Radiação de dipolo

- Qual a função de onda esférica?
- Existe uma função de onda esférica para ondas transversais?
- O que é uma fonte puntiforme para o eletromagnetismo?
- Onde nasce uma onda?

Ondas esféricas: Acústica x Ótica

Som: onda de pressão

Solução esférica: $p(\vec{r},t) = p_0 \frac{1}{r} \cos(kr - \omega t + \varphi) =$

$$= Re\left[\frac{\tilde{p}_0}{r} e^{i(kr-\omega t)}\right] = Re[\tilde{p}]$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial}{\partial x}\tilde{p} = \frac{\partial}{\partial r}\tilde{p} \cdot \frac{\partial r}{\partial x}$$

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} = \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$= \left[-\frac{\tilde{p}_0}{r^2} e^{i(kr-\omega t)} + ik\,\tilde{p}\right]\frac{x}{r}$$

$$= \left(ik - \frac{1}{r}\right)\frac{x}{r}\,\tilde{p}$$

$$\frac{\partial^2}{\partial x^2}\tilde{p} = \frac{\partial}{\partial x}\tilde{p} \cdot \left(ik - \frac{1}{r}\right) \cdot \frac{x}{r} + \tilde{p}\,\frac{\partial}{\partial x}\left(ik\frac{x}{r} - \frac{x}{r^2}\right)$$

$$\frac{\partial}{\partial x}\left(ik\frac{x}{r}\right) = \frac{ik}{r} + ikx\frac{\partial}{\partial x}\left(\frac{1}{r}\right)$$

$$= \frac{ik}{r} - ik\frac{x^2}{r^3}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial r}{\partial x} =$$

$$= -\frac{1}{r^2} \cdot \frac{x}{r} = -\frac{x}{r^3}$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^2}\right) = \frac{1}{r^2} + x\frac{\partial}{\partial x}\left(\frac{1}{r^2}\right)$$

$$= \frac{1}{r^2} - 2\frac{x^2}{r^4}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r^2}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) \cdot \frac{\partial r}{\partial x}$$

$$= -\frac{2}{r^3} \cdot \frac{x}{r}$$

$$\frac{\partial^2}{\partial x^2}\tilde{p} = \tilde{p}\left[\left(ik - \frac{1}{r}\right)^2 \frac{x^2}{r^2} + \frac{ik}{r} - ik\frac{x^2}{r^3} - \frac{1}{r^2} + 2\frac{x^2}{r^4}\right]$$

$$= \tilde{p}\left[-k^2\frac{x^2}{r^2} - 2ik\frac{x^2}{r^3} + \frac{x^2}{r^4} + i\frac{k}{r} - ik\frac{x^2}{r^3} - \frac{1}{r^2} + \frac{2x^2}{r^4}\right]$$

$$= \tilde{p}\left[-k^2\frac{x^2}{r^2} - 3ik\frac{x^2}{r^3} + \frac{3x^2}{r^4} + i\frac{k}{r} - \frac{1}{r^2}\right]$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\tilde{p} = \tilde{p}\left[-k^2 - 3i\frac{k}{r} + \frac{3}{r^2} + 3i\frac{k}{r} - \frac{3}{r^2}\right] = \underline{-k^2\tilde{p}}$$

Coordenadas esfericas: $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2}{\partial\varphi^2}$

$$\nabla^2\tilde{p}(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\tilde{p}(r)\right) = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\, r^2\frac{\partial}{\partial r}\left(\frac{e^{ikr}}{r}\right)\right]\tilde{p}_0\, e^{-i\omega t}$$

$$\frac{\partial}{\partial r}\left(\frac{e^{ikr}}{r}\right) = -\frac{1}{r^2}e^{ikr} + ik\frac{e^{ikr}}{r}$$

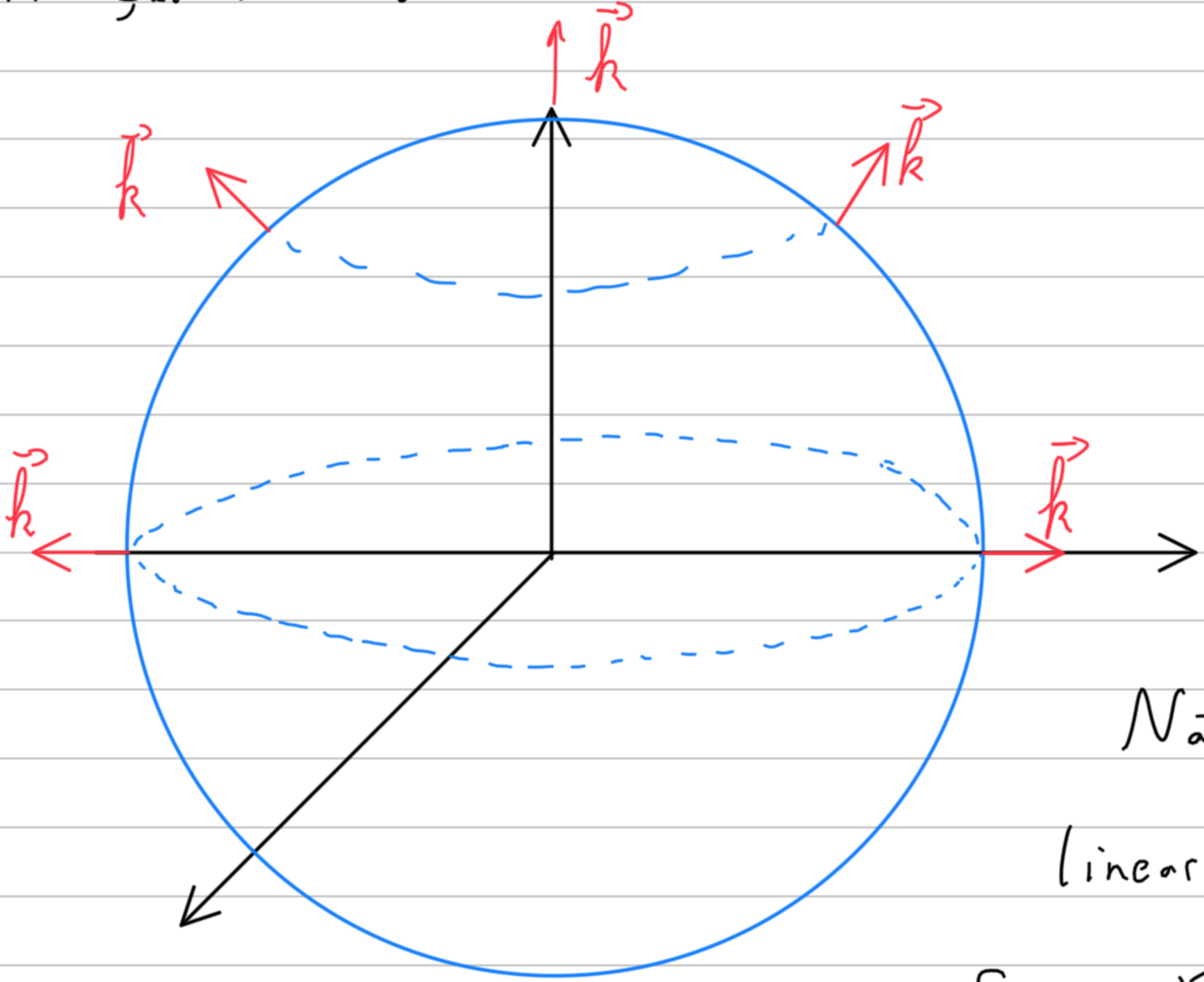
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\left(\frac{e^{ikr}}{r}\right)\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[-e^{ikr} + ikre^{ikr}\right]$$

$$= \frac{1}{r^2}\left[-ik\,e^{ikr} + ik\,e^{ikr} + (ik)^2 r\,e^{ikr}\right] = -k^2\frac{e^{ikr}}{r}$$

$$\left.\begin{aligned} \therefore\ \nabla^2\tilde{p}(r) &= -k^2\,\tilde{p}(r) \\ \frac{\partial^2}{\partial t^2}\tilde{p}(r) &= -\omega^2\,\tilde{p}(r) \end{aligned}\right\}\quad \boxed{\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\tilde{p} = 0}$$

Ondas transversais: $\vec{E} = \frac{E}{r} \hat{n} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$?

polarização varia!

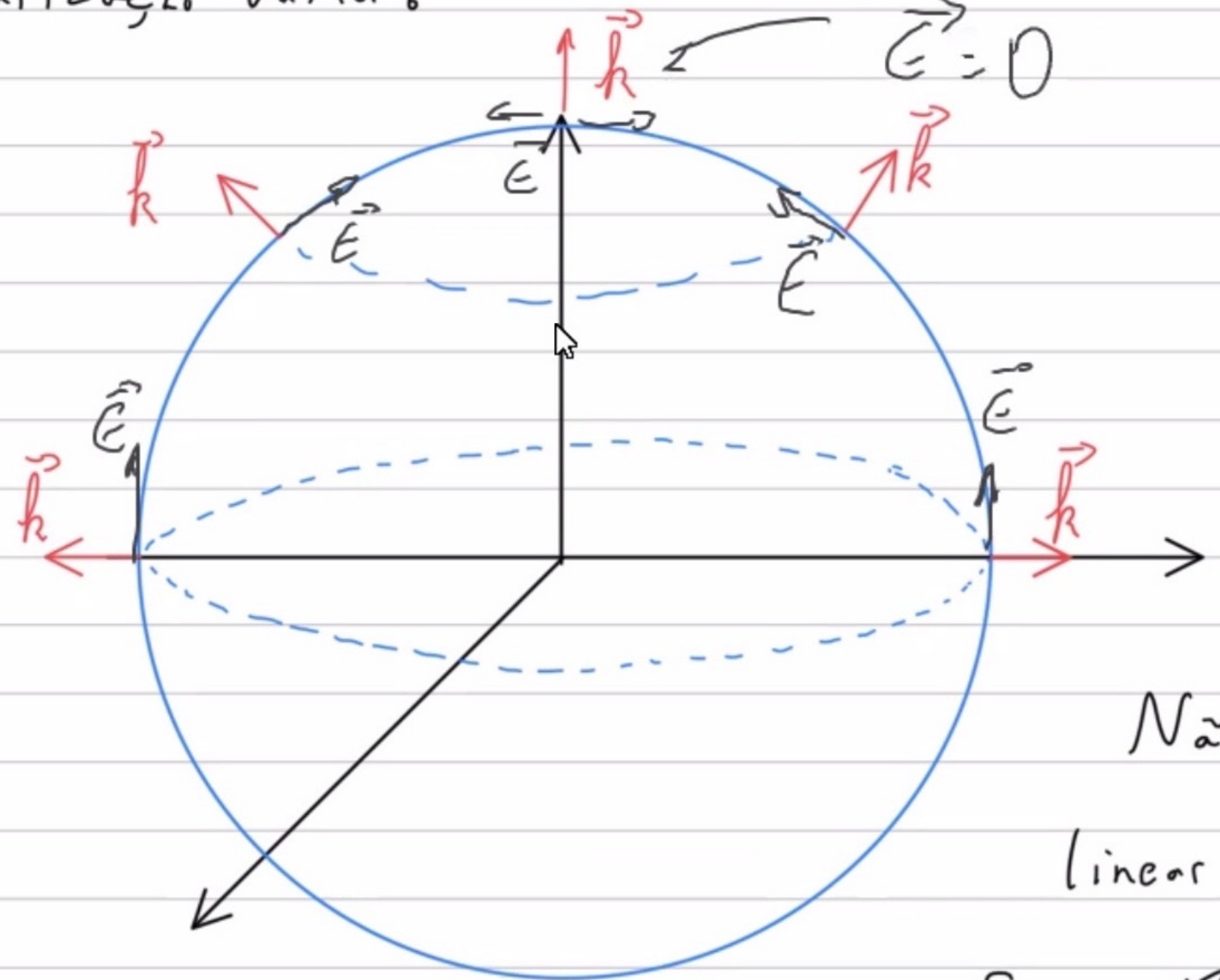


Não pode ser linear em toda a esfera → $\nabla\cdot\vec{E} \neq 0$ no pólo!

Veremos o truque mais tarde...

Ondas transversais: $\vec{E} = \frac{E}{r}\hat{n}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$?

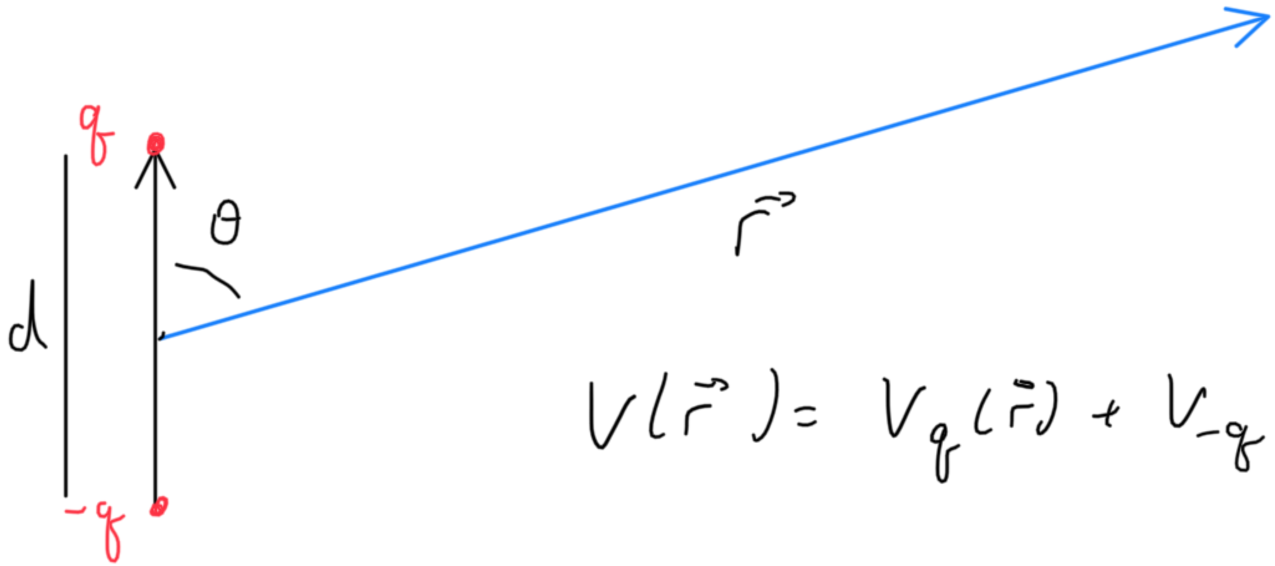
polarização varia!



Não pode ser linear em toda a esfera → $\underline{\nabla\cdot\vec{E}}\neq 0$ no pólo!

Veremos o truque mais tarde...

Potencial elétrico de um dipolo



$$V(\vec{r}) = V_q(\vec{r}) + V_{-q}(\vec{r})$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{|\vec{r}-\vec{d}/2|} - \frac{1}{|\vec{r}+\vec{d}/2|}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{|\vec{r}-\vec{d}/2|} - \frac{1}{|\vec{r}+\vec{d}/2|}\right]$$

$$(\vec{r}-\vec{d})^2 = r^2 \operatorname{sen}^2\theta + \left(r\cos\theta - \frac{d}{2}\right)^2 = r^2\operatorname{sen}^2\theta + r^2\cos\theta - dr\cos\theta + d^2/4$$

$$= r^2 + d^2/4 - dr\cos\theta$$

$$(\vec{r}+\vec{d})^2 = r^2 + d^2/4 + dr\cos\theta$$

$r \gg d$ $$(\vec{r}-\vec{d})^2 \simeq r^2\left(1 - \frac{d}{r}\cos\theta\right)$$

$$(\vec{r}+\vec{d}) \simeq r^2\left(1 + \frac{d}{r}\cos\theta\right)$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{1}{\sqrt{1-\frac{d}{r}\cos\theta}} - \frac{1}{\sqrt{1+\frac{d}{r}\cos\theta}}\right]$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{1}{\sqrt{1-\frac{d}{r}\cos\theta}} - \frac{1}{\sqrt{1+\frac{d}{r}\cos\theta}}\right]$$

$$\frac{1}{\sqrt{1+\alpha}} \simeq 1 - \frac{1}{2}\alpha$$

$$\Rightarrow \quad V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r} \cdot \frac{d\cos\theta}{r} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

com $\vec{p} = q \cdot \vec{d}$

Lembra o que vimos na aula passada:

esfera metálica em campo elétrico

$$\vec{A}(\vec{r}) \simeq \frac{\mu_0}{4\pi r^2} \int \vec{J}(\vec{r}\,')\, r' \cos\theta \; r'^2 \, dr' \, d\theta \, d\varphi$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \qquad \vec{m} = \frac{1}{2} \int_V \left[\vec{r}\,' \times \vec{J}(\vec{r}\,')\right] dv$$

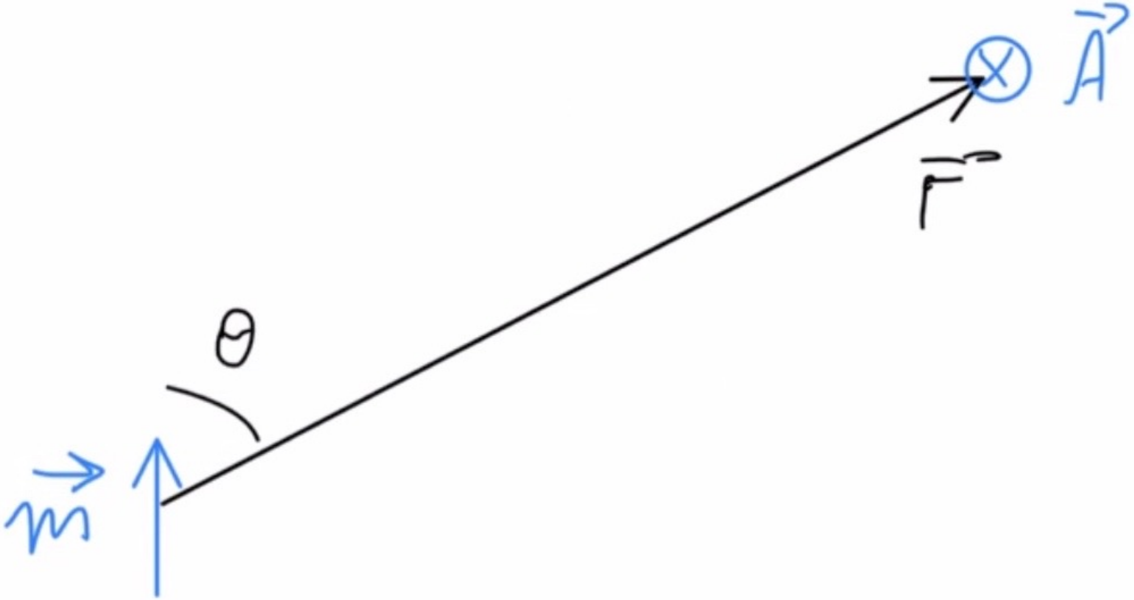
Potencial vetor de dipolo magnético

Momento de dipolo magnético

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2} \qquad \vec{p} = \int \vec{r}\,' \rho(\vec{r}\,') \, dv$$

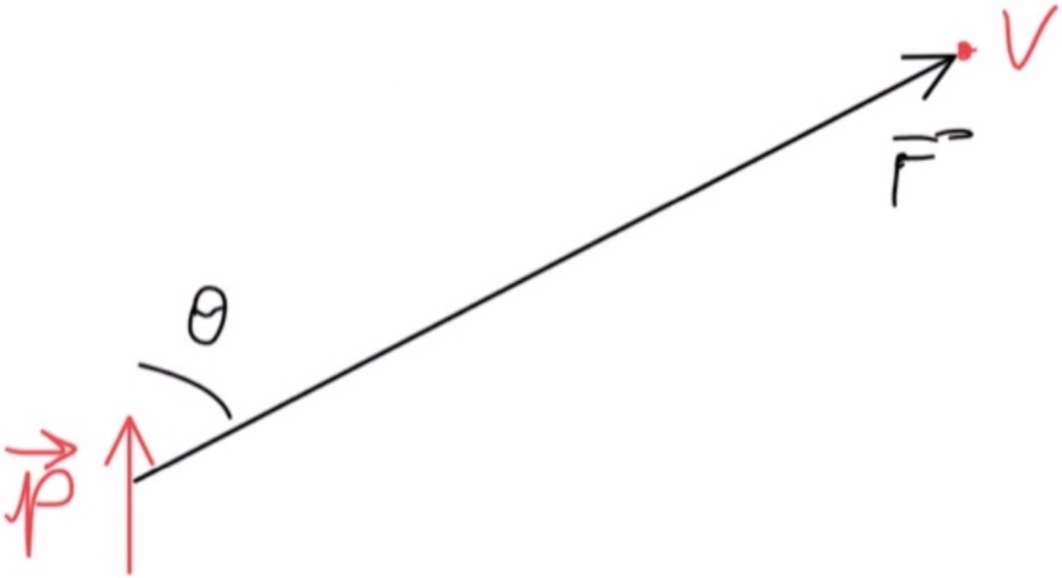
Potencial de dipolo elétrico

Momento de dipolo elétrico

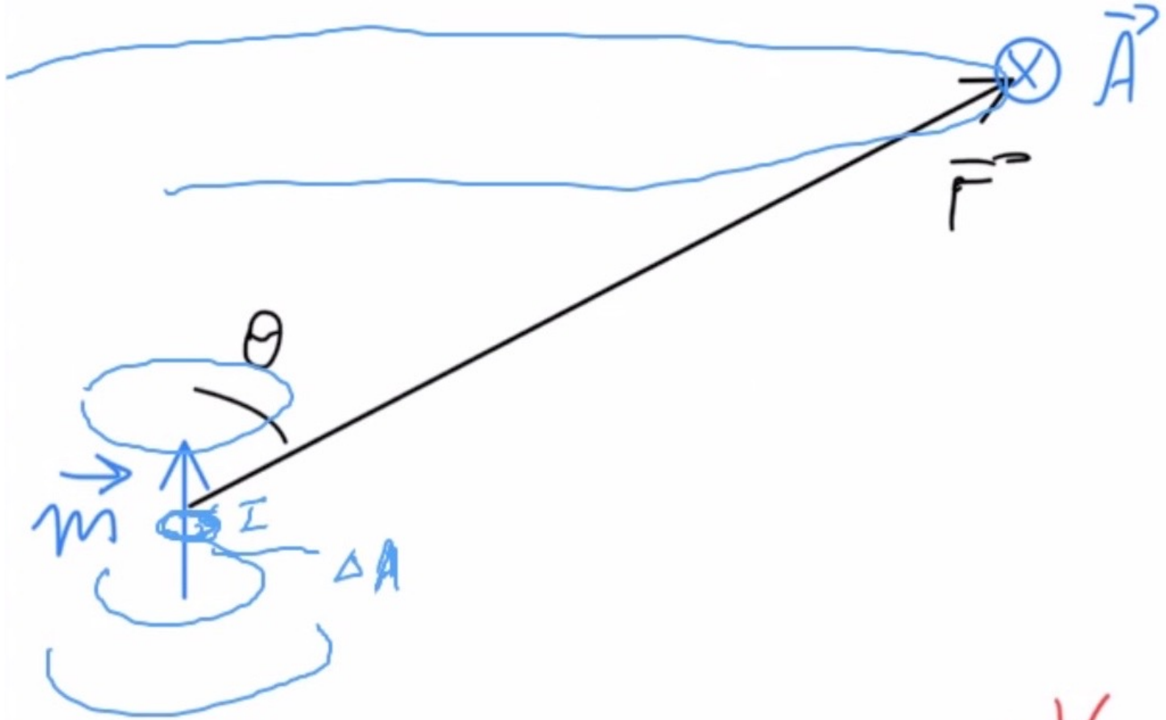


$$A = \frac{\mu_0 m}{4\pi r^2} \operatorname{sen}\theta$$

(caneis)

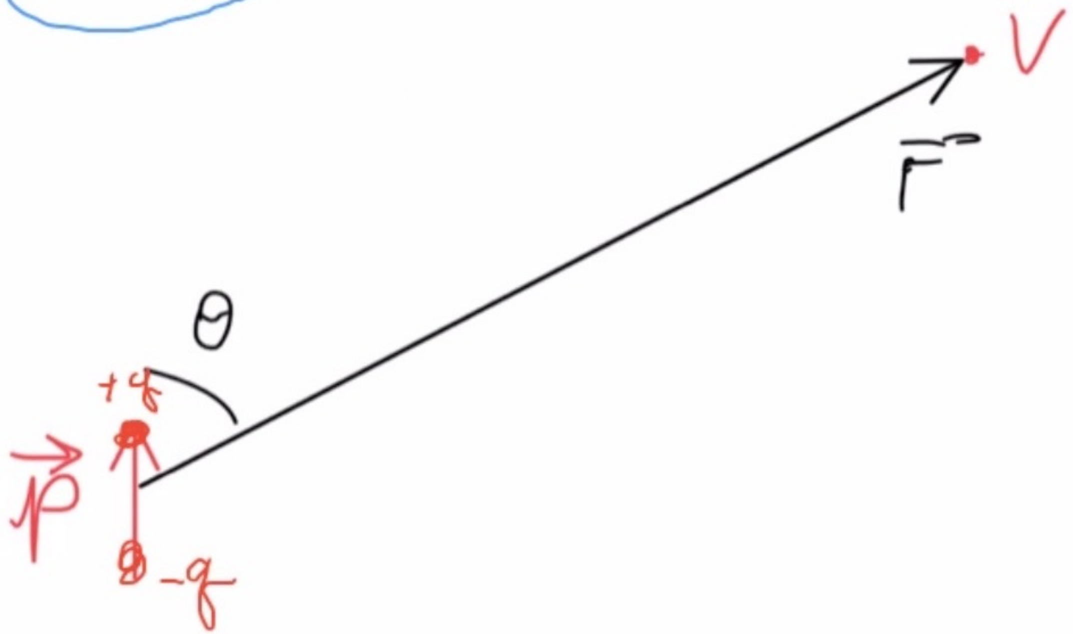


$$V = \frac{p}{4\pi \varepsilon_0 r^2} \cos\theta$$



$$A = \frac{\mu_0 m}{4\pi r^2} \operatorname{sen}\theta$$

(anéis)



$$V = \frac{p}{4\pi \varepsilon_0 r^2} \cos\theta$$

Aula 27: Radiação

Fontes: Cargas em movimento

É mais fácil usar os potenciais:

$$\vec{E} = -\nabla V - \frac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \nabla \times \vec{A}$$

Calibre de Lorentz: $\nabla \cdot \vec{A} = -\frac{1}{c^2}\frac{\partial}{\partial t} V$

$\Rightarrow$ Das eqs. de Maxwell

$$\begin{cases} \nabla^2 V - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} V = -\frac{1}{\epsilon_0}\rho \\ \nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = -\mu_0 \vec{J} \end{cases}$$

Caso estacionário: eqs. de Poisson: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$; $\nabla^2 \vec{A} = -\mu_0 \vec{J}$

$$\Rightarrow \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r}\, dv \quad , \quad r = |\vec{r} - \vec{r}'|$$

↳ fonte

↳ espaço $\forall$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r}\, dv$$

Cargas se movem: o potencial visto se refere à configuração <u>antiga</u> - tempo regresso t_r

$$t_r = t - \frac{r}{c}$$

Generalizando: $$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}', t_r)}{r}\, dv$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}', t_r)}{r}\, dv$$

Generalizando: $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,';t_r)}{r}\, dv$

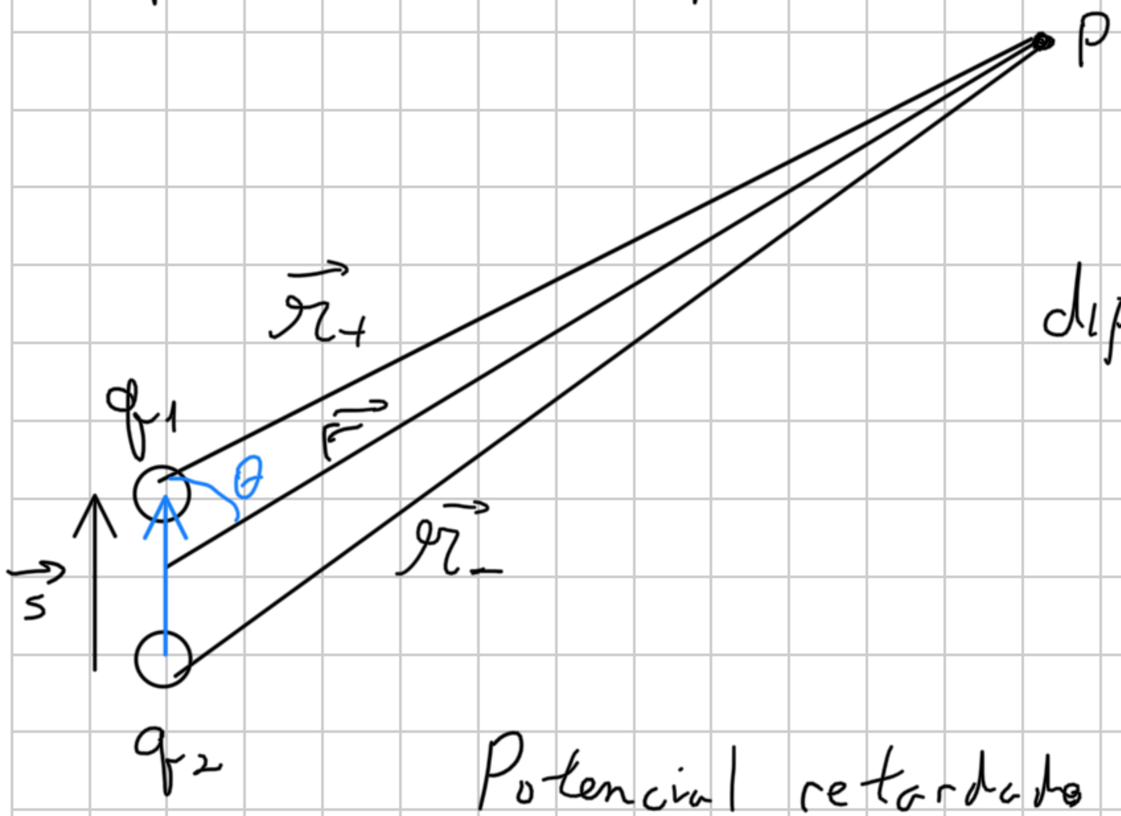
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,';t_r)}{r}\, dv$$

Potenciais retardados de um tempo r/c no integrando

Precisamos conferir se isso satisfaz o Calibre de Lorentz

Leis de Coulomb e Biot-Savart não satisfazem!

Aplicando ao potencial de dipolo



$$q_1 = q(t) = q_0 \cos(\omega t) = -q_2$$

dipolo $\vec{p}(t) = q(t)\,\vec{s}$

$$= p_0 \cos(\omega t)\,\vec{k}$$

$$p_0 = q_0 s$$

Potencial retardado em P

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0\cos\omega(t - r_+/c)}{r_+} - \frac{q_0\cos\omega(t - r_-/c)}{r_-}\right]$$

$$r_\pm = \sqrt{r^2 \mp r s \cos\theta + (s/2)^2}$$

$$r_{\pm} = \sqrt{r^2 \mp r s \cos\theta + (s/2)^2}$$

Dipolo ideal : $s \ll r \Rightarrow r_{\pm} \simeq r\left(1 \mp \frac{1}{2}\frac{s}{r}\cos\theta\right)$

$$\frac{1}{r_{\pm}} \simeq \frac{1}{r}\left(1 \pm \frac{1}{2}\frac{s}{r}\cos\theta\right)$$

$$\cos\omega\left(t - \frac{r_{\pm}}{c}\right) \simeq \cos\left[\omega\left(t - \frac{r}{c}\right) \pm \frac{\omega s}{2c}\cos\theta\right]$$

$$= \cos\omega(t - r/c)\cos\left(\frac{\omega s}{2c}\cos\theta\right) \mp \operatorname{sen}\omega(t - r/c)\operatorname{sen}\left(\frac{\omega s}{2c}\cos\theta\right)$$

Fonte << comprimento de onda : $s << \lambda = \frac{2\pi c}{\omega}$

$$\Rightarrow \frac{\omega s}{c} << 1$$

$$\cos\left(\frac{\omega s}{2c}\cos\theta\right) \simeq 1 \; ; \; \operatorname{sen}\left(\frac{\omega s}{2c}\cos\theta\right) \simeq \frac{\omega s}{2c}\cos\theta$$

$$\left.\begin{array}{l} \Rightarrow \cos\omega\left(t - \frac{r_\pm}{c}\right) \simeq \cos\omega(t - r/c) \mp \frac{\omega s}{2c}\cos\theta \operatorname{sen}\omega(t - r/c) \\ \frac{1}{r_\pm} \simeq \frac{1}{r}\left(1 \pm \frac{1}{2}\frac{s}{r}\cos\theta\right) \end{array}\right\}$$

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0\cos\omega(t - r_+/c)}{r_+} - \frac{q_0\cos\omega(t - r_-/c)}{r_-}\right]$$

$$\simeq \frac{q_0}{4\pi\epsilon_0}\left[-\frac{\omega s}{cr}\cos\theta \operatorname{sen}\omega(t - r/c) + \frac{s}{r^2}\cos\theta\cos\omega(t - r/c)\right]$$

$$\boxed{V(\vec{r},t) = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r}\left[-\frac{\omega}{c}\,\mathrm{sen}\,\omega(t-r/c) + \frac{1}{r}\cos\omega(t-r/c)\right]}$$

$\omega = 0 \rightarrow$ $V(\vec{r}) = \dfrac{p_0 \cos\theta}{4\pi\epsilon_0 r^2}$ $\rightarrow$ de volta ao começo ▽

Distante da fonte: $\underline{r \gg \dfrac{c}{\omega} = 2\pi\lambda}$

$$\boxed{V(\vec{r},t) = -\frac{p_0\,\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\mathrm{sen}\,\omega(t-r/c)}$$

Potencial vetor: $\vec{J} = \frac{dq}{dt}\hat{k} = -q_0 \omega \operatorname{sen}(\omega t)\hat{k}$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{-s/2}^{s/2} \frac{-q_0 \omega \operatorname{sen} \omega(t - r/c)}{r}\hat{k}\, dz$$

$$\simeq \frac{\mu_0}{4\pi} \cdot -q_0 \omega \frac{\operatorname{sen} \omega(t - r/c)}{r}\hat{k} \underbrace{\int_{-s/2}^{s/2} dz}_{s}$$

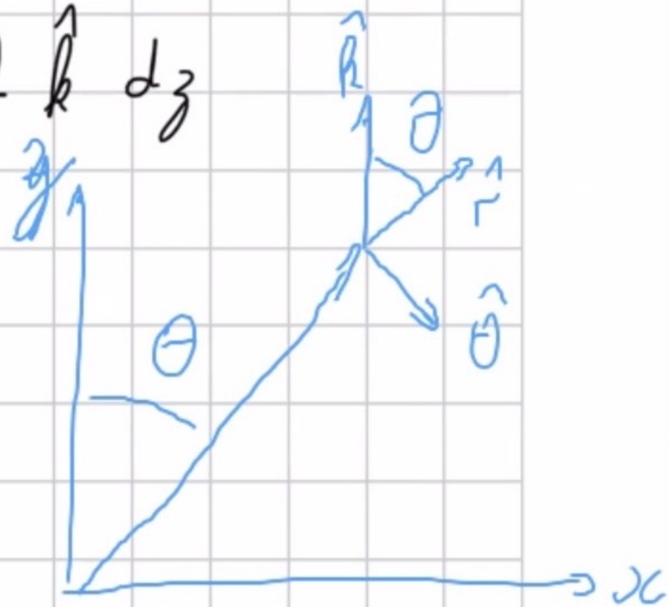
$$\boxed{\vec{A}(\vec{r},t) = -\frac{\mu_0 p_0 \omega}{4\pi}\frac{1}{r}\operatorname{sen}\omega(t - r/c)\hat{k}}$$

$$\hat{k} = \cos\theta\,\hat{r} - \operatorname{sen}\theta\,\hat{\theta}$$

Potencial vetor: $\vec{J} = \frac{dq}{dt}\hat{k} = -q_0 \omega \,\mathrm{sen}(\omega t)\,\hat{k}$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{-s/2}^{s/2} \frac{-q_0\omega \,\mathrm{sen}\,\omega(t - r/c)}{r}\hat{k}\,dz$$

$$\simeq \frac{\mu_0}{4\pi}\cdot -q_0\omega \frac{\mathrm{sen}\,\omega(t-r/c)}{r}\hat{k}\underbrace{\int_{-s/2}^{s/2} dz}_{s}$$



$$\boxed{\vec{A}(\vec{r},t) = -\frac{\mu_0 p_0 \omega}{4\pi}\frac{1}{r}\,\mathrm{sen}\,\omega(t-r/c)\,\hat{k}}$$

$$\hat{k} = \cos\theta\,\hat{r} - \mathrm{sen}\,\theta\,\hat{\theta}$$

Dados os potenciais, vamos aos campos!

$$\nabla V = \left(\hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} \right) V$$

$$= - \frac{p_0 \omega}{4 \pi \epsilon_0 c} \left[\hat{r} \frac{\partial}{\partial r} \left(\frac{\cos \theta}{r} \right) \operatorname{sen} \omega (t - r/c) + \hat{\theta} \frac{1}{r^2} \operatorname{sen} \omega (t - r/c) \frac{\partial}{\partial \theta} \cos \theta \right]$$

$$= - \frac{p_0 \omega}{4 \pi \epsilon_0 c} \left\{ \cos \theta \left[\cancel{- \frac{1}{r^2} \operatorname{sen} \omega (t - r/c)} - \frac{\omega}{r c} \cos \omega (t - r/c) \right] \hat{r} \cancel{- \frac{\operatorname{sen} \theta}{r^2} \operatorname{sen} \omega (t - r/c) \hat{\theta}} \right\}$$

$$\simeq \frac{p_0 \omega^2}{4 \pi \epsilon_0 c^2} \left(\frac{\cos \theta}{r} \right) \cos \omega (t - r/c) \, \hat{r}$$

$$\frac{\partial}{\partial t} A = - \frac{\mu_0 p_0 \omega}{4\pi} \frac{\cos\theta\, \hat{r} - \operatorname{sen}\theta\, \hat{\theta}}{r} \frac{\partial}{\partial t} \operatorname{sen} \omega (t - r/c)$$

$$= - \frac{p_0 \omega^2}{4\pi \epsilon_0 c^2} \frac{\cos\theta\, \hat{r} - \operatorname{sen}\theta\, \hat{\theta}}{r} \cos \omega (t - r/c)$$

$$\Rightarrow \boxed{\vec{E} = - \nabla V - \frac{\partial}{\partial t} \vec{A} = - \frac{\mu_0 p_0 \omega^2}{4\pi} \frac{\operatorname{sen}\theta}{r} \cos \omega (t - r/c)\, \hat{\theta}}$$

$$\nabla \times \vec{A} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} A_r \right] \hat{\varphi}$$

$$= - \frac{p_0 \omega}{4\pi \epsilon_0 c^2} \frac{1}{r} \left[\frac{\omega}{c} \operatorname{sen}\theta \cos \omega (t - r/c) + \cancel{\frac{\operatorname{sen}\theta}{r}} \operatorname{sen} \omega (t - r/c) \right] \hat{\varphi}$$

$$\boxed{\vec{B} = \nabla \times \vec{A} = - \frac{\mu_0 p_0 \omega^2}{4\pi c} \frac{\operatorname{sen}\theta}{r} \cos \omega (t - r/c)\, \hat{\varphi}}$$

$$\vec{k} = \frac{\omega}{c}\hat{r} \qquad \hat{\theta}\times\hat{\varphi} = \hat{k}$$

$$\boxed{\begin{aligned} \vec{E} &= -\frac{\mu_0 p_0 \omega^2}{4\pi}\frac{\operatorname{sen}\theta}{r}\cos(\vec{k}\cdot\vec{r}-\omega t)\,\hat{\theta} \\ \vec{B} &= -\frac{\mu_0 p_0 \omega^2}{4\pi c}\frac{\operatorname{sen}\theta}{r}\cos(\vec{k}\cdot\vec{r}-\omega t)\,\hat{\varphi}\end{aligned}}$$

Ondas esféricas moduladas por $\underline{\operatorname{sen}\theta}$

Vetor de Poynting

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\mu_0}{c}\left(\frac{p_0\omega^2}{4\pi}\right)^2\frac{\operatorname{sen}^2\theta}{r^2}\cos^2(\vec{k}\cdot\vec{r}-\omega t)\,\hat{r}$$

$$\langle\vec{S}\rangle = \frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\frac{\operatorname{sen}^2\theta}{r^2}\,\hat{r}$$

Potência irradiada pelo dipolo

$$\langle P \rangle = \int \langle \vec{S} \rangle \cdot d\vec{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int \frac{\text{sen}^2\theta}{r^2} r^2 \text{sen}\theta \, d\theta \, d\varphi$$

$$\int_0^{\pi} \text{sen}^2\theta \, \text{sen}\theta \, d\theta = \int_{-1}^{1} (1 - x^2) \, dx = \left(x - \frac{1}{3} x^3 \right) \Big|_{-1}^{1} = 2 - \frac{2}{3} = \frac{4}{3}$$

$$x = \cos\theta \qquad \frac{dx}{d\theta} = -\text{sen}\theta$$

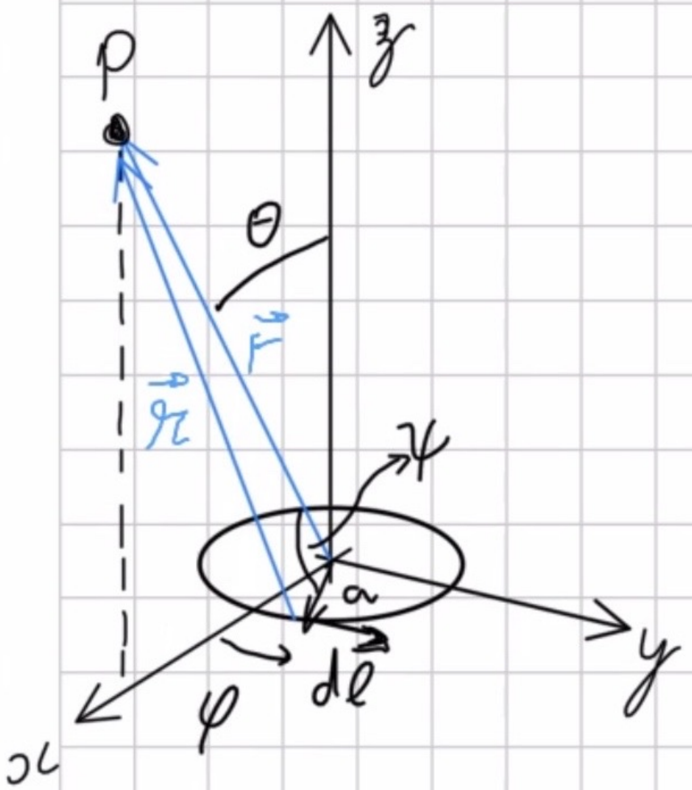
$$\boxed{\langle P \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \cdot \frac{4}{3} \cdot 2\pi = \frac{\mu_0 p_0^2}{12\pi c} \omega^4}$$

Qualquer dipolo oscilante vai ter dissipação por irradiação!

$\langle P \rangle \propto \omega^4$ → espalhamento maior em altas frequências

espalhamento Rayleigh → céu azul

Dipolo magnético



$$I = I_0 \cos(\omega t)$$

$$\vec{m}(t) = \pi a^2 I(t)\hat{k} = m_0 \cos \omega t \,\hat{k}$$

$$\rho = 0 \Rightarrow V = 0$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\, dv$$

$$= \frac{\mu_0}{4\pi}\int I_0 \frac{\cos \omega(t - \mathscr{r}/c)}{\mathscr{r}}\, d\vec{l}$$

Em $\vec{P} = (x,y,z) = (x,0,z)$; $\vec{A} = A\,\hat{j}$ (componentes em $\hat{i}$ se cancelam)

$$\Rightarrow \vec{A}(\vec{r},t) = \frac{\mu_0 I_0}{4\pi} \cdot \hat{j} \cdot a \int_0^{2\pi} \frac{\cos w(t - \mathscr{R}/c)}{\mathscr{R}} \cos\varphi \, d\varphi$$

$$\mathscr{R} = \sqrt{r^2 + a^2 - 2 r a \cos\psi}$$

$$\vec{r} = r \operatorname{sen}\theta \, \hat{i} + r \cos\theta \, \hat{k} \, ; \quad \vec{a} = a \cos\varphi \, \hat{i} + a \operatorname{sen}\varphi \, \hat{j}$$

$$\vec{r} \cdot \vec{a} = r \cdot a \cdot \cos\psi = r \, a \operatorname{sen}\theta \cos\varphi$$

$$\mathscr{R} = \sqrt{r^2 - 2 r a \cos\varphi \operatorname{sen}\theta + a^2} \simeq r \left(1 - \frac{a}{r} \cos\varphi \operatorname{sen}\theta\right)$$

se $a << r$

$$\frac{1}{\mathscr{R}} = \frac{1}{r} \left(1 + \frac{a}{r} \cos\varphi \operatorname{sen}\theta\right)$$

$$\cos \omega(t - r/c) = \cos\left[\omega\left(t - r/c\right) + \frac{\omega a}{c} \operatorname{sen}\theta \cos\varphi\right]$$

$$= \cos \omega(t - r/c) \cos\left(\frac{\omega a}{c} \operatorname{sen}\theta \cos\varphi\right) - \operatorname{sen} \omega(t - r/c) \operatorname{sen}\left(\frac{\omega a}{c} \operatorname{sen}\theta \cos\varphi\right)$$

$$\simeq \cos \omega(t - r/c) - \operatorname{sen} \omega(t - r/c) \cdot \frac{\omega a}{c} \operatorname{sen}\theta \cos\varphi$$

se $a \ll \lambda = 2\pi \frac{c}{\omega}$

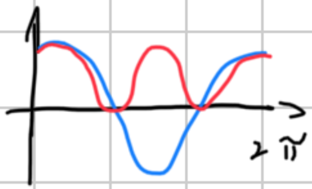
$$\Rightarrow \vec{A}(\vec{r},t) = \frac{\mu_0 I_0 a}{4\pi r}\,\hat{j}\left\{ \int_0^{2\pi} \cos \omega(t - r/c)\cos\varphi \, d\varphi \right.$$

$$+ \int_0^{2\pi} \frac{a}{r} \operatorname{sen}\theta \cos \omega(t - r/c) \cdot \cos^2\varphi \, d\varphi$$

$$\left. - \int_0^{2\pi} \frac{\omega a}{c} \operatorname{sen}\theta \operatorname{sen}(\omega t - r/c) \cos^2\varphi \, d\varphi \right\}$$

retendo termos em 1ª ordem

$$\int_0^{2\pi} \cos\varphi \, d\varphi = 0 \qquad \int_0^{2\pi} \cos^2\varphi \, d\varphi = \pi$$



$$\Rightarrow \vec{A}(\vec{r},t) = \frac{\mu_0 \pi a^2 I_0}{4\pi r}\left(\frac{\operatorname{sen}\theta}{r}\right)\left[\frac{1}{r}\cos\omega(t-r/c) - \frac{\omega}{c}\operatorname{sen}\omega(t-r/c)\right]\hat{\varphi}$$

onde, por simetria, passamos $\hat{j} \to \hat{\varphi}$

$$\omega = 0 \Rightarrow \vec{A}(r,\theta) = \frac{\mu_0}{4\pi}\frac{m_0 \operatorname{sen}\theta}{r^2}\hat{\varphi}$$

$$r \gg \lambda = \frac{2\pi c}{\omega} \Rightarrow \vec{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\frac{\operatorname{sen}\theta}{r}\operatorname{sen}\left(\omega t - \frac{\omega r}{c}\right)\hat{\varphi}$$

$$\Rightarrow \boxed{\begin{aligned} \vec{E} &= -\frac{\partial}{\partial t}\vec{A} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\operatorname{sen}\theta}{r}\right)\cos(\vec{k}\cdot\vec{r} - \omega t)\,\hat{\varphi} \\ \vec{B} &= \nabla\times\vec{A} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\operatorname{sen}\theta}{r}\right)\cos(\vec{k}\cdot\vec{r} - \omega t)\,\hat{\theta} \end{aligned}}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{\mu_0}{c}\left[\frac{m_0 \omega^2}{4\pi c}\left(\frac{\text{sen}\,\theta}{r}\right)\cos\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]^2 \hat{r}$$

$$\langle \vec{S} \rangle = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\left(\frac{\text{sen}^2\theta}{r^2}\right)\hat{r} \; ; \; \langle P \rangle = \left(\frac{\mu_0 m_0^2}{12\pi c^3}\right)\omega^4$$

$\vec{E}, \vec{B} \to$ trocam orientações $\to$ similares?

Nem tanto: $$\frac{P_{mag}}{P_{el}} = \left(\frac{m_0}{p_0 c}\right)^2 = \left(\frac{\pi a^2 I_0}{q_0 s} \cdot \frac{1}{c}\right)^2$$

se $I_0 \sim q_0 \omega$, $s \sim \pi a$

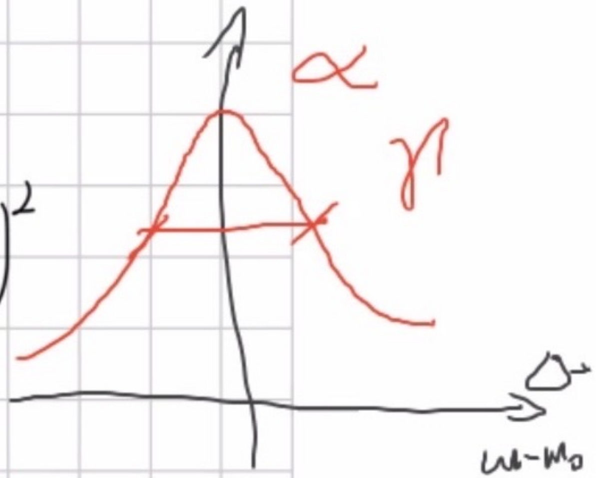
$$\frac{P_{mag}}{P_{el}} = \left(\frac{a\omega}{c}\right)^2 = \left(2\pi \frac{a}{\lambda}\right)^2$$

Acoplamento de dipolo magnético $\ll$ Acoplamento dipolo elétrico!

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\mu_0}{c}\left[\frac{m_0\omega^2}{4\pi c}\left(\frac{\operatorname{sen}\theta}{r}\right)\cos(\vec{k}\cdot\vec{r}-\omega t)\right]^2\hat{r}$$

$$\langle\vec{S}\rangle = \frac{\mu_0 m_0^2\omega^4}{32\pi^2 c^3}\left(\frac{\operatorname{sen}^2\theta}{r^2}\right)\hat{r} \quad ; \quad \langle P\rangle = \left(\frac{\mu_0 m_0^2}{12\pi c^3}\right)\omega^4$$

$\vec{E}, \vec{B}$ → trocam orientações → similares?



Nem tanto: $$\frac{P_{mag}}{P_{el}} = \left(\frac{m_0}{p_0 c}\right)^2 = \left(\frac{\pi a^2 I_0}{q_0 s}\cdot\frac{1}{c}\right)^2$$

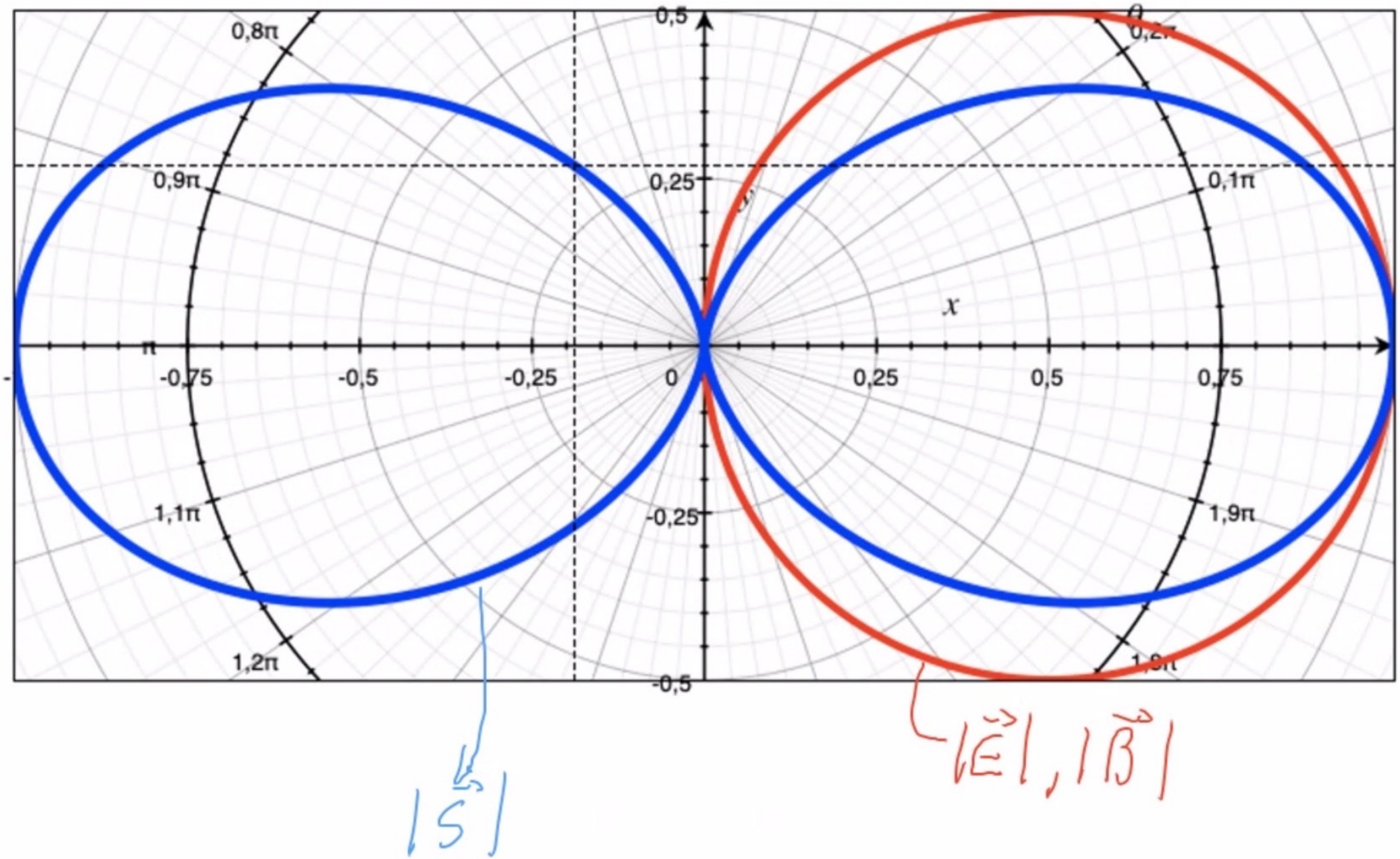
se $I_0 \sim q_0\omega$, $s \sim \pi a$

$$\frac{d}{dt}E = -\gamma E$$

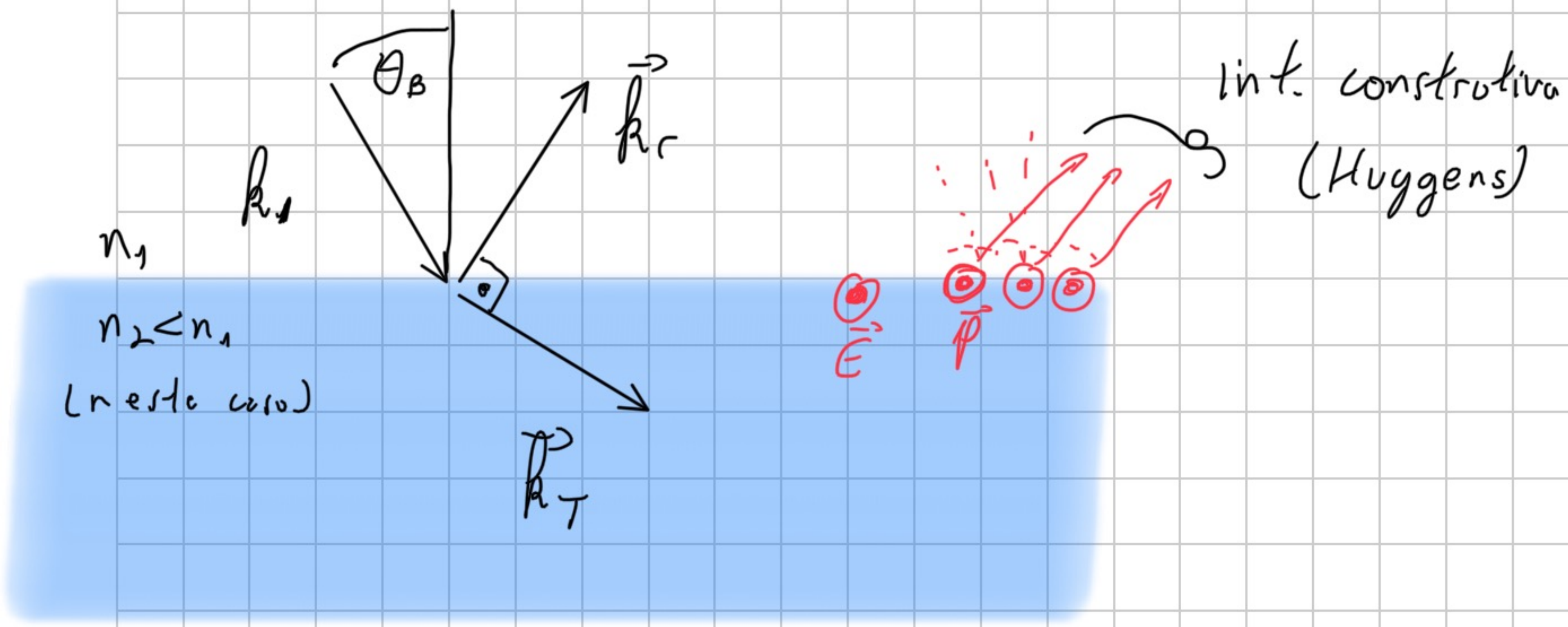
$$\frac{P}{E} = \gamma$$

$$\frac{P_{mag}}{P_{el}} = \left(\frac{a\omega}{c}\right)^2 = \left(2\pi\frac{a}{\lambda}\right)^2$$

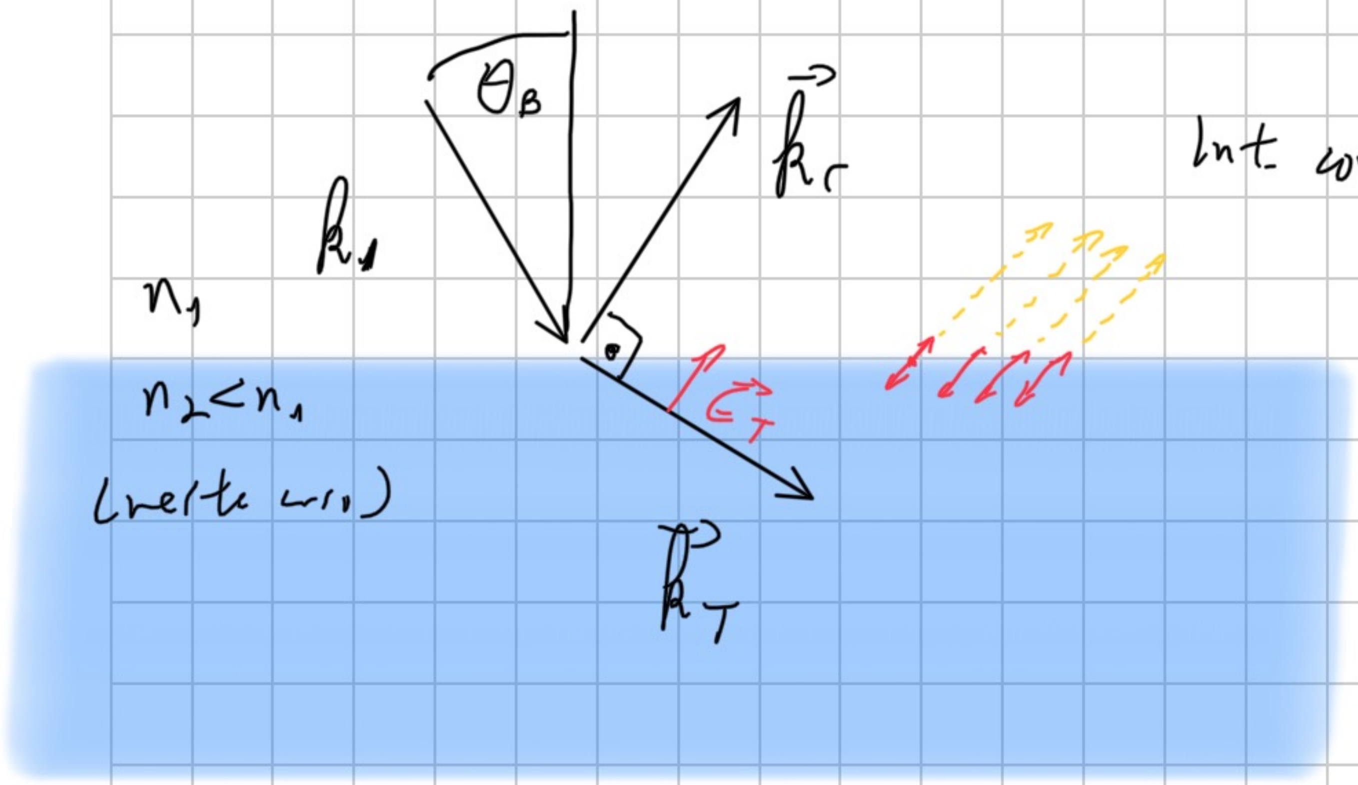
Acoplamento de dipolo magnético $\ll$ Acoplamento dipolo elétrico!


0,5
0,25
-0,25
-0,5
-0,75
-0,5
-0,25
0
0,25
0,5
0,75
0,8π
0,9π
1,1π
1,2π
0,1π
1,9π
π
x
y
$|\vec{S}|$
$|\vec{E}|, |\vec{B}|$

Reflexão Polarizada



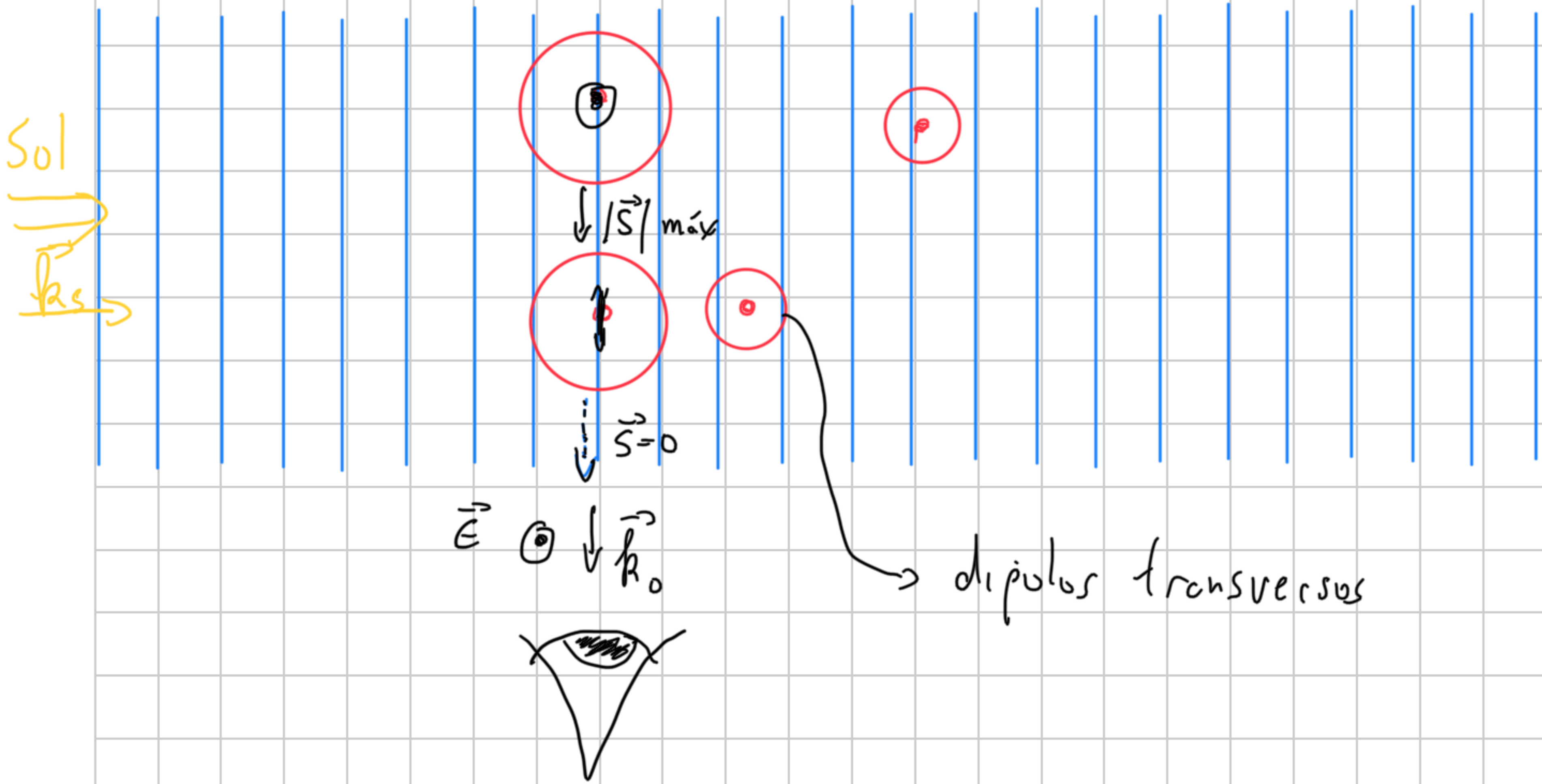
Reflexão Polarizada



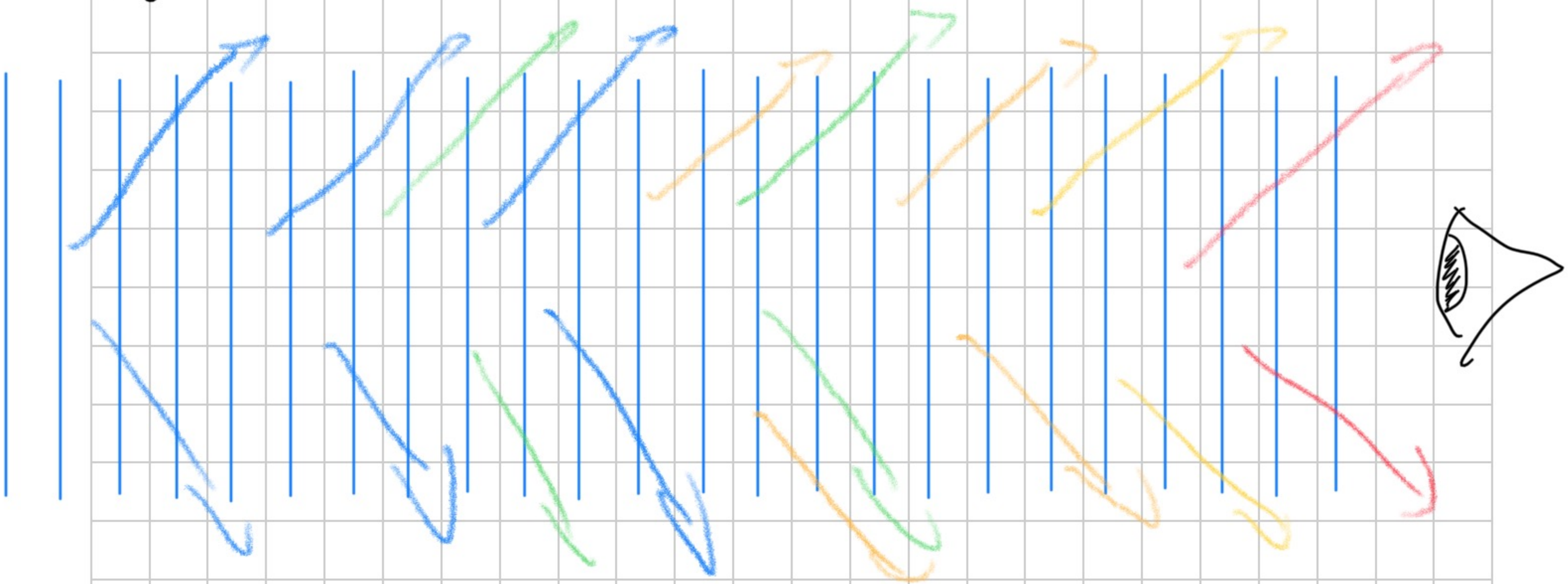
Int. construtiva

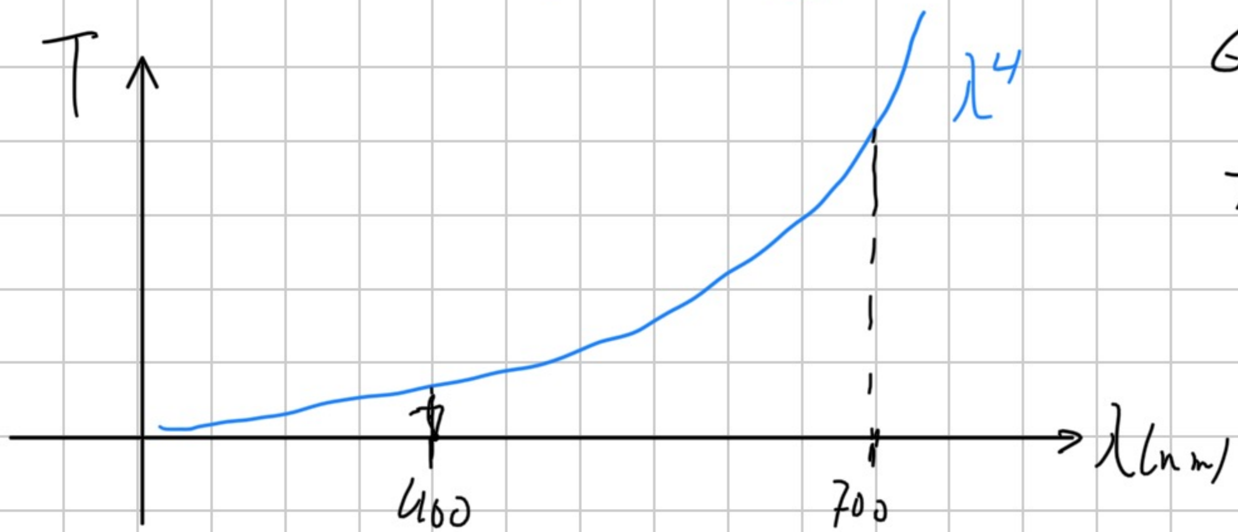
$\langle \vec{S} \rangle = 0 \to$ sem irradiação de dipolo

Espalhamento Rayleigh



Por do Sol





Espalhamento $\propto \omega^4 = 1-T$

$$T = 1 - K \cdot \omega^4$$

$$= 1 - \frac{K' c^4}{\lambda^4}$$

$$= \frac{\lambda^4 - K' c^4}{\lambda^4}$$