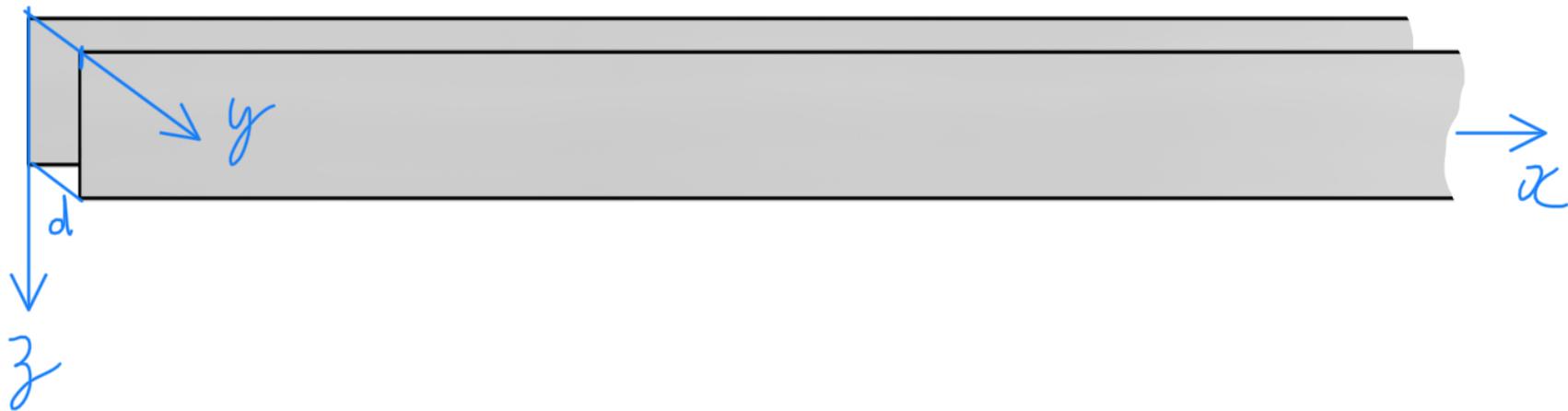


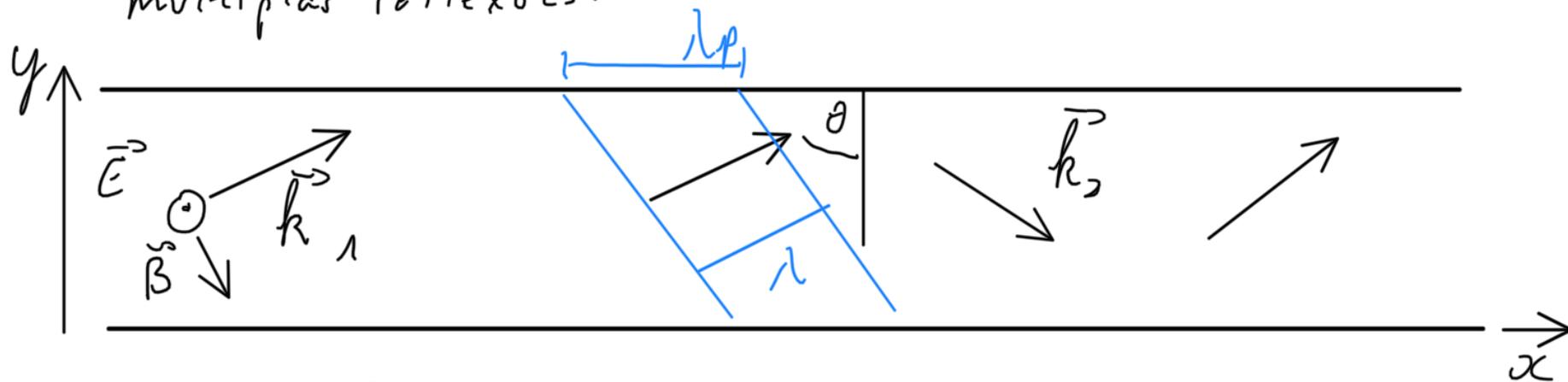
## Eletrromagnetismo Guias de onda

- Ondas planas são uma abstração matemática.
- No curso de Ótica vimos que ondas limitadas espacialmente difratam.
- Como podemos concentrar uma onda sem que sua energia se perca?
- Podemos encontrar **condições de contorno?**

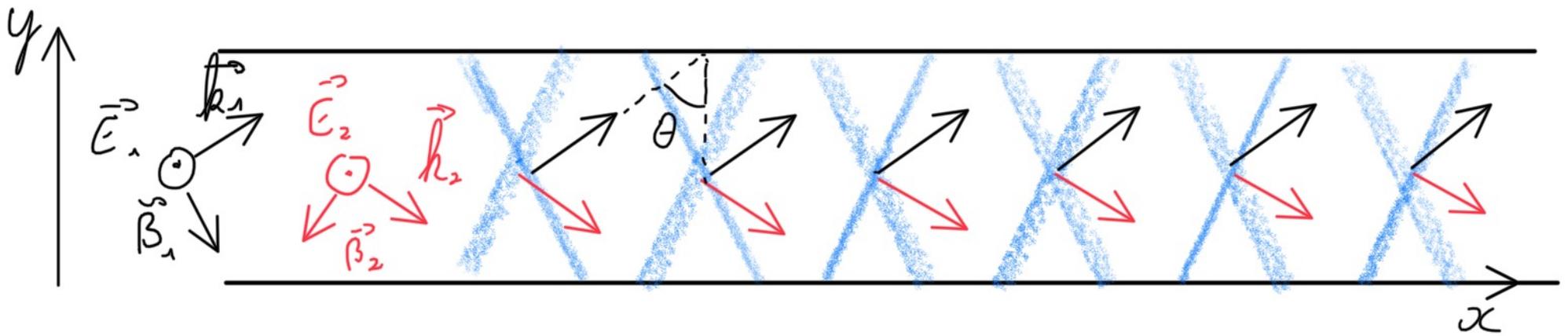
Case simpler: Duas placas planas longas, perfeitamente condutoras.



Uma onda pode se propagar entre os condutores, com múltiplas reflexões.



Compo  $\vec{E}$  transversal (T.E.)



No intervalo, temos a combinação de múltiplas

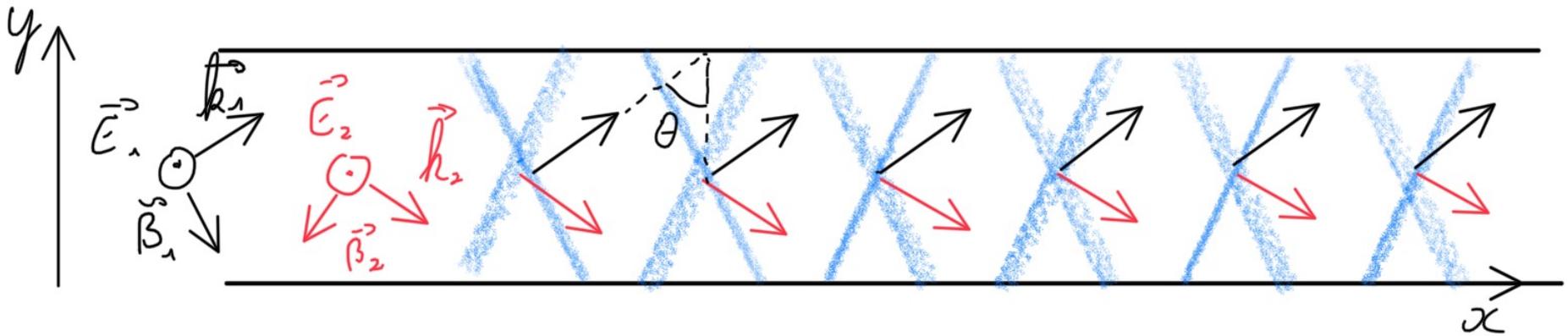
reflexões:  $\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)}$

$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$$

$$\vec{r} = x \hat{x} + y \hat{y}$$

Com  $\vec{k}_1 = k (\sin \theta \hat{x} + \cos \theta \hat{y})$

$$\vec{k}_2 = k (\sin \theta \hat{x} - \cos \theta \hat{y})$$



1ª condição: no plano de reflexão (condutor), o campo

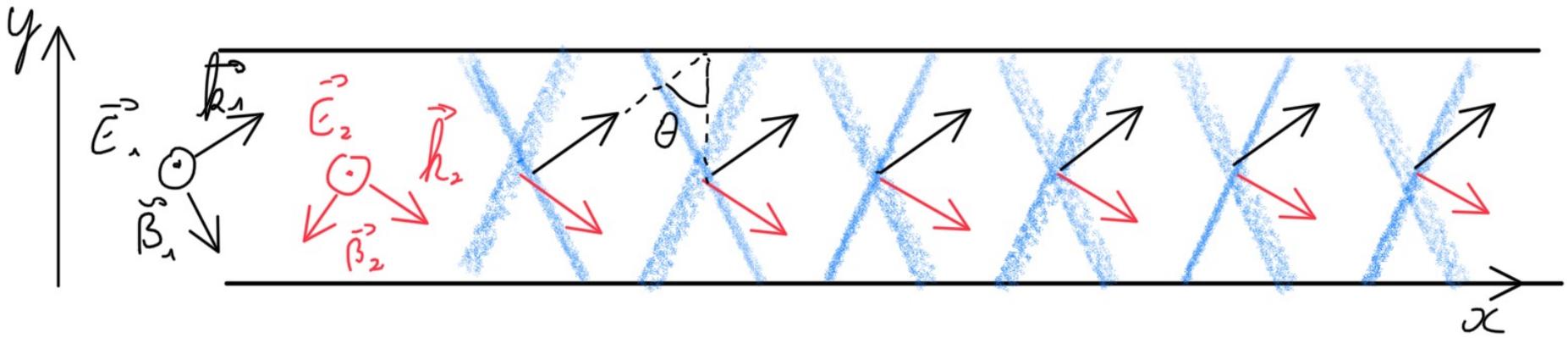
paralelo é nulo.  $\vec{E}_1(x, 0, t) = \vec{E}_{01} e^{i(k \sin \theta x - \omega t)}$

$$\vec{E}_2(x, 0, t) = \vec{E}_{02} e^{i(k \sin \theta x - \omega t)}$$

$$\vec{E}_1 + \vec{E}_2 = 0 \Rightarrow \vec{E}_{01} = -\vec{E}_{02} \equiv \vec{E}_0$$

Campo resultante:

$$\vec{E}_1 + \vec{E}_2 = \vec{E}_0 \left[ e^{i(k \sin \theta \cdot x + k \cos \theta y - \omega t)} - e^{i(k \sin \theta \cdot x - k \cos \theta y - \omega t)} \right]$$



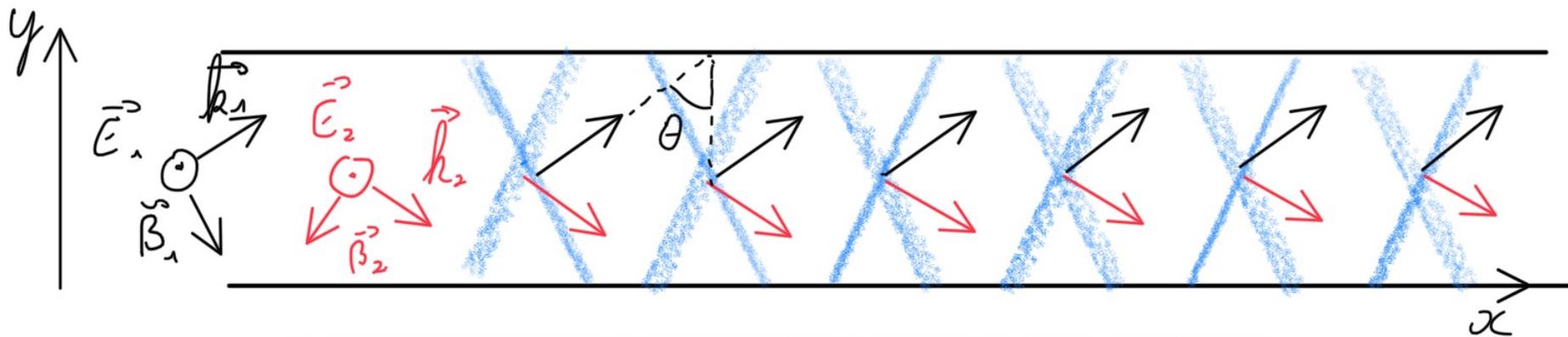
$$\Rightarrow \vec{E} = \vec{E}_0 e^{i(k \sin \theta x - \omega t)} \underbrace{(e^{i k \cos \theta y} - e^{-i k \cos \theta y})}_{2i \sin(k y \cos \theta)}$$

$$\vec{E} = 2 \vec{E}_0 i \sin(k y \cos \theta) \cdot e^{i(k \sin \theta x - \omega t)}$$

Modulação em  $y$   
 $\rightarrow$  condição de contorno

$$\vec{E}_0 = E_0 \hat{z}$$

Onda propagante  
 com comprimento de onda  
 $\lambda' = \frac{2\pi}{k \cos \theta} = \frac{2\pi}{k \sin \theta} = \frac{\lambda}{\sin \theta}$



$$\vec{E} = 2 E_0 i \sin(k y \cos \theta) \cdot e^{i(k \sin \theta x - \omega t)} \hat{j}$$

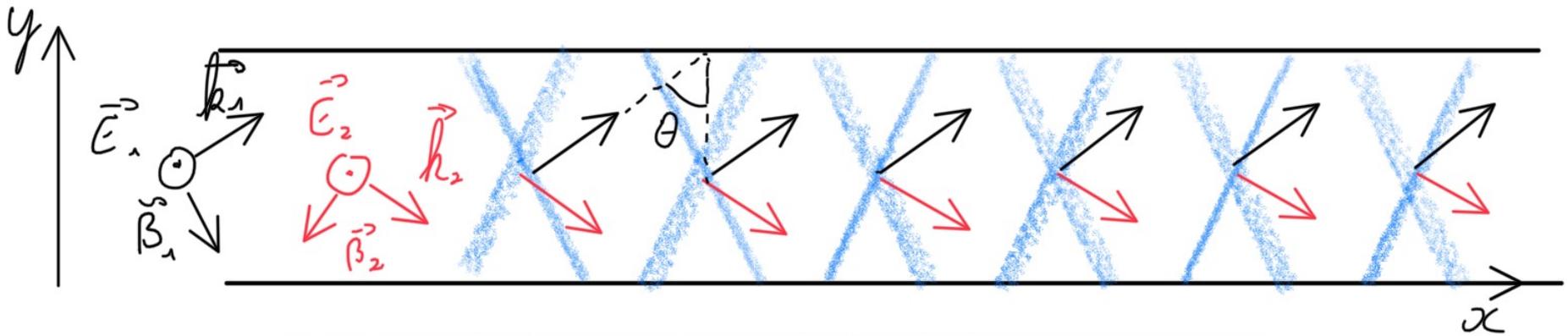
Condição de contorno:  $\vec{E}(y=0) = 0$  OK

Condutor  $\vec{E}(y=d) = 0$

$$\sin(k d \cos \theta) = 0 \Rightarrow k_n d \cos \theta = n \pi$$

$$\frac{2\pi}{\lambda_n} d \cos \theta = n \pi$$

$$\lambda_n = \frac{2}{n} d \cos \theta$$



$$\vec{E} = 2 E_0 i \sin(k y \cos \theta) \cdot e^{i(k \sin \theta x - \omega t)} \hat{z}$$

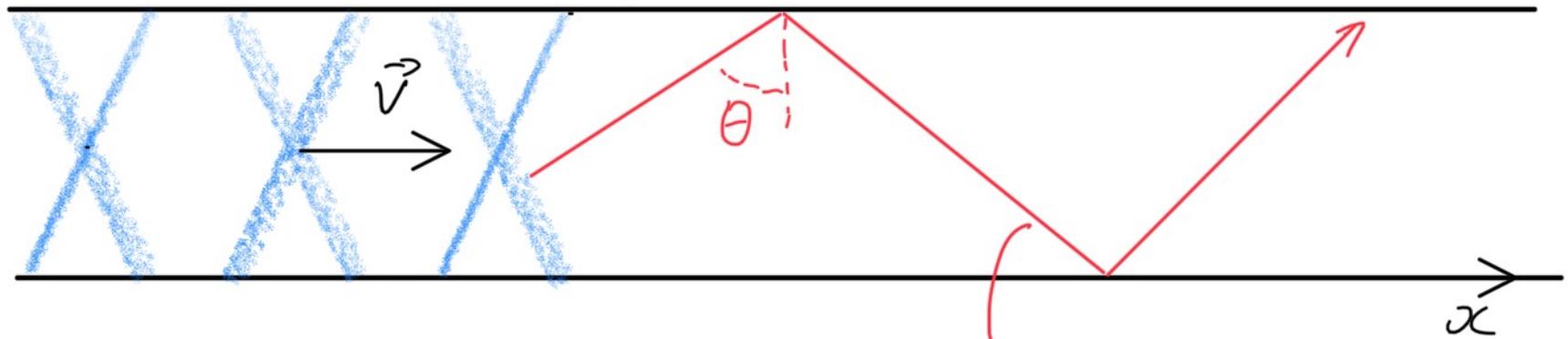
$$dn = \frac{2}{n} d \cos \theta$$

$$\omega_n = \frac{2 \tilde{\pi} c}{dn} = \frac{n \tilde{\pi} c}{d \cos \theta} > \frac{\tilde{\pi} c}{d} = \omega_0$$

OU  $\rightarrow$  dada a frequência  $\omega \Rightarrow \theta_n = \arccos\left(\frac{n \tilde{\pi} c}{d \omega}\right) = \arccos\left(n \frac{\omega_0}{\omega}\right)$

Conjunto discreto de ângulos de excitação, dado  $n < \frac{\omega}{\omega_0}$

Velocidade da onda na guia  $\rightarrow v = \frac{\omega}{k_{ef}} = \frac{\omega}{k \sin \theta} = \frac{c}{\sin \theta} > c$   
 $\lambda \rightarrow \lambda / \sin \theta$



velocidade de fase:

$$\lambda \rightarrow \lambda_f = \lambda / \sin \theta$$

$$k_{ef} = k \sin \theta$$

$\rightarrow$  número de onda da guia ( $g$ )

$\theta$  "raio" se propaga com velocidade  $c \cdot \sin \theta$   
 Velocidade de grupo:  
 a onda estacionária vai sendo construída

Note que, dado  $\theta = \arccos\left(\frac{\tilde{\pi}c}{d\omega}\right) \rightarrow \omega_c = \frac{\tilde{\pi}c}{d \cos\theta} \leq \omega_n$

→ Frequência de corte

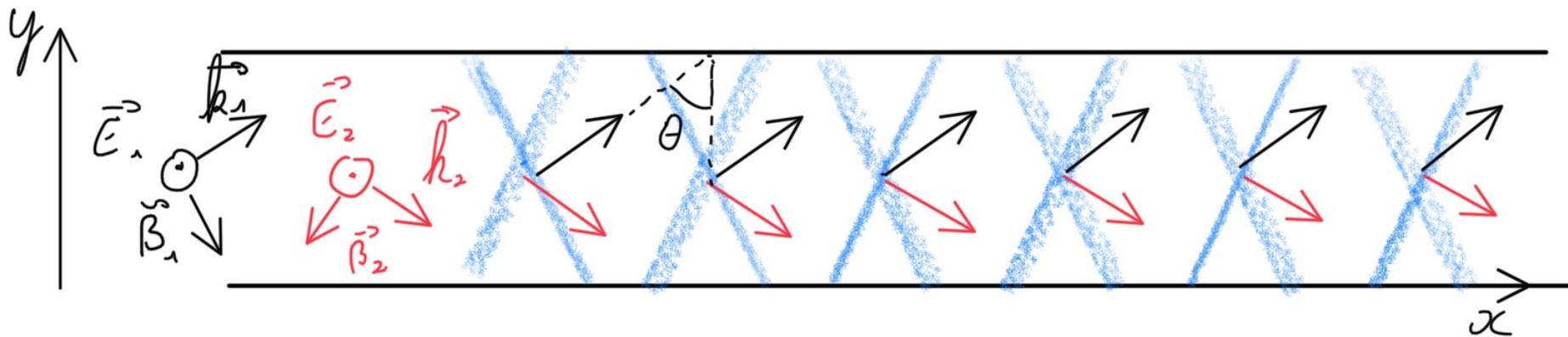
$$\omega_c = 2\tilde{\pi}f_c = \frac{\tilde{\pi}c}{d \cos\theta} \quad ; \quad \omega_n = 2\tilde{\pi}f_n = n\frac{\tilde{\pi}c}{d \cos\theta}$$

Como  $\lambda = c/f_n \Rightarrow \lambda_c \equiv \frac{2d}{n} = \frac{\lambda}{\cos\theta}$

$$\Rightarrow \frac{1}{\lambda_c^2} + \frac{1}{\lambda_d^2} = \frac{1}{\lambda^2} \Rightarrow k^2 = k_d^2 + k_c^2$$

↳ vetor de onda de grupo  
↳ vetor de onda de corte

$$k_d = 2\tilde{\pi} \sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}}$$



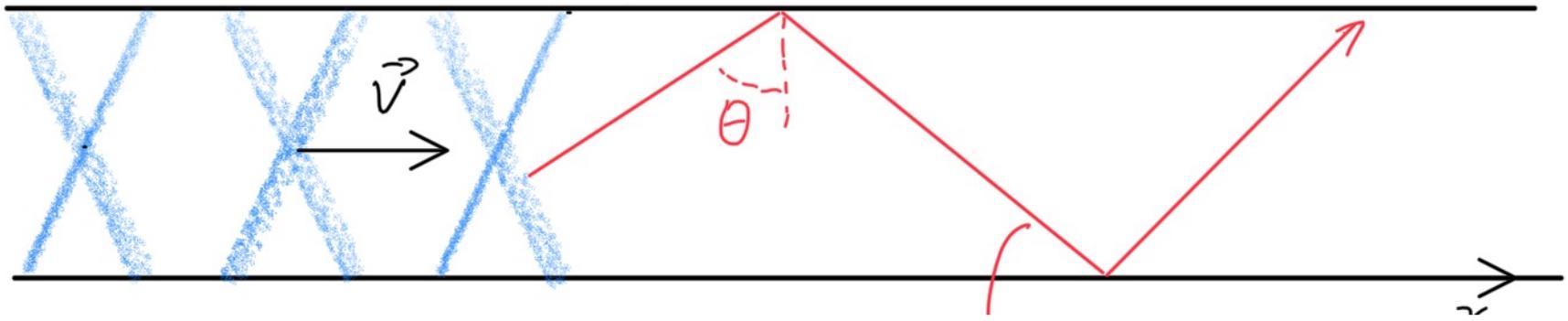
$$k_d = 2\pi \sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}}$$

$$\vec{E} = -2i E_0 \sin\left(\frac{n\pi y}{d}\right) e^{i(k_d x - \omega t)}$$

$\Rightarrow \lambda < \lambda_c \Rightarrow k_d \in \mathbb{R} \rightarrow$  propagação

$$\lambda > \lambda_c \Rightarrow k_d \in \mathbb{C} \Rightarrow \vec{E} = -2i E_0 \sin\left(\frac{n\pi y}{d}\right) e^{-|k_d|bc} e^{-i\omega t}$$

Atenuação exponencial!  $\checkmark$



Velocidade de grupo:  $v_g = \frac{dm}{dk_{ef}}$

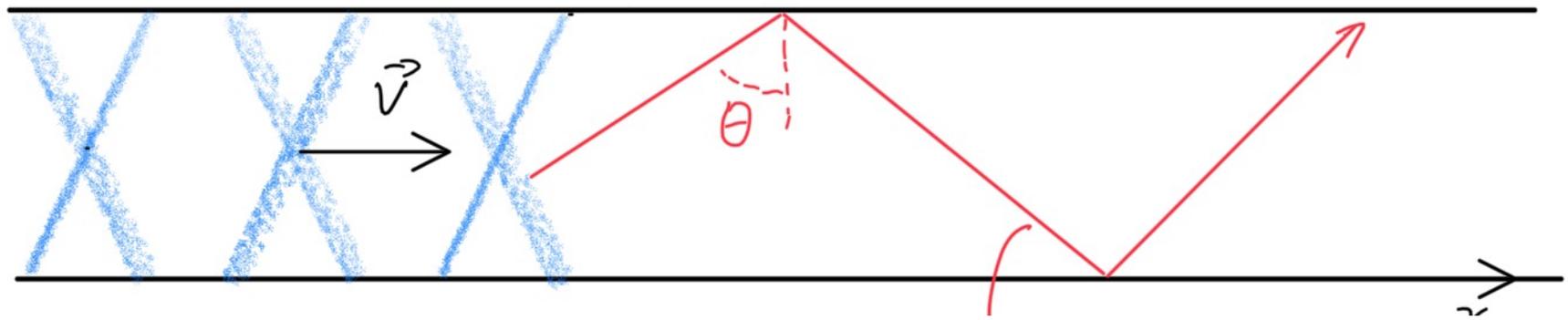
$$k = \frac{m}{c} = \sqrt{k_{ef}^2 + k_i^2}$$

$$\Rightarrow \frac{dm}{dk_{ef}} = c \frac{k_{ef}}{\sqrt{k_{ef}^2 + k_i^2}} = c \frac{k_{ef}}{k}$$

$$v_g = c \sin \theta$$

$$v_{ef} = \frac{c}{\sin \theta}$$

$$v_g \cdot v_{ef} = c^2$$



$$v_g = c \sin \theta$$

$$v_f = \frac{c}{\sin \theta}$$

$$v_g \cdot v_f = c^2$$

→ As consequências: frequência limite inferior

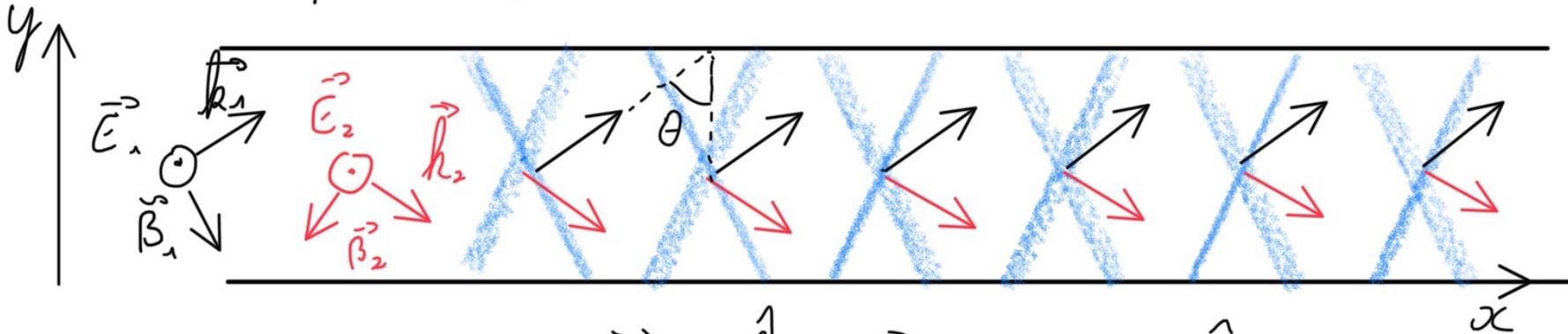
ângulo de incidência

velocidade de grupo e fase

são observadas em guias mais elaborados

(as duas placas não confinam a onda na direção z)

# Campo Magnético



Campo magnético:  $\vec{B}_1 = \frac{\hat{k}_1}{c} \times \vec{E}_1$        $\hat{k}_1 = \text{sen } \theta \hat{x} + \text{cos } \theta \hat{y}$

$$\vec{B}_1 = (\text{sen } \theta \hat{x} + \text{cos } \theta \hat{y}) \times \frac{\underline{\epsilon}_0}{c} \hat{z} e^{i(\hat{k}_1 \cdot \vec{r} - \omega t)}$$

$$\vec{B}_1 = (-\text{sen } \theta \hat{y} + \text{cos } \theta \hat{x}) \cdot \frac{\underline{\epsilon}_0}{c} e^{i(\hat{k}_1 \cdot \vec{r} - \omega t)}$$

$$\begin{aligned} \hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y} \end{aligned}$$

$$\vec{B}_2 = \frac{\hat{k}_2}{c} \times \vec{E}_2$$

$$\hat{k}_2 = \text{sen } \theta \hat{x} - \text{cos } \theta \hat{y}$$

$$\vec{B}_2 = (\text{sen } \theta \hat{x} - \text{cos } \theta \hat{y}) \times (-\underline{\epsilon}_0 \hat{z}) e^{i(\hat{k}_2 \cdot \vec{r} - \omega t)}$$

$$\vec{B}_2 = (\text{sen } \theta \hat{y} + \text{cos } \theta \hat{x}) \cdot \frac{\underline{\epsilon}_0}{c} e^{i(\hat{k}_2 \cdot \vec{r} - \omega t)}$$

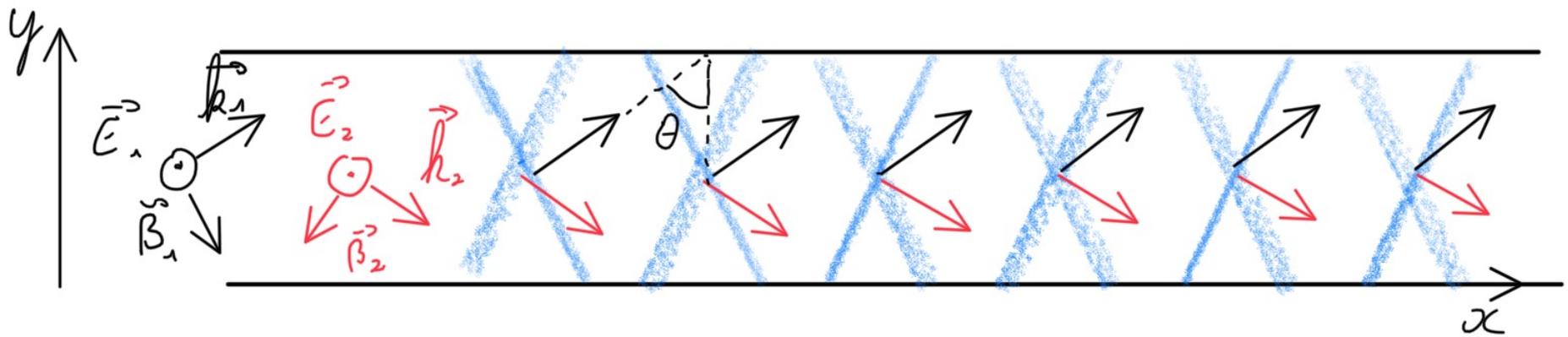
$$\Rightarrow \mathcal{B}_{zc} = \frac{\epsilon_0}{c} \omega r \theta e^{-i\omega t} (e^{i\vec{k}_1 \cdot \vec{r}} + e^{i\vec{k}_2 \cdot \vec{r}})$$

Lembrando:  $\vec{k}_1 = k (\sin \theta \hat{x} + \cos \theta \hat{y})$ ;  $\vec{r} = x \hat{x} + y \hat{y}$   
 $\vec{k}_2 = k (\sin \theta \hat{x} - \cos \theta \hat{y})$ ;

$$e^{i\vec{k}_1 \cdot \vec{r}} + e^{i\vec{k}_2 \cdot \vec{r}} = e^{i k x \sin \theta} \underbrace{(e^{i k y \cos \theta} + e^{-i k y \cos \theta})}_{2 \cos(k y \cos \theta)}$$

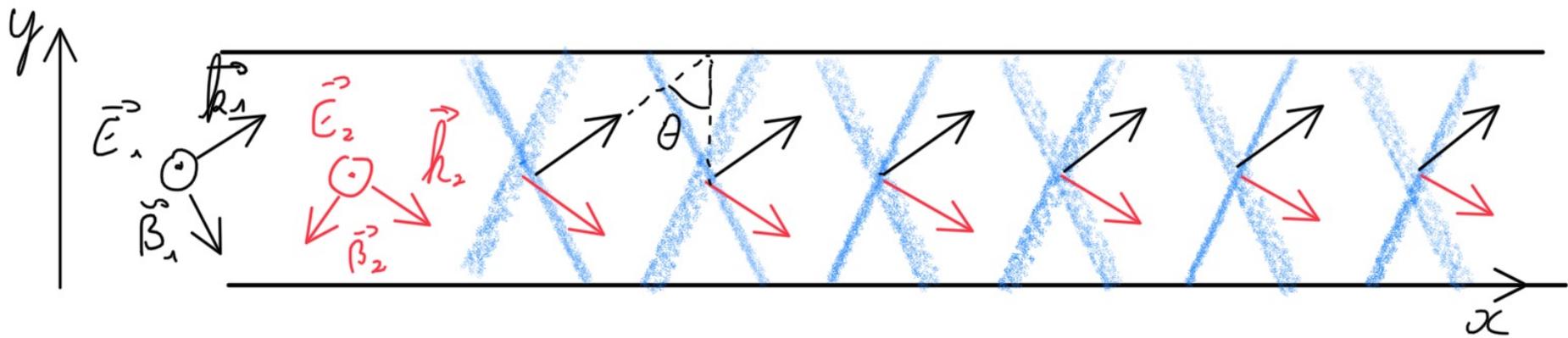
$$\Rightarrow \mathcal{B}_y = \underline{\underline{-\frac{\epsilon_0}{c} \sin \theta e^{-i\omega t} (e^{i\vec{k}_1 \cdot \vec{r}} - e^{i\vec{k}_2 \cdot \vec{r}})}}$$

$$e^{i\vec{k}_1 \cdot \vec{r}} - e^{i\vec{k}_2 \cdot \vec{r}} = e^{i k x \sin \theta} \cdot \underbrace{(e^{i k y \cos \theta} - e^{-i k y \cos \theta})}_{2 i \sin(k y \cos \theta)}$$



$$\therefore \vec{B} = 2 \frac{\epsilon_0}{c} \left[ \cos \theta \cos(ky \cos \theta) \hat{x} - i \sin \theta \sin(ky \cos \theta) \hat{y} \right] e^{i(kx \cos \theta - \omega t)}$$

Surpresa!   
 Componente longitudinal de  $\vec{B}$ !



Condições de contorno:  $\vec{E}_{\parallel} = 0$ ,  $\vec{B}_{\perp} = 0$

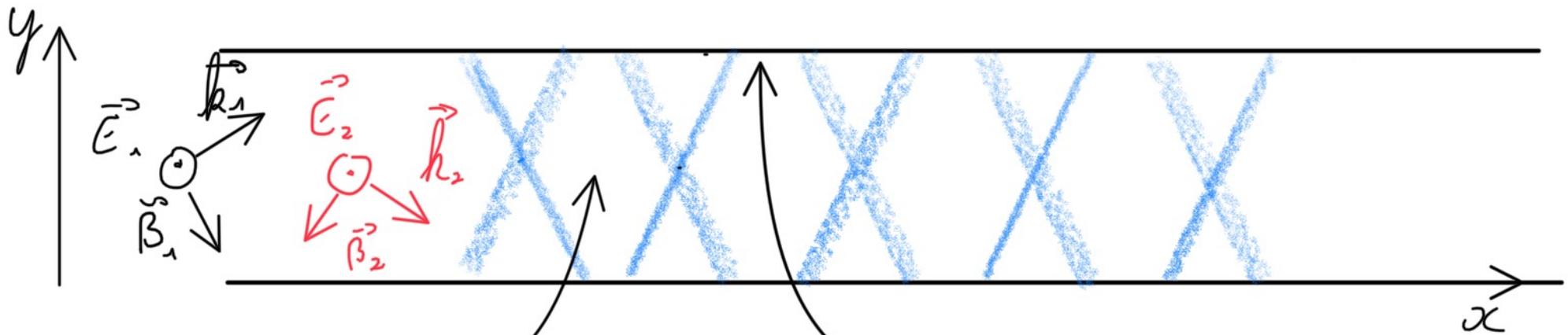
$$\vec{E} = 2 E_0 i \sin(ky \cos \theta) \cdot e^{i(kx \sin \theta - \omega t)} \hat{z}$$

$$\vec{B} = 2 \frac{E_0}{c} [\cos \theta \cos(ky \cos \theta) \hat{x} - i \sin \theta \sin(ky \cos \theta) \hat{y}] e^{i(kx \sin \theta - \omega t)}$$

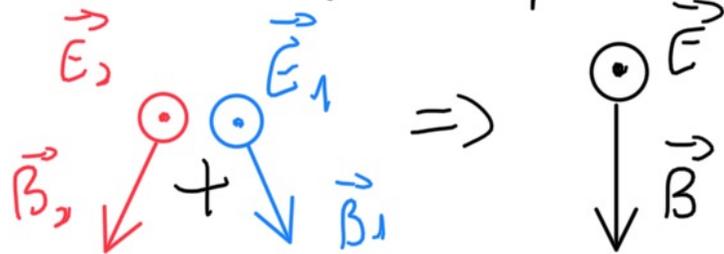
Onde  $|\vec{E}| = 0$ ;  $B_y = 0$  (componente transversal de  $\vec{B}$ )

mas  $|B_z|$  é máximo (componente longitudinal de  $\vec{B}$ )

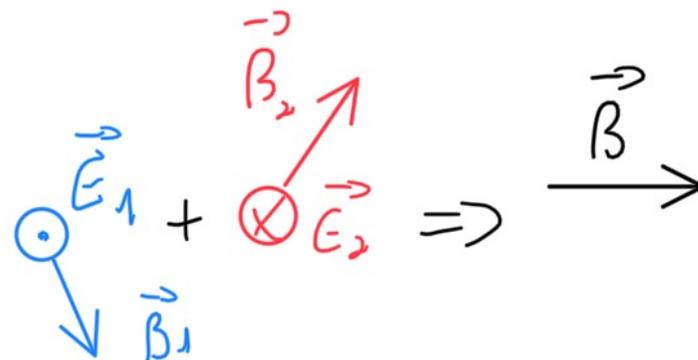
Onde  $|\vec{E}|=0$  ;  $B_y=0$  (componente transversa de  $\vec{B}$ )  
 mas  $|B_z|$  é máximo (componente longitudinal de  $\vec{B}$ )



Interferência construtiva  $\forall \vec{E}$

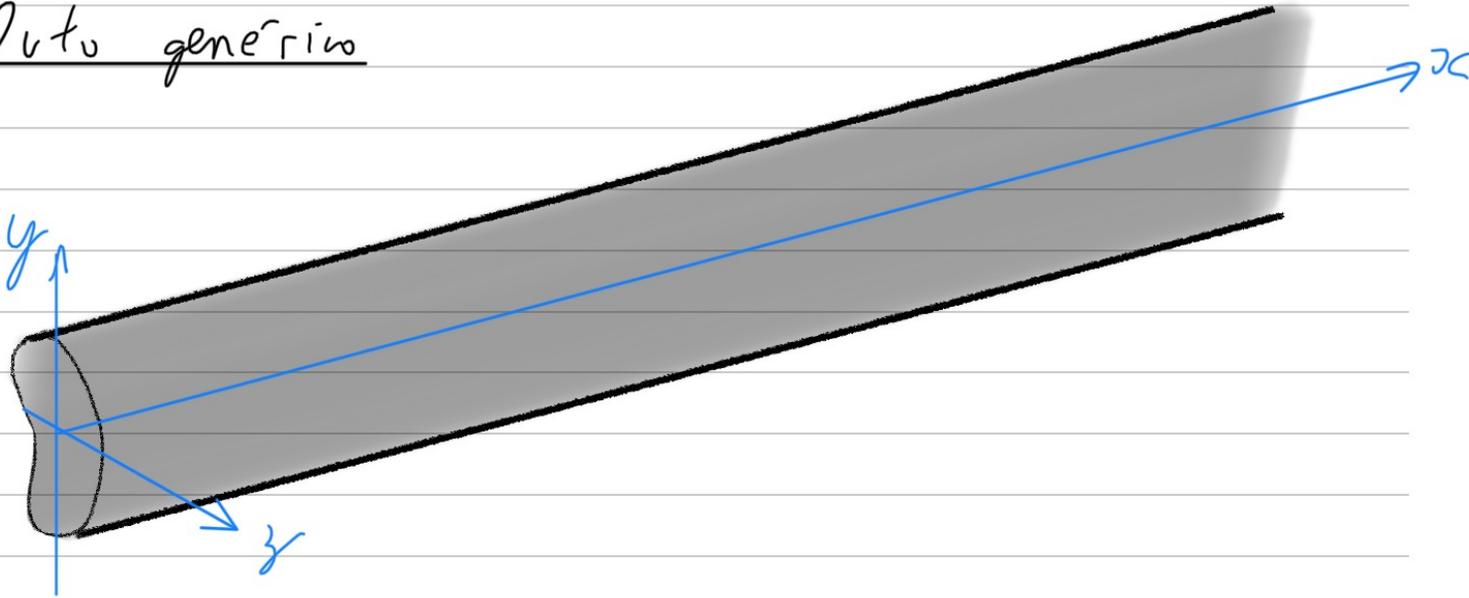


Interferência destrutiva  $\forall \vec{E}$



# Dutos como Guias

## Duto genérico



+ Ondas monocromáticas:  $\vec{E}(x, y, z, t) = \vec{E}_0(y, z) e^{i(k_x x - \omega t)}$   
 $\vec{B}(x, y, z, t) = \vec{B}_0(y, z) e^{i(k_x x - \omega t)}$

Parte lenta  $\nearrow$  termo harmônico

$$\Rightarrow \frac{\partial}{\partial t} \vec{E} = -i\omega \vec{E} ; \frac{\partial}{\partial x} \vec{E} = ik_x \vec{E}$$

$$\vec{E}_0(y, z) = E_x \hat{x} + E_y \hat{y} + E_z \hat{z} ; \vec{B}_0 = \dots$$

Eqs. de Maxwell :  $\nabla \cdot \vec{E} = 0$      $\nabla \cdot \vec{B} = 0$

$$\nabla \times \vec{E} = i\omega \vec{B} \quad \nabla \times \vec{B} = -i\omega \mu \epsilon \vec{E}$$

### Rotacionais

$$\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = i\omega B_{xc} \checkmark$$

$$\frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y = -i \frac{\omega}{c^2} E_{xc} \checkmark$$

$$\frac{\partial}{\partial z} E_x - ik_y E_z = i\omega B_y$$

$$\frac{\partial}{\partial z} B_{xc} - ik_y B_z = -i \frac{\omega}{c^2} E_y$$

$$ik_y E_y - \frac{\partial}{\partial y} E_{xc} = i\omega B_z$$

$$ik_y B_y - \frac{\partial}{\partial y} B_{xc} = -i \frac{\omega}{c^2} E_z$$

Resolvemos nas componentes transversais

$$E_y = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial y} E_x + m \frac{\partial}{\partial z} B_x \right)$$

$$E_z = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial z} E_x - m \frac{\partial}{\partial y} B_x \right)$$

$$B_y = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial y} B_x - \frac{m}{c^2} \frac{\partial}{\partial z} E_x \right)$$

$$B_z = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial z} B_x + \frac{m}{c^2} \frac{\partial}{\partial y} E_x \right)$$

Que dependem de componentes longitudinais  $E_x, B_x$

Tomando as componentes transversais e derivando, obtemos

$$\left[ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + (k^2 - k_g^2) \right] E_x = 0 \quad / \quad \left[ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + (k^2 - k_g^2) \right] B_x = 0$$

Quando  $E_x = 0$ ,  $\vec{E}$  transversal a  $\hat{x}$  (TE)

$B_x = 0$ ,  $\vec{B}$  transversal a  $\hat{x}$  (TM)

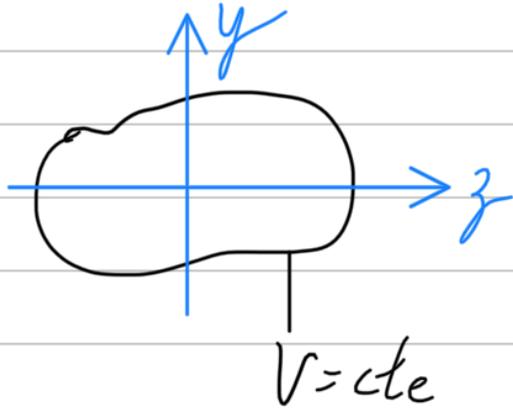
→ Não nulas (simultaneamente) em um duto vazio

$$E_x = 0 \Rightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z = 0$$

$$B_x = 0 \Rightarrow \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = i m B_x = 0 \Rightarrow \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = 0$$

$\nabla \cdot \vec{E} = 0$ ,  $\nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla V \rightarrow$  pode ser descrito por um potencial

Condição de contorno no duto: não permite máximo local



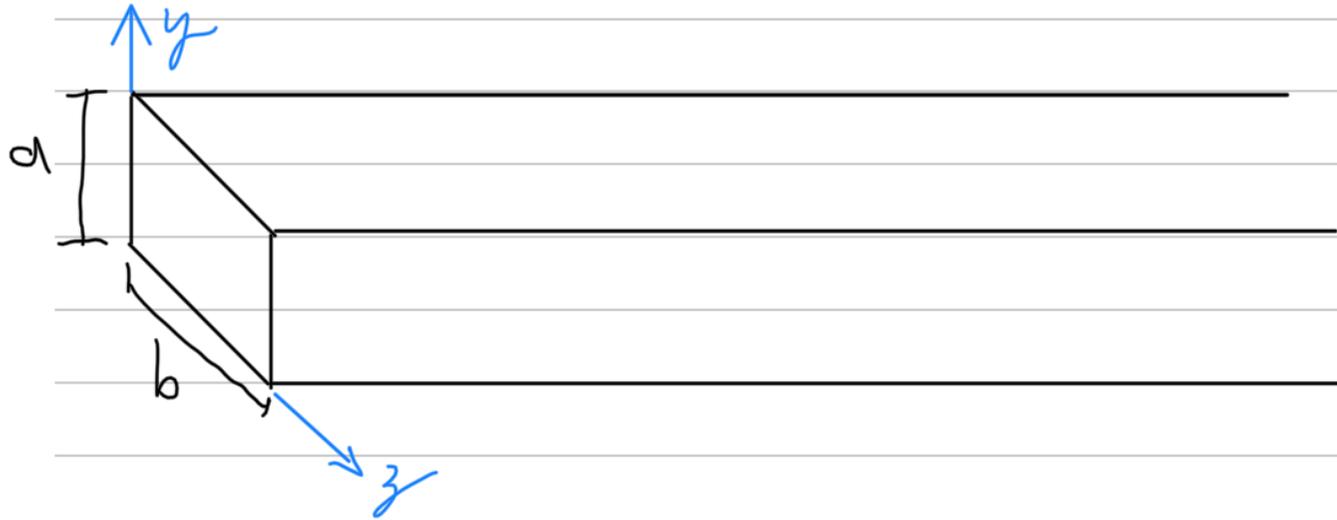
$\Rightarrow$  potencial uniforme dentro

do duto  $\Rightarrow \vec{E} = -\nabla V = 0 \checkmark$

- Veremos que um fio faz toda diferença!

- Diferente das placas paralelas

# Guia de Onda Retangular



Dado as condições de contorno de um condutor ideal:  $\vec{E} = 0$   
 $\vec{B} = 0$

$$\left. \begin{array}{l} \text{Como } \nabla \cdot \vec{B} = 0 \Rightarrow B_{\perp} = 0 \\ \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow E_{\parallel} = 0 \end{array} \right\} \text{Na superfície}$$

$\Rightarrow$  Condições de contorno para

$$\left[ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + (k^2 - k_g^2) \right] E_x = 0$$

$$\left[ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + (k^2 - k_g^2) \right] B_x = 0$$

Separação de variáveis  $B_x(y, z) = Y(y)Z(z)$

$$\Rightarrow Z \frac{\partial^2}{\partial y^2} Y + Y \frac{\partial^2}{\partial z^2} Z + (k^2 - k_g^2) \cdot YZ = 0 \quad \left( \frac{1}{YZ} \right)$$

$$\Rightarrow \frac{1}{Y} \frac{\partial^2}{\partial y^2} Y + \frac{1}{Z} \frac{\partial^2}{\partial z^2} Z + (k^2 - k_g^2) = 0 \quad \forall y, z$$

$$-k_y^2 - k_z^2 + (k^2 - k_g^2) = 0 \quad \leftarrow$$

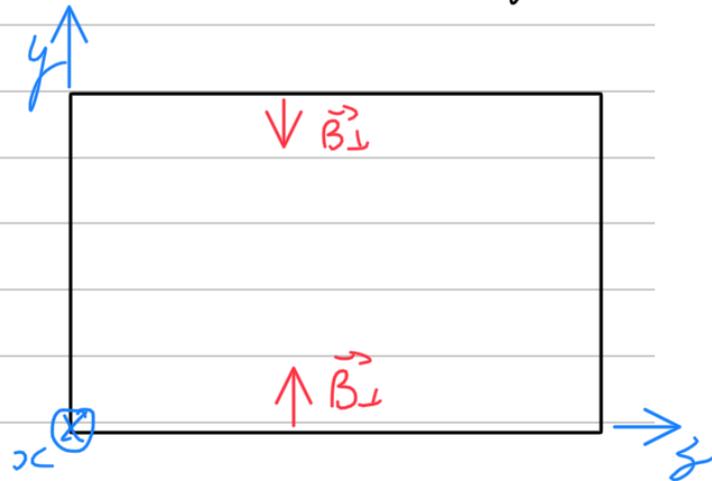
$$\Rightarrow \frac{\partial^2}{\partial y^2} Y = k_y^2 Y$$

$\Rightarrow Y(y) = A \sin(k_y y) + B \cos(k_y y)$

$$B_y(0, z) = B_y(a, z) = 0$$

Como vimos, no duto:

$$B_y = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial y} B_x - \frac{m}{c^2} \frac{\partial}{\partial z} E_x \right)$$



$$\text{Modo TE} \rightarrow \varepsilon_x = 0 \Rightarrow \frac{\partial B_x}{\partial y} \Big|_{y=0} = \frac{\partial B_x}{\partial y} \Big|_{y=a} = 0$$

$$\frac{\partial Y}{\partial y} = k_y A \cos(k_y y) - k_y B \sin(k_y y) = 0$$

$$y=0 \Rightarrow A=0$$

$$y=a \Rightarrow \sin(k_y a) = 0 \quad \underline{k_y = m\pi/a} \quad (m \in \mathbb{N})$$

Aplicando o mesmo raciocínio em  $z \Rightarrow$

$$k_z = n\pi/b \quad (n \in \mathbb{N})$$

$$\begin{cases} B_x(y, z) = B \cos\left(m\pi \frac{y}{a}\right) \cos\left(n\pi \frac{z}{b}\right) \\ \varepsilon_x(y, z) = 0 \end{cases}$$

Componentes transversais obtidas por derivação:

$$E_y = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial y} E_x + \omega \frac{\partial}{\partial z} B_x \right) \quad B_y = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial y} B_x - \frac{\omega}{c^2} \frac{\partial}{\partial z} E_x \right)$$

$$E_z = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial z} E_x - \omega \frac{\partial}{\partial y} B_x \right) \quad B_z = \frac{i}{k^2 - k_g^2} \left( k_g \frac{\partial}{\partial z} B_x + \frac{\omega}{c^2} \frac{\partial}{\partial y} E_x \right)$$

Modo  $TE_{mn} \rightarrow$  Transverso Elétrico,  $m, n$

onde  $(m+n) > 0$

$$k_g = \sqrt{k^2 - k_y^2 - k_z^2}$$

$$k = \frac{\omega}{c} \Rightarrow \omega > \omega_{mn} = c \sqrt{(m/a)^2 + (n/b)^2}$$

$\omega_{mn} \rightarrow$  frequência de corte

$$k_g = \frac{1}{c} \sqrt{\omega^2 - \omega_{mn}^2} \quad ; \quad v_g = \frac{\omega}{k_g} = \frac{c}{\sqrt{1 - (\omega_{mn}/\omega)^2}} > c$$

$$v_g = c^2 / v_\phi$$

Podemos repetir o processo para  $B_{xc} = 0, E_{xc} = YZ$

Condição de contorno  $E_{||} = 0 \Rightarrow E_{xc}(0, z) = E_{xc}(a, z) = 0$

$$E_{xc}(y, 0) = E_{xc}(y, b) = 0$$

$$\Rightarrow E_{xc} = E_0 \sin\left(n\pi \frac{y}{a}\right) \sin\left(m\pi \frac{z}{b}\right) \quad m \cdot n > 1$$

Frequência de corte mínima maior!

$$w_{mn} = c\pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

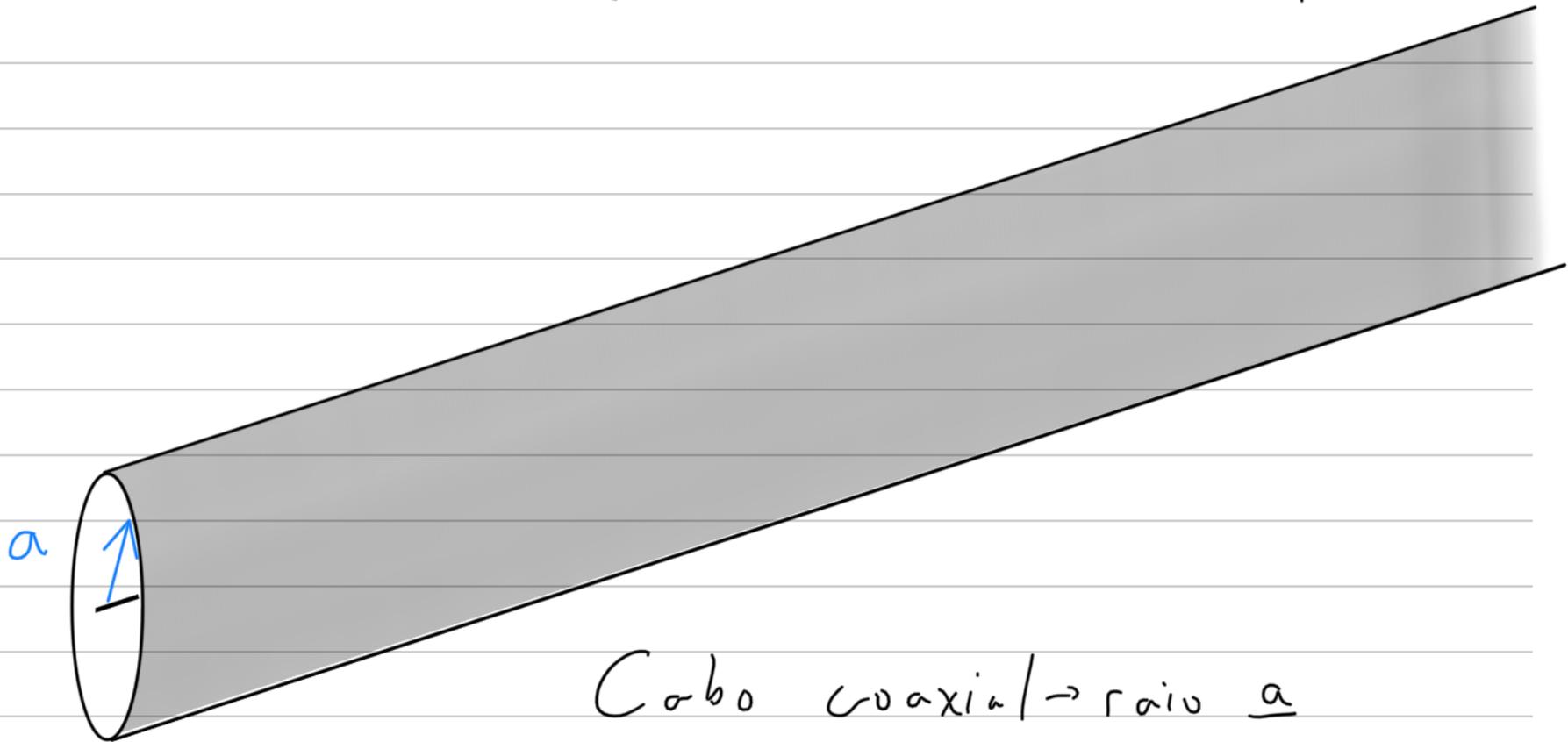
→ Discretização das frequências aceitas

→ Casamento do modo inserido no guia

Cabo coaxial: Um duto vazio não suporta

ondas TEM, somente TE ou TM

E se tem um condutor em seu interior?



Como se enrolássemos as placas paralelas?

Podemos ter  $E_x = 0$  &  $B_x = 0 \rightarrow k_x = k = \omega/c$

$$\Rightarrow \vec{B} = \frac{\hat{k}}{c} \times \vec{E} = \frac{\hat{x}}{c} \times (E_y \hat{y} + E_z \hat{z}) = \frac{E_y}{c} \hat{z} - \frac{E_z}{c} \hat{y}$$

$$\Rightarrow B_y = -E_z/c \quad B_z = E_y/c$$

$\nabla \cdot \vec{E} = 0$	$\nabla \cdot \vec{B} = 0$
----------------------------	----------------------------

$$\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\frac{\partial B_z}{\partial y} + \frac{\partial B_y}{\partial z} = 0$$

$\nabla \times \vec{E} = i\omega \vec{B}$	$\nabla \times \vec{B} = -i\omega \mu \epsilon \vec{E}$
---	---

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\omega B_x \quad \rightarrow 0$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = -i \frac{\omega}{c^2} E_x \quad \rightarrow 0$$

$$\frac{\partial E_x}{\partial z} - i k_y E_z = i\omega B_y \quad \rightarrow 0$$

$$\frac{\partial B_x}{\partial z} - i k_y B_z = -i \frac{\omega}{c^2} E_y$$

$$i k_y E_y - \frac{\partial E_x}{\partial y} = i\omega B_z \quad \rightarrow 0$$

$$i k_y B_y - \frac{\partial B_x}{\partial y} = -i \frac{\omega}{c^2} E_z$$

$$\frac{\partial \mathcal{E}_y}{\partial y} + \frac{\partial \mathcal{E}_z}{\partial z} = 0$$

$$\frac{\partial \mathcal{B}_z}{\partial y} + \frac{\partial \mathcal{B}_y}{\partial z} = 0$$

$$\frac{\partial \mathcal{E}_z}{\partial y} - \frac{\partial \mathcal{E}_y}{\partial z} = 0$$

$$\frac{\partial \mathcal{B}_z}{\partial y} - \frac{\partial \mathcal{B}_y}{\partial z} = 0$$

Equivalente ao caso eletrostático 2D  $\nabla$

$$\vec{E}_0 = E_0 \frac{1}{r} \hat{r} \quad ; \quad \vec{B}_0 = \frac{E_0}{c} \frac{1}{r} \hat{\varphi}$$

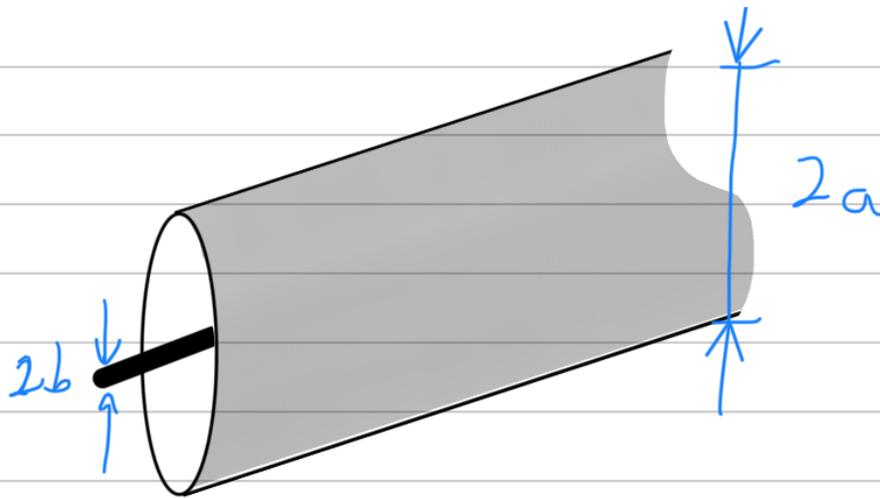


# Impedância

Campo magnético:

$$\int \vec{B} \cdot d\vec{l} = \mu I + \mu \epsilon \int \frac{d\vec{E}}{dt} \cdot d\vec{\alpha}$$

$\rightarrow 0, \vec{E} \cdot d\vec{\alpha} = 0$



$$B = \frac{\mu I}{2\pi r} \quad b < r < a$$

Energia/comprimento  $\rightarrow U_B = \int \frac{B^2}{2\mu} dv = \int_0^l \int_0^{2\pi} \int_b^a \frac{\mu I^2}{2} \frac{1}{(2\pi r)^2} \cdot r dr d\theta dz$

$$U_B = \frac{\mu I^2}{2 \cdot (2\pi)^2} \int_b^a \frac{1}{r} dr \cdot l \cdot 2\pi = \frac{\mu l}{2\pi} \ln\left(\frac{a}{b}\right) \cdot \frac{I^2}{2} = \frac{L \cdot I^2}{2}$$

$$L = \frac{\mu}{2\pi} \ln\left(\frac{a}{b}\right) \cdot l$$

## Campo elétrico

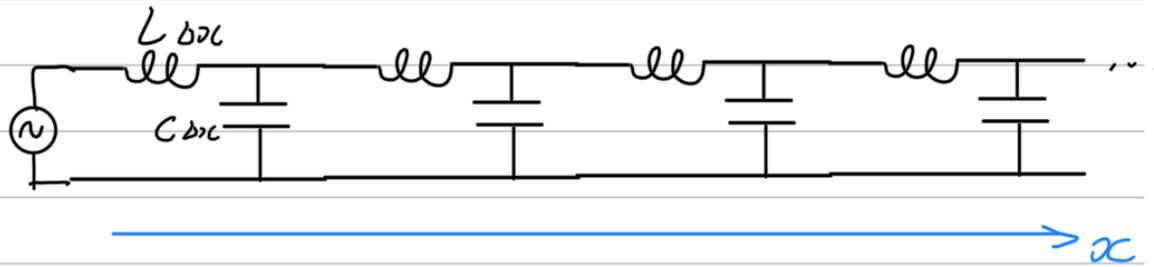
$$\int \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon} \Rightarrow E \cdot 2\pi r \cdot l = \frac{q}{\epsilon} \Rightarrow E = \frac{q}{2\pi r l \epsilon}$$

$$U_{\epsilon} = \int \frac{\epsilon E^2}{2} dV = \int_0^l \int_0^{2\pi} \int_0^a \frac{q^2}{4\pi^2 r^2 l^2 \epsilon^2} \cdot \frac{\epsilon}{2} dz r dr d\theta$$

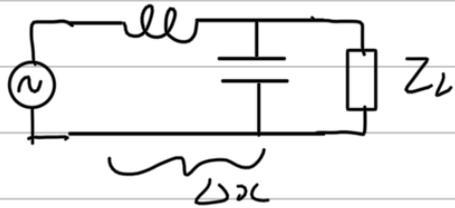
$$= \frac{1}{\epsilon \cdot 2\pi l} \ln\left(\frac{a}{b}\right) \cdot \frac{q^2}{2} = \frac{1}{C} \cdot \frac{q^2}{2}$$

$$C = \frac{\epsilon \cdot 2\pi}{\ln\left(\frac{a}{b}\right)} \cdot l$$

Cabo Coaxial:



Série Infinita: Modelo de Heaviside para linha de transmissão



$$\frac{\Delta V}{\Delta x} = X_L \cdot I = -j\omega L I$$

"Queda" de tensão na reatância  $X_L$

$$\frac{\Delta I}{\Delta x} = \frac{V}{X_C} = -j\omega C \cdot V$$

"Equação do telegrafista"  $\frac{\partial}{\partial x} V = -L \frac{\partial I}{\partial t} ; \frac{\partial}{\partial x} I = -C \frac{\partial V}{\partial t}$

$$\frac{\partial^2}{\partial x^2} V = -L \frac{\partial}{\partial t} \frac{\partial I}{\partial x} = LC \frac{\partial^2 V}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2} I = LC \frac{\partial^2 I}{\partial t^2} \quad v = \frac{1}{\sqrt{LC}} \quad (\text{porisso } \Delta x)$$

$$I(x,t) = I_0 e^{i(kx - \omega t)} \quad \frac{\partial I}{\partial t} = -i\omega I$$

$$V(x,t) = V_0 e^{i(kx - \omega t)} \quad \frac{\partial V}{\partial x} = ikV$$

$$\frac{\partial V}{\partial x} = -L \frac{\partial I}{\partial t} \Rightarrow ikV = i\omega L I \Rightarrow \frac{V}{I} = \frac{\omega}{k} L = \frac{L}{C}$$

$$Z = \frac{V}{I} = \sqrt{\frac{L}{C}} = \text{Impedância da linha}$$

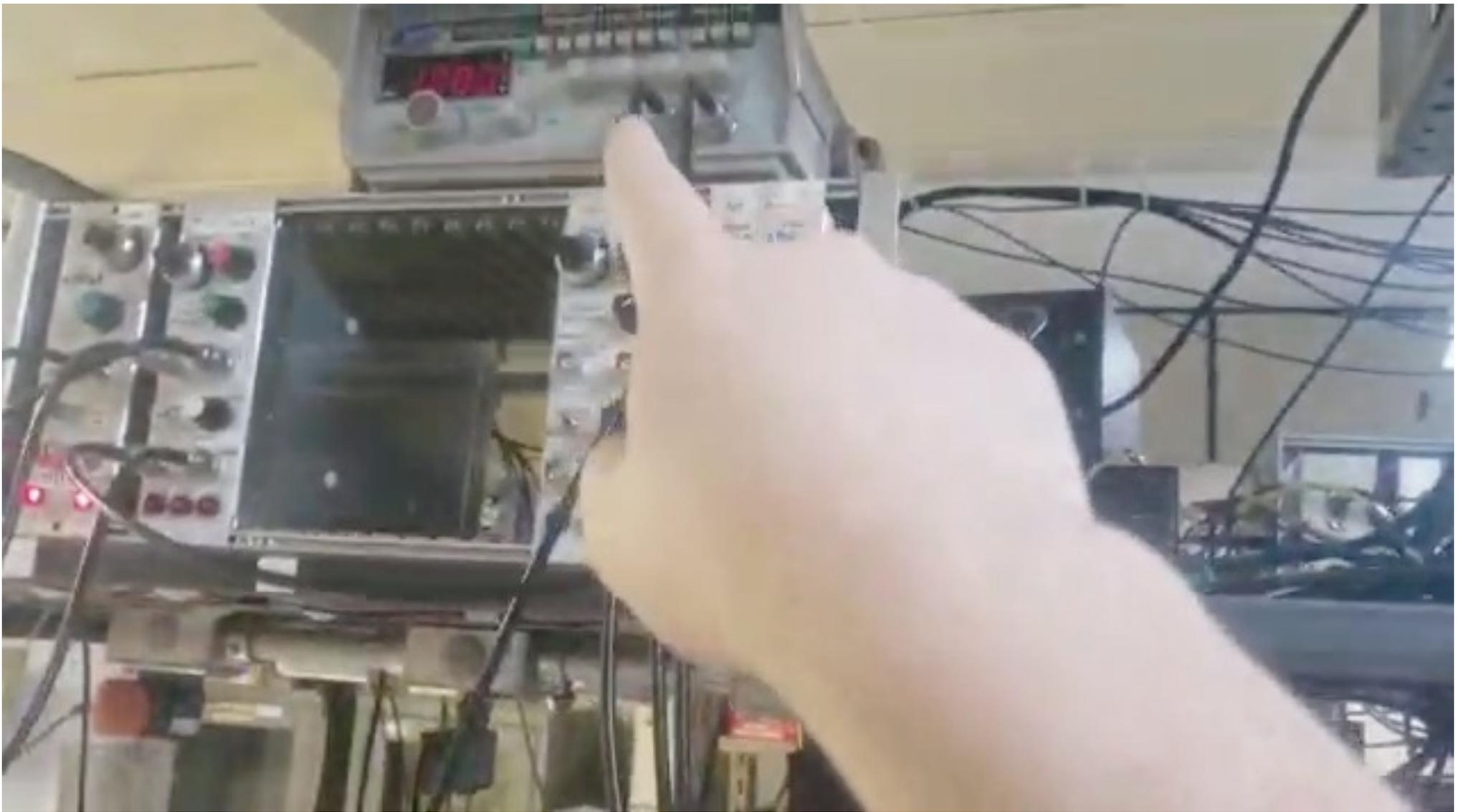
Cabo coaxial:

$$C = \frac{\epsilon \cdot 2\pi}{\ln(a/b)} \cdot l \quad ; \quad L = \frac{\mu}{2\pi} \ln(a/b) \cdot l$$

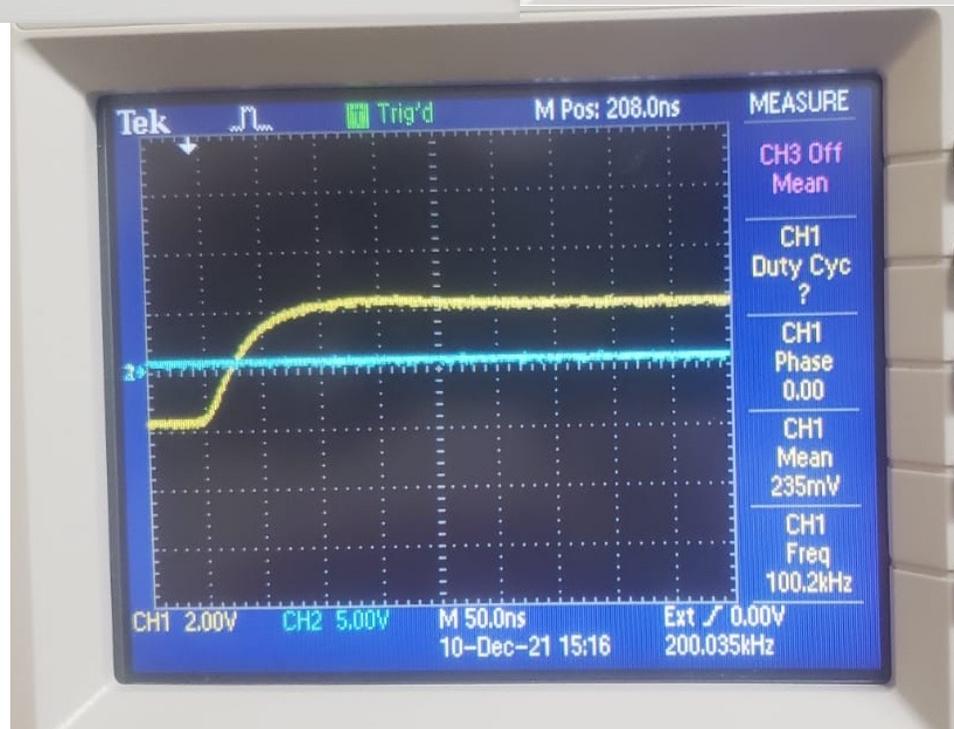
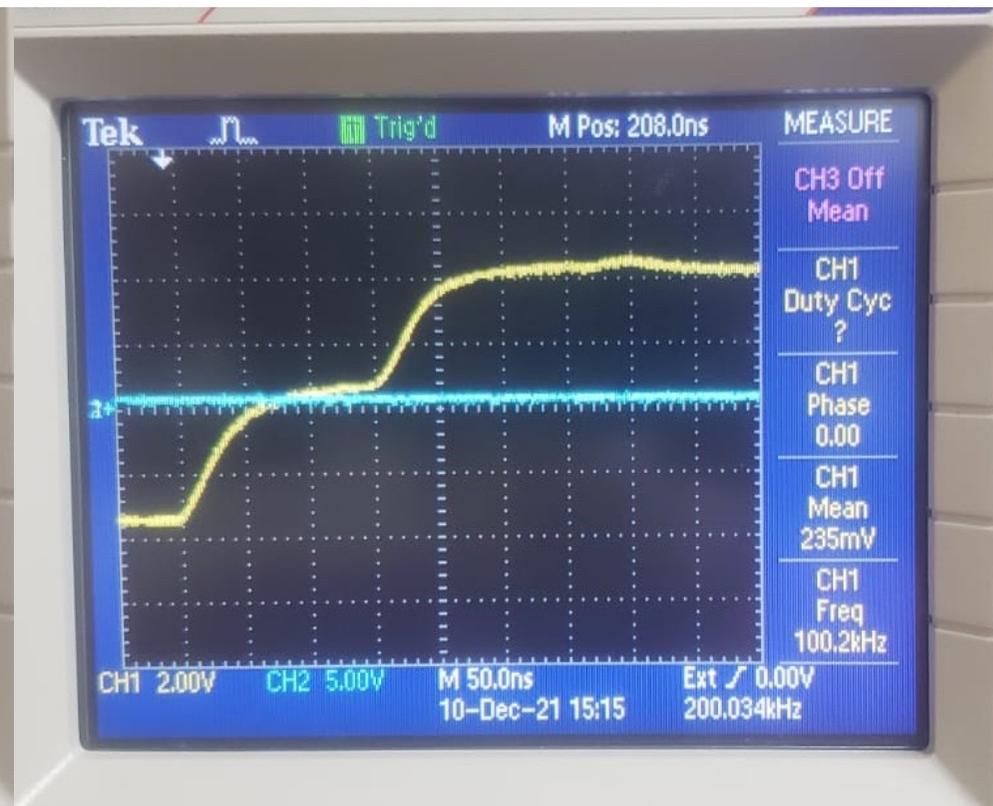
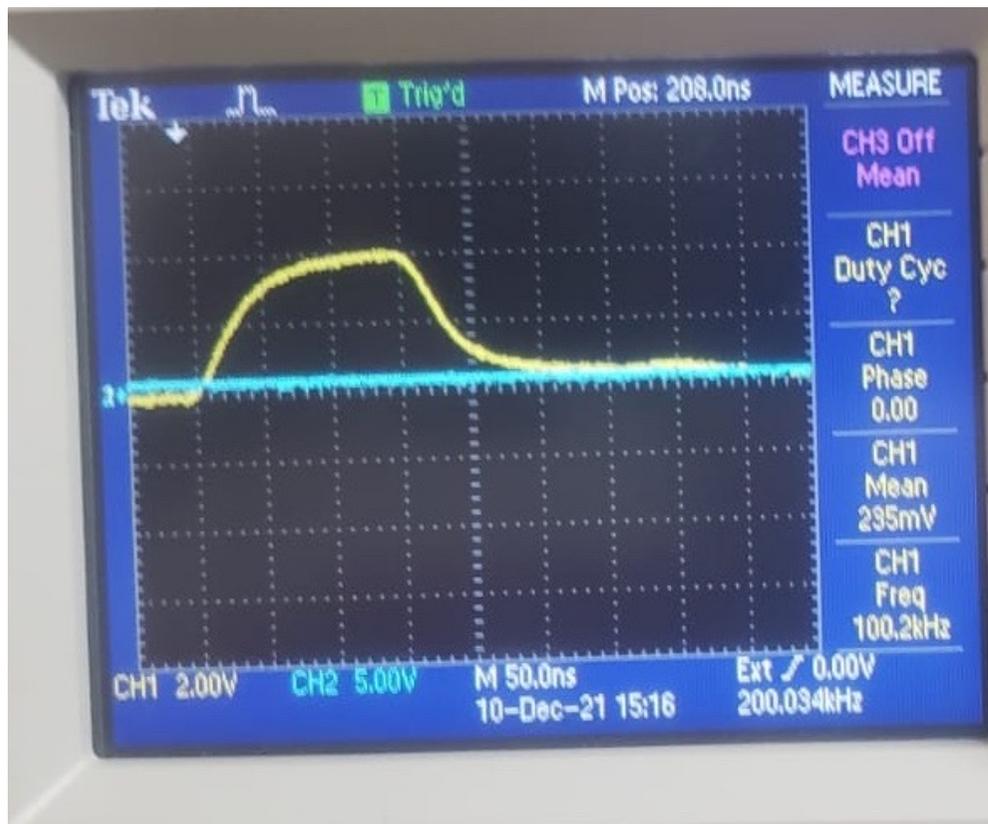
$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}} \cdot \frac{1}{l} \quad (\text{para } l) \Rightarrow v_p = \frac{1}{\sqrt{\mu\epsilon}}$$

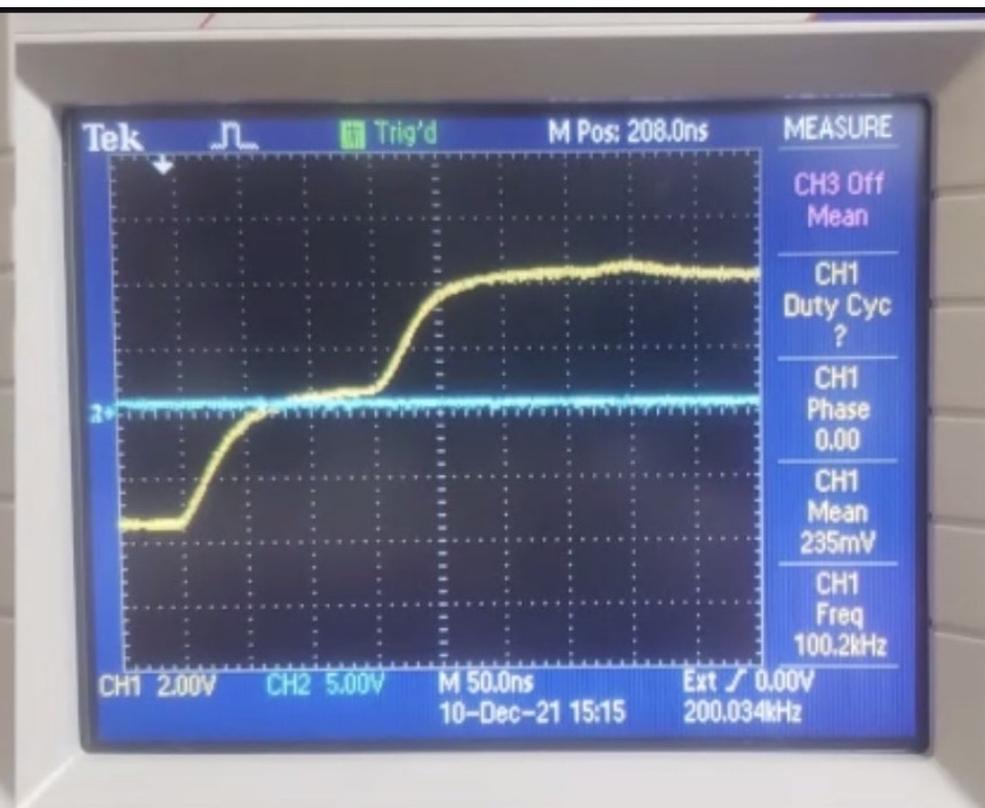
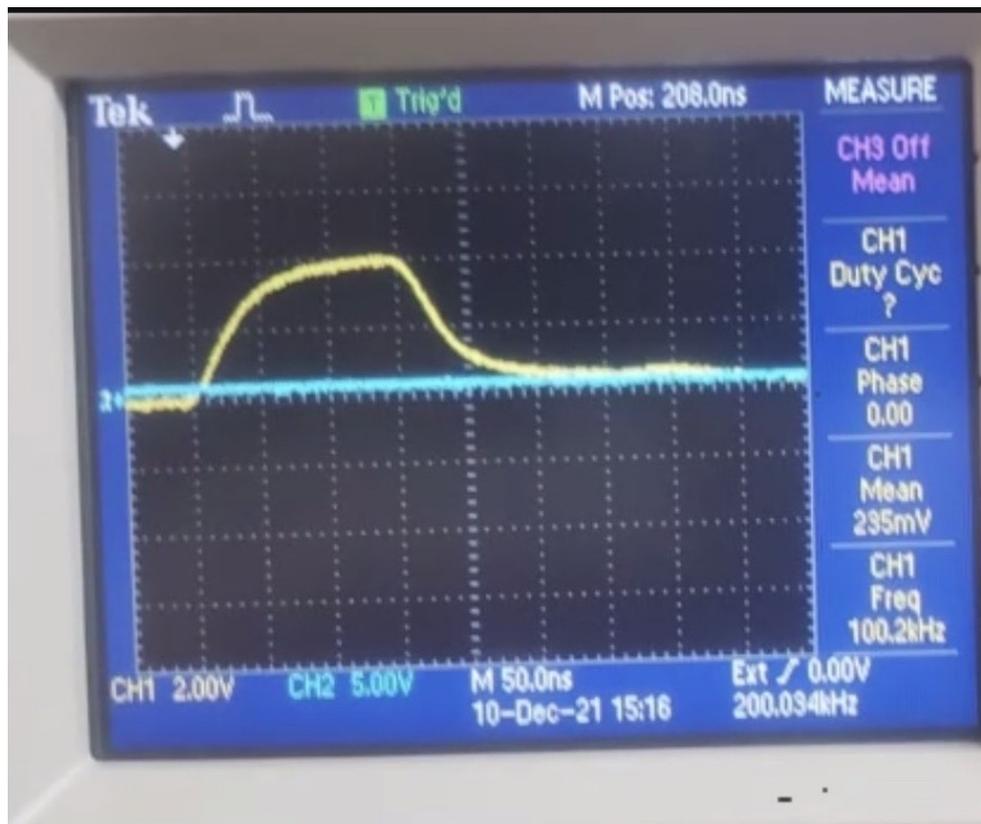
$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{a}{b}\right) \quad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega$$

$$RG-58 \Rightarrow Z = 50 \Omega \quad RG-59 \Rightarrow Z = 75 \Omega$$

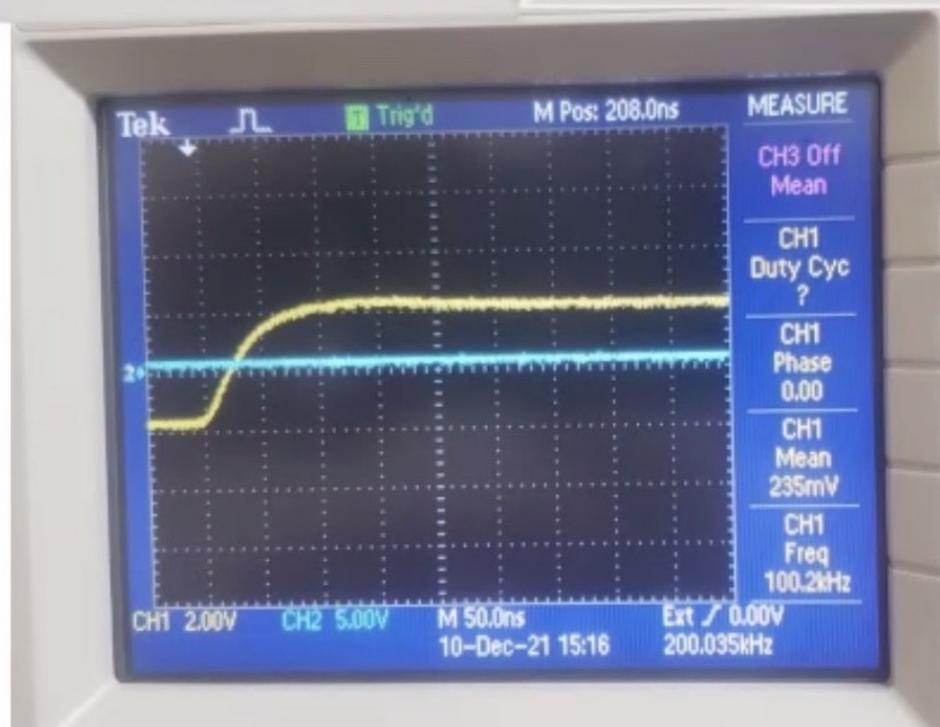


$L=15\text{ m}$





$L = 15 \text{ m}$   
 $\Delta x = 2L = 30 \text{ m}$   
 $\Delta t = 150 \text{ ns}$   
 $= 0,15 \mu\text{s}$



$$v = \frac{\Delta x}{\Delta t} = \frac{30 \text{ m}}{0,15 \mu\text{s}}$$

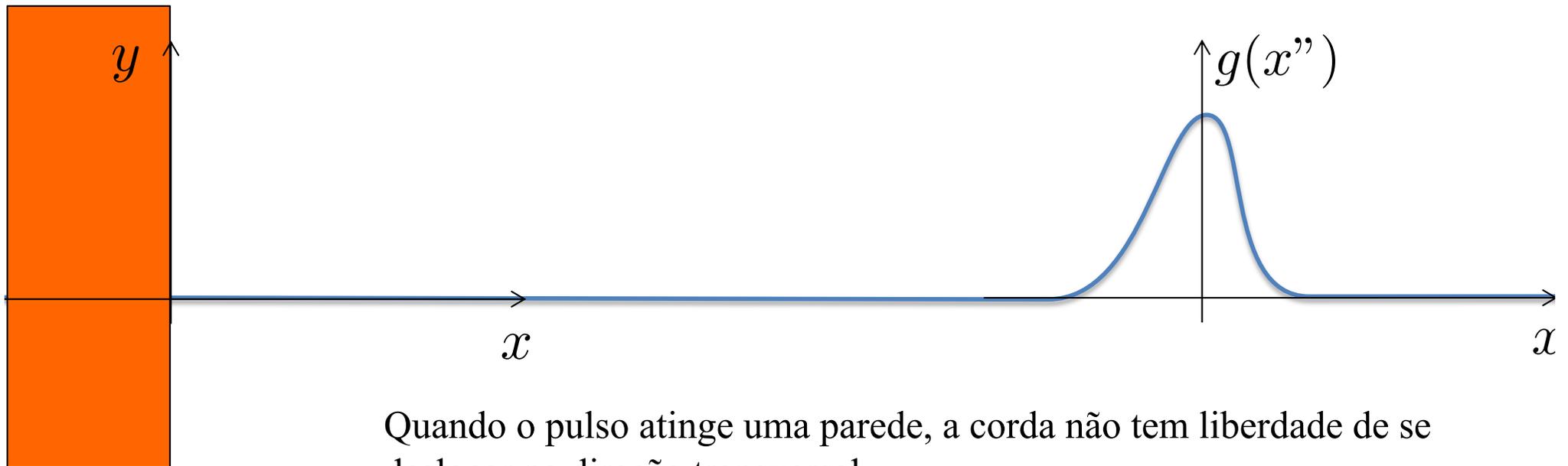
$$= 200 \cdot 10^6 \text{ m/s}$$

$$= \frac{c}{3} \cdot 2$$

$$\epsilon \rightarrow \text{meiri dielektro}$$

$$\frac{\epsilon}{\epsilon_0} \sim \left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2,25$$

# Reflexão de ondas



Quando o pulso atinge uma parede, a corda não tem liberdade de se deslocar na direção transversal.

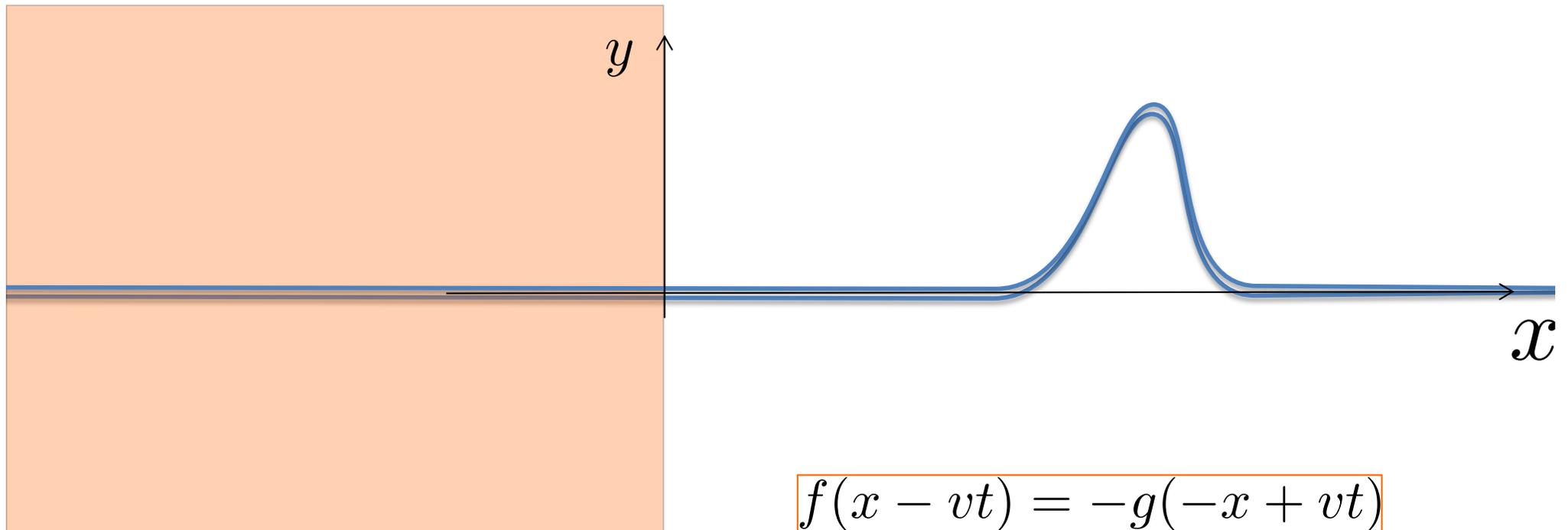
A solução da equação de onda deve levar isto em conta:

$$y(x, t) = f(x - vt) + g(x + vt) \quad y(0, t) = 0$$

$$f(-vt) = -g(vt) \quad f(-x') = -g(x')$$

$$f(x - vt) = -g(-x + vt)$$

## Reflexão de ondas

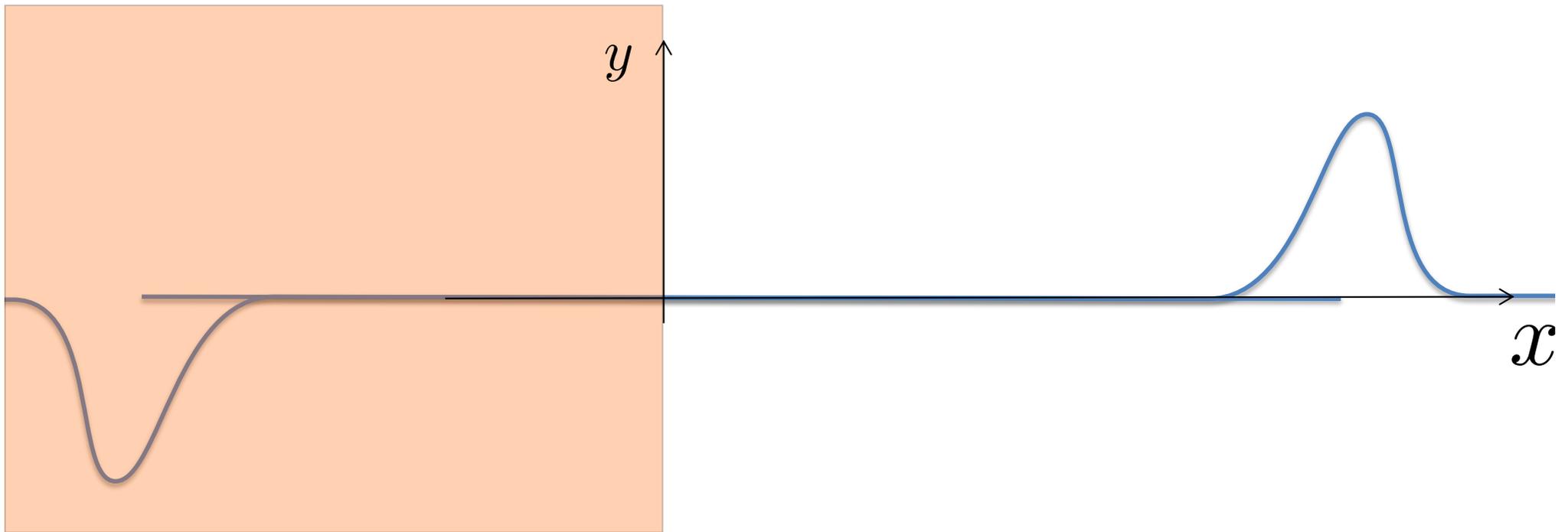


$$y(x, t) = g(vt + x) - g(vt - x)$$

A função complementar (propagante) é o espelho da função incidente após duas reflexões: uma no eixo  $x$ , outra no eixo  $y$ . Ou ainda, ao giro desta em  $180^\circ$  em torno da origem.

A solução é uma função ímpar em  $x$ .

## Reflexão de ondas



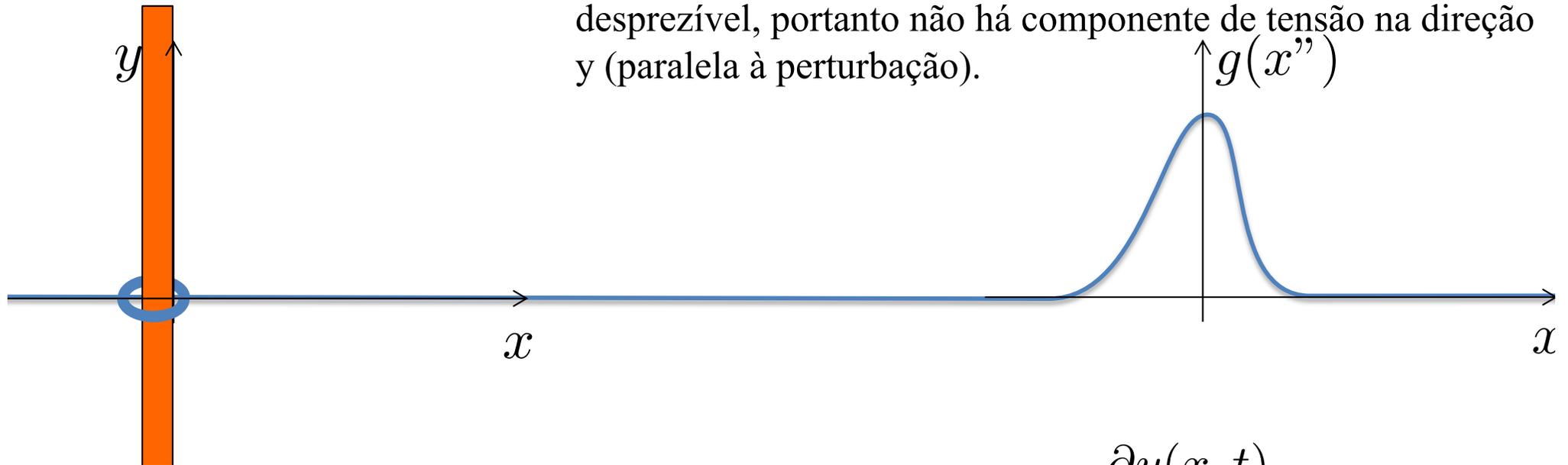
$$y(x, t) = g(vt + x) - g(vt - x)$$

Na reflexão, a onda retorna com a mesma amplitude, mas com uma inversão de sinal.

Inversão de fase

# Reflexão de ondas

Quando o pulso atinge uma extremidade livre, a tensão é apenas normal: não há atrito no anel, que tem massa desprezível, portanto não há componente de tensão na direção  $y$  (paralela à perturbação).



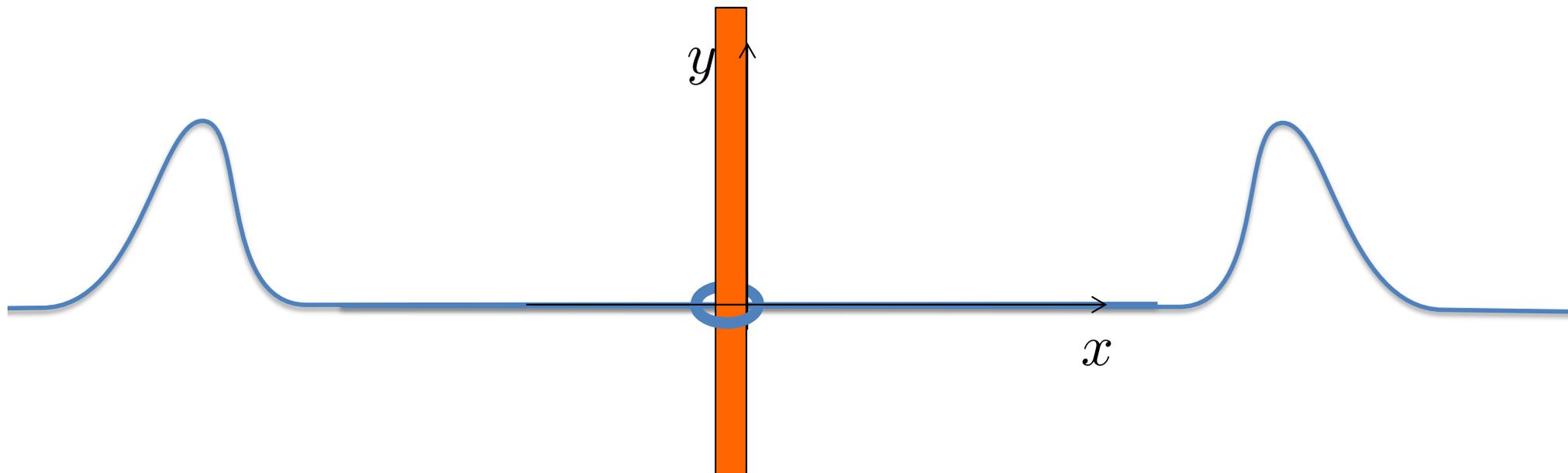
$$y(x, t) = f(x - vt) + g(x + vt) \quad F_y(0, t) = -T \frac{\partial y(x, t)}{\partial x} \Big|_{x=0} = 0$$

$$f'(x) = \frac{df(x)}{dx} \quad \frac{\partial y(x, t)}{\partial x} \Big|_{x=0} = f'(-vt) + g'(vt) = 0$$

$$\frac{\partial x'(x, t)}{\partial x} = \frac{\partial x''(x, t)}{\partial x} = 1$$

$$f'(x) = -g'(-x)$$

# Reflexão de ondas



$$y(x, t) = f(x - vt) + g(x + vt)$$

$$f'(x) = -g'(-x)$$

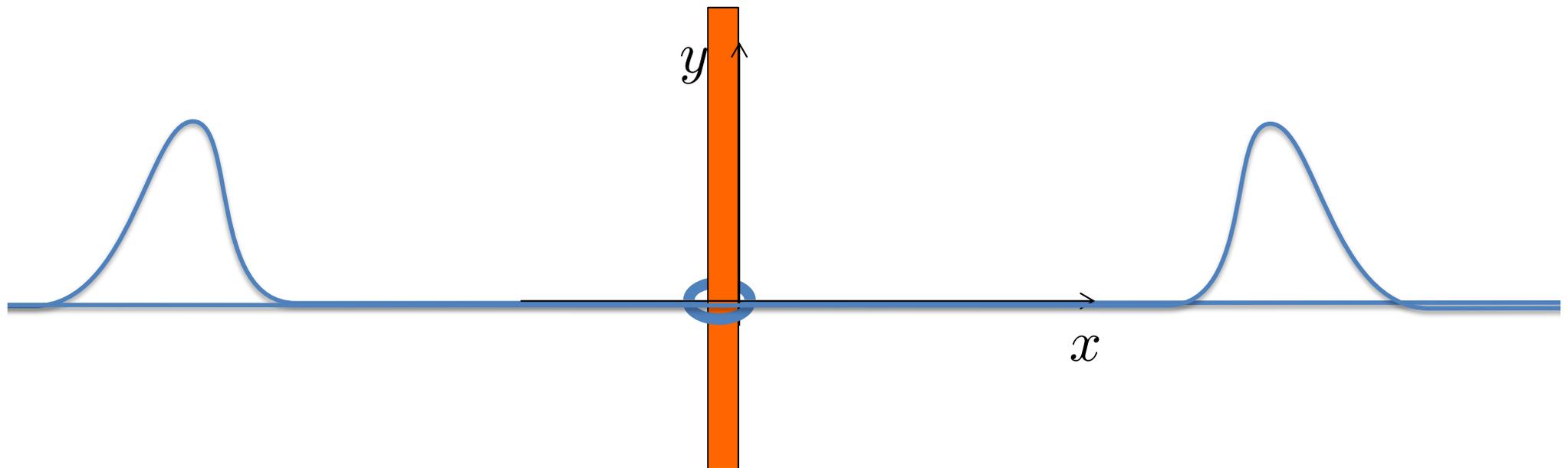
Se a derivada é uma função ímpar, a função primitiva é par em  $x$ !

$$f(x) = g(-x)$$

A solução corresponde a uma única reflexão em torno do eixo  $y$ .

$$y(x, t) = g(vt + x) + g(vt - x)$$

# Reflexão de ondas



$$y(x, t) = f(x - vt) + g(x + vt)$$

$$f'(x) = -g'(-x)$$

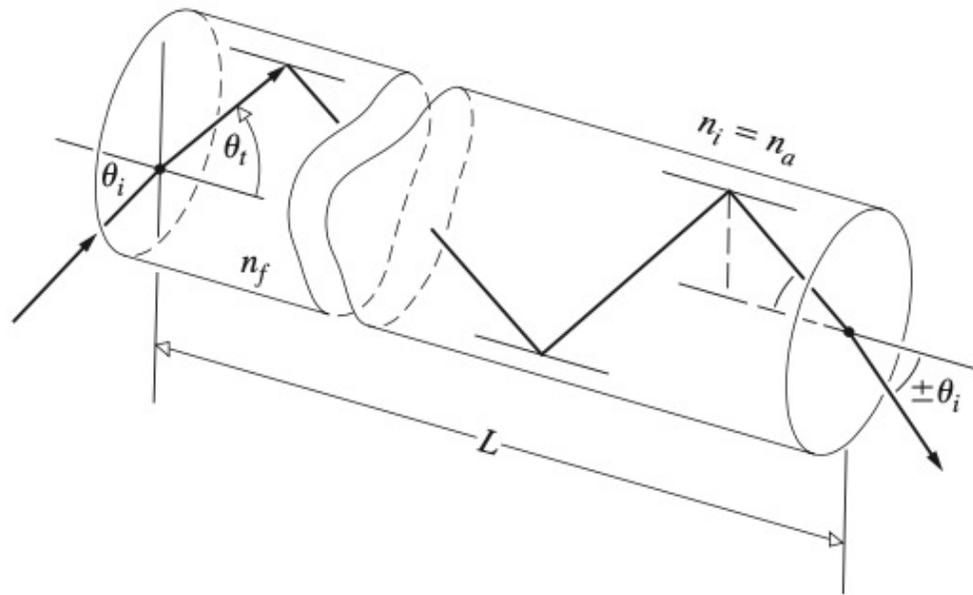
Se a derivada é uma função ímpar, a função primitiva é par em  $x$ !

$$f(x) = g(-x)$$

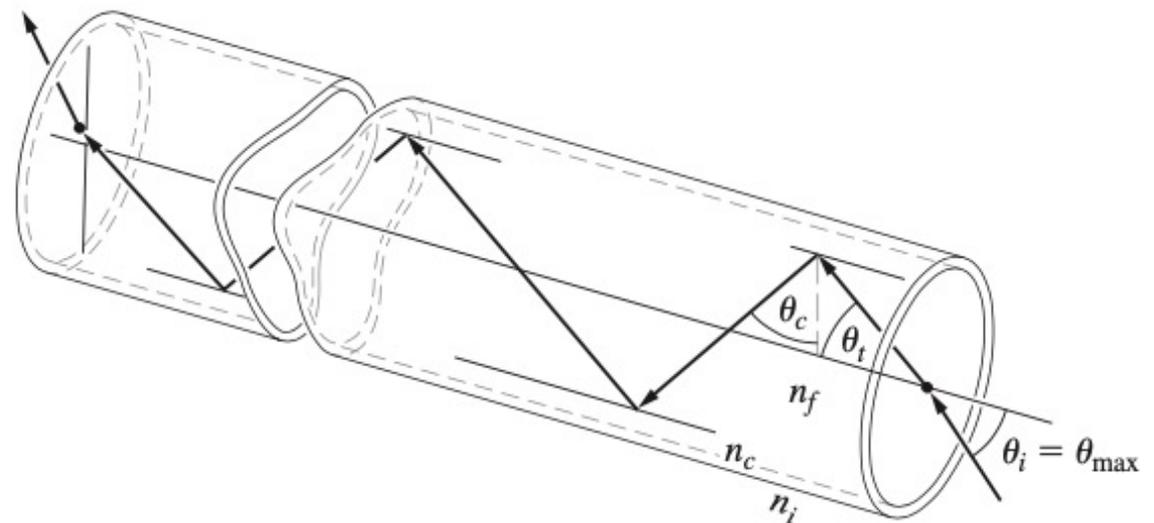
A solução corresponde a uma única reflexão em torno do eixo  $y$ .

$$y(x, t) = g(vt + x) + g(vt - x)$$

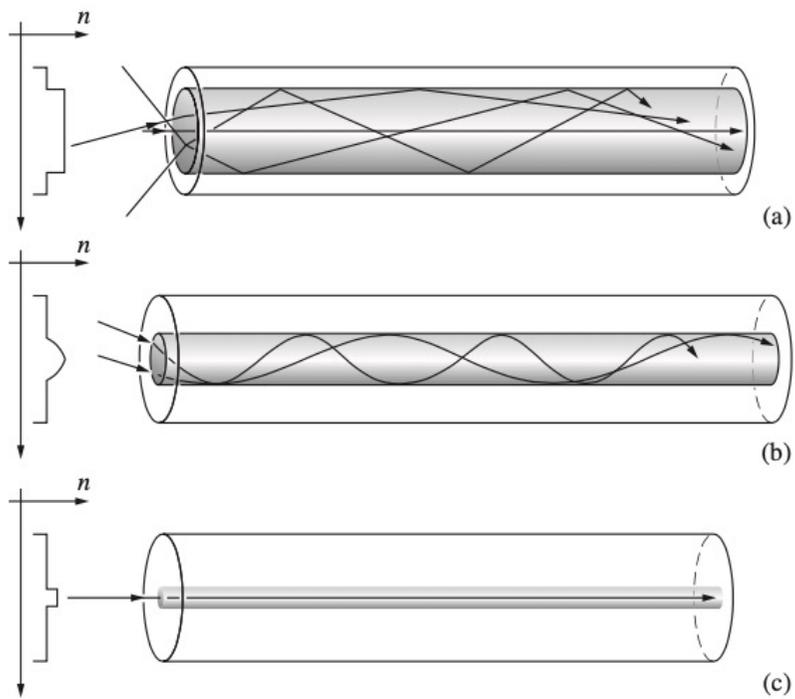
# Fibras Óticas (ver Hecht, cap. 5)



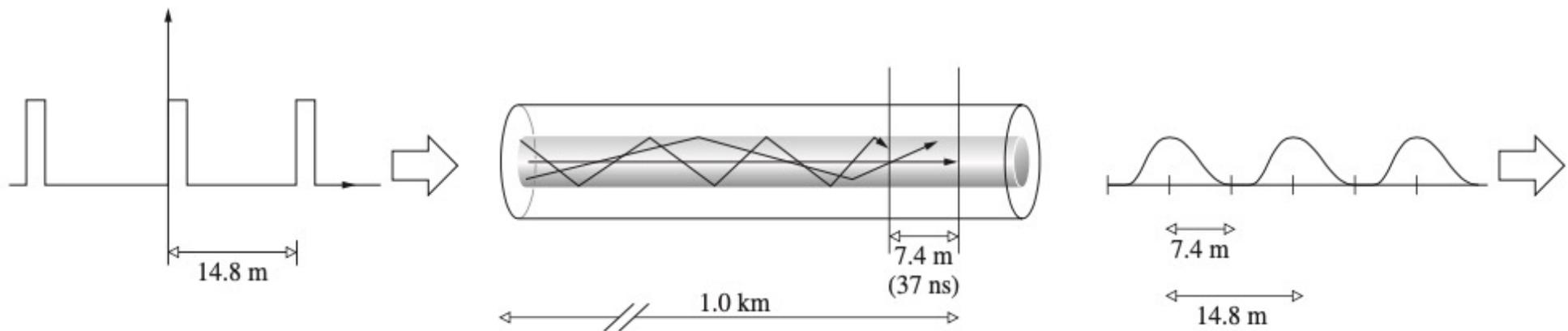
**Figure 5.80** Rays reflected within a dielectric cylinder.



**Figure 5.81** Rays in a clad optical fiber.



**Figure 5.82** The three major fiberoptic configurations and their index profiles. (a) Multimode stepped-index fiber. (b) Multimode graded-index fiber. (c) Single-mode stepped-index fiber.



**Figure 5.85** The spreading of an input signal due to intermodal dispersion.

# Além do ângulo crítico!

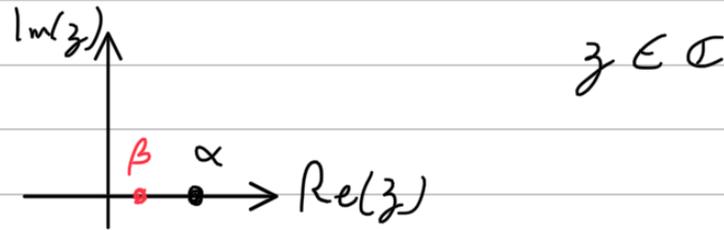
$$r = \frac{\alpha - \beta}{\alpha + \beta}$$

$$\alpha = \sqrt{\frac{1 - \frac{\sin^2 \theta_2}{\beta^2}}{1 - \sin^2 \theta_1}} \quad / \quad \beta = \frac{n_1}{n_2}$$

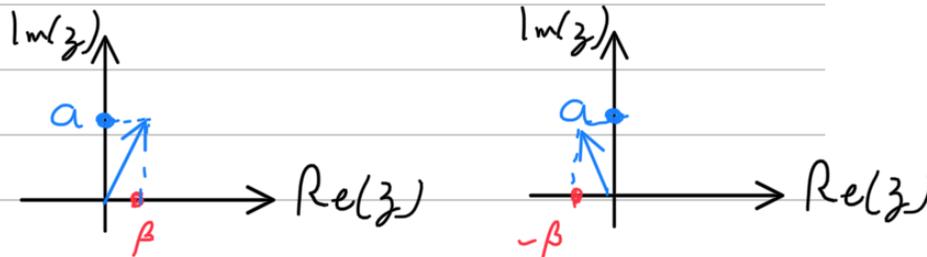
$$1 - \frac{\sin^2 \theta_1}{\beta^2} < 0 \Rightarrow \alpha \in \mathbb{C} \rightarrow \alpha = ia$$

$$a \in \mathbb{R}$$

$$\Gamma = \frac{ia - \beta}{ia + \beta}$$



$$\begin{aligned} |\Gamma|^2 &= \frac{ia - \beta}{ia + \beta} \cdot \frac{-ia - \beta}{-ia + \beta} \\ &= \frac{a^2 + \beta^2}{a^2 + \beta^2} = 1 \end{aligned}$$



mesmo módulo!

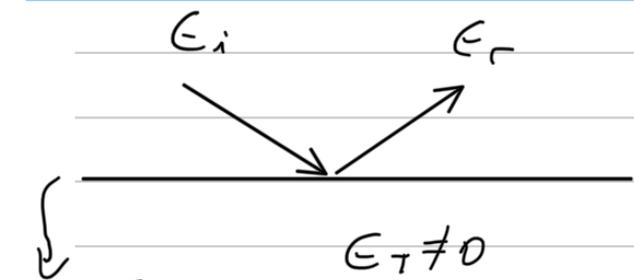
$$R = |\Gamma|^2 = 1$$

$$T = 1 - R = 0$$

$$\Gamma = |\Gamma| e^{i\theta} = \frac{ia - \beta}{i\omega + \beta} \cdot \frac{-i\omega + \beta}{-ja + \beta} =$$

$$= \frac{(i\omega - \beta)^2}{\beta^2 + \omega^2} = \frac{-\beta^2 + \omega^2 + 2i\omega\beta}{\beta^2 + \omega^2}$$

$$\tan \theta = \left( \frac{2\omega\beta}{\omega^2 - \beta^2} \right)$$



descontínuo:  $\epsilon_1 E_{1n} = \epsilon_2 n \cdot E_2$

$$E_{1||} = E_{2||}$$

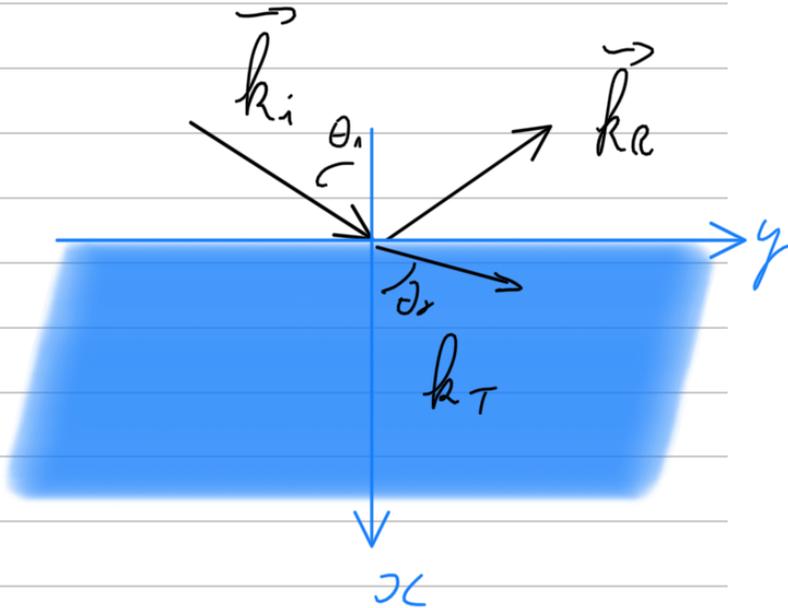
Onda evanescente  $\nabla$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 > 1$$

$$1 - \cos^2 \theta_2 = \frac{n_1^2 \sin^2 \theta_1}{n_2^2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_c}$$

$$\cos \theta_2 = \sqrt{1 - \frac{\sin^2 \theta_1}{\sin^2 \theta_c}} = i \sqrt{\frac{\sin^2 \theta_1}{\sin^2 \theta_c} - 1} = i H$$



$$k_{2c} = k_T \cdot \cos \theta_2 \Rightarrow \vec{E}_T = \vec{E}_T e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

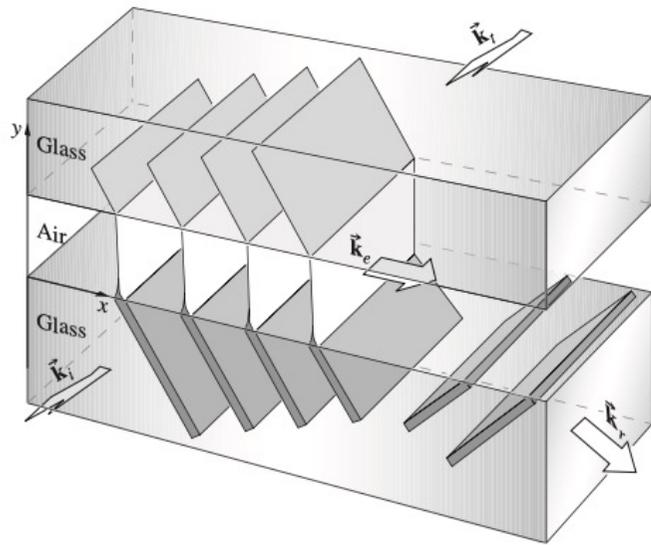
$$= \vec{E}_T e^{i k_{2c} x} e^{i(k_y y - \omega t)}$$

$$= \vec{E}_T e^{-k_{2c} x} e^{i(k_y y - \omega t)}$$

$$k_y = k_T \sin \theta_2 = k_T \frac{\sin \theta_1}{\sin \theta_2} = M k_T$$

velocidade de fase:  $v = \frac{\omega}{M k_T} = \frac{1}{v_2} \frac{n_2}{n_1 \sin \theta_1} = \frac{c}{n_1 \sin \theta_1}$

Como  $\vec{k} = k_y \hat{y} + i k_{2c} \hat{x}$ , não há fluxo de energia.  $\nabla$



**Figure 4.65** Frustrated total internal reflection.