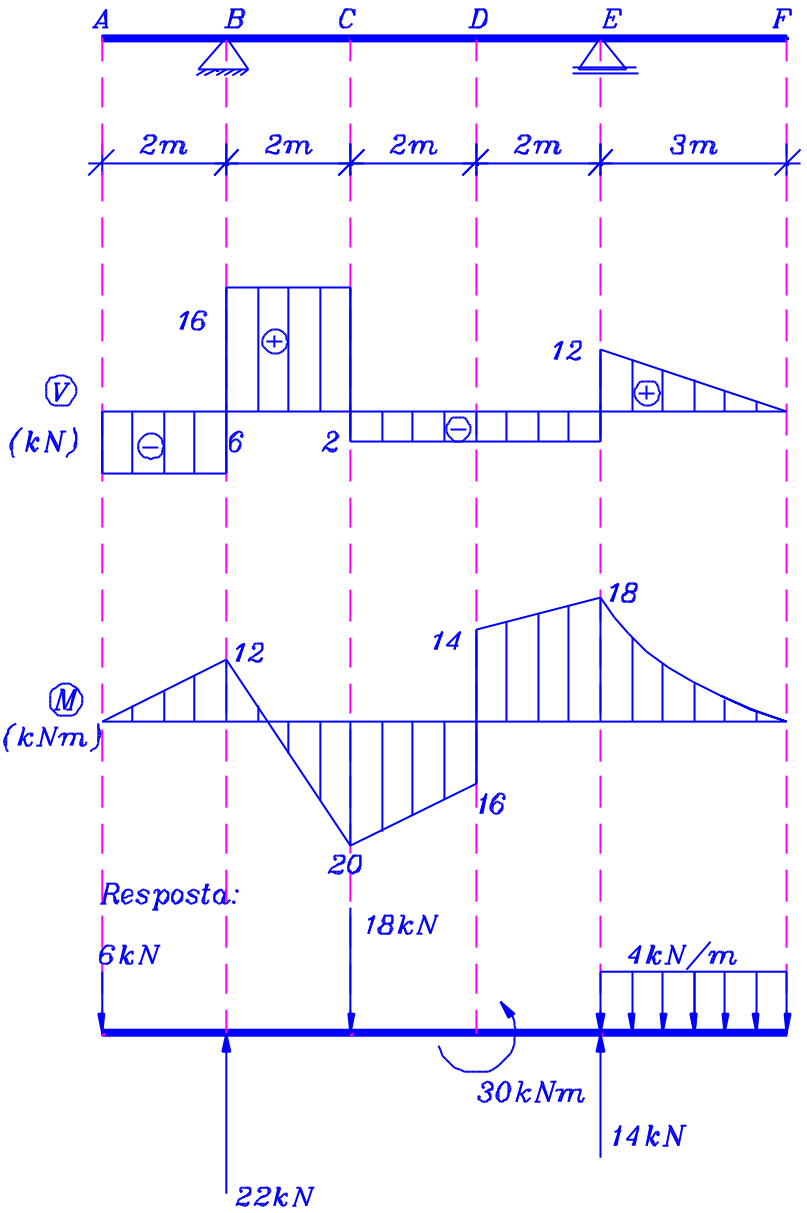
Sugestão: Para conferir os resultados obtidos, determine agora os esforços solicitantes vindo pela direita
Deverão ser obtidos os mesmos esforços solicitantes nos dois casos.

4 - A partir dos diagramas de V e M, determinar os esforços externos ativos e reativos:

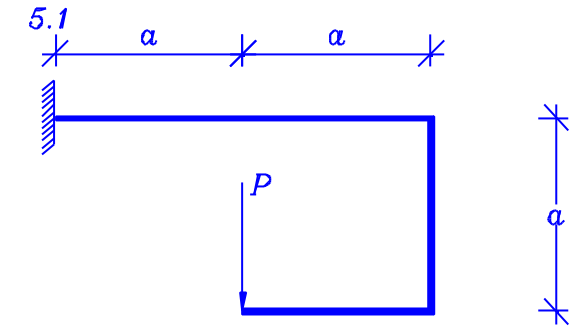
1.1)

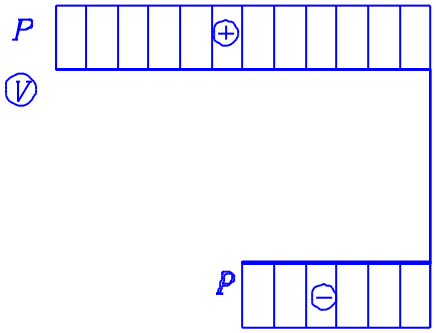


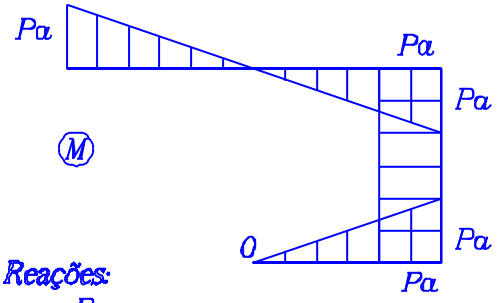
4.2)

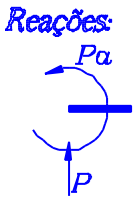


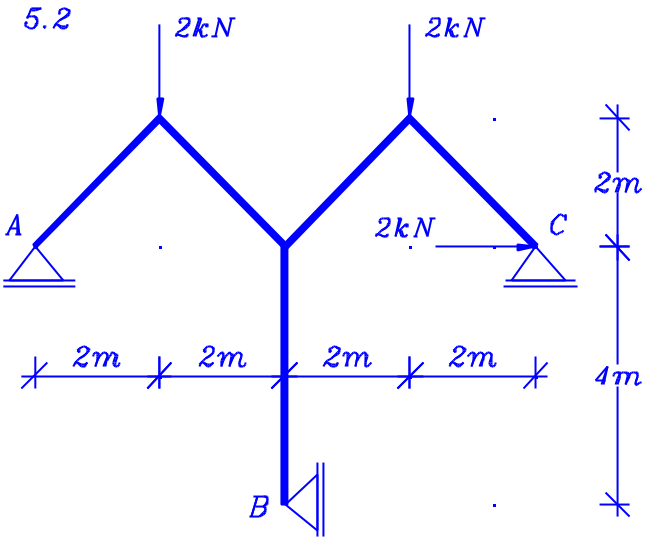
5 - Traçar os diagramas de esforços solicitantes:

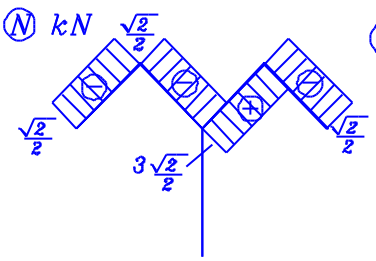


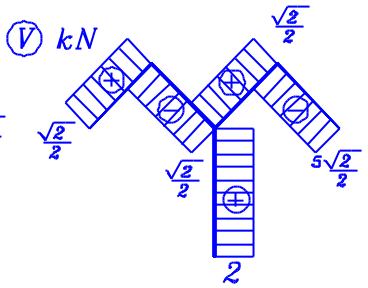


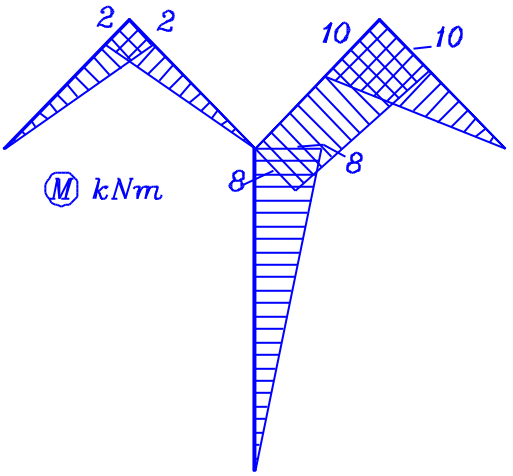












Reações de apoio (para 5.2):

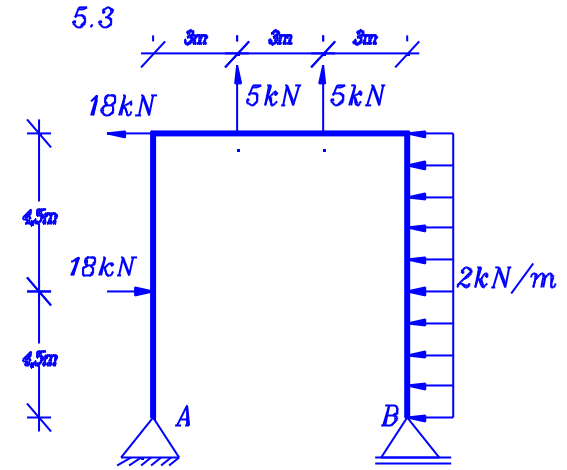
$\Sigma M_A = 0 \Rightarrow -2.2 - 2.6 + Y_C.8 + X_B.4 = 0$

$2.Y_C + X_B = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

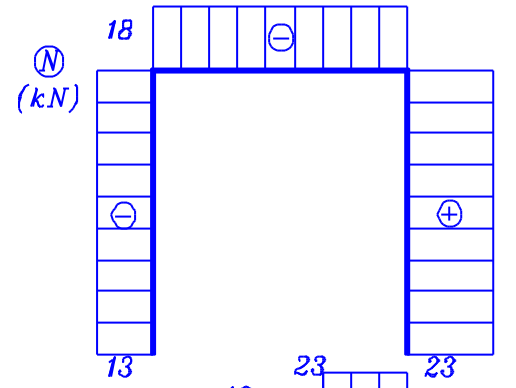
$\Sigma X = 0 \Rightarrow \boldsymbol{X_B} = \boldsymbol{-2kN} \leftarrow \ldots [em(1)] \Rightarrow \boldsymbol{Y_C} = \boldsymbol{3kN} \uparrow$

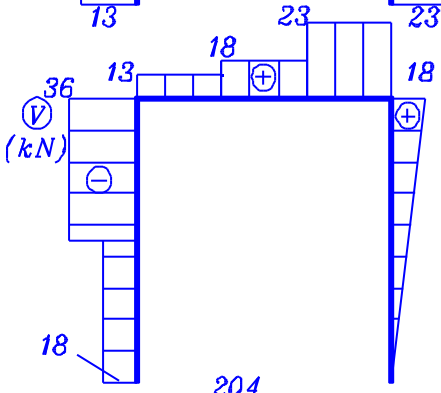
$\Sigma Y = 0 \Rightarrow Y_A + Y_C = 4 \Rightarrow \boldsymbol{Y_A} = \boldsymbol{1kN} \uparrow$

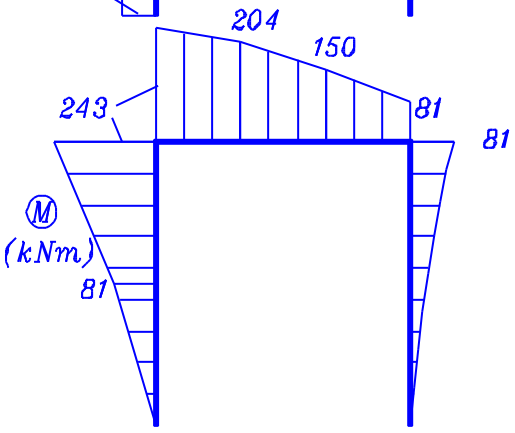
5.3



R:







Reações de apoio:

$\Sigma X = O \Rightarrow X_A = 2.9 + 18 - 18 = \boldsymbol{18kN} \rightarrow$

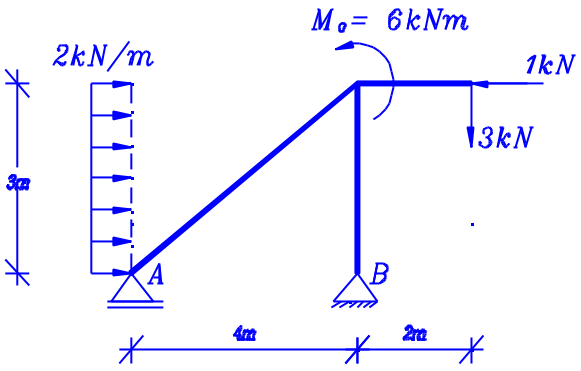
$\Sigma Y = 0 \Rightarrow Y_A + 5 + 5 + Y_B = 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$

$\Sigma M_A = 0 \Rightarrow 18 \cdot 4{,}5 + 18 \cdot 9 + 5 \cdot 3 + 5 \cdot 6 +$

$+18 \cdot 4{,}5 + Y_B \cdot 9 = 0 \Rightarrow \boldsymbol{Y_B} = \boldsymbol{-23kN} \downarrow$

$\left[em(1)\right] \Rightarrow \boldsymbol{Y_A} = \boldsymbol{13kN} \uparrow$

5.4



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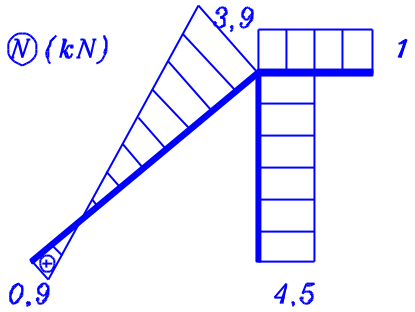
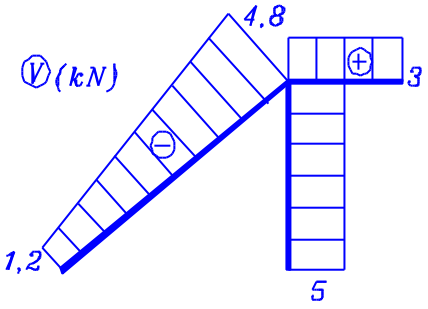
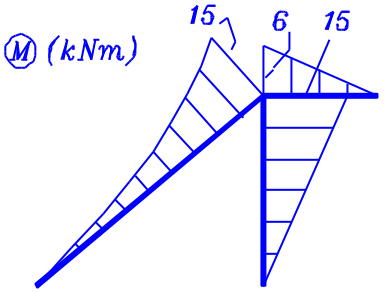






Reações de apoio:

$\Sigma X = 0 \Rightarrow 3{.}2 - 1 + X_B = 0$

$\boldsymbol{X_B} = \boldsymbol{-5kN} \leftarrow$

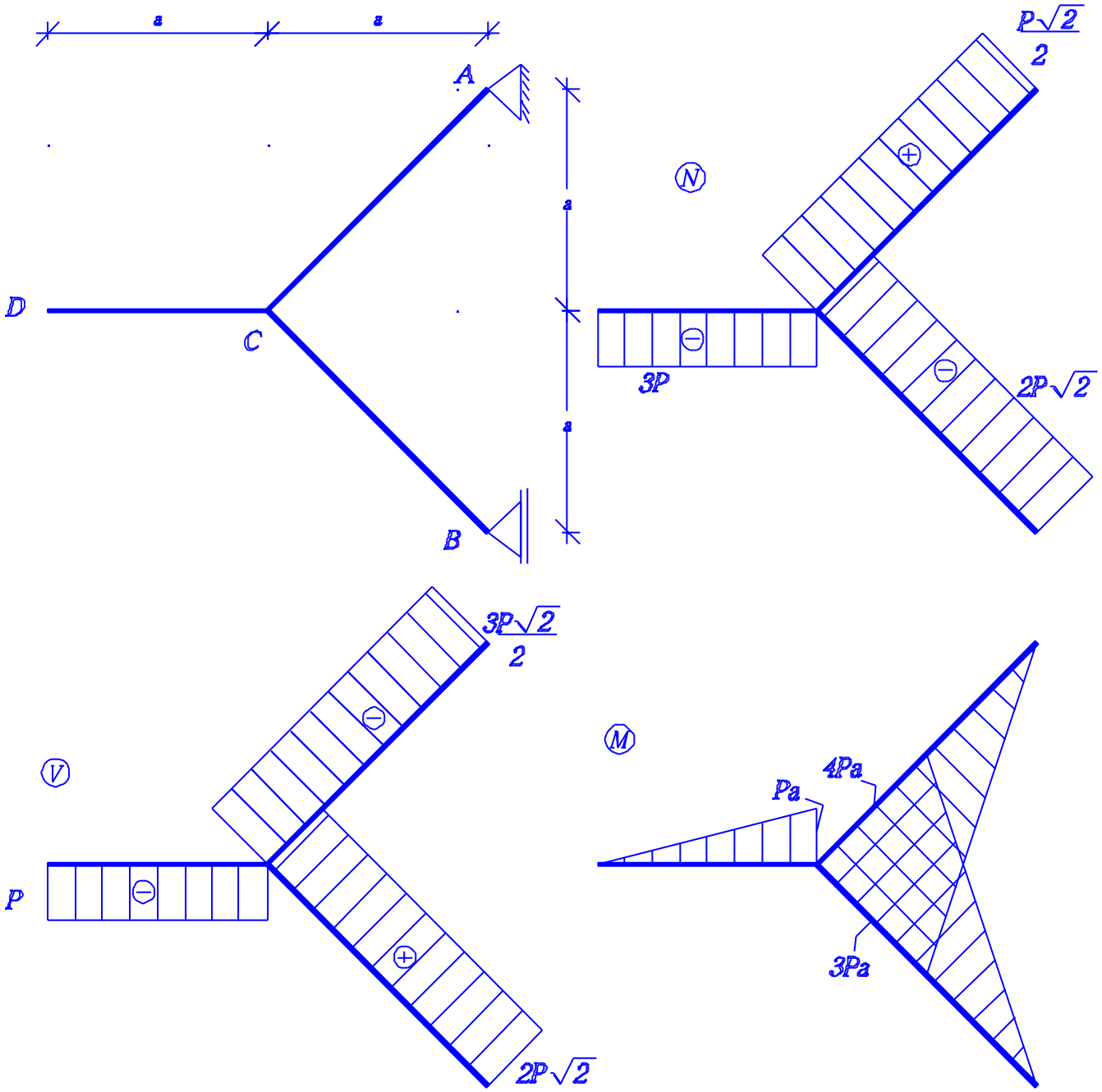
$\Sigma Y = 0 \Rightarrow Y_A + Y_B = 3 \ldots\ldots\ldots\ldots (1)$

$\Sigma M_A = 0 \Rightarrow -6 \cdot 1{,}5 + 6 + Y_B \cdot 4 - 3 \cdot 6 +$

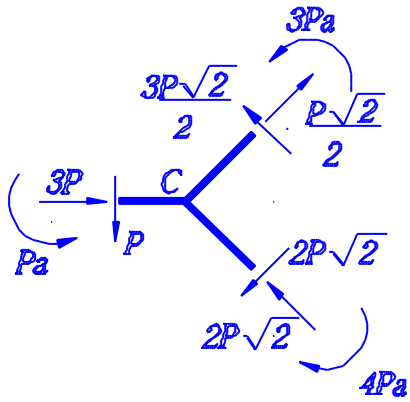
$+1 \cdot 3 = 0 \Rightarrow \boldsymbol{Y_B} = \boldsymbol{4{,}5kN} \uparrow$

$\left[em(1)\right] \Rightarrow \boldsymbol{Y_A} = \boldsymbol{-1{,}5kN} \downarrow$

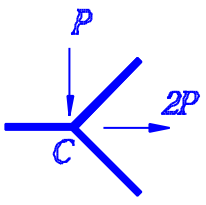
6 - A estrutura da figura está em equilíbrio sob a ação de um determinado carregamento. Conhecidos os diagramas de esforços solicitantes, pede-se isolar o **nó C** e indicar os esforços solicitantes e os esforços externos que nele atuam:



Respostas:

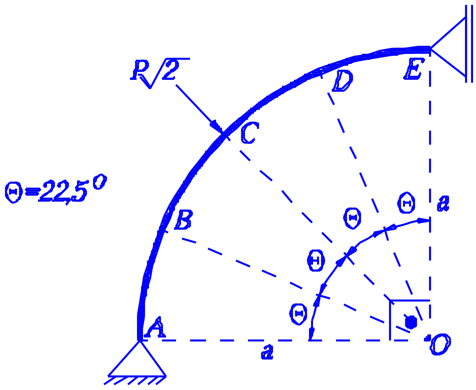


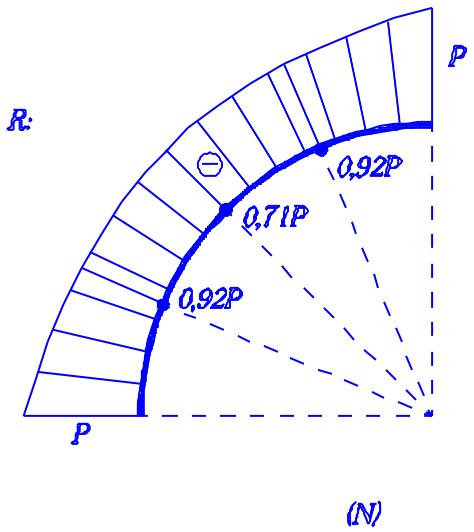
Esforços solicitantes

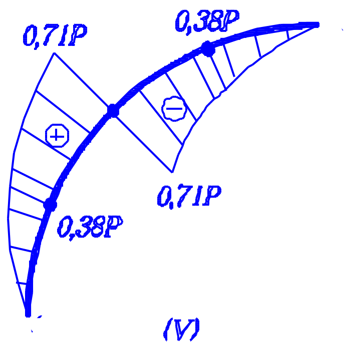


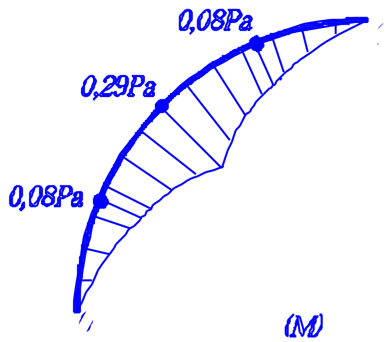
Esforços externos

7 - Determinar N, V e M nas seções A,B,C,D e E e traçar os diagramas de esforços solicitantes:









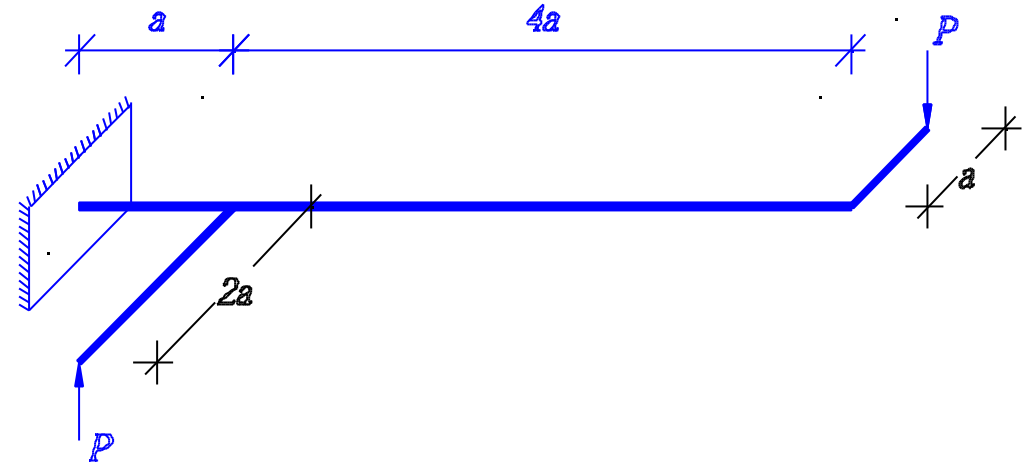
Reações de apoio:

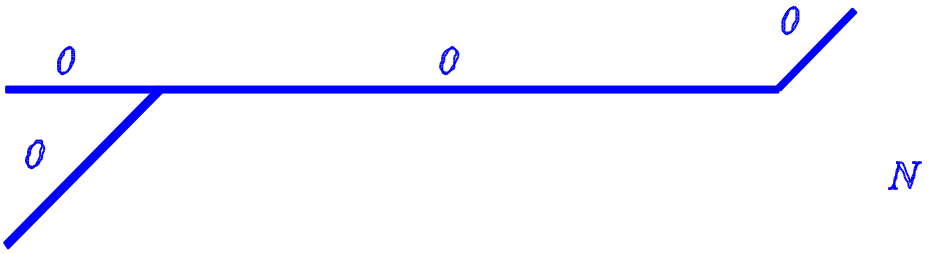
$$\Sigma X = 0 \Rightarrow P\sqrt{2}.\frac{\sqrt{2}}{2} + X_A + X_E = O \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

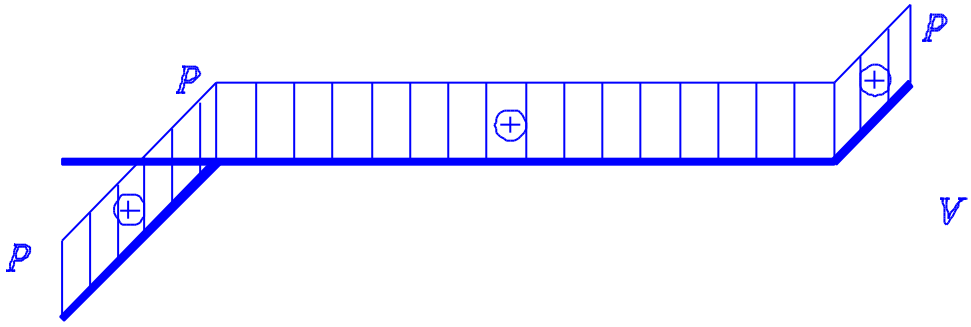
$$\Sigma Y = 0 \Rightarrow Y_A - P\sqrt{2}\,\frac{\sqrt{2}}{2} = 0 \Leftrightarrow \boldsymbol{Y_A = P} \uparrow$$

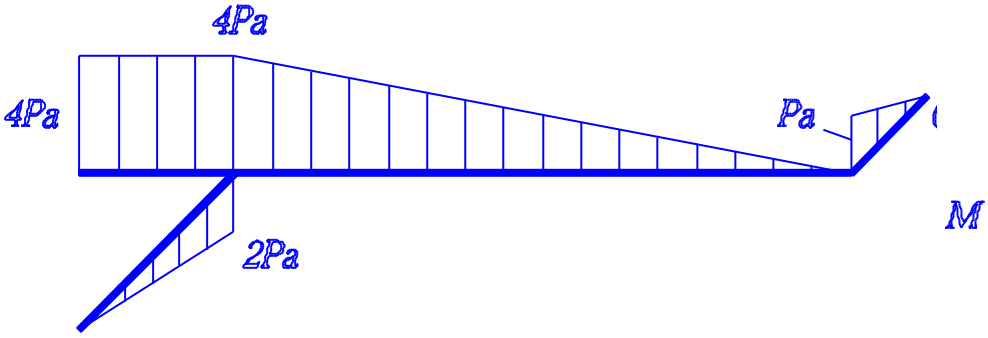
$$\Sigma M_O = 0 \Rightarrow -X_E.a - Y_A.a = 0 \Leftrightarrow X_E = -Y_A \Rightarrow \boldsymbol{X_E = P} \leftarrow \ldots[em(1)] \Rightarrow \boldsymbol{X_A = 0}$$

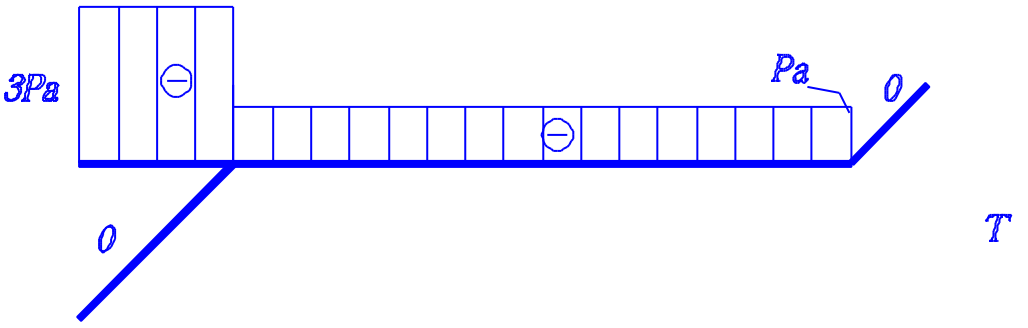
8 - Traçar os diagramas de esforços solicitantes:



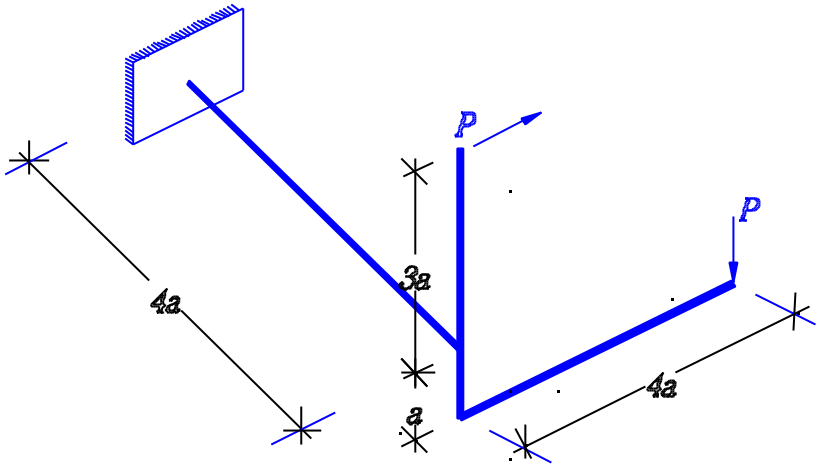


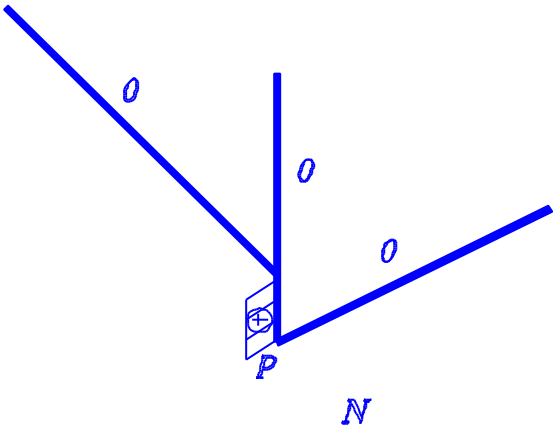




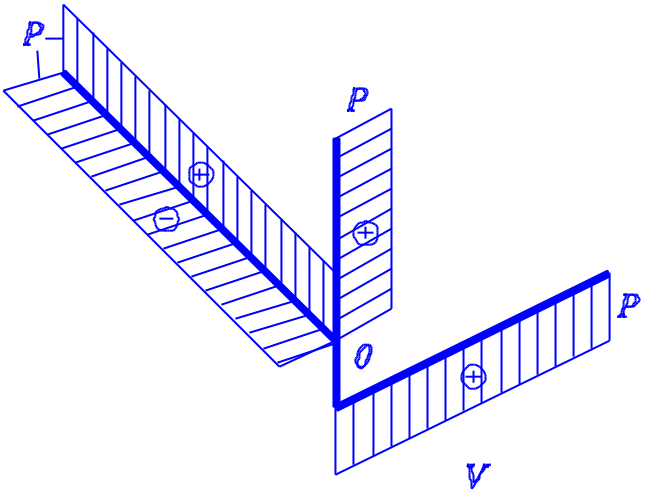


8.2)

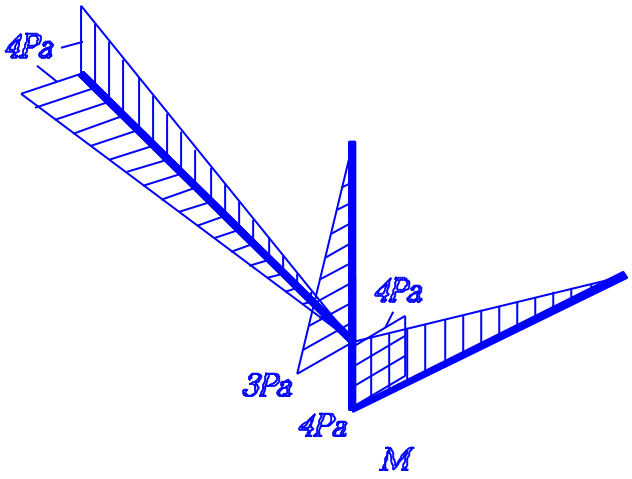




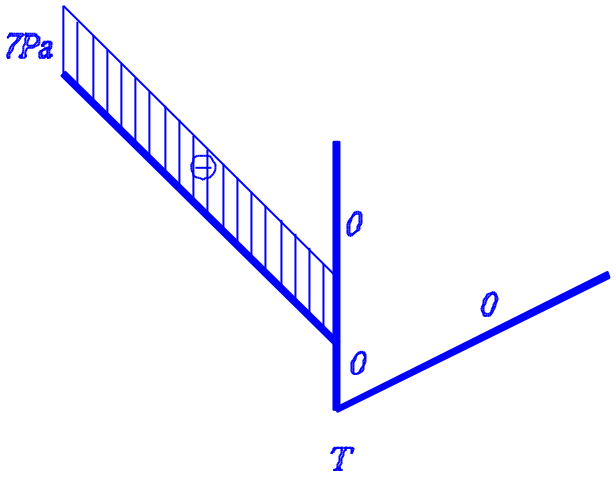
N



V

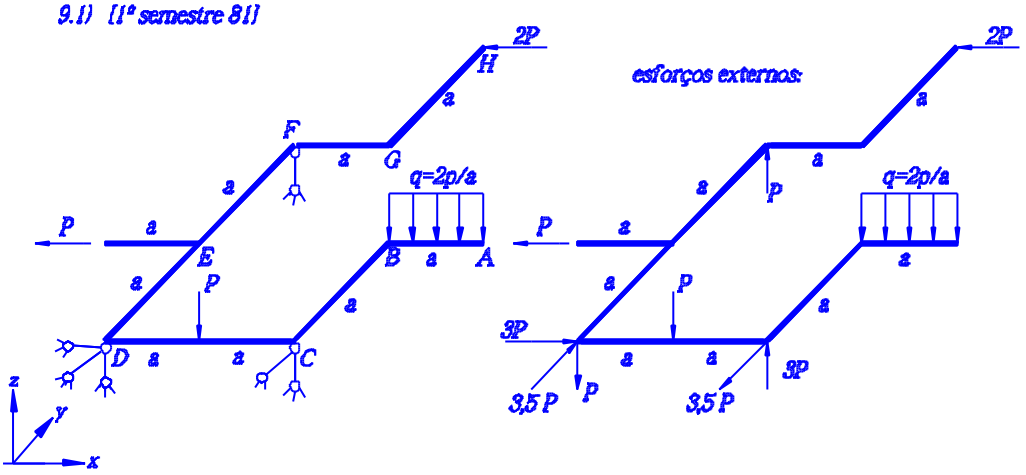


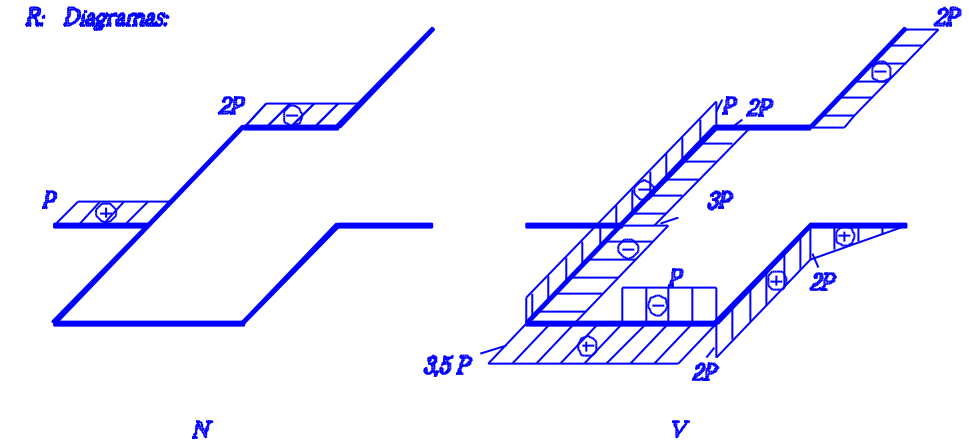
M

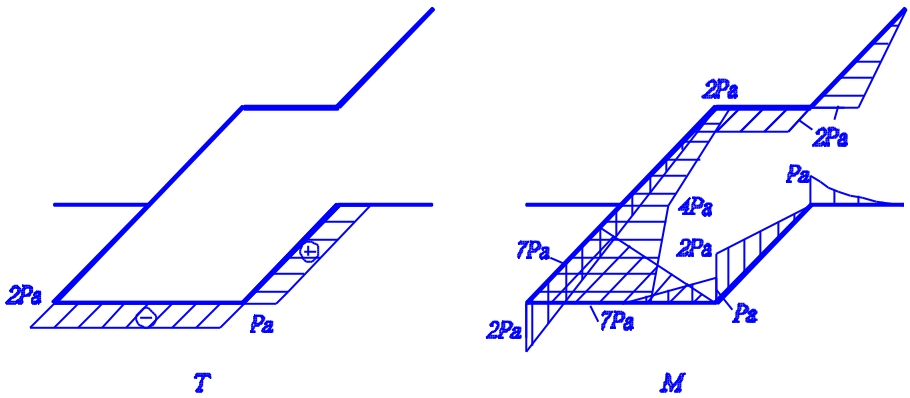


T

9 - Determinar as reações de apoio e traçar os diagramas de esforços solicitantes:







Cálculo das reações:

Adotando-se inicialmente todas as reações com sentido concordando com a orientação de seus respectivos eixos, tem-se:

$\vec{R} = 0 \Rightarrow X_D = 3P$

$\Rightarrow Y_C + Y_D = 0$

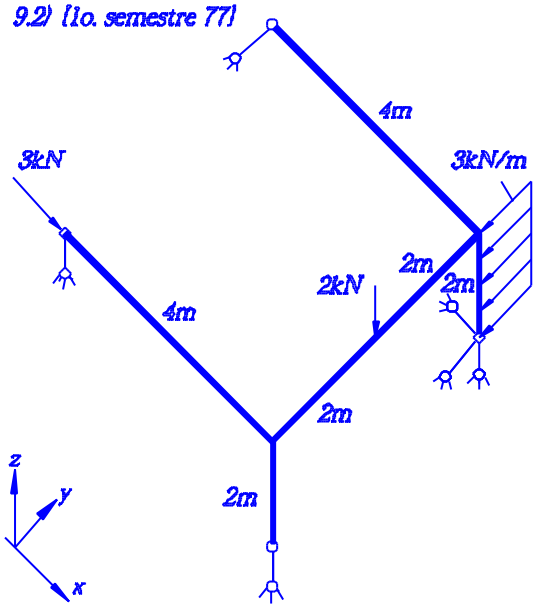
$\Rightarrow Z_C + Z_D + Z_F = 3P$

$\bullet \sum M_{D,x} = 0 \Rightarrow 2Pa = 2a.Z_F \Rightarrow Z_F = P$

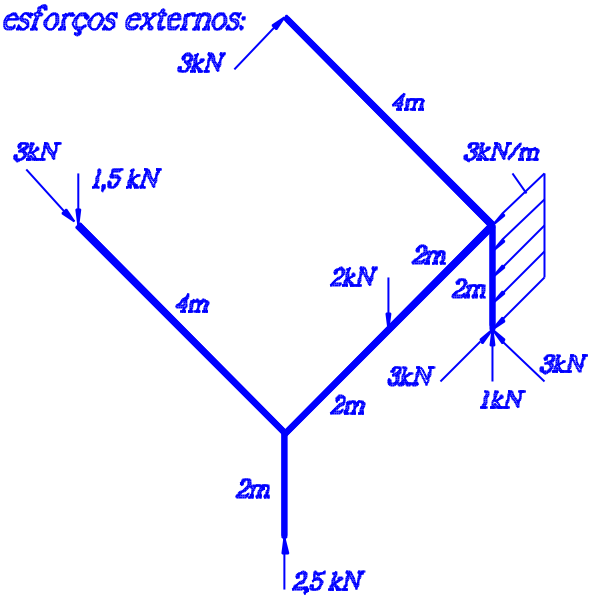
$\bullet \sum M_{D,y} = 0 \Rightarrow Pa + 2P\frac{5}{2}a = Z_C.2a \Rightarrow \boldsymbol{Z_C = 3P} \Rightarrow \boldsymbol{Z_D = -P}$

$\bullet \sum M_{D,z} = 0 \Rightarrow PA + 2P.3a + Y_C.2a = O \Rightarrow \boldsymbol{Y_C = -3{,}5P} \Rightarrow \boldsymbol{Y_D = 3{,}5P}$

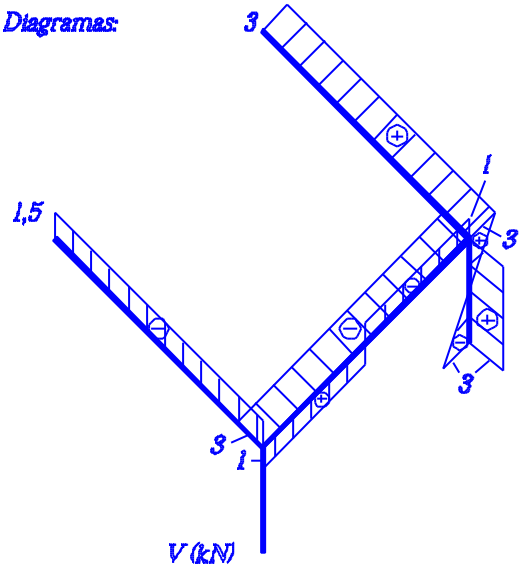
9.2) (1o. semestre 77)



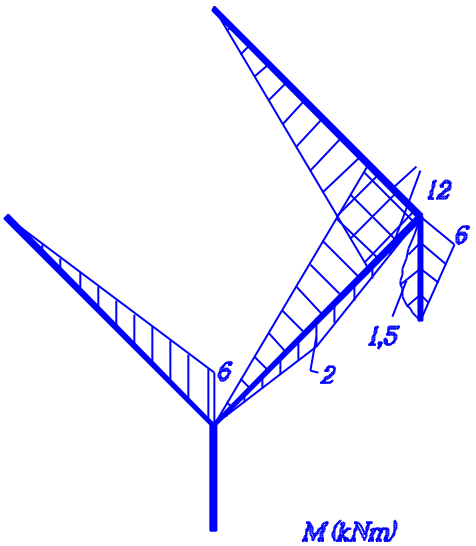
esforços externos:



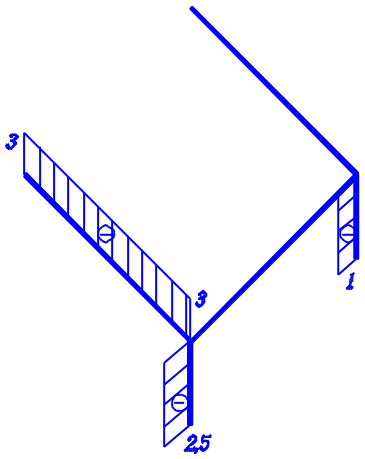
Diagramas:



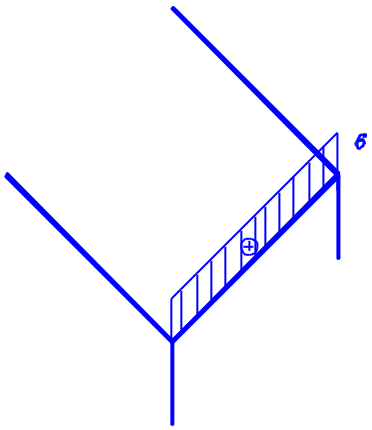
V (kN)



M (kNm)



N (kN)



T (kNm)

Cálculo das reações:

$\vec{R} = 0 \Rightarrow X_B + 3 = 0 \Rightarrow \boldsymbol{X_B = -3kN}$

$\Rightarrow Y_B + Y_D = 6$

$\Rightarrow Z_A + Z_B + Z_C = 2$

- $\sum M_{B,Y} = 0 \Rightarrow Z_C.4 + 3.2 = 0 \Rightarrow \boldsymbol{Z_C = -1,5kN}$
- $\sum M_{B,Z} = 0 \Rightarrow Y_D.4 = 3.4 \Rightarrow \boldsymbol{Y_D = 3kN} \Rightarrow \boldsymbol{Y_B = 3kN}$
- $\sum M_{B,X} = 0 \Rightarrow Z_A.4 + Z_C.4 + Y_D.2 = 2.2 + 6.1 \Rightarrow \boldsymbol{Z_A = 2,5kN} \Rightarrow \boldsymbol{Z_B = 1kN}$