Eletromagnetismo

Reflexão de Ondas eletromagnéticas

- Mais algumas leis de ótica explicadas pelo eletromagnetismo...
- Reflexão e refração
- Lei de Snell Descartes
- Lei de Brewster
- O que acontece em uma interface?

Condições de Contorno

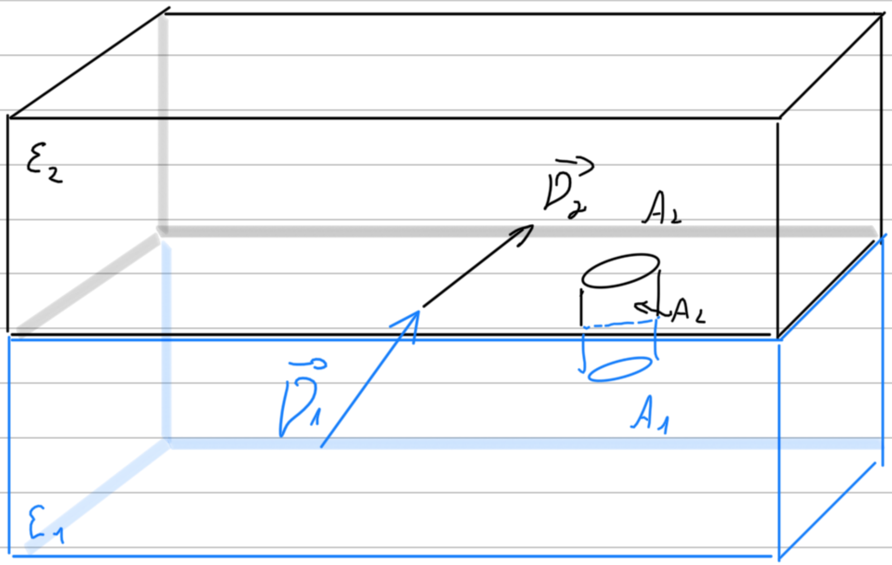
Campo do Vetor Deslocamento $\vec{D} \Rightarrow$ cargas livres

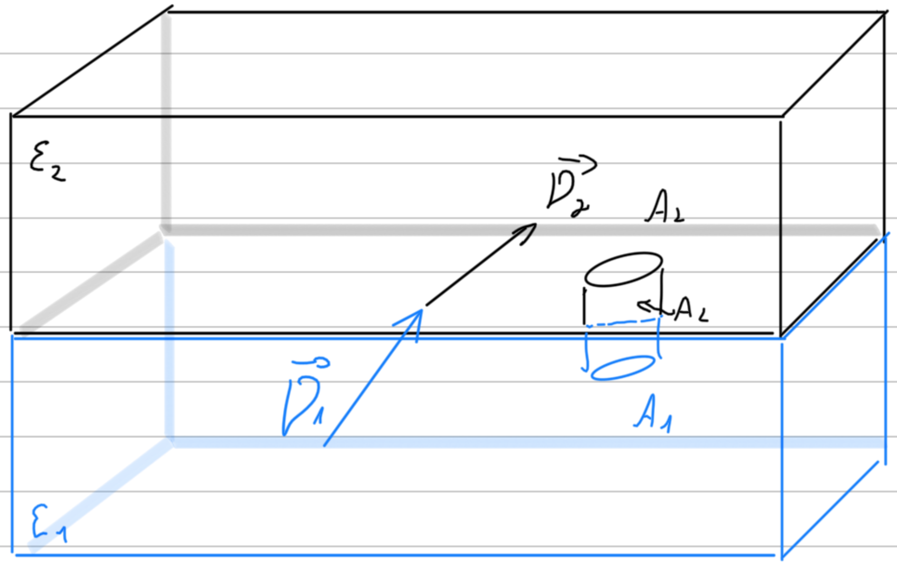
$$\nabla \cdot \vec{D} = \rho \Rightarrow \oint_S \vec{D} \cdot \hat{n} \, da = 0$$

Campo Elétrico $\vec{E} = -\nabla V \Rightarrow \nabla \times \vec{E} = 0$

$$\Rightarrow V(\vec{r}_B) - V(\vec{r}_A) = -\int_{r_A}^{r_B} \vec{E} \cdot d\vec{\ell}$$

O que ocorre entre dois dielétricos?





$$\oint \vec{D}\cdot\vec{n}\, da = \int_{A_1} \vec{D}_1 \cdot \hat{n}_1\, da_1 + \int_{A_2} \vec{D}_2 \cdot \hat{n}_2\, da_2 + \int_{A_L} \vec{D}_L \cdot \vec{n}_L \cdot da_L$$

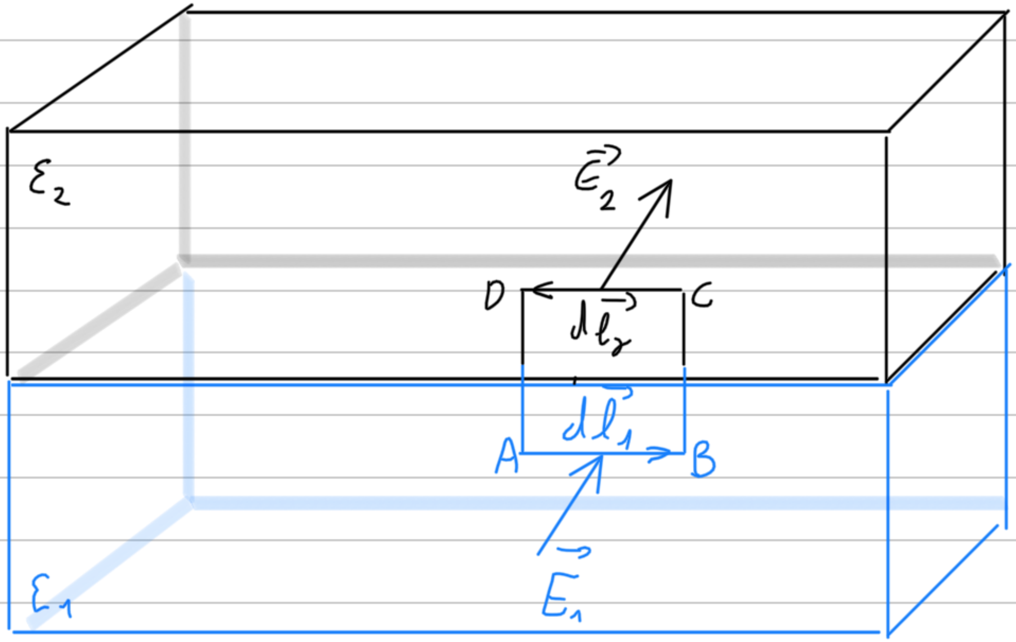
$$\rightarrow 0$$

$$\hat{n}_1 = -\hat{n}_2 \;;\; da_1 = da_2$$

$$\oint \vec{D}\cdot\hat{n}\, da = \int_{A_2} (\vec{D}_2 - \vec{D}_1)\, \hat{n}_2\, da_2$$

Carga nula (livre) $\Rightarrow$ $\underline{(\vec{D}_2 - \vec{D}_1)\cdot\hat{n}_2 = 0}$

$$\Rightarrow (\varepsilon_2 \vec{E}_2 - \varepsilon_1 \vec{E}_1)\, \hat{n}_2 = 0$$



$$V_{BA} = V_B - V_A = -\int_A^B \vec{E}_1 \cdot d\vec{l}_1 \quad ; \quad V_{CB} = -\int_B^C \vec{E} \cdot d\vec{l}$$

$$V_{DC} = V_D - V_C = -\int_C^D \vec{E}_2 \, d\vec{l}_2 \quad ; \quad V_{AD} = -\int_D^A \vec{E} \, d\vec{l}$$

$$V_{BA} = V_B - V_A = -\int_A^B \vec{E}_1 \cdot d\vec{l}_1 \quad ; \quad V_{CB} = -\int_B^C \vec{E} \cdot d\vec{L}$$

$$V_{DC} = V_D - V_C = -\int_C^D \vec{E}_2 \, d\vec{l}_2 \quad ; \quad V_{AD} = -\int_D^A \vec{E} \, d\vec{L}$$

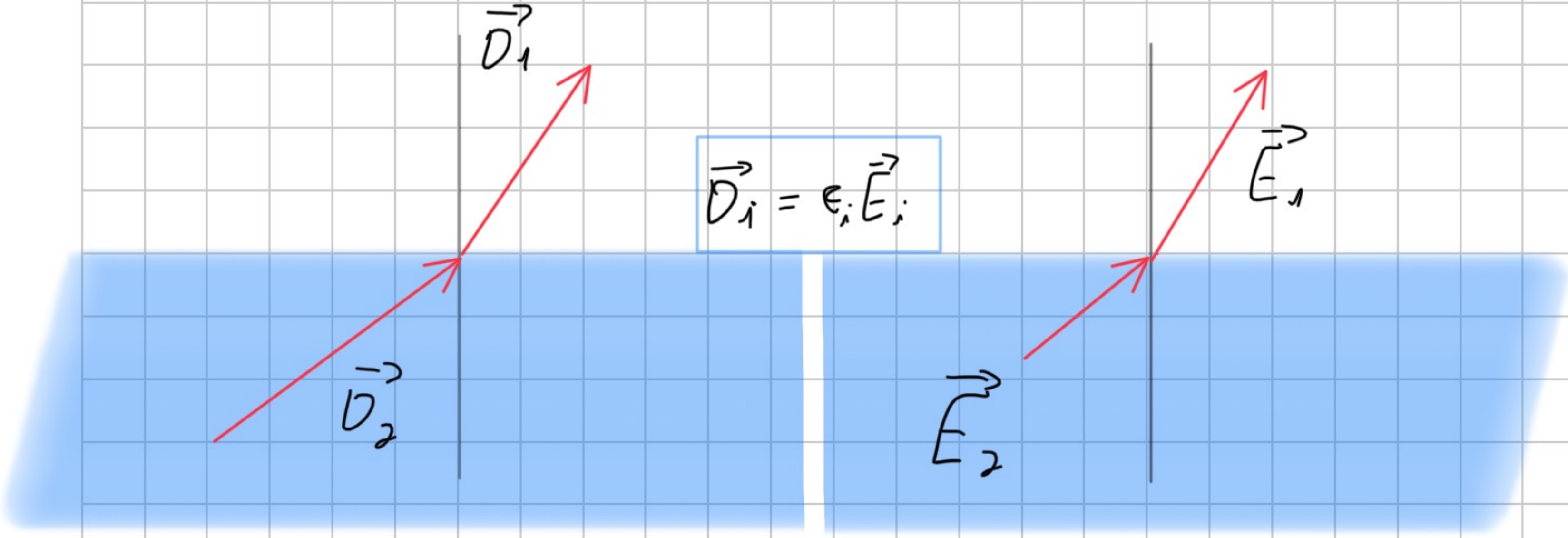
$$L \to 0 \Rightarrow V_{CB} = V_{AD} = 0$$

$V_{BA} + V_{CB} + V_{DC} + V_{AD} = 0 \to$ volta completa

$$\Rightarrow V_{BA} - V_{DC} = 0 \qquad d\vec{l}_1 = -d\vec{l}_2$$

$$\Rightarrow \int_A^B (\vec{E}_1 - \vec{E}_2) \, d\vec{l}_2 = 0$$

$$\begin{cases} \text{Componente transversal: } & \vec{E}_1\, d\vec{l}_2 = \vec{E}_2\, d\vec{l}_2 \\ \text{Componente perpendicular: } & \vec{D}_1 \cdot \hat{n} = \vec{D}_2 \cdot \hat{n} \end{cases}$$



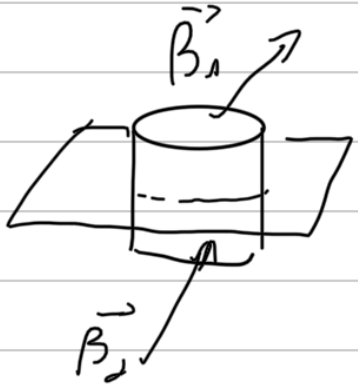
Outra consequência: $\oint \vec{D} \cdot d\vec{l}$ não é nulo na interface

No metal: Se $\vec{E}_1 \cdot d\vec{l}_1 = \vec{E}_2 \cdot d\vec{l}_2$

temos que no meio condutor: $\vec{E} = 0$ $\therefore$

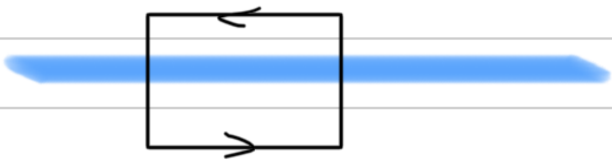
meio 1 $\to$ condutor $\to$ $\vec{E}_1 = 0 \Rightarrow \vec{E}_2 \cdot d\vec{l}_2 = 0 \Rightarrow \vec{E}_2 = E\, \hat{n}_2$

Interface : $\nabla \cdot \vec{B} = 0$ $\qquad$ $\nabla \times \vec{H} = \vec{J}$



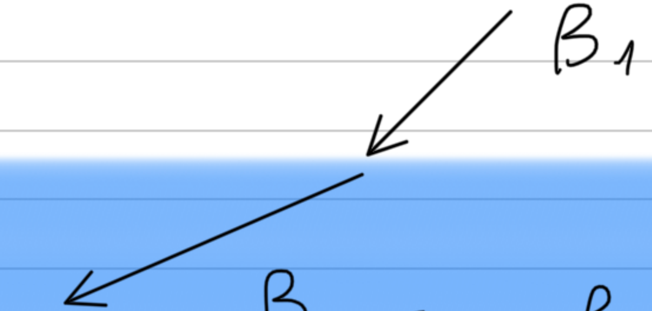
$$\vec{B}_1 \cdot \vec{n}_1 = \vec{B}_2 \cdot \vec{n}_2$$

$$B_{1n} = B_{2n}$$



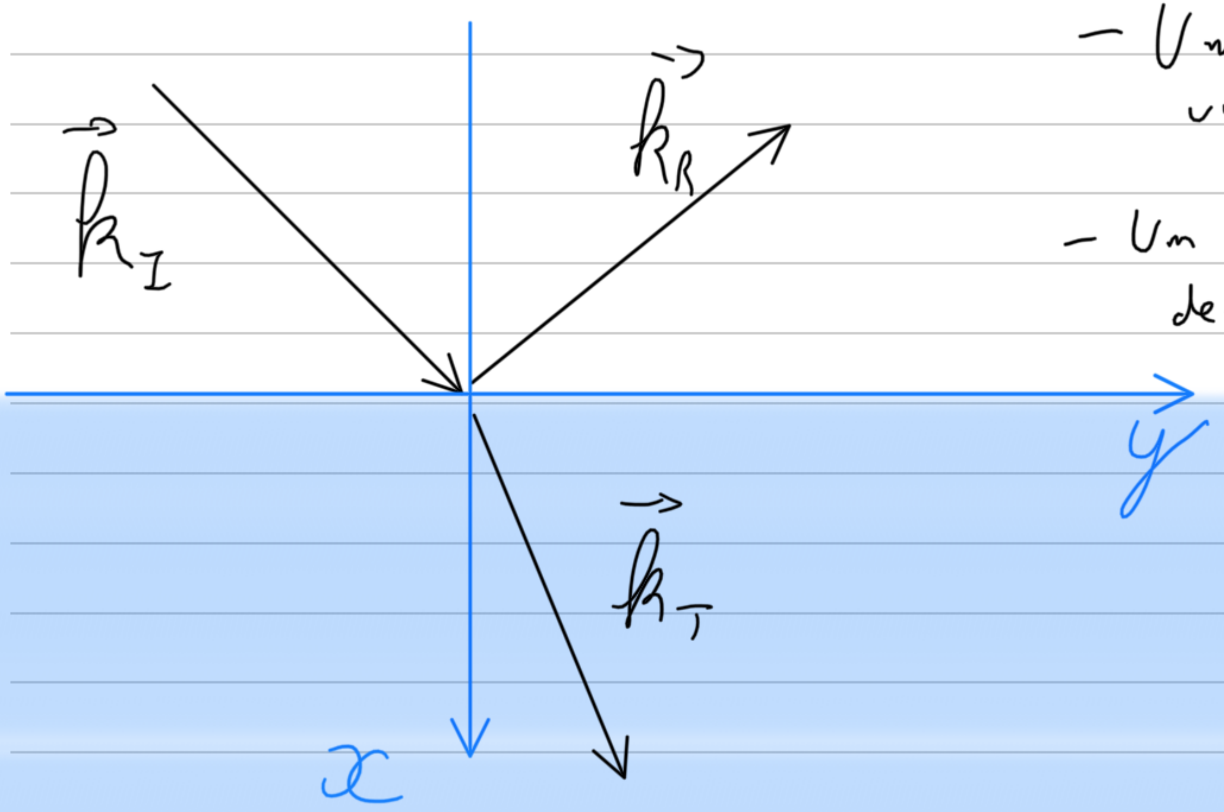
$$H_{1\parallel} = H_{2\parallel}$$

$$\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$$



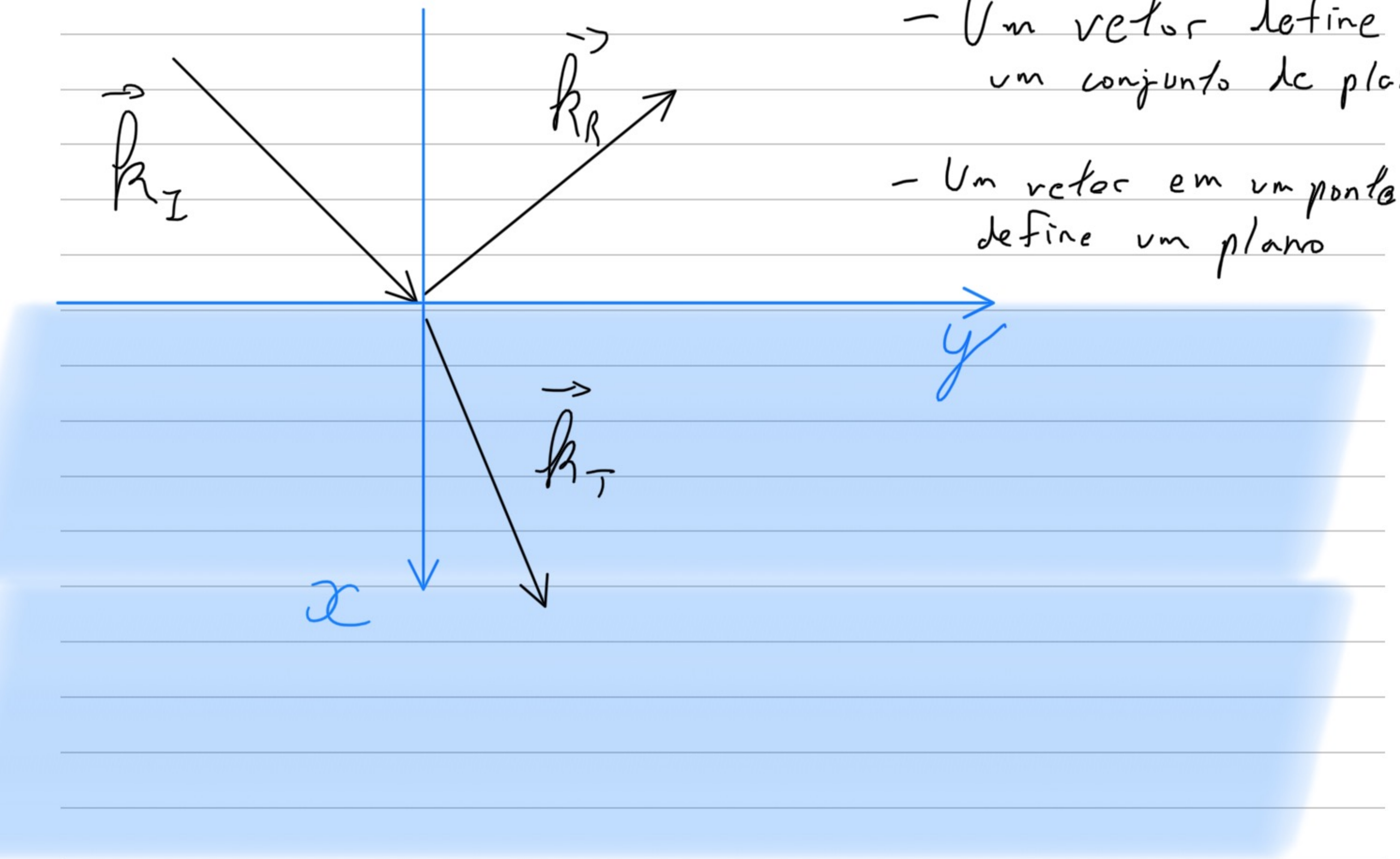
$$B_{2\parallel} = \frac{\mu_2}{\mu_1} B_{1\parallel} \; ; \; B_{2n} = B_{1n}$$

Interface: $\boxed{\begin{array}{l} E_{1\parallel} = E_{2\parallel} \ ; \ \epsilon_1 E_{1n} = \epsilon_2 E_{2n} \\ \dfrac{B_{1\parallel}}{\mu_1} = \dfrac{B_{2\parallel}}{\mu_2} \ ; \ B_{1n} = B_{2n} \end{array}}$



- Um vetor define um conjunto de planos

- Um vetor em um ponto define um plano



- Um vetor define um conjunto de planos

- Um vetor em um ponto define um plano

$$\vec{E}_I(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{I0}\, e^{i(\vec{k}_I\cdot\vec{r}-\omega_I t)}\right] ; \quad \vec{B}_I = \frac{\hat{k}_I}{v_I} \times \vec{E}_I$$

$$\vec{E}_R(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{R0}\, e^{i(\vec{k}_R\cdot\vec{r}-\omega_R t)}\right] ; \quad \vec{B}_R = \frac{\hat{k}_R}{v_R} \times \vec{E}_R$$

$$\vec{E}_T(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{T0}\, e^{i(\vec{k}_T\cdot\vec{r}-\omega_T t)}\right] ; \quad \vec{B}_T = \frac{\hat{k}_T}{v_T} \times \vec{E}_T$$

1) Na fronteira, a resposta temporal deve ser a <u>mesma</u> de um lado e de outro: $\omega_1 = \omega_2 = \omega_3 = \omega$

2) Os módulos de $\vec{k}$ se relacionam:

$$\omega = k_I \cdot v_I = k_R v_R = k_T v_T$$

3) A onda incidente e a refletida estão no mesmo meio

$$v_I = v_R = v_1 \qquad v_T = v_2$$

4) Condições dos campos na fronteira: $\vec{r} = y\hat{y} + z\hat{z}$ $(x=0)$

$$\boxed{\begin{array}{l} E_{1\parallel} = E_{2\parallel} \quad ; \quad \epsilon_1 E_{1n} = \epsilon_2 E_{2n} \\[1em] \dfrac{B_{1\parallel}}{\mu_1} = \dfrac{B_{2\parallel}}{\mu_2} \quad ; \quad B_{1n} = B_{2n} \end{array}}$$

$\forall t$

$$\Rightarrow \alpha \vec{E}_1 + \beta \vec{E}_3 = \gamma \vec{E}_2$$

$$\vec{E}_I(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{I0}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega_I t)}\right]$$

$$\vec{E}_R(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{R0}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega_R t)}\right]$$

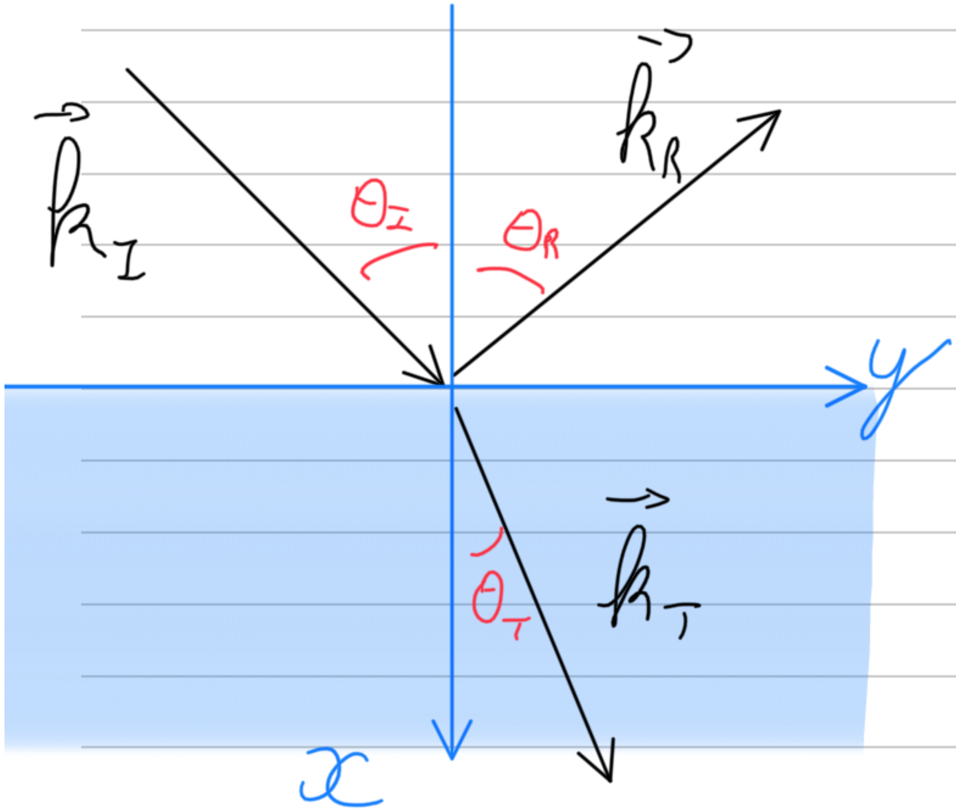
$$\vec{E}_T(\vec{r},t) = \mathrm{Re}\left[\vec{\mathcal{E}}_{T0}\, e^{i(\vec{k}_T\cdot\vec{r} - \omega_T t)}\right]$$

$$\vec{k}_I\cdot\vec{r} = \vec{k}_R\cdot\vec{r} = \vec{k}_T\cdot\vec{r}$$

@ $x = 0$!

$$\vec{k}_I \cdot \vec{r} = k_{Ix} \cdot x + k_{Iy} \cdot y + k_{Iz} \cdot z$$

$$\Rightarrow k_{Iy} = k_{Ry} = k_{Ty} \; ; \; k_{Iz} = k_{Rz} = k_{Tz}$$



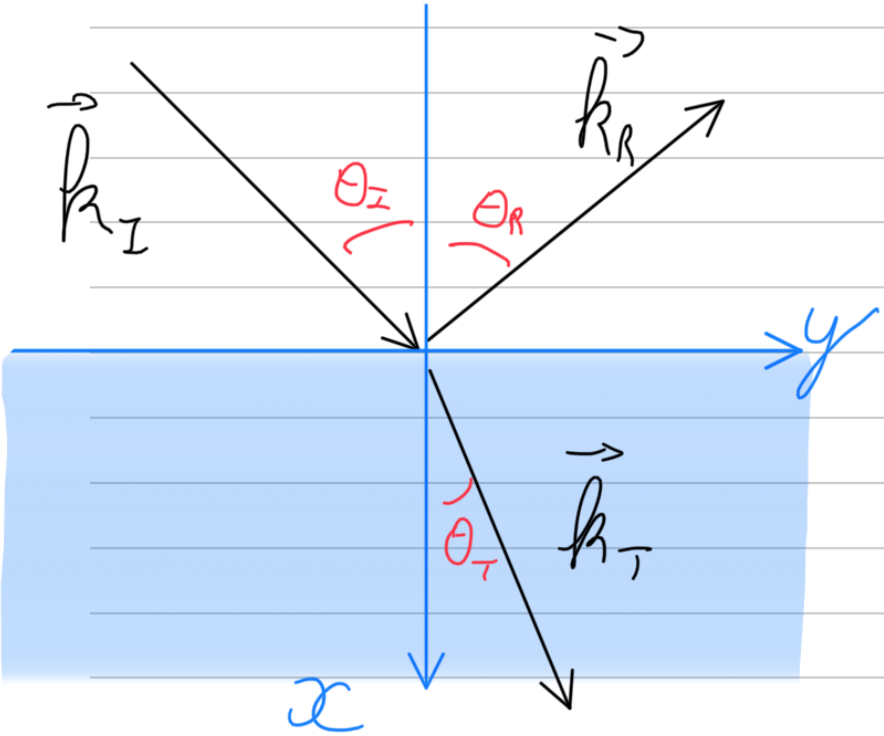
$$\Rightarrow k_I \operatorname{sen}\theta_I = k_R \operatorname{sen}\theta_R = k_T \operatorname{sen}\theta_T$$

$$k_I = \frac{\omega}{v_1} = k_T = k_1$$

$$\Rightarrow \operatorname{sen}\theta_I = \operatorname{sen}\theta_R$$

$$\Rightarrow \boxed{\theta_I = \theta_R = \theta_1}$$

Lei da Reflexão



$$\Rightarrow k_I \operatorname{sen}\theta_I = k_R \operatorname{sen}\theta_R = k_T \operatorname{sen}\theta_T$$

$$k_I = \frac{w}{v_1} = k_R = k_1$$

$$\Rightarrow \operatorname{sen}\theta_I = \operatorname{sen}\theta_R$$

$$\Rightarrow \theta_I = \theta_R = \theta_1$$

Lei da Reflexão

$$k_1 \operatorname{sen}\theta_1 = k_T \operatorname{sen}\theta_T$$

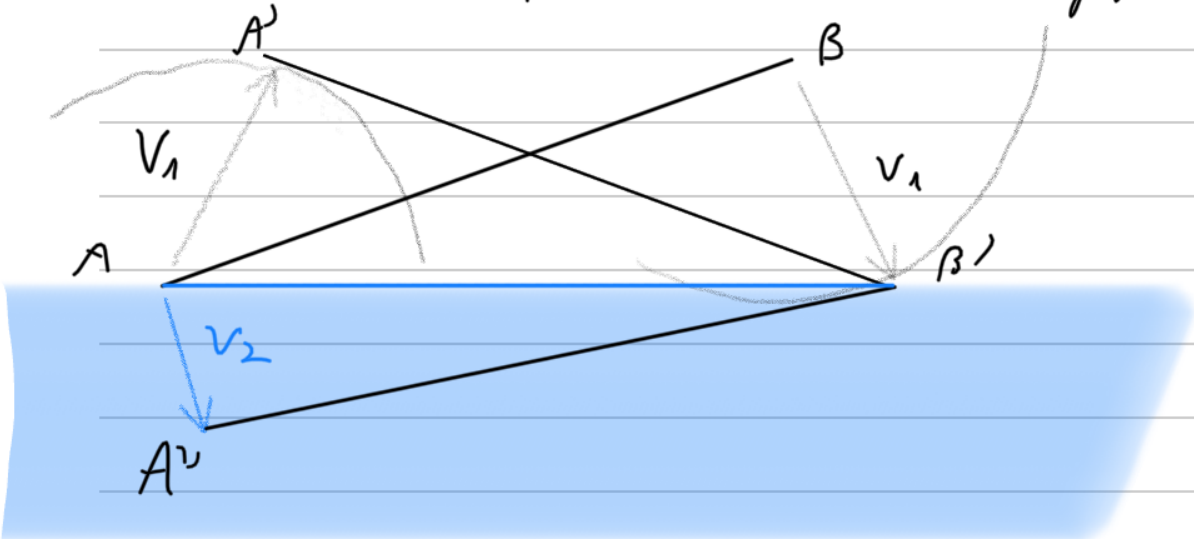
$$c \cdot \frac{w}{v_1} \operatorname{sen}\theta_1 = \frac{w}{v_2} \cdot \operatorname{sen}\theta_2 \; c. \quad \Rightarrow \quad n_1 \operatorname{sen}\theta_1 = n_2 \operatorname{sen}\theta_2$$

Eqs. Maxwell $\longrightarrow$ Lei de Snell-Descartes

Princípio de Huygens

Eqs. Maxwell $\longrightarrow$ <u>Lei de Snell-Descartes</u>

Princípio de Huygens



$$AA' = v_1 t = BB'$$

$$AA' = AB' \operatorname{sen}\theta_I$$

$$BB' = AB' \operatorname{sen}\theta_R$$

$$\underline{\operatorname{sen}\theta_I = \operatorname{sen}\theta_R}$$

$$AA'' = v_2 t \Rightarrow AA'' = AB' \operatorname{sen}\theta_T$$

$$\Rightarrow \frac{1}{c}\frac{v_2 t}{\operatorname{sen}\theta_T} = \frac{v_1 t}{\operatorname{sen}\theta_I}\frac{1}{c} \Rightarrow \underline{\operatorname{sen}\theta_I \cdot n_1 = n_2 \operatorname{sen}\theta_T}$$

Como ficam as amplitudes?

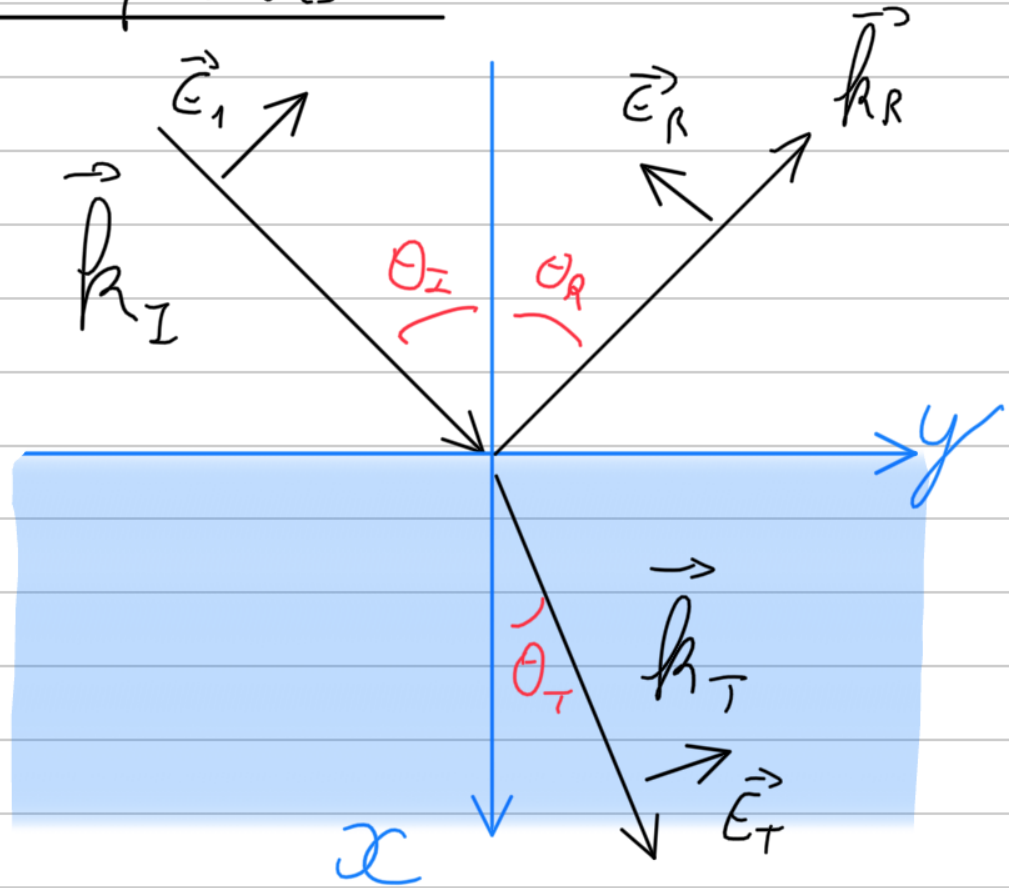
$$E_{1\parallel} = E_{2\parallel}$$

$$\Rightarrow \mathcal{E}_{Iy} + \mathcal{E}_{Ry} = \mathcal{E}_{Ty}$$

$$\mathcal{E}_{Iz} + \mathcal{E}_{Rz} = \mathcal{E}_{Tz}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

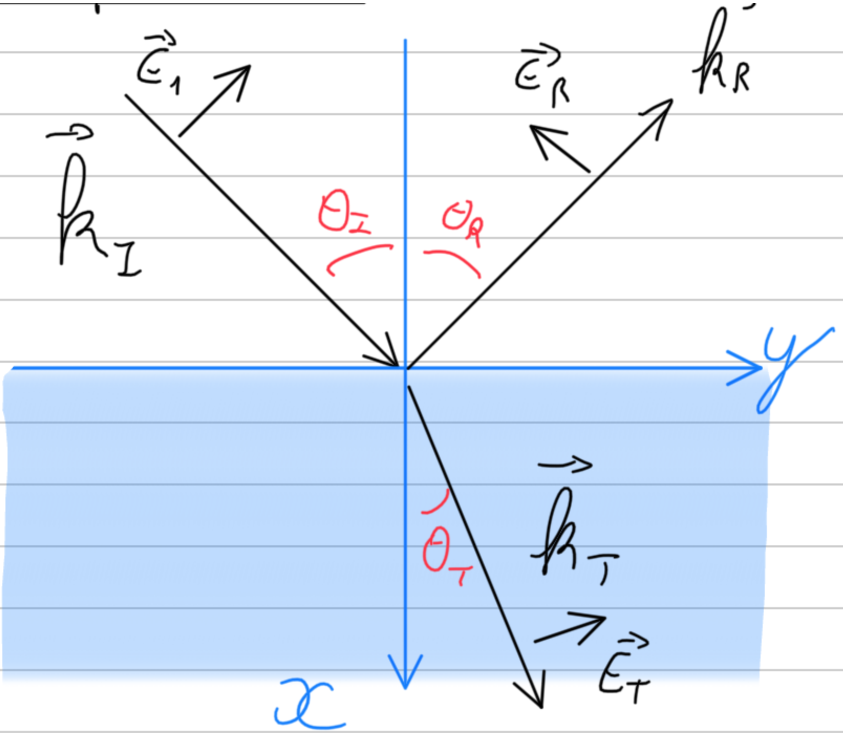
$$\epsilon_1 (\mathcal{E}_{Ix} + \mathcal{E}_{Rx}) = \mathcal{E}_{Tx} \cdot \epsilon_2$$



$$\underline{E_{1\parallel} = E_{2\parallel}}$$

$$\Rightarrow \mathcal{E}_{Iy} + \mathcal{E}_{Ry} = \mathcal{E}_{Ty}$$

$$\mathcal{E}_{Iz} + \mathcal{E}_{Rz} = \mathcal{E}_{Tz}$$



$$\underline{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$$\epsilon_1(\mathcal{E}_{Ix} + \mathcal{E}_{Rx}) = \mathcal{E}_{Tx} \cdot \epsilon_2$$

Para o campo magnético:

$$\underline{B_{1n} = B_{2n}} \Rightarrow B_{Ix} + B_{Rx} = B_{Tx}$$

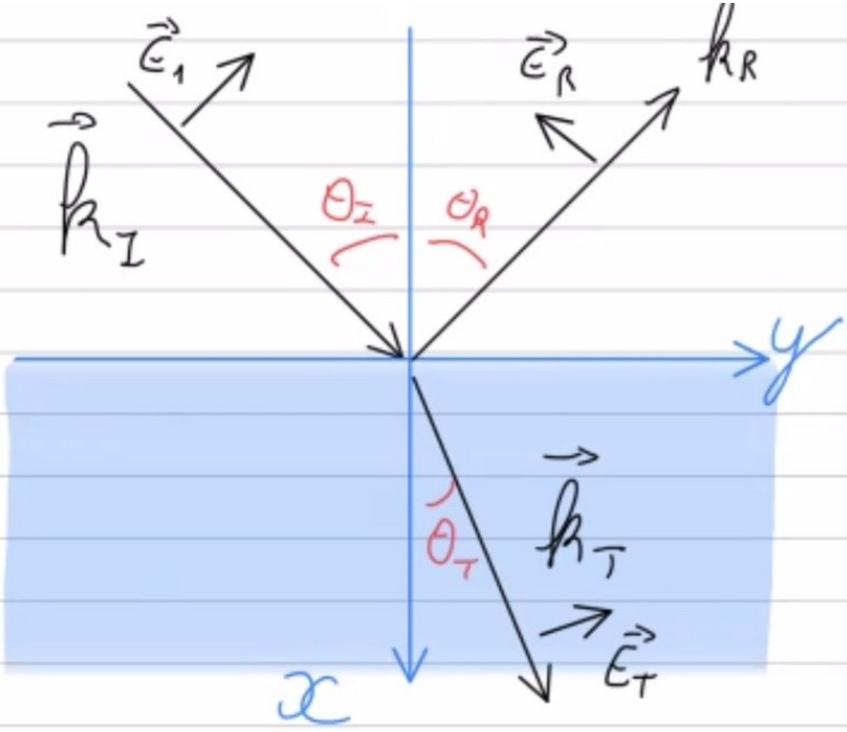
$$\underline{\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}} \Rightarrow \frac{1}{\mu_1}[B_{Iy} + B_{Ry}] = \frac{1}{\mu_2}[B_{Ty}]$$

$$\frac{1}{\mu_1}[B_{Iz} + B_{Rz}] = \frac{1}{\mu_2}[B_{Tz}]$$

$$\underline{E_{1\parallel} = E_{2\parallel}}$$

$$\Rightarrow \underline{\mathcal{E}_{Iy} + \mathcal{E}_{Ry} = \mathcal{E}_{Ty}}$$

$$\cancel{\mathcal{E}_{Iz} + \mathcal{E}_{Rz} = \mathcal{E}_{Tz}}$$



$$\underline{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

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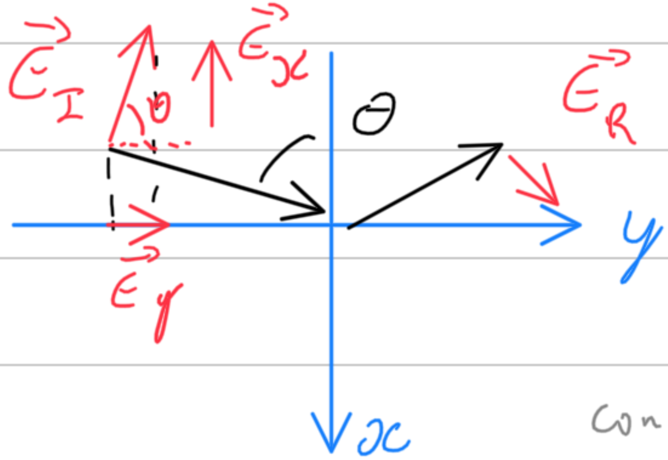
Para o campo magnético:

$$\underline{B_{1n} = B_{2n}} \Rightarrow \cancel{B_{Ix} + B_{Rx} = B_{Tx}}$$

$$\underline{\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}} \Rightarrow \cancel{\frac{1}{\mu_1}\left[B_{Iy} + B_{Ry}\right] = \frac{1}{\mu_2}\left[B_{Ty}\right]}$$

$$\underline{\frac{1}{\mu_1}\left[B_{Iz} + B_{Rz}\right] = \frac{1}{\mu_2}\left[B_{Tz}\right]}$$

Comp. $\vec{E}$ no plano xy ->



escolha conveniente

$$\vec{E} = E_x \hat{x} + E_y \hat{y}$$

$$= E \cdot \text{sen}\,\theta\, \hat{x} + E \cos\theta\, \hat{y}$$

$$x \Rightarrow \epsilon_1 \left[\mathcal{E}_I \,\text{sen}\,\theta_I - \mathcal{E}_R \,\text{sen}\,\theta_R \right] = \epsilon_2 \left[\mathcal{E}_T \cdot \text{sen}\,\theta_T \right]$$

$$\mathcal{E}_I - \mathcal{E}_R = \left(\frac{\epsilon_2}{\epsilon_1} \cdot \frac{\text{sen}\,\theta_T}{\text{sen}\,\theta_I} \right) \mathcal{E}_T$$

$$y \Rightarrow \left[\mathcal{E}_I \cos\theta_I + \mathcal{E}_R \cos\theta_R \right] = \mathcal{E}_T \cos\theta_T \Rightarrow (\mathcal{E}_I + \mathcal{E}_R) = \frac{\cos\theta_T}{\cos\theta_I} \mathcal{E}_T$$

$$z \Rightarrow \text{Como } B_z = \frac{k}{\omega} \mathcal{E} = \frac{1}{v} \mathcal{E}$$

$$\Rightarrow \frac{1}{\mu_1 v_1} (\mathcal{E}_I - \mathcal{E}_R) = \frac{1}{\mu_2 v_2} (\mathcal{E}_T)$$

$$(\mathcal{E}_I - \mathcal{E}_R) = \frac{\mu_1 v_1}{\mu_2 v_2} \mathcal{E}_T = \frac{\mu_1}{\mu_2} \frac{n_2}{n_1} \mathcal{E}_T$$

$$\mathcal{E}_I + \mathcal{E}_R = \alpha \mathcal{E}_T \qquad / \qquad \mathcal{E}_I - \mathcal{E}_R = \beta \mathcal{E}_T$$

$$\beta = \frac{\epsilon_2}{\epsilon_1} \cdot \frac{\operatorname{sen}\theta_T}{\operatorname{sen}\theta_I} = \frac{\epsilon_2}{\epsilon_1} \cdot \frac{n_1}{n_2} = \frac{\mu_1 v_1^2}{\mu_2 v_2^2} \cdot \frac{c}{v_1} \cdot \frac{v_2}{c} = \frac{\mu_1 \cdot v_1}{\mu_2 v_2} = \frac{\mu_1}{\mu_2} \frac{n_2}{n_1}$$

$$\epsilon_i = \frac{1}{\mu_i v_i^2}$$

Redundância

$$2 \mathcal{E}_I = (\alpha + \beta) \mathcal{E}_T \Rightarrow \boxed{t = \frac{\mathcal{E}_T}{\mathcal{E}_I} = \frac{2}{\alpha + \beta}}$$

$$\mathcal{E}_R = \alpha \mathcal{E}_T - \mathcal{E}_I = \left(\frac{2\alpha}{\alpha+\beta} - 1\right) \mathcal{E}_I \Rightarrow \boxed{r = \frac{\mathcal{E}_R}{\mathcal{E}_I} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)}$$

Equações de Fresnel

Note que tipicamente $\mu_1 = \mu_2$, $\beta = \frac{n_2}{n_1}$

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1-\operatorname{sen}^2\theta_T}}{\cos\theta_I} = \sqrt{\frac{1-\left(\frac{n_1}{n_2}\operatorname{sen}\theta_I\right)^2}{1-\operatorname{sen}^2\theta_I}}$$

$$n_1 \operatorname{sen}\theta_I = n_2 \operatorname{sen}\theta_T$$

$$\Rightarrow \boxed{\alpha = \sqrt{\frac{1-\frac{\operatorname{sen}^2\theta_I}{\beta^2}}{1-\operatorname{sen}^2\theta_I}} \qquad \beta = \frac{n_T}{n_I}}$$

$$t_{\parallel} = \frac{2}{\frac{\cos\theta_T}{\cos\theta_I} + \frac{n_T}{n_I}} = \frac{2\, n_I \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I}$$

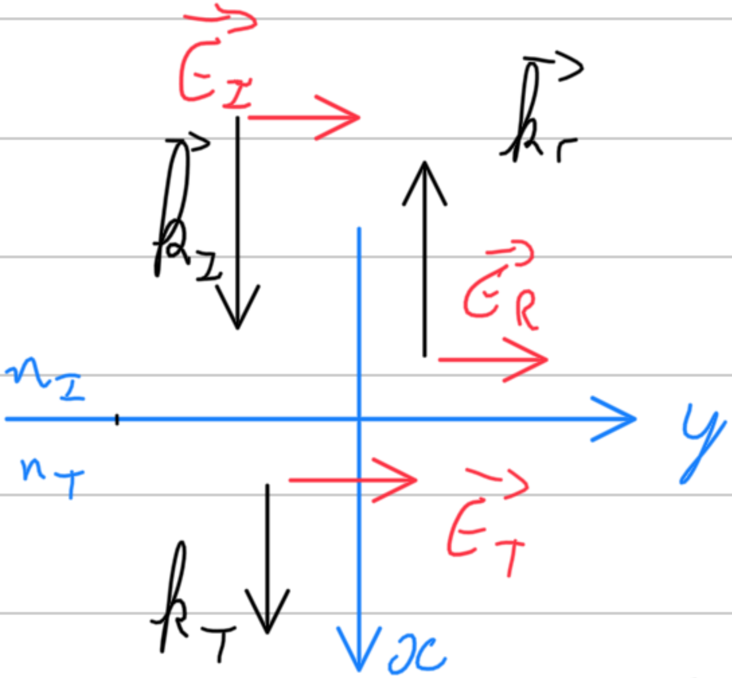
$$r_{\parallel} = \frac{n_I \cos\theta_T - n_T \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I}$$

Incidência normal: $\theta_I = 0 \Rightarrow \alpha = 1$

$$\Gamma = \frac{1-\beta}{1+\beta} = \frac{n_I - n_T}{n_I + n_T} \quad , \quad t = \frac{2n_I}{n_I + n_T}$$

$$n_I > n_T \rightarrow \Gamma > 0 \quad ; \quad n_I < n_T \rightarrow \Gamma < 0$$

Inversão!

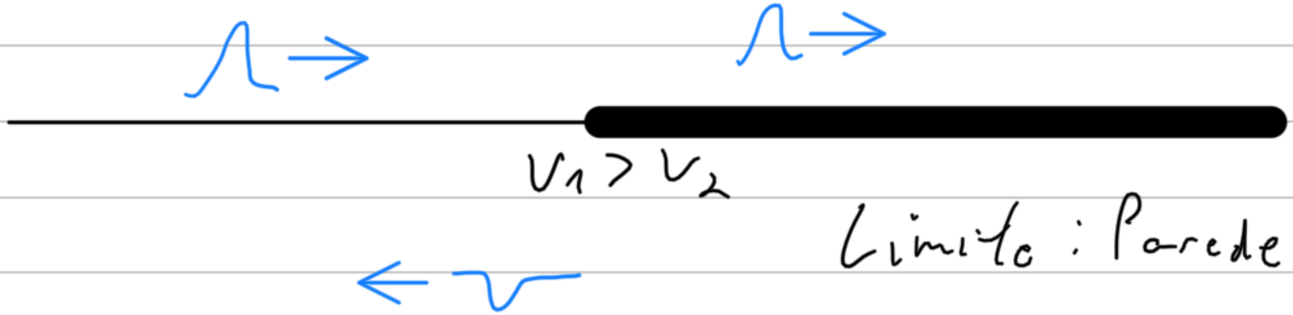


$$n_I = \frac{c}{v_I} \qquad n_T = \frac{c}{v_T}$$

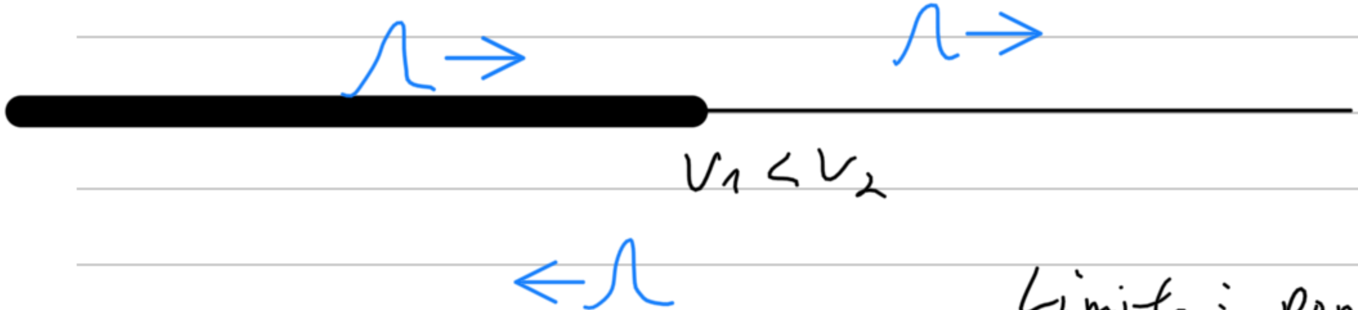
$$\Gamma = \frac{v_T - v_I}{v_T + v_I}$$

→ Conveniência da escolha: $\theta = 0 \Rightarrow \vec{E}_I \propto \vec{E}_R$!

Onda em corda: $v = \sqrt{T/\mu}$ (tensão / dens. linear)



Limite: Parede



Limite: ponta livre

Reflexão nula: $\alpha = \beta \Rightarrow r = 0!$

$$\boxed{r = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)}$$

$$\boxed{\alpha = \sqrt{\frac{1-\frac{\operatorname{sen}^2\theta_I}{\beta^2}}{1-\operatorname{sen}^2\theta_I}} \quad / \quad \beta = \frac{n_T}{n_I}}$$

$$\Rightarrow \quad 1-\frac{\operatorname{sen}^2\theta_I}{\beta^2} = \beta^2\left(1-\operatorname{sen}^2\theta_I\right)$$

$$\beta^2 - \operatorname{sen}^2\theta_I = \beta^4 - \beta^4\operatorname{sen}^2\theta_I$$

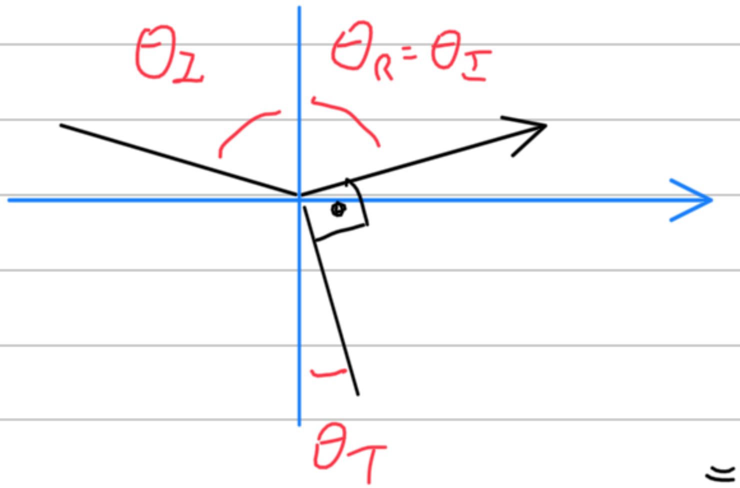
$$\operatorname{sen}^2\theta_I\left(1-\beta^4\right) = \beta^2-\beta^4 \Rightarrow \operatorname{sen}^2\theta_I = \frac{\beta^2-\beta^4}{1-\beta^4}$$

Note porém que $\operatorname{tg}^2\theta = \frac{\operatorname{sen}^2\theta}{\cos^2\theta} = \frac{\operatorname{sen}^2\theta}{1-\operatorname{sen}^2\theta}$

$$\cos^2\theta = 1-\operatorname{sen}^2\theta = \frac{1-\beta^4-\beta^2+\beta^4}{1-\beta^4} = \frac{1-\beta^2}{1-\beta^4}$$

$$\operatorname{tg}^2\theta = \beta^2\frac{(1-\beta^2)}{1-\beta^4}\cdot\frac{1-\beta^4}{1-\beta^2} = \beta^2 ! \quad \therefore \quad \boxed{\operatorname{tg}\theta_\beta = \frac{n_T}{n_I}}$$

Ângulo de Brewster (1781-1868)



$$\operatorname{sen}\theta_T = \frac{n_I}{n_T}\operatorname{sen}\theta_I$$

$$= \frac{1}{\operatorname{tg}\theta_B}\operatorname{sen}\theta_B = \cos\theta_B !$$

$\Rightarrow \operatorname{sen}\theta_T = \cos\theta_B \Rightarrow$ ângulo reto!

Por quê? Qual a origem física?

Reflexão rasante: $\theta_I \to \pi/2 \Rightarrow \text{sen}\,\theta_2 = 1$, $\cos\theta_1 \to 0$

$$\boxed{r = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)}$$

$$\boxed{\alpha = \frac{\sqrt{1-\frac{\text{sen}^2\theta_2}{\beta^2}}}{\cos\theta_2} \quad \Big/ \quad \beta = \frac{n_T}{n_I}}$$

$$\lim_{\theta\to\pi/2} r = \lim_{\theta\to\pi/2}\left(\frac{\alpha-\beta}{\alpha+\beta}\right) = \lim_{\theta\to\pi/2}\left(\frac{\sqrt{\beta^2-\text{sen}^2\theta}-\beta\cos\theta}{\sqrt{\beta^2-\text{sen}^2\theta}+\beta\cos\theta}\right)$$

$$\underline{r \simeq 1}$$

Reflexão total

$$\boxed{r = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)}$$

$$\boxed{\alpha = \sqrt{\frac{1-\frac{\operatorname{sen}^2\theta_I}{\beta^2}}{1-\operatorname{sen}^2\theta_I}} \quad / \quad \beta = \frac{n_T}{n_I}}$$

$$\alpha \in \mathbb{R} \Rightarrow 1-\frac{\operatorname{sen}^2\theta_I}{\beta^2} > 0 \rightarrow \operatorname{sen}^2\theta_I < \beta^2$$

se $n_T > n_I \rightarrow \theta_I \subset [0, \pi/2]$

se $n_T < n_I \rightarrow \operatorname{sen}\theta_I = \beta$ é o limite para haver transmissão?

Acima do ângulo crítico $\theta_c = \arcsin\left(\frac{n_T}{n_I}\right)$

a condição de propagação deve ser revista,

$|r| = 1$

Além do ângulo crítico!

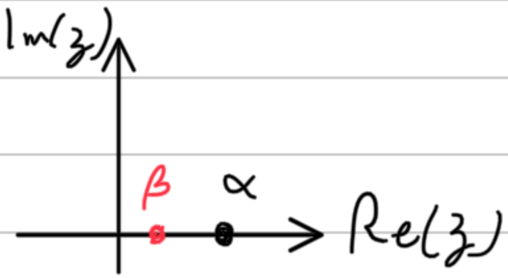
$$\boxed{r = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)}$$

$$\boxed{\alpha = \sqrt{\frac{1-\frac{\operatorname{sen}^2\theta_I}{\beta^2}}{1-\operatorname{sen}^2\theta_I}} \quad / \quad \beta = \frac{n_T}{n_I}}$$

$$1-\frac{\operatorname{sen}^2\theta_I}{\beta^2} < 0 \implies \alpha \in \mathbb{C} \rightarrow \alpha = ia$$

$$a \in \mathbb{R}$$

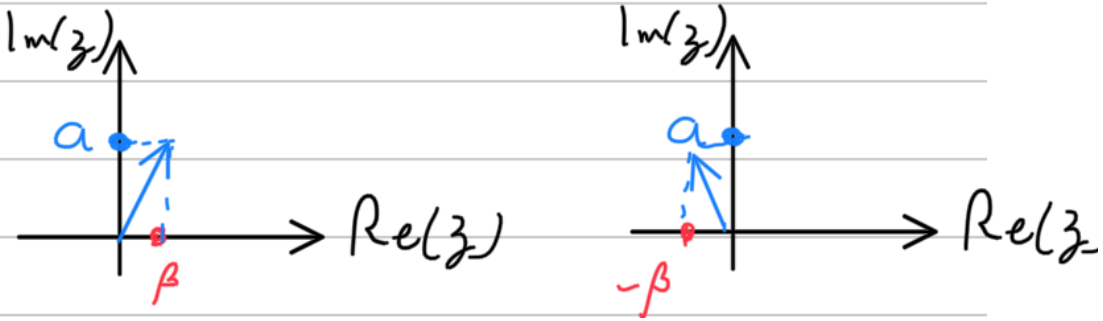
$z \in \mathbb{C}$



$$\Gamma = \frac{ia-\beta}{ia+\beta}$$

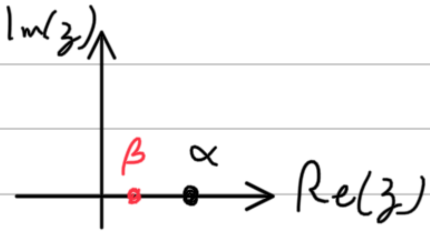
$$|\Gamma|^2 = \frac{ia-\beta}{ia+\beta} \cdot \frac{-ia-\beta}{-ia+\beta}$$

$$= \frac{a^2+\beta^2}{a^2+\beta^2} = 1$$



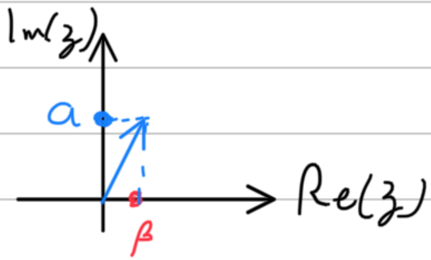
$z \in \mathbb{C}$

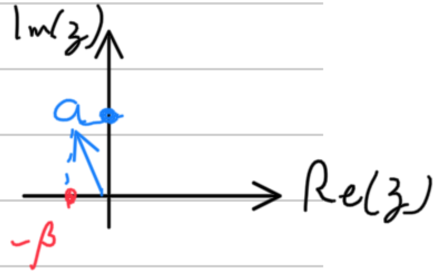
$$\Gamma = \frac{ia - \beta}{ia + \beta}$$



$$|\Gamma|^2 = \frac{ia-\beta}{ia+\beta} \cdot \frac{-ia-\beta}{-ia+\beta}$$

$$= \frac{a^2+\beta^2}{a^2+\beta^2} = \boxed{1}$$

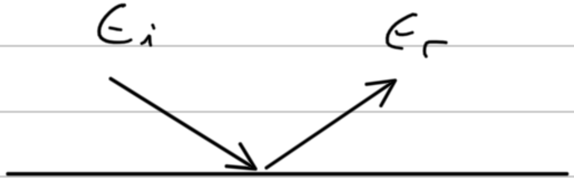




mesmo módulo!

$$R = |\Gamma|^2 = 1$$

$$T = 1 - R = 0$$



$E_T \neq 0$

não é descontínuo: $\epsilon_1 E_{1n} = E_{2n} \cdot \epsilon_2$

$$E_{1\parallel} = E_{2\parallel}$$

Onda evanescente!

$$\Gamma = |\Gamma| e^{i\theta} = \frac{ia-\beta}{ia+\beta} \cdot \frac{-ia+\beta}{-ia+\beta} =$$

$$= \frac{-(ia-\beta)^2}{\beta^2+a^2} = \frac{-\beta^2+a^2+2\beta ia}{\beta^2+a^2}$$

$$\operatorname{tg}\theta = \left(\frac{2a}{a^2-\beta^2}\right)$$

Campo elétrico transverso!

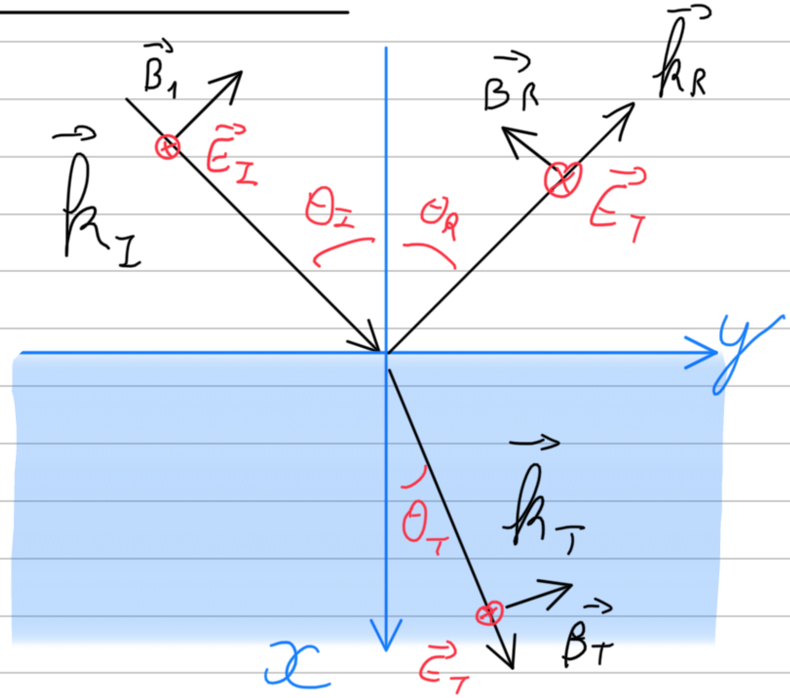
$$E_{1//} = E_{2//}$$

$$\Rightarrow \cancel{\mathcal{E}_{Iy}+\mathcal{E}_{Ry}=\mathcal{E}_{Ty}}$$

$$\mathcal{E}_{Iz}+\mathcal{E}_{Rz}=\mathcal{E}_{Tz}$$

$$\cancel{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$$\cancel{\epsilon_1(\mathcal{E}_{Ix}+\mathcal{E}_{Rx}) = \mathcal{E}_{Tx}\cdot\epsilon_2}$$



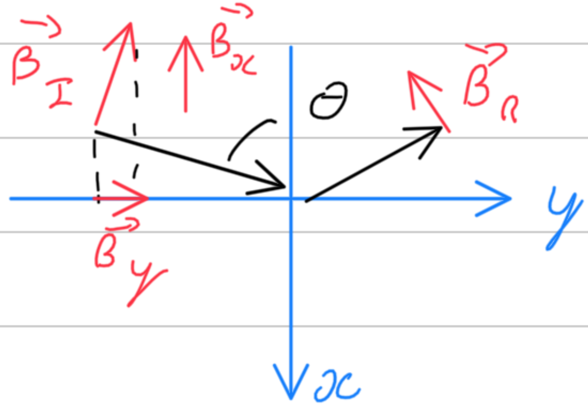
Para o campo magnético:

$$B_{1n}=B_{2n} \Rightarrow B_{Ix}+B_{Rx}=B_{Tx}$$

$$\frac{B_{1//}}{\mu_1}=\frac{B_{2//}}{\mu_2} \Rightarrow \frac{1}{\mu_1}\left[B_{Iy}+B_{Ry}\right] = \frac{1}{\mu_2}\left[B_{Ty}\right]$$

$$\cancel{\frac{1}{\mu_1}\left[B_{Iz}+B_{Rz}\right]=\frac{1}{\mu_2}\left[B_{Tz}\right]}$$

Campo $\vec{B}$ no plano xy $\rightarrow$



$$\vec{B} = B_x \hat{x} + B_y \hat{y}$$

$$= B \cdot \text{sen}\,\theta \, \hat{x} + B \cos\theta \, \hat{y}$$

$$\Rightarrow x \Rightarrow B_I \,\text{sen}\,\theta_I + B_R \,\text{sen}\,\theta_R = B_T \,\text{sen}\,\theta_T$$

$$B_I + B_R = \frac{\text{sen}\,\theta_T}{\text{sen}\,\theta_I} B_T = \frac{n_I}{n_T} B_T \qquad \text{I}$$

$$y \Rightarrow (B_I - B_R)\cos\theta_I = \frac{\mu_I}{\mu_T} B_T \cos\theta_T \qquad \text{II}$$

$$\mathcal{E}_I + \mathcal{E}_R = \mathcal{E}_T$$

$$B = \frac{1}{v}\mathcal{E} = \frac{n}{c}\mathcal{E} \Rightarrow \frac{\mathcal{E}_I + \mathcal{E}_R}{c} \cdot n_I = \mathcal{E}_T \frac{n_T}{c} \cdot \frac{n_I}{n_T} \quad \text{de I}$$

(redundante)

$$\text{II} \Rightarrow (\mathcal{E}_I - \mathcal{E}_R)\cos\theta_I \cdot \frac{n_I}{c} = \frac{\mu_I}{\mu_T}\cos\theta_T \frac{n_T}{c}\mathcal{E}_T$$

$$\therefore\ \mathcal{E}_I + \mathcal{E}_R = \mathcal{E}_T$$

$$\mathcal{E}_I - \mathcal{E}_R = K\,\mathcal{E}_T \qquad K = \frac{\mu_I}{\mu_T} \cdot \frac{n_T \cdot \cos\theta_T}{n_I \cos\theta_I}$$

$$\mathcal{E}_T = \frac{2}{1+K}\,\mathcal{E}_I \quad \Rightarrow \quad \boxed{t_\perp = \frac{2 n_I \cos\theta_I}{n_T \cos\theta_T + n_I \cos\theta_I}}$$

$$\mathcal{E}_R = \frac{1-K}{2}\,\mathcal{E}_T = \frac{1-K}{1+K} \quad \Rightarrow \quad \boxed{r_\perp = \frac{n_I \cos\theta_I - n_T \cos\theta_T}{n_T \cos\theta_T + n_I \cos\theta_I}}$$

Campo Paralelo ao plano de incidência

$$\boxed{t_{\parallel} = \frac{2 n_I \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I}} \qquad \boxed{r_{\parallel} = \frac{n_I \cos\theta_T - n_T \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I}}$$

$$E_T = \frac{2}{1+K} E_I \Rightarrow t_\perp = \frac{2 n_I \cos\theta_I}{n_T \cos\theta_T + n_I \cos\theta_I}$$

$$E_R = \frac{1-K}{2} E_T = \frac{1-K}{1+K} \Rightarrow r_\perp = \frac{n_I \cos\theta_I - n_T \cos\theta_T}{n_T \cos\theta_T + n_I \cos\theta_I}$$

Campo Paralelo ao plano de incidência

$$t_{\parallel} = \frac{2 n_I \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I} \qquad r_{\parallel} = \frac{n_I \cos\theta_T - n_T \cos\theta_I}{n_I \cos\theta_T + n_T \cos\theta_I}$$

Parecido? Nem tanto

$$r_\perp = 0 \Rightarrow n_I \cos\theta_I = n_T \cos\theta_T$$

mas $n_I \operatorname{sen}\theta_I = n_T \operatorname{sen}\theta_T$

$$\Rightarrow \cos\theta_I \cdot n_T \frac{\operatorname{sen}\theta_T}{\operatorname{sen}\theta_I} = n_T \cos\theta_T$$

$$\operatorname{tg}\theta_I = \operatorname{tg}\theta_T \rightarrow \theta_I = \theta_T \quad \nexists$$

Fluxo de Energia: $\vec{S} = \vec{E} \times \vec{H}$ $\qquad B = E/v$

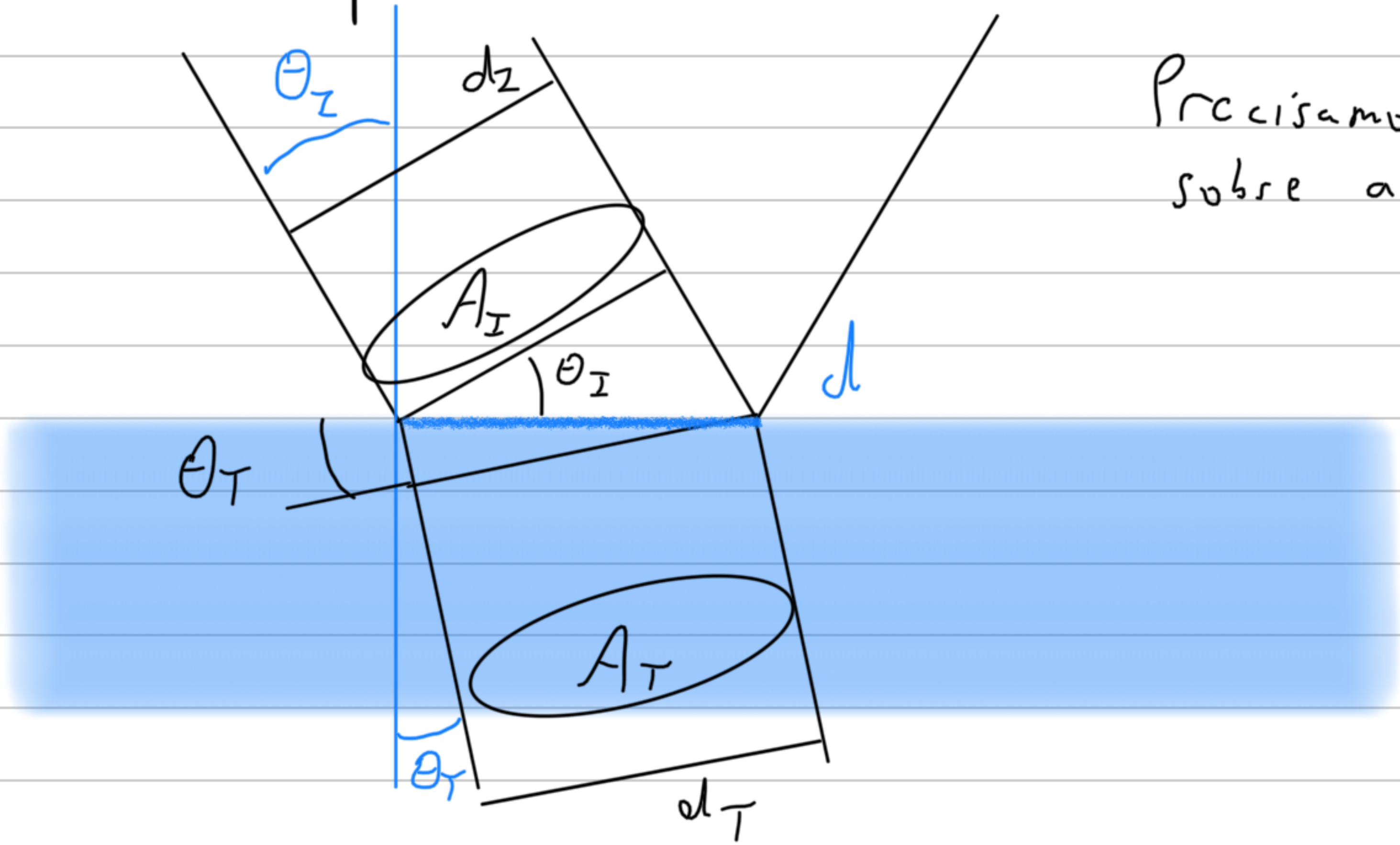
$$I = \langle |\vec{S}| \rangle = \frac{\langle |\vec{E} \times \vec{B}| \rangle}{\mu_0} = \frac{1}{v}\frac{1}{\mu_0} \langle E^2 \rangle = \frac{c^2 \epsilon_0}{v} \frac{|E|^2}{2}$$

$$\mu_0 = \frac{1}{\epsilon_0 c^2} \qquad I = n \cdot \frac{c\epsilon_0}{2} |E|^2$$

$$\frac{I_T}{I_I} = \frac{n_T}{n_I} \cdot |t|^2 \qquad \frac{I_R}{I_I} = |r|^2$$

Qual a potência total transmitida?

Qual a potência total transmitida?



Precisamos integrar sobre a área!

Potência Incidente: $P_I = \int_{A_I} I_I \cdot da_i = I_I \cdot A_I$

Potência Transmitida: $P_T = \int_{A_T} I_T \, da_T = I_T \cdot A_T$

Potência Incidente: $P_I = \int_{A_I} I_I \cdot da_i = I_I \cdot A_I$

Potência Transmitida: $P_T = \int_{A_T} I_T \, da_T = I_T \cdot A_T$

Usando uma área pequena o suficiente para ter $\underline{I = cte}$ só para facilitar a dedução

$$T = \frac{P_T}{P_I} = \frac{I_T}{I_I} \cdot \frac{A_T}{A_I} = \frac{n_T}{n_I} |t|^2 \frac{A_T}{A_I}$$

Note que no processo de refração temos uma contração ou extensão na dimensão paralela ao plano. Pela figura, vemos que o segmento d é comum aos dois triângulos, o que leva a

$$\cos\theta_I = \frac{d_I}{d} \; ; \; \cos\theta_T = \frac{d_T}{d} \; ; \; A_I \propto d_I \, , \; A_T \propto d_T$$

Note que no processo de refração temos uma contração ou extensão na dimensão paralela ao plano. Pela figura, vemos que o segmento d é comum aos dois triângulos, o que leva a

$$\cos\theta_I = \frac{d_I}{d} \; ; \; \cos\theta_T = \frac{d_T}{d} \; ; \; A_I \propto d_I \, , \, A_T \propto d_T$$

$$\Rightarrow \frac{A_T}{A_I} = \frac{\cos\theta_T}{\cos\theta_I}$$

$$\therefore \; T = \frac{n_T \cos\theta_T}{n_I \cos\theta_I} \cdot |t|^2$$

Como $R = \dfrac{P_R}{P_I} = |\Gamma|^2 \Rightarrow T + R = 1$

Para $\perp$ & $//$ Verifique!

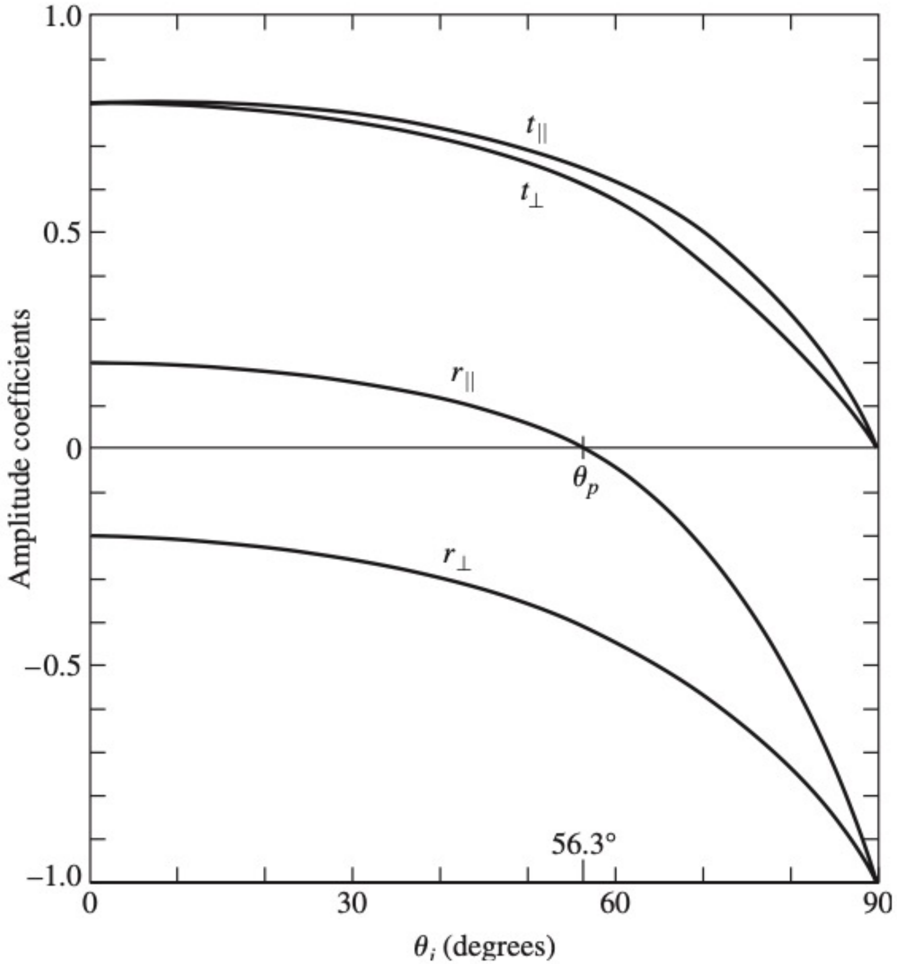


Figure 4.49 The amplitude coefficients of reflection and transmission as a function of incident angle. These correspond to external reflection $n_t > n_i$ at an air–glass interface ($n_{ti} = 1.5$).

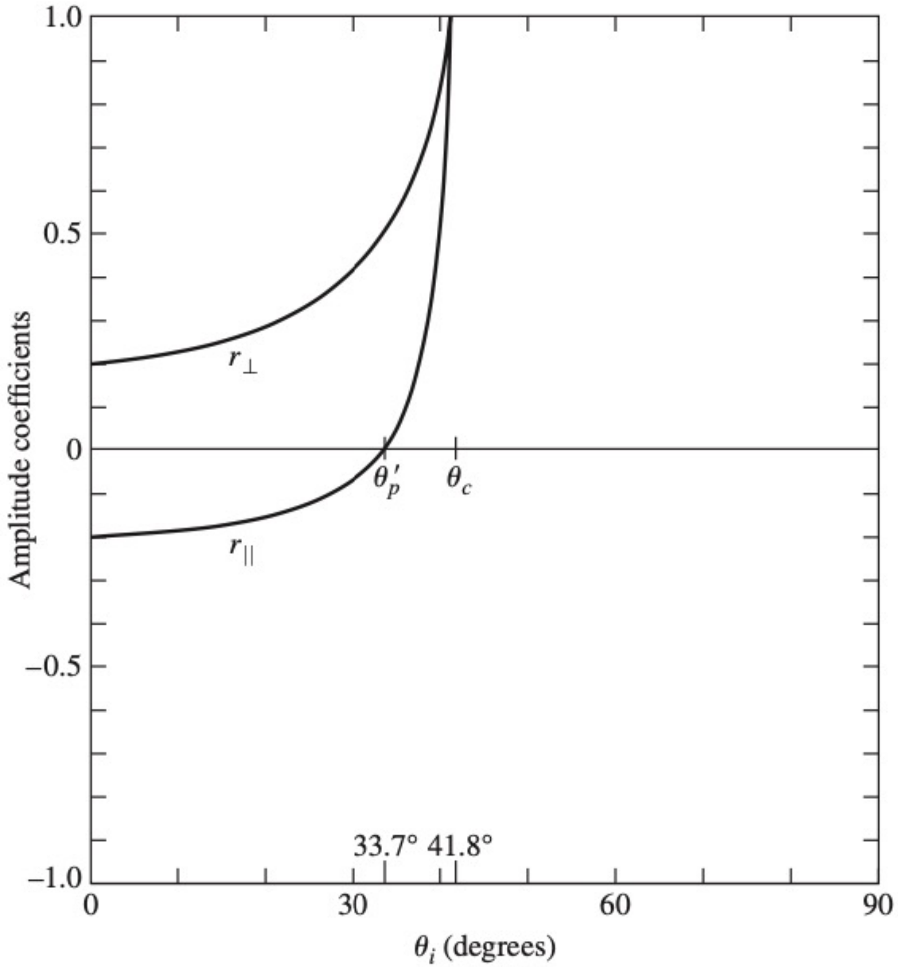


Figure 4.50 The amplitude coefficients of reflection as a function of incident angle. These correspond to internal reflection $n_t < n_i$ at an air-glass interface ($n_{ti} = 1/1.5$).

Hecht, Optics, Cap. 4