PGF5112 - Plasma Physics I

By

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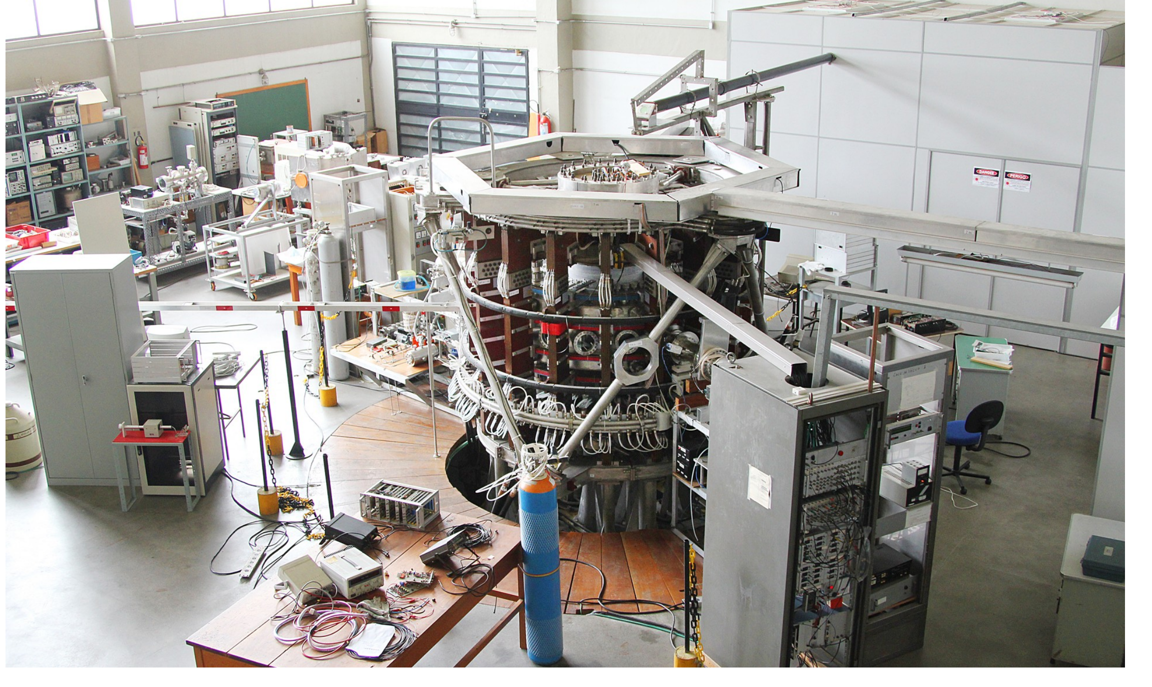

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São Paulo - SP, 13 April 2023

- **Nuclear fusion and tokamak physics**
  - *Nuclear fusion reactions*
  - *Thermonuclear fusion*
  - *Breakeven and the Lawson criterion for ignition*
  - *Scaling laws*

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Nuclear fusion reactions

- **Among nuclear fusion reactions, the exothermic are the ones of interest**

$$_1D^2 \;+\; _1D^2 \;\rightarrow\; _2He^3\ (0.82\ \text{MeV}) \;+\; _0n^1\ (2.45\ \text{MeV}):\ \ \Delta E_{\text{fus}} = 3.27\ \text{MeV}\ (50\,\%\ \text{Probability})$$

$$_1D^2 \;+\; _1D^2 \;\rightarrow\; _1T^3\ (1.01\ \text{MeV}) \;+\; _1p^1\ (3.02\ \text{MeV}):\ \ \Delta E_{\text{fus}} = 4.03\ \text{MeV}\ (50\,\%\ \text{Probability})$$

$$_1D^2 + \; _2He^3 \;\rightarrow\; _2He^4\ (3.6\ \text{MeV}) \;+\; _1p^1\ (14.7\ \text{MeV}):\ \ \Delta E_{\text{fus}} = 18.3\ \text{MeV}$$

$$_1D^2 \;+\; _1T^3 \;\rightarrow\; _2He^4\ (3.52\ \text{MeV}) \;+\; _0n^1\ (14.1\ \text{MeV}):\ \ \Delta E_{\text{fus}} = 17.6\ \text{MeV}$$

  - *The energies given here are the kinetic energies of the reaction products*
  - *By far, the most promising reactions are the D-T and D-He*^{3}

- **The mass-energy balance from the mass deficit δm in the reaction below**

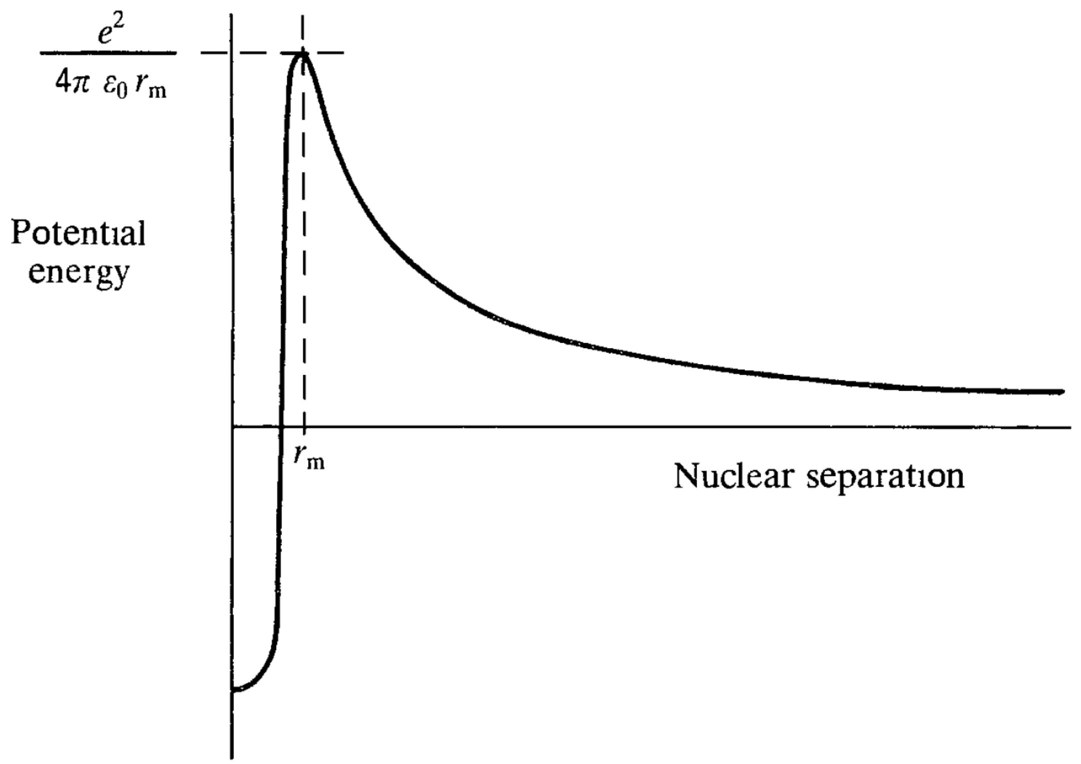
$$\begin{array}{ccccccc} _1D^2 & + & _1T^3 & \rightarrow & _2He^4 & + & _0n^1 \\ (2-0.000994)m_p & & (3-0.006284)m_p & & (4-0.027404)m_p & & (1+0.001378)m_p \end{array}$$

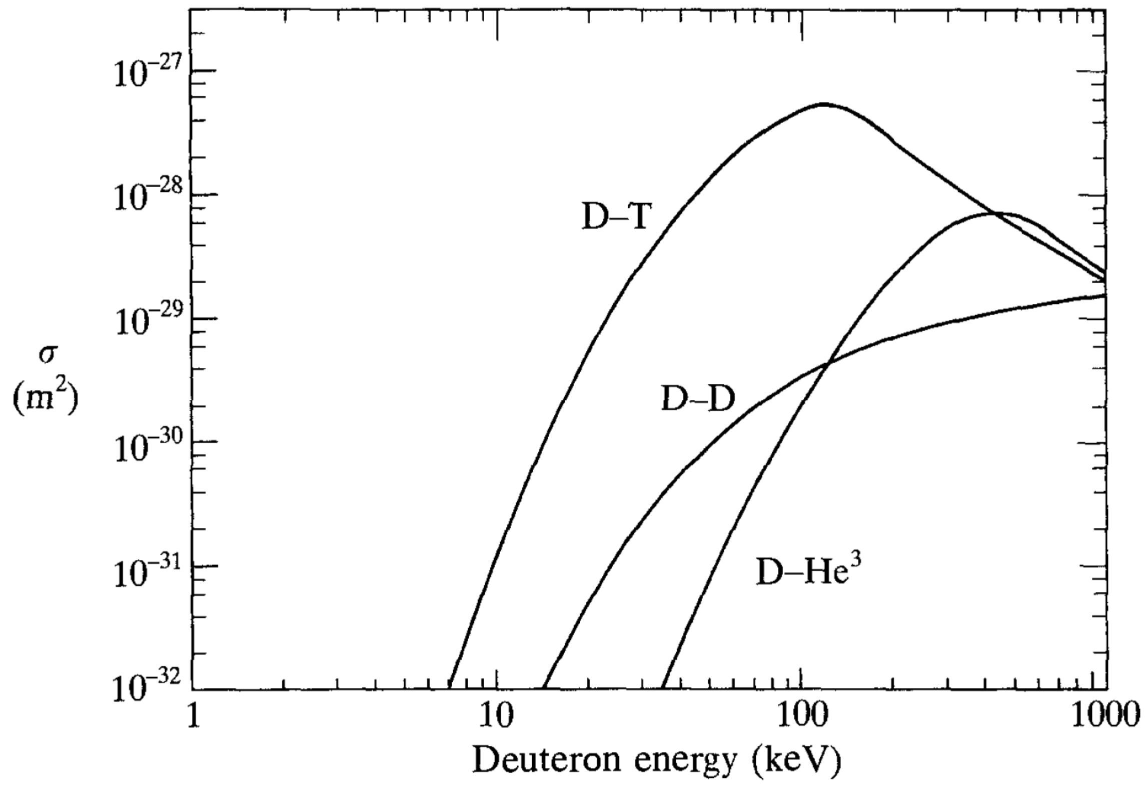
Here, $\delta m = 0.01875\, m_p$ *with* $m_p = 1.6726\times 10^{-27}$ *kg being the proton mass*

$$\Delta E_{\text{fus}} = \delta m\, c^2 = 0.01875\times 1.6726\times 10^{-27}\times (2.9979\times 10^8)^2 = 2.8186\times 10^{-12}\ \text{J} = 17.59\ \text{MeV}$$

Nuclear fusion reactions

- **Among the nuclear fusion reactions just mentioned, the D-T reaction is the one that has the largest cross section at lower energies**
  - *Quantum tunneling allows fusion to occur at significantly lower energies*





Nuclear fusion reactions

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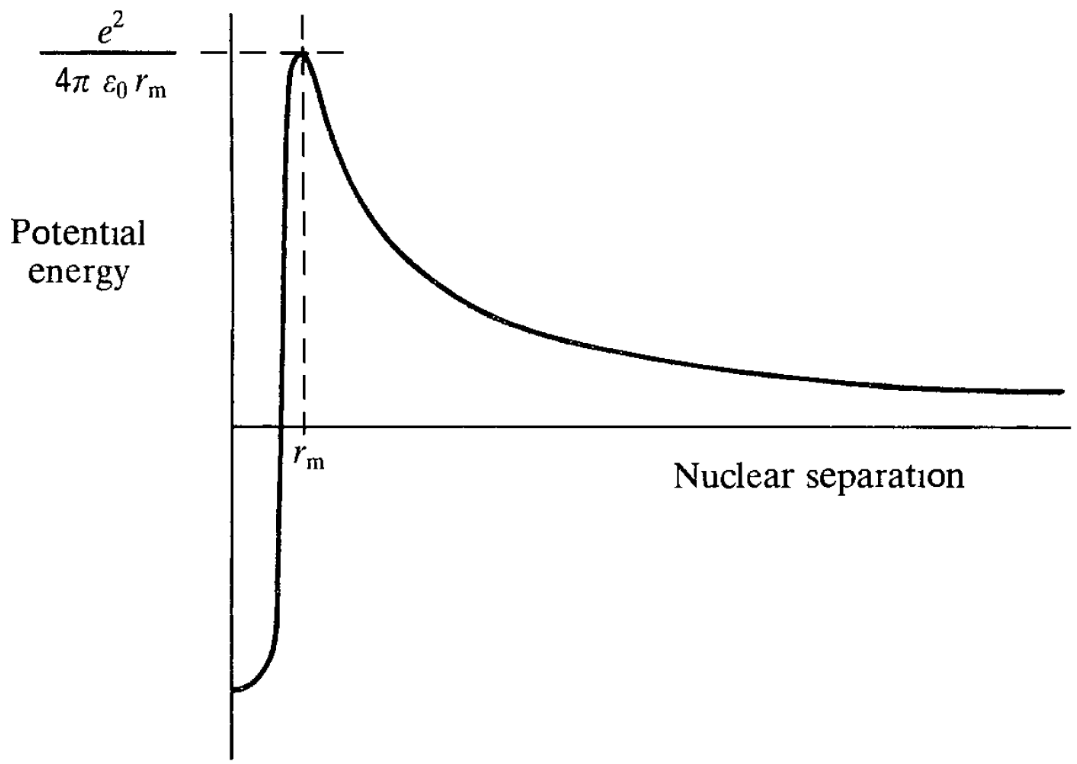

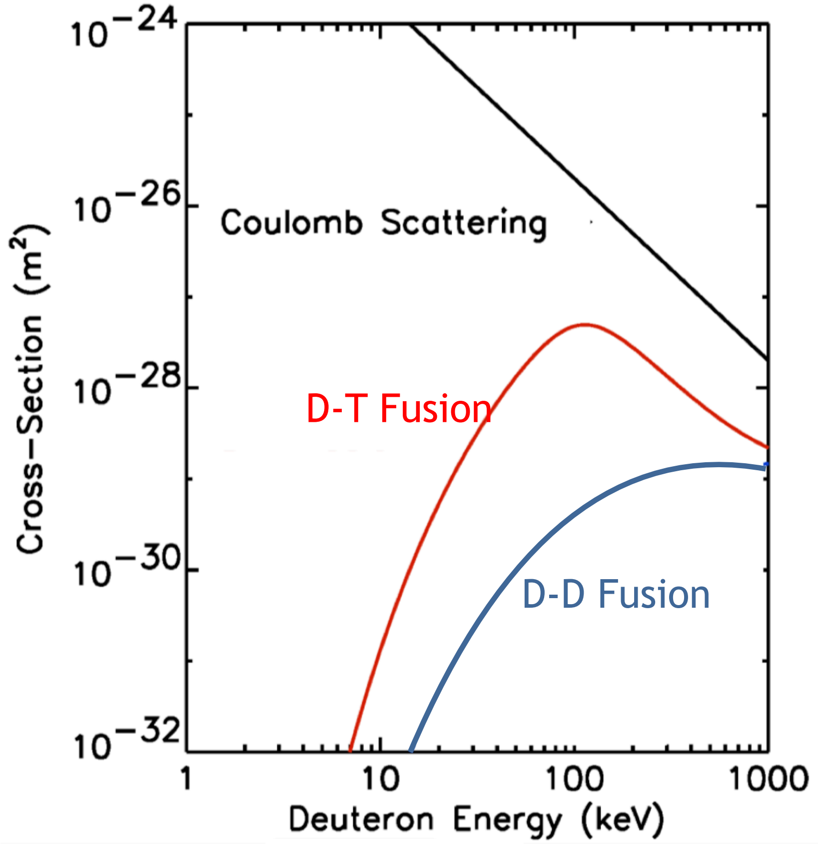

  - *The cross section of a D-T reaction is, for an ion average energy of 10 keV, about 10^7 time smaller than that associated with a Coulomb scattering*

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Thermonuclear fusion

- **The most promising approach to fuse D and T is by heating a mixture of these to hight enough temperature**
  - *Particles in the velocity distribution tail can undergo fusion reactions*

- **Let's calculate the D-T fusion reaction rate,**

$$R_{DT} = \int_{v_D}\int_{v_T} \sigma_{DT}\left(\,|\mathbf{v_D} - \mathbf{v_T}|\,\right)\,|\mathbf{v_D} - \mathbf{v_T}|\, f_D(v_D) f_T(v_T)\; d^3v_D\, d^3v_T$$

- **In addition, let's suppose that D and T have a Maxwellian distribution**

$$f_j(v_j) = n_j \left(\frac{m_j}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v_j^2}{2k_B T}\right)$$

Thermonuclear fusion

- **By writing**

$$u = |\mathbf{v_D} - \mathbf{v_T}| \qquad \textit{(relative velocity)}$$

$$\mu = \frac{m_D + m_T}{m_D\, m_T} \qquad \textit{(reduced mass)}$$

and $$V = \frac{v_D + v_T}{2}$$

the reaction rate can be rewritten as

$$R_{DT} = n_D n_T \langle \sigma u \rangle_{DT} = n_D n_T \frac{(m_D\, m_T)^{3/2}}{(2\pi k_B T)^3} \int_u \int_V \exp\left[-\frac{m_D + m_T}{2k_B T}\left(V + \frac{(m_D - m_T)u}{2(m_D + m_T)}\right)^2\right] \times$$

$$\times\, \sigma_{DT}(u)\, u\, \exp\left(-\frac{\mu u^2}{2k_B T}\right) d^3V\, d^3u$$

- **The integral in V can be carried out and results in**

$$R_{DT} = 4\pi n_D n_T \left(\frac{\mu}{2\pi k_B T}\right)^{3/2} \int \sigma_{DT}(u)\, u^3 \exp\left(-\frac{\mu u^2}{2k_B T}\right) d^3u$$

Thermonuclear fusion

- **Cross sections measured in laboratory experiments are usually given in terms of the energy of the projectile particles. Therefore, taking D as the projectile particle (we could have chosen T without any problem), and defining**

$$\epsilon = \frac{1}{2} m_D u^2$$

allows us to write the reaction rate as

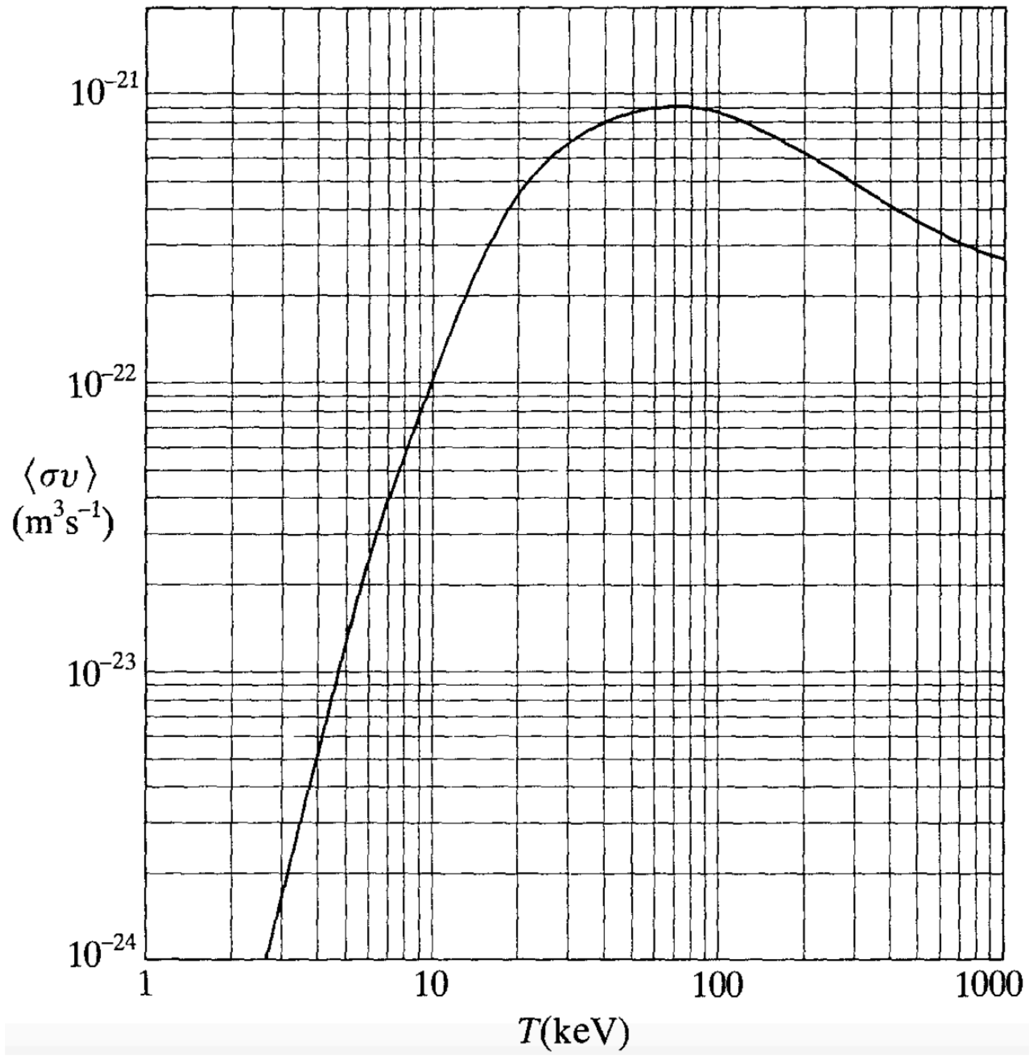
$$R_{DT} = n_D n_T \langle \sigma u \rangle_{DT}$$

with

$$\langle \sigma u \rangle_{DT} = \sqrt{\frac{8}{\pi}} \frac{1}{m_D^2} \left(\frac{\mu}{k_B T} \right)^{3/2} \times$$

$$\times \int_0^\infty \sigma_{DT}(\epsilon)\, \epsilon \, \exp\left(- \frac{\mu \epsilon}{m_D k_B T} \right) d\epsilon$$

D-T reaction



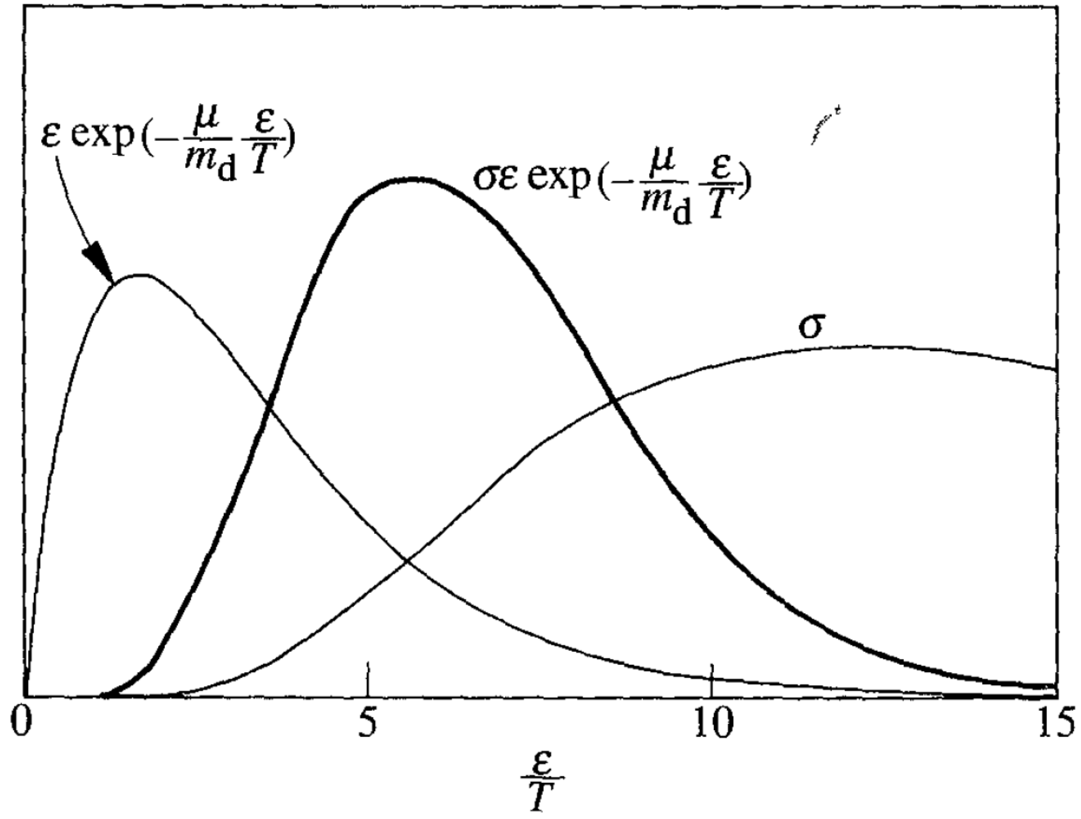
  – *For a given ion density the maximum rate is achieved for $n_D = n_T$*

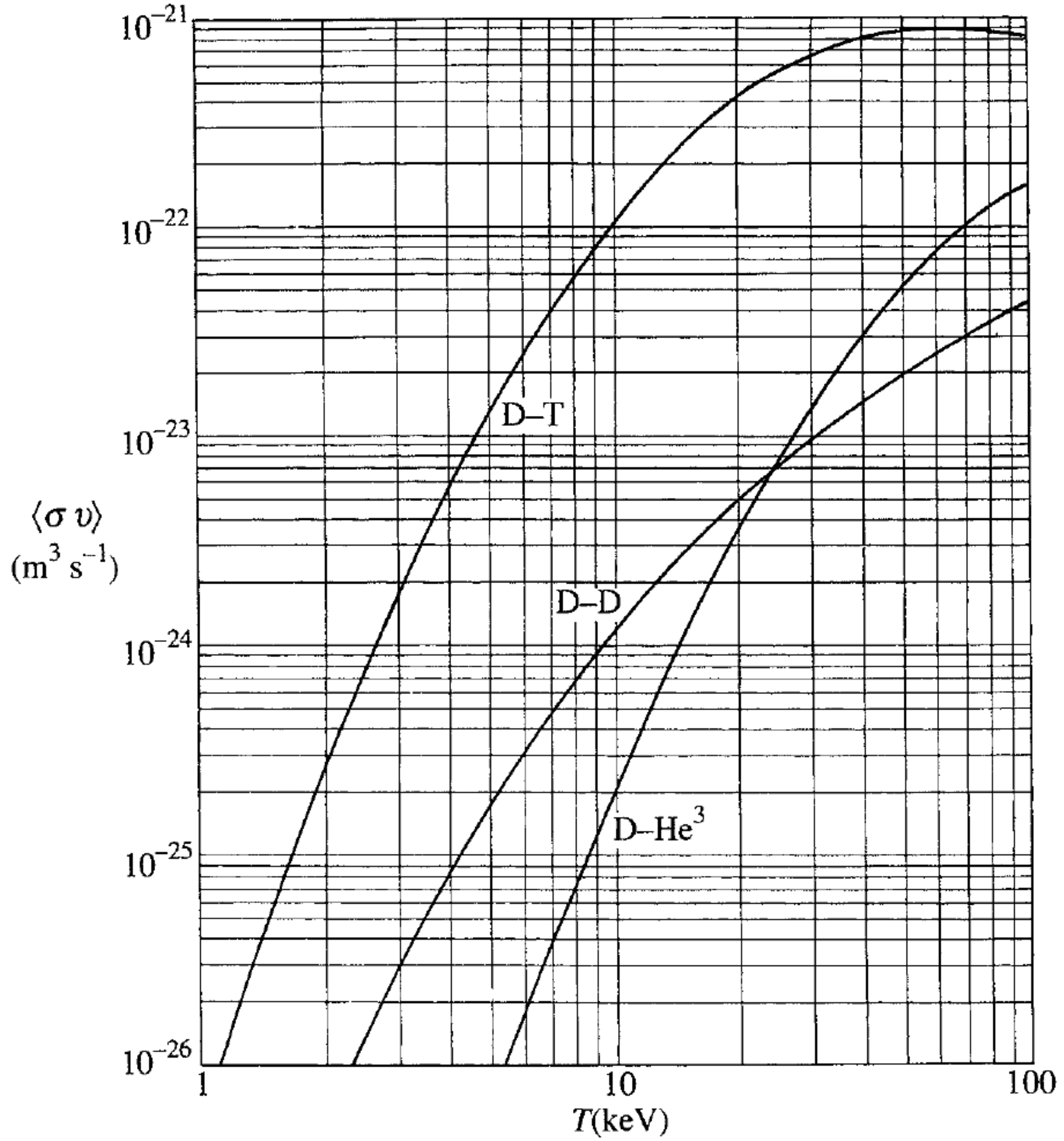
Thermonuclear fusion

- **The $\langle \sigma\, u \rangle$ parameter can also be calculated for other nuclear fusion reactions**

$$\langle \sigma\, u \rangle_{12} = \sqrt{\frac{8}{\pi}}\, \frac{1}{m_1^2} \left(\frac{\mu}{k_B T} \right)^{3/2} \times$$

$$\times \int_0^\infty \sigma_{12}(\epsilon)\, \epsilon\, \exp\left(- \frac{\mu \epsilon}{m_1\, k_B T} \right) d\epsilon$$





Thermonuclear fusion power density

- **With the $\langle \sigma\, u \rangle$ parameter in hands, one can now calculate the fusion power density produced by the any fusion reaction by**

$$p_{\text{fus}} = R_{12}\, \Delta E_{\text{fus}} = n_1\, n_2\, \langle \sigma\, u \rangle_{12} \Delta E_{\text{fus}}$$

- **For the case a D-T reaction, and writing the ion density $n_D = n_T = n/2$, one has**

$$p_{\text{fus}} = \frac{n^2}{4} \langle \sigma\, u \rangle_{DT} \Delta E_{DT}$$

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Breakeven and the Lawson criterion for ignition

- **An important aspect that must be taken into account in designing a fusion reactor is the so-called power balance**
  - *In a tokamak, there is a continuous loss of energy from the plasma, which has to be replenished through additional plasma heating*

- **Using the equipartition energy theorem, we have that the local energy density in the plasma is**

$$w = n_D \frac{3}{2} k_B T + n_T \frac{3}{2} k_B T + n_e \frac{3}{2} k_B T = 3 n k_B T$$

  - *Here, one used that* $n_e = n_D/2 = n_T/2 = n$ *(quasi-neutrality condition)*

- **Therefore, the total plasma stored energy is**

$$W_P = \int w\, dV = \int 3 n k_B T\, dV = 3 \bar{p} V$$

where $\bar{p} = \overline{n k_B T} = \frac{1}{V} \int n k_B T\, dV$ **is the volume-averaged plasma pressure**

Breakeven and the Lawson criterion for ignition

- **The power P_{fus} has contributions from two species: α-particles and neutrons**
  - *Neutrons leave the plasma (not charged particles) while the α-particles (charged particles) remain confined by the magnetic field*

$$ {}_1\mathrm{D}^2 \;+\; {}_1\mathrm{T}^3 \;\rightarrow\; {}_2\mathrm{He}^4\;(3.52\ \mathrm{MeV}) \;+\; {}_0\mathrm{n}^1\;(14.1\ \mathrm{MeV}) : \;\; \Delta E_{\text{fus}} = 17.6\ \mathrm{MeV} $$

  - *Note that for a D-T reaction, $\Delta E_{\text{fus}} = \Delta E_n + \Delta E_\alpha$*
  - *Note that $\Delta E_\alpha = \Delta E_{\text{fus}}/5$*
  - *The α-particles, therefore, transfer 1/5 of total fusion energy to the plasma*

- **The α-particles heating density is, therefore,**

$$ p_\alpha = \frac{p_{\text{fus}}}{5} = \frac{n^2}{4}\langle \sigma\, u\rangle \frac{\Delta E_{\text{fus}}}{5} = \frac{n^2}{4}\langle \sigma\, u\rangle \Delta E_\alpha $$

and the total α-heating is

$$ P_\alpha = \int p_\alpha\, dV = \frac{1}{4}\overline{n^2\langle \sigma\, u\rangle}\Delta E_\alpha V \quad \textbf{with} \quad \overline{n^2\langle \sigma\, u\rangle} = \frac{1}{V}\int n^2 \langle \sigma\, u\rangle\, dV $$

Breakeven and the Lawson criterion for ignition

- **The rate of energy loss (power loss) is characterized in terms of an energy confinement time, which is defined as**

$$P_L = \frac{W_P}{\tau_E}$$

- **In a tokamak, external/auxiliary heating must be applied to compensate P_L**

- **In most of present day tokamaks, the fusion power is negligible**
  - *Therefore, under steady state, one usually have*

$$P_L = \frac{W_P}{\tau_E} = P_H \quad \textit{and, therefore,} \quad \tau_E \equiv \frac{W_P}{P_H}$$

  - *This relation allows us to determine τ_E from experimentally know quantities*

- **In the overall power balance in future fusion reactors, however, the power loss is balanced by external/auxiliary heating plus the α-particles heating**

$$P_H + P_\alpha = P_L$$

Breakeven and the Lawson criterion for ignition

- **Substitution into the previous equation yields**

$$P_H + \frac{1}{4}\overline{n^2\langle \sigma u\rangle}\,\Delta E_\alpha V = \frac{3\,\overline{nk_BT}\,V}{\tau_E}$$

- **Assuming constant plasma density and temperature profiles, for simplicity, yields**

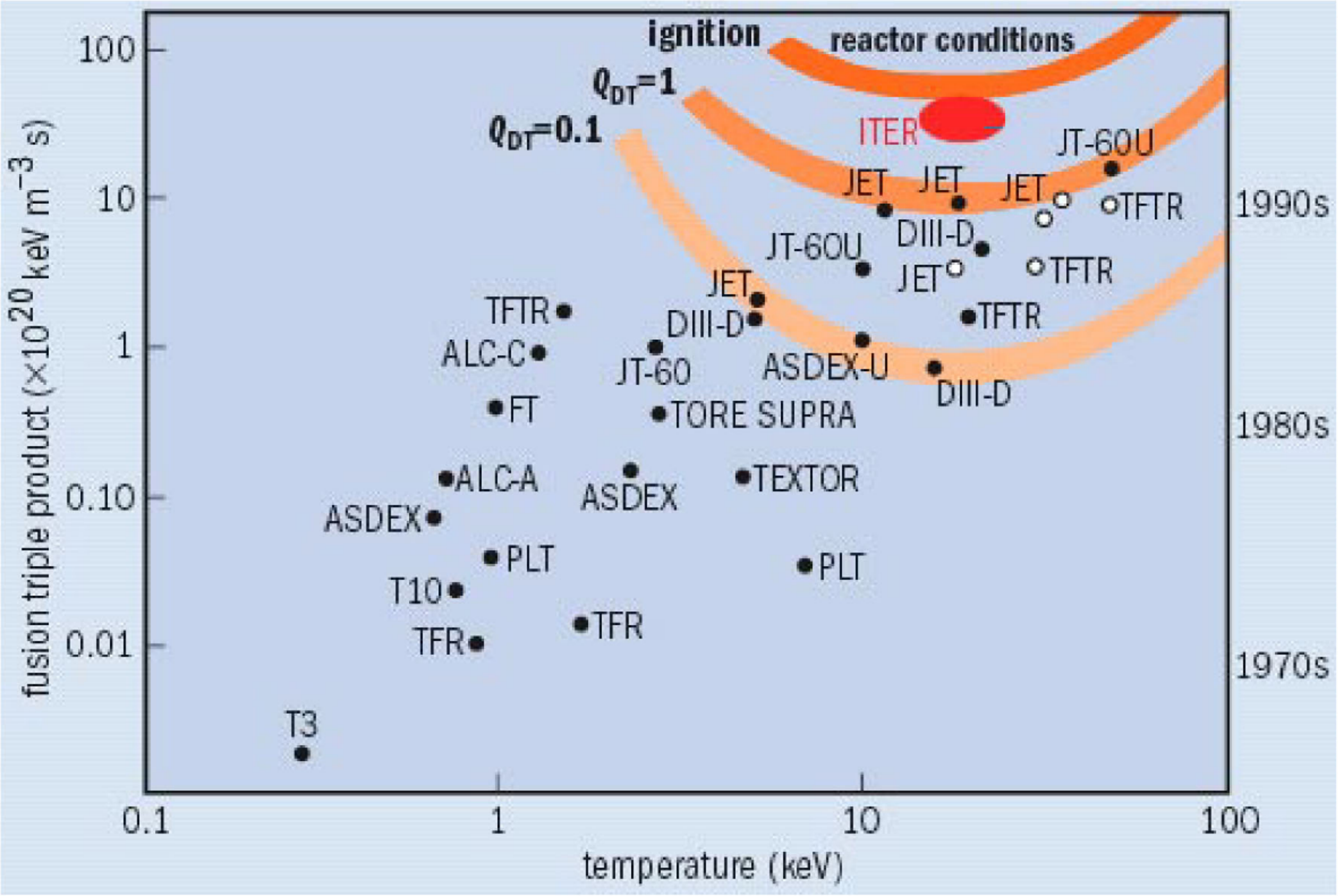
$$P_H = \left(\frac{3\,nk_BT}{\tau_E} - \frac{n^2\langle \sigma u\rangle\,\Delta E_\alpha}{4}\right)V$$

- **An important parameter used to measured the performance of a fusion plasma is the fusion factor gain as** $Q = P_{\mathrm{fus}}/P_H$
  - *The condition at which* $P_H = P_{\mathrm{fus}}$ *, i.e.* $Q = 1$*, is called the breakeven*
  - *The breakeven condition corresponds to the point at which the total (neutron plus* α*-particles) fusion power produced equals the necessary external/auxiliary heating power*

- **Since** $P_{\mathrm{fus}} = 5P_\alpha$**, the** $Q = 1$ **condition corresponds to an** α**-particle heating power that is only 20% of the externally applied heating power**

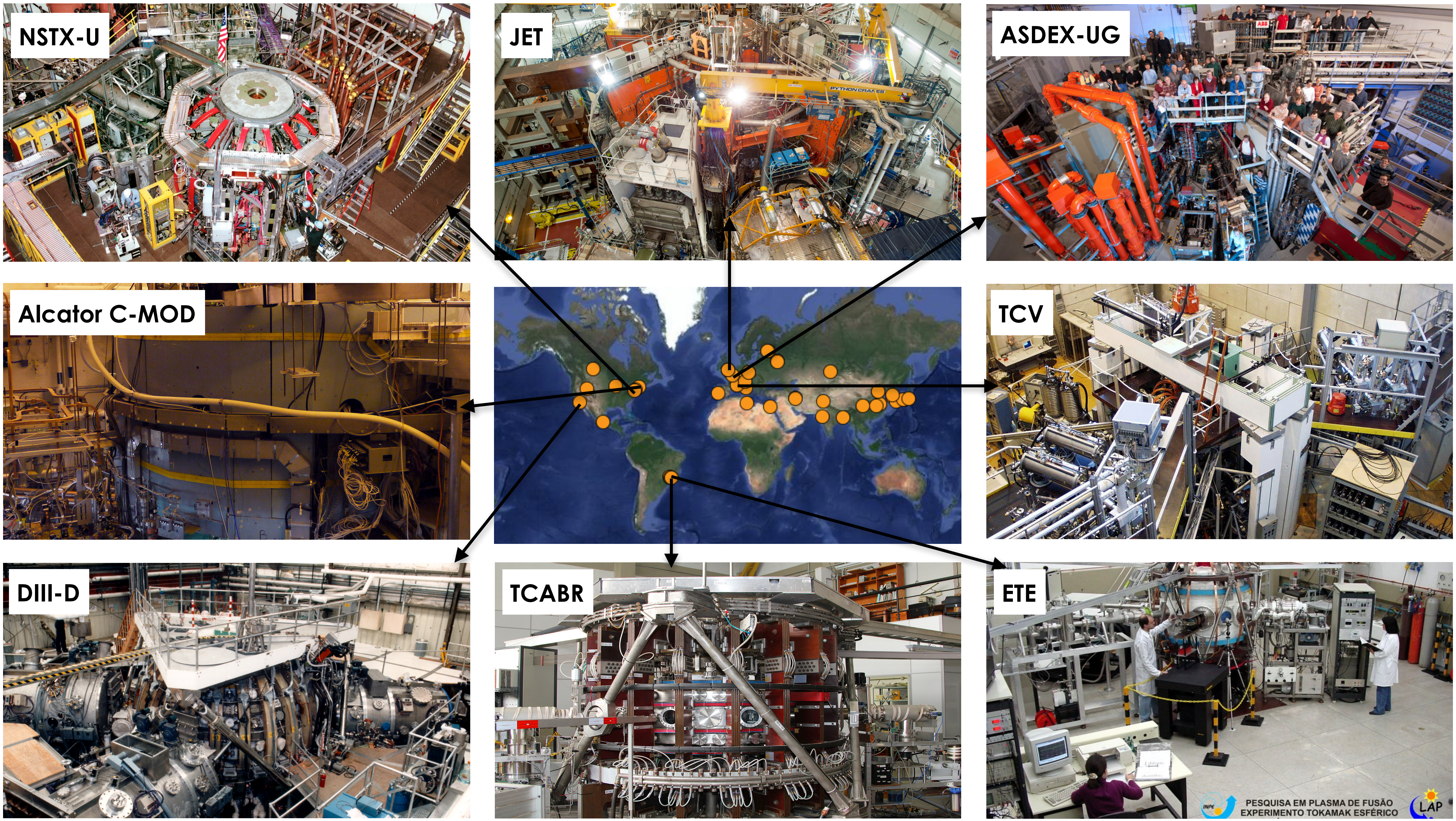
Breakeven and the Lawson criterion for ignition

- **Several tokamaks were built over the past aiming at the breakeven condition**



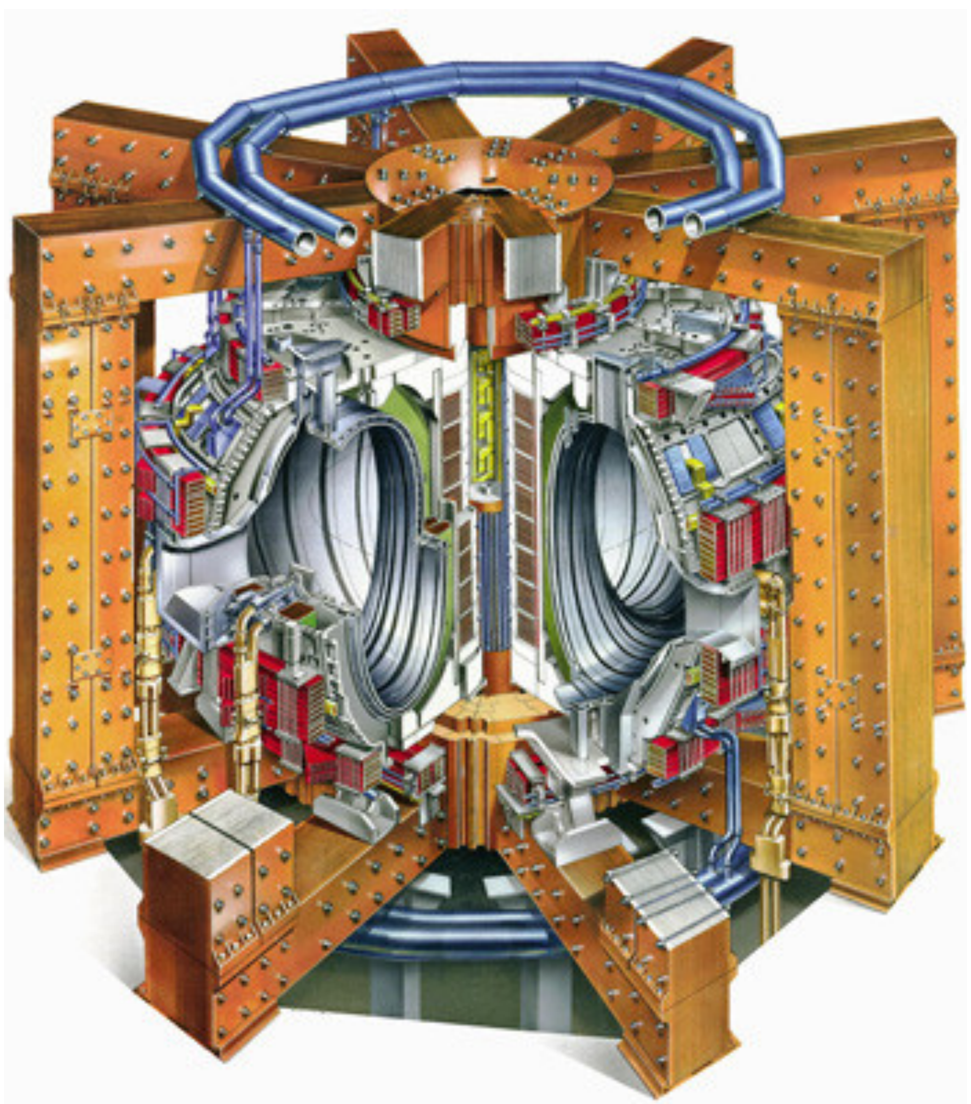
- **The International Thermonuclear Experimental Reactor (ITER) aims at** $Q = 10$

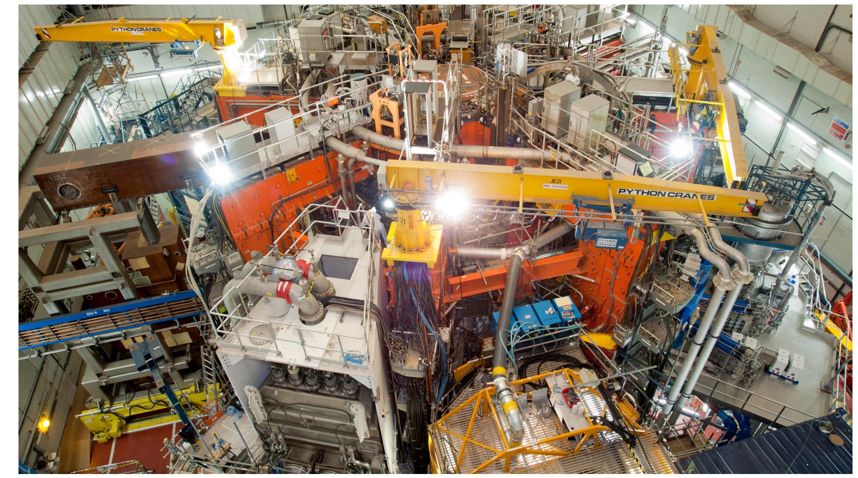
Tokamaks in operation around the world

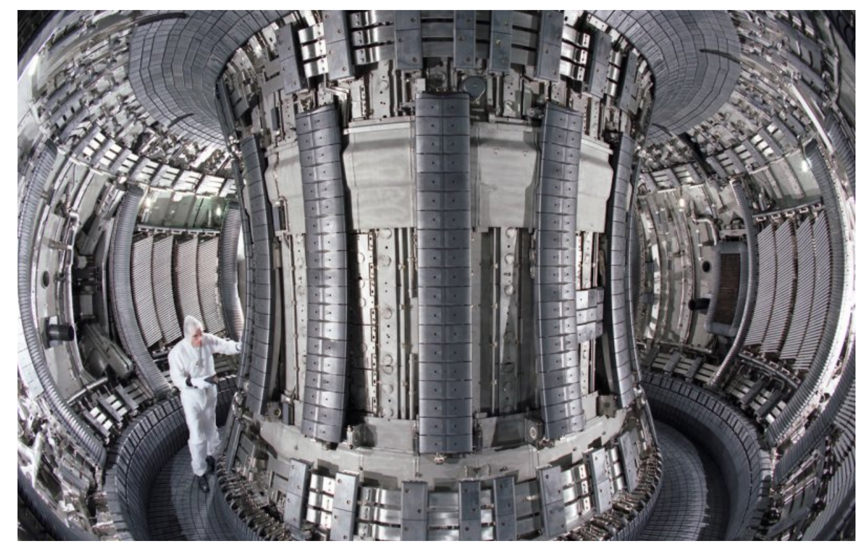


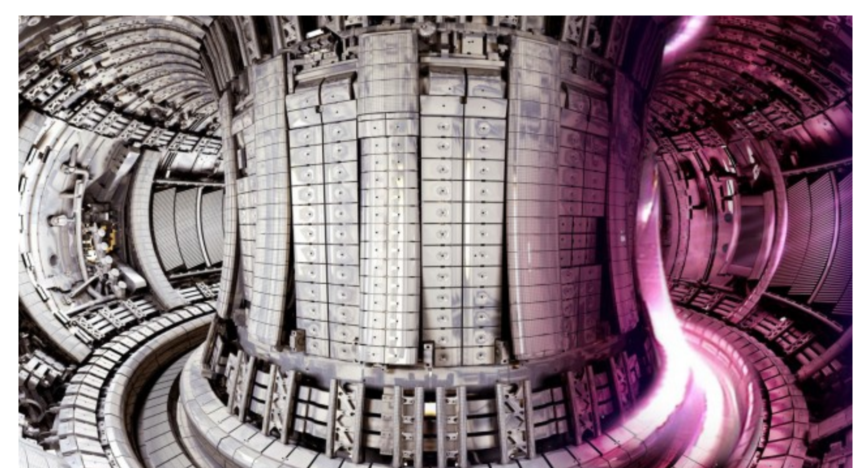
The Joint European Torus (JET) is the largest tokamak ever built and it is located in Oxfordshire - UK

- **JET features R_0 = 2.96 m, a = 1.25-2.10 m, I_P = 4.8 MA, B_0 = 3.45 T and pulse duration 10 s**
  - JET holds the world record for fusion power produced. In 1997, JET achieved $Q = 0.67$









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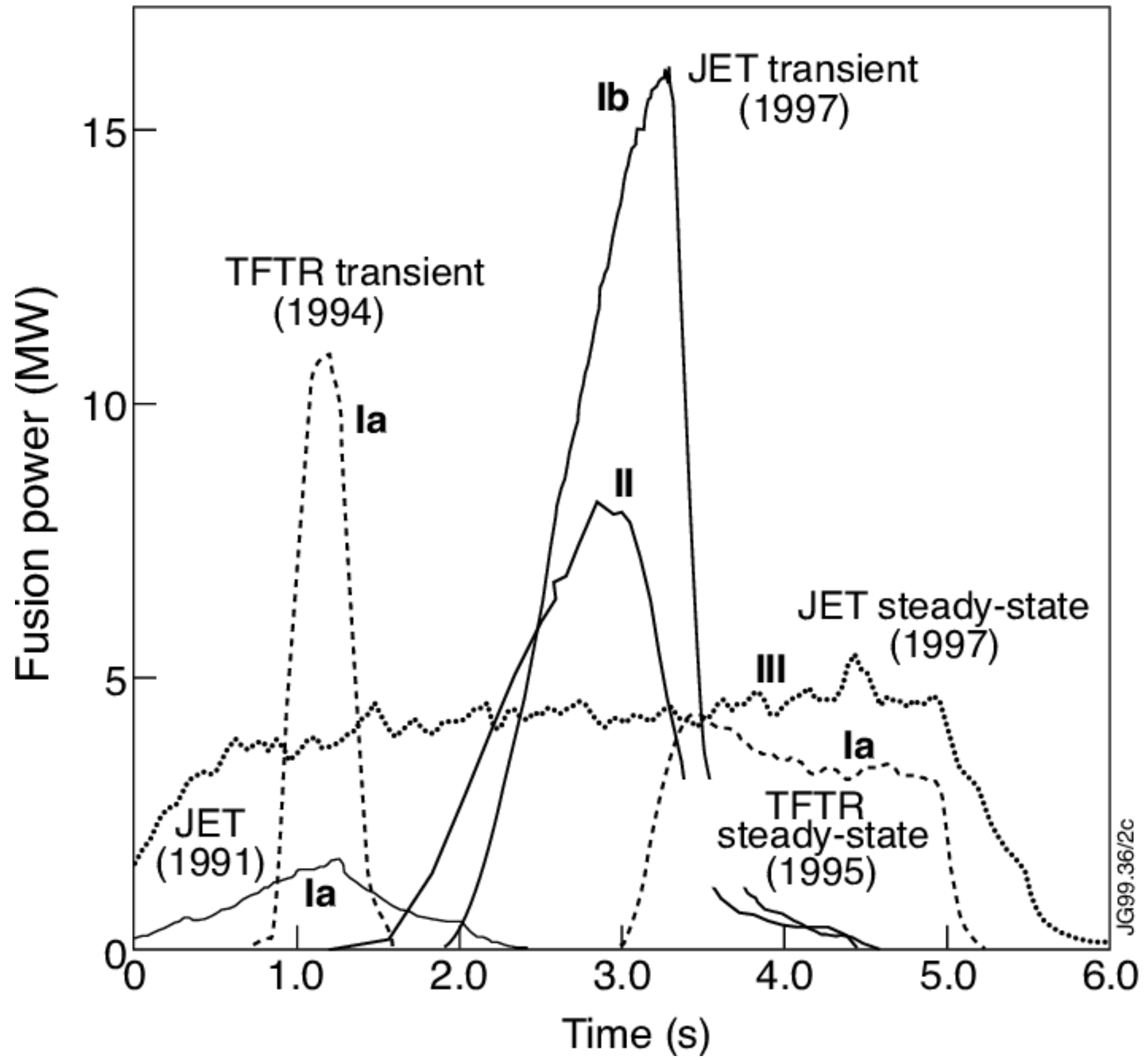
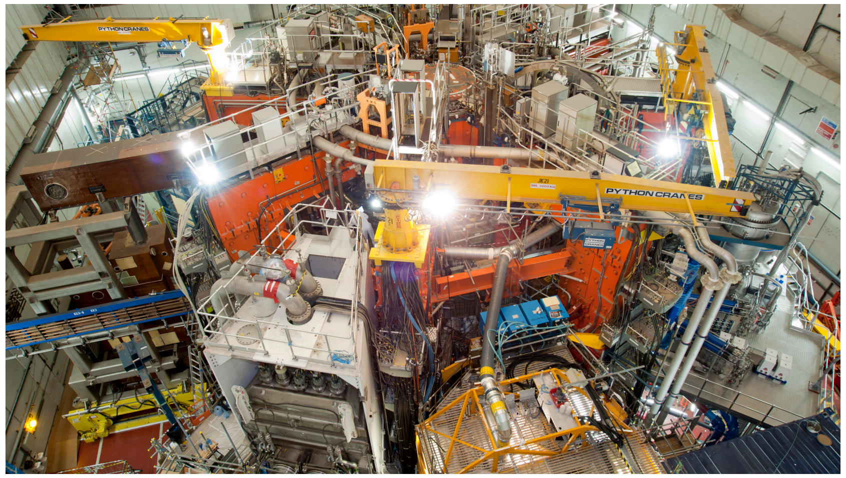
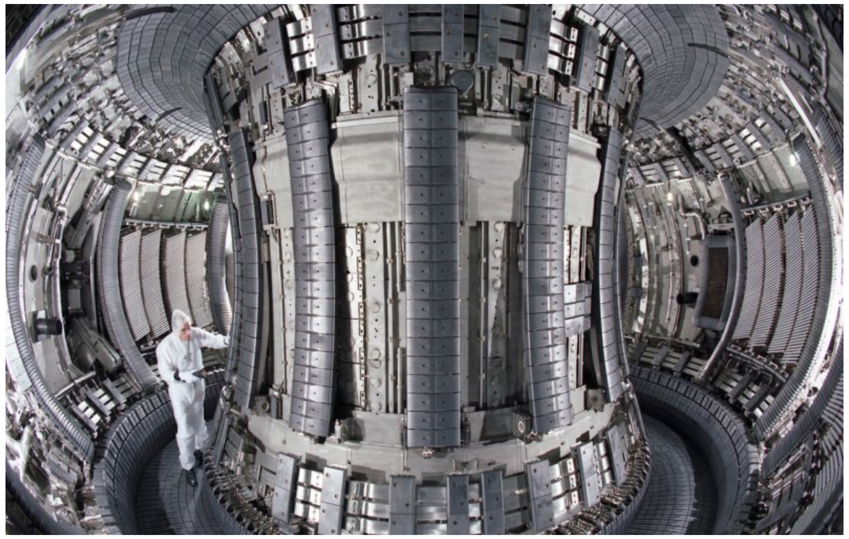
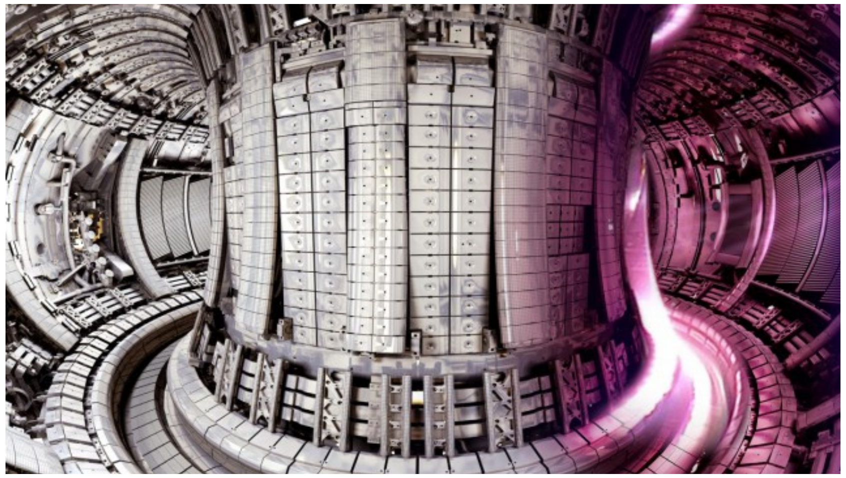


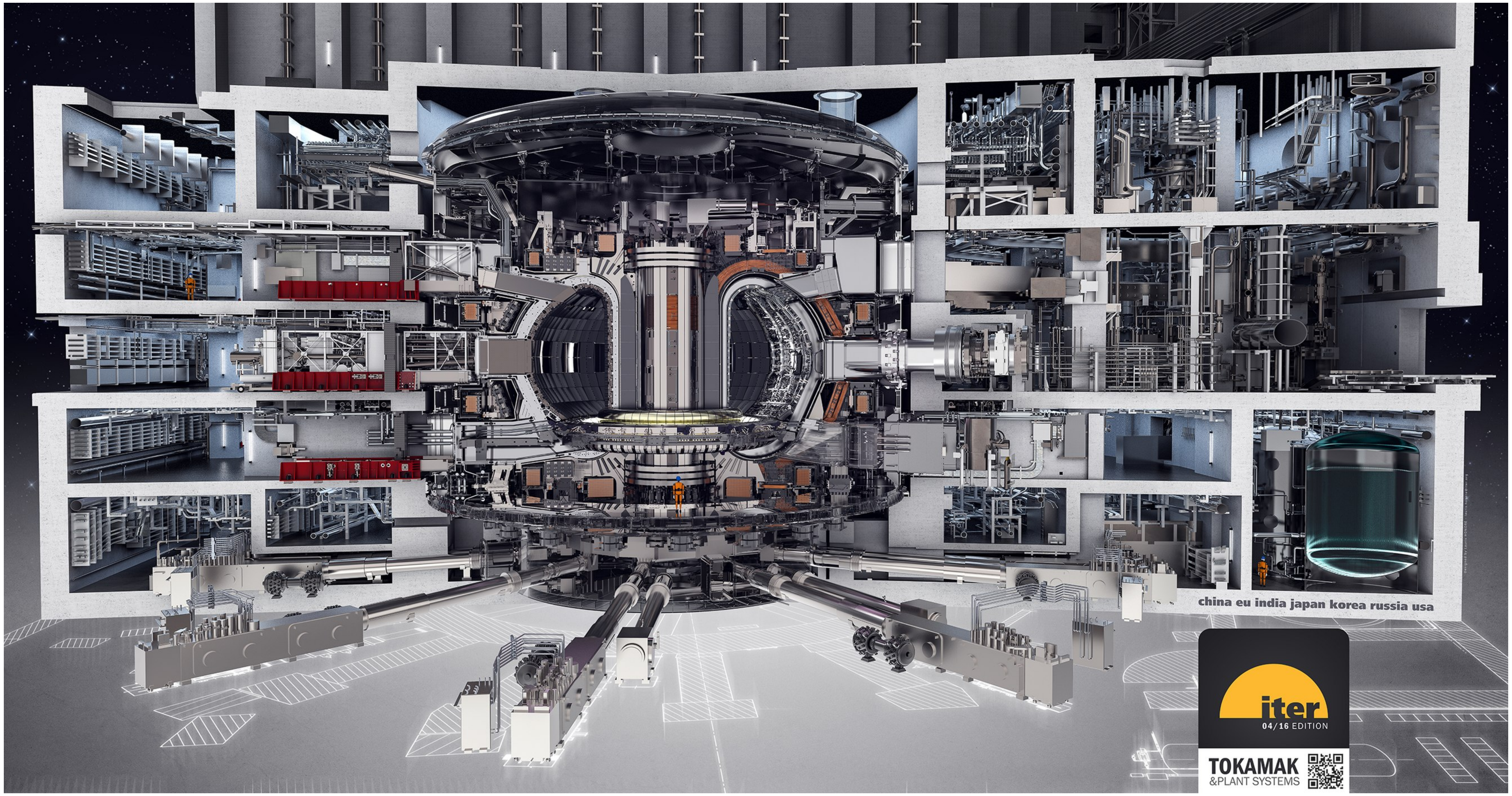






ITER will be the largest, and most complex, experiment ever built by mankind

- **ITER is being constructed in France through a consortium composed by several countries: European Union, India, Japan, China, Russia, South Korea e USA**
  - ITER features R_0 = 6.2 m, a = 2.0 m, I_P = 15 MA, B_0 = 12 T, pulse duration up to 1000 s and it was designed to achieve $Q = 10$



Breakeven and the Lawson criterion for ignition

- **From the power balance equation**

$$P_H = \left(\frac{3\, n k_B T}{\tau_E} - \frac{n^2 \langle \sigma\, u \rangle\, \Delta E_\alpha}{4} \right) V$$

the condition at which $P_H = 0$ is called the ignition condition. To have the plasma temperature being set alone by self-heating due to α-particles heating,

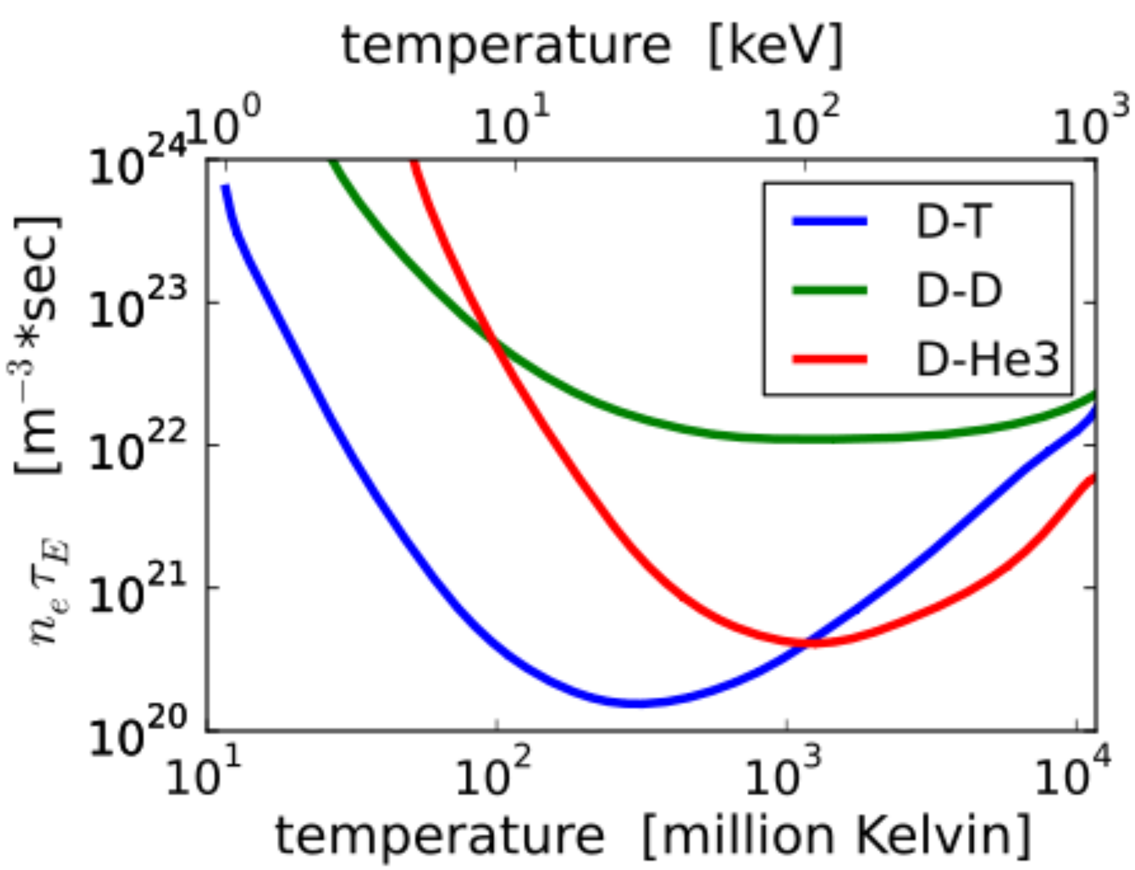
$$n\tau_E \geq \frac{12 k_B}{\Delta E_\alpha} \frac{T}{\langle \sigma\, u \rangle}$$

- **In the range 10-20 keV, the $\langle \sigma\, u \rangle$ parameter can be approximated (to within 10%) by**

$$\langle \sigma\, u \rangle = 1.1 \times 10^{-24}\ T^2\ m^3/s \quad \text{(with T in keV)}$$

- **The expression $T/\langle \sigma u \rangle$ has a minimum at T = 26 keV. At this temperature,**

$$n\tau_E \geq 1.5 \times 10^{20} \quad \text{s/m}^3 \quad \text{(The Lawson Criterion)}$$



Breakeven and the Lawson criterion for ignition

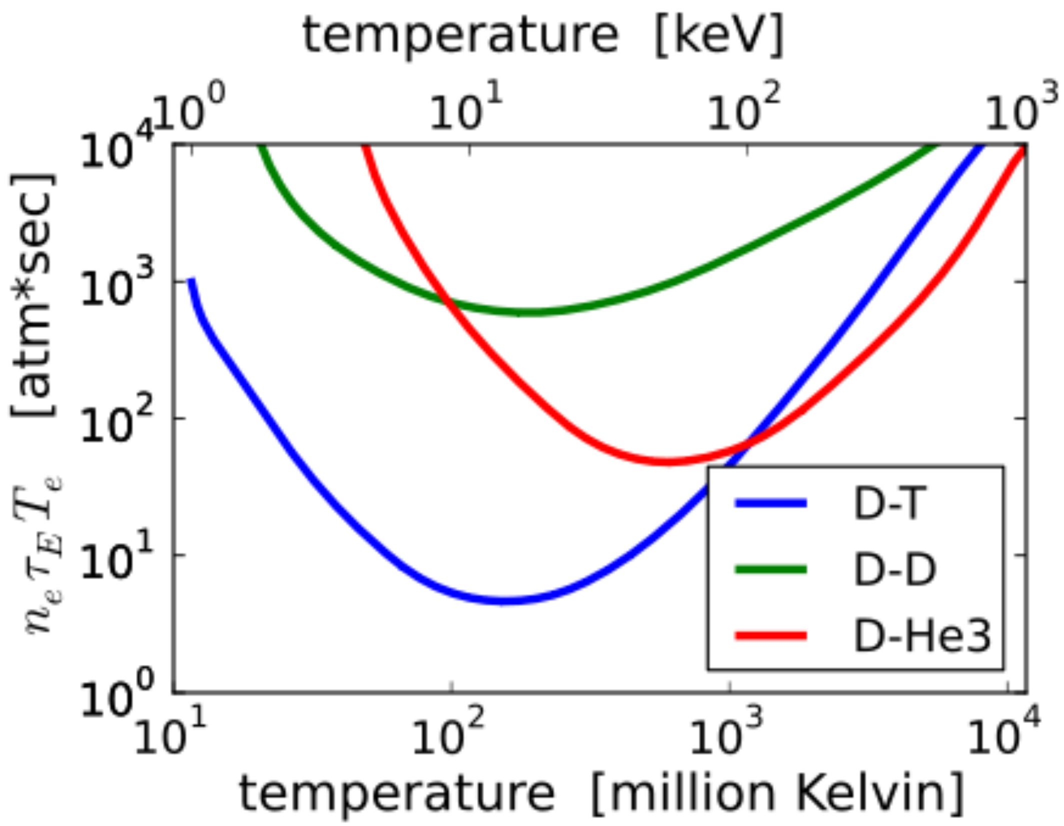
- **In tokamaks, the maximum achievable plasma pressure is limited, to a major extent, by its toroidal magnetic field. Therefore, to account for this limitation, the Lawson criterion is multiplied by the plasma temperature leading to**

$$nT\tau_E \geq \frac{12k_B}{\Delta E_\alpha}\frac{T^2}{\langle \sigma\, u\rangle}$$

- **The expression $T^2/\langle \sigma u\rangle$ also has a minimum, but at a slightly lower temperature, i.e. at T ~ 10 keV. At this temperature,**

$$nT\tau_E \geq 3\times 10^{21}\ \ \text{keV . s/m}^3$$

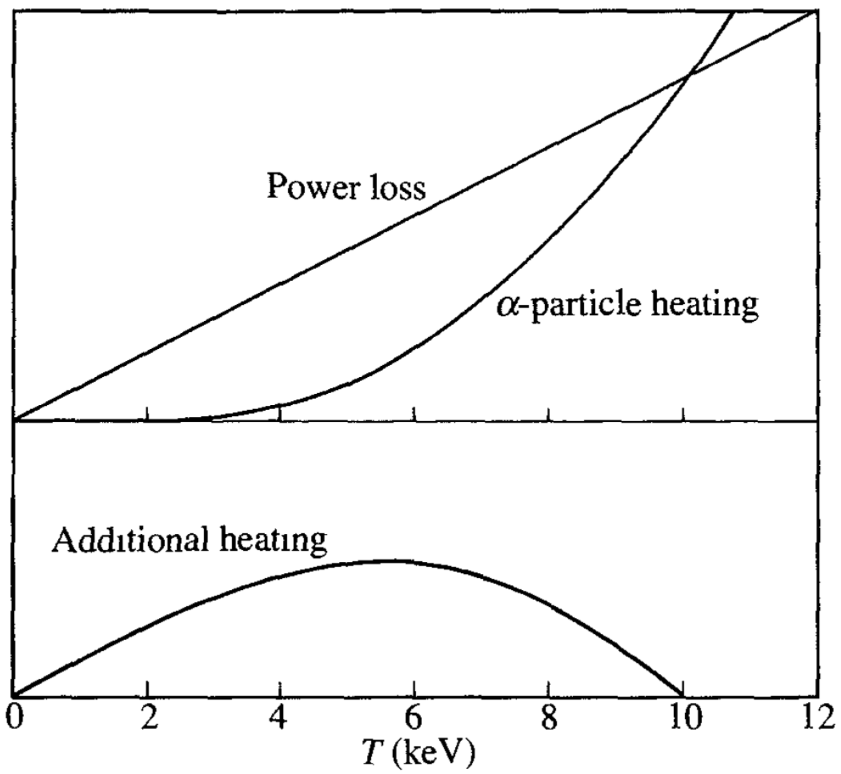
(The Triple Product)

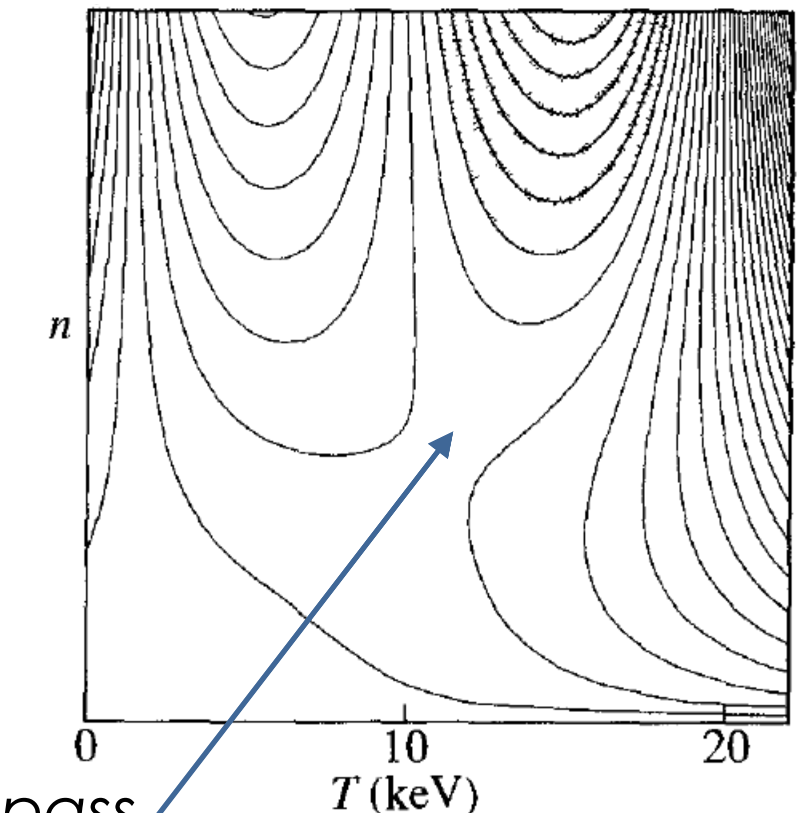


- **For a tokamak plasma with**
  - *n = 1x10*^{20} *m*$^{-3}$
  - *T = 10 keV*
  - *The energy confinement time must be, at least,* $\tau_E = 3\ s$

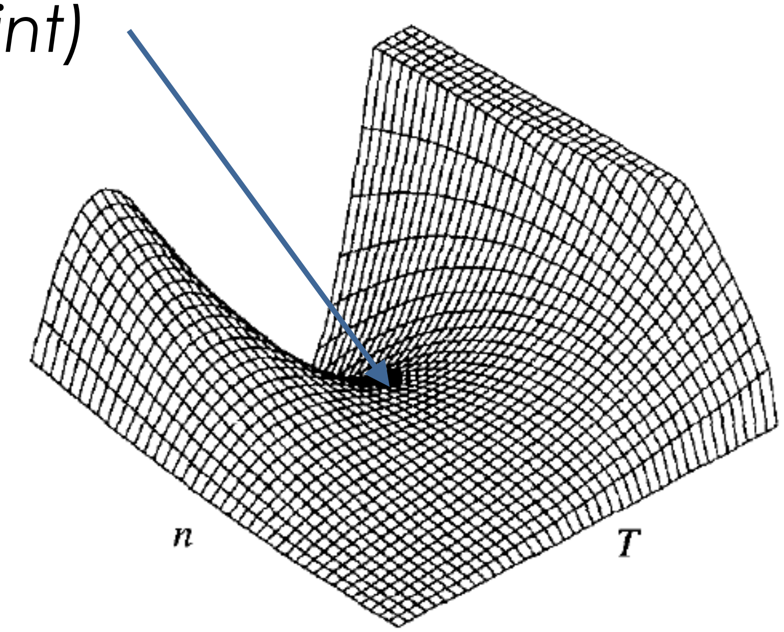
Breakeven and the Lawson criterion for ignition

- **As the ignition condition is approached, the fusion gain factor tends to infinity, i.e. as $P_H \rightarrow 0$ one has that $Q = P_{\text{fus}}/P_H \rightarrow \infty$**





The Cordey pass
(Saddle point)



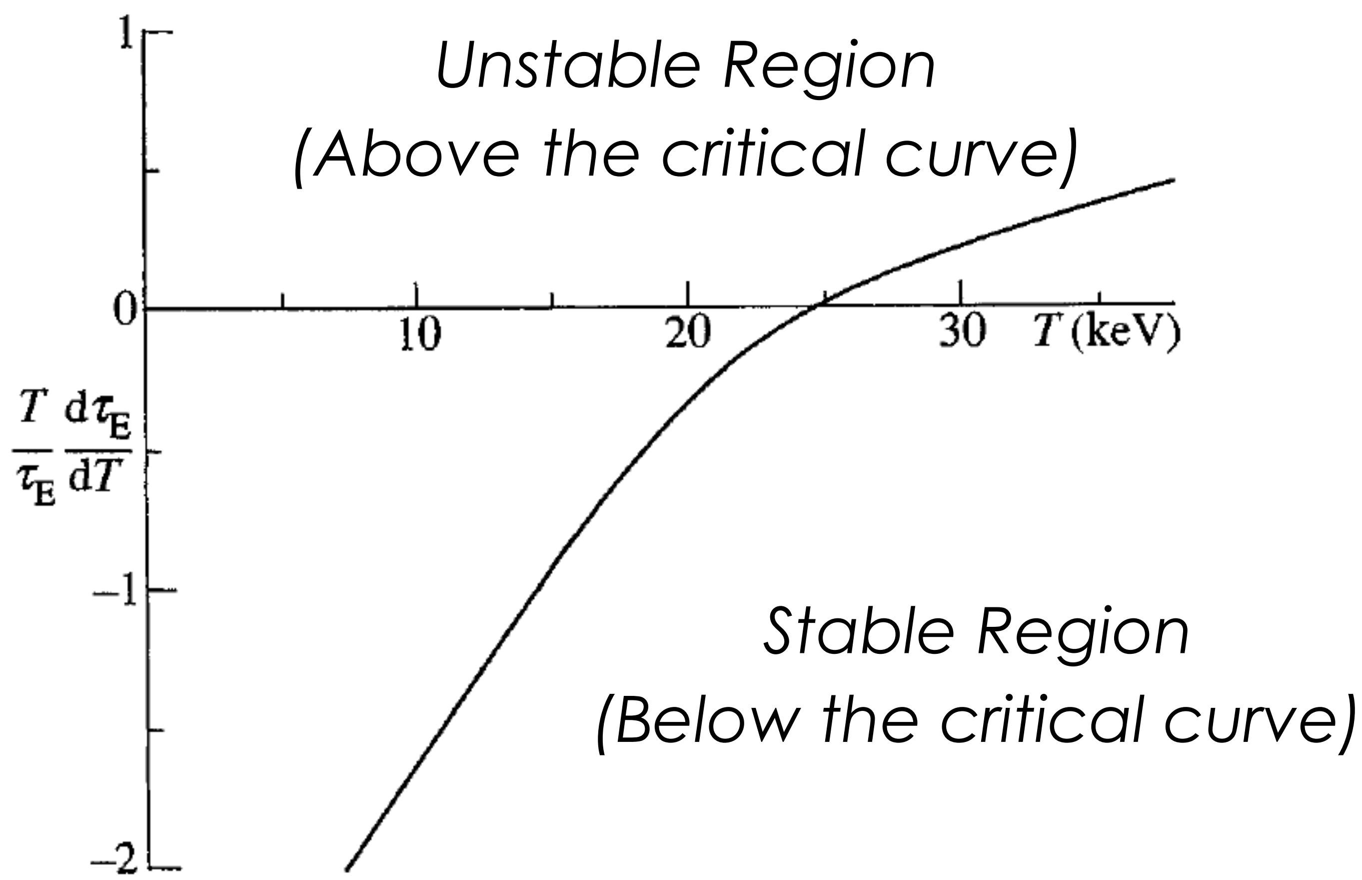
- **However, operation beyond ignition is not desired due to a thermal instability**

- **Although an ignited plasma does not need external heating to maintain the plasma, high fusion powers can be achieved at still finite values of Q**

Exercise

- **Show that the condition for thermal stability of a plasma with α-particle heating is given by (see Wesson, Ch. 1, at the end of section 5):**

$$\frac{T}{\tau_E}\frac{d\tau_E}{dT} < 1 - \frac{T}{\langle\sigma v\rangle}\frac{d\langle\sigma v\rangle}{dT}$$



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Scaling laws

- **From kinetic theory, the Vlasov equation must be invariant under just 3 scaling transformations [J.W. Connor and J.B. Taylor, Nuclear Fusion 17, 1047 (1977)]:**
  - *T1:* $f_j \rightarrow \alpha f_j$
  - *T2:* $t \rightarrow \beta^{-1} t\,,\quad v \rightarrow \beta v\,,\quad B \rightarrow \beta B\,,\quad E \rightarrow \beta^2 E$
  - *T3:* $t \rightarrow \gamma t\,,\quad x \rightarrow \gamma x\,,\quad E \rightarrow \gamma^{-1} E\,,\quad B \rightarrow \gamma^{-1} B$

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j + \frac{q_j}{m_j} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_j = 0$$

- **Let's suppose that the thermal energy confinement time depends on engineering parameters, for example, as**

$$\tau_{E,th} = C\, n^p\, B^q\, T^r\, a^s$$

- **Remember that** $n = \int f\, d^3v$ **and** $T = \dfrac{1}{n} \int \dfrac{1}{2} m v^2\, f\, d^3v$

Scaling laws

- **Equating the powers α, β and γ on the two sides of the resulting equation leads to 3 different constraints for the powers p, q r, and s**
  - *T1:* $\tau_{E,th} = C\,(\alpha\, n)^p\ B^q\ T^r\ a^s \quad \rightarrow \quad \alpha^p = 1$
  - *T2:* $\beta^{-1}\tau_{E,th} = C\,(\beta^3\, n)^p\ (\beta\, B)^q\ (\beta^2\, T)^r\ a^s \quad \rightarrow \quad \beta^{-1} = \beta^{3p+q+2r}$
  - *T3:* $\gamma\tau_{E,th} = C\, n^p\ (\gamma^{-1}\, B)^q\ T^r\ (\gamma\, a)^s \quad \rightarrow \quad \gamma = \gamma^{s-q}$
  - *This set of equations has as solution:* $p = 0,\ q + 2r = -1$ and $s - q = 1$
- **Substitution of these constraints in the proposed form of $\tau_{E,th}$ yields**

$$\tau_{E,th} = C\, B^q\, T^{-(1+q)/2}\, a^{1+q}$$

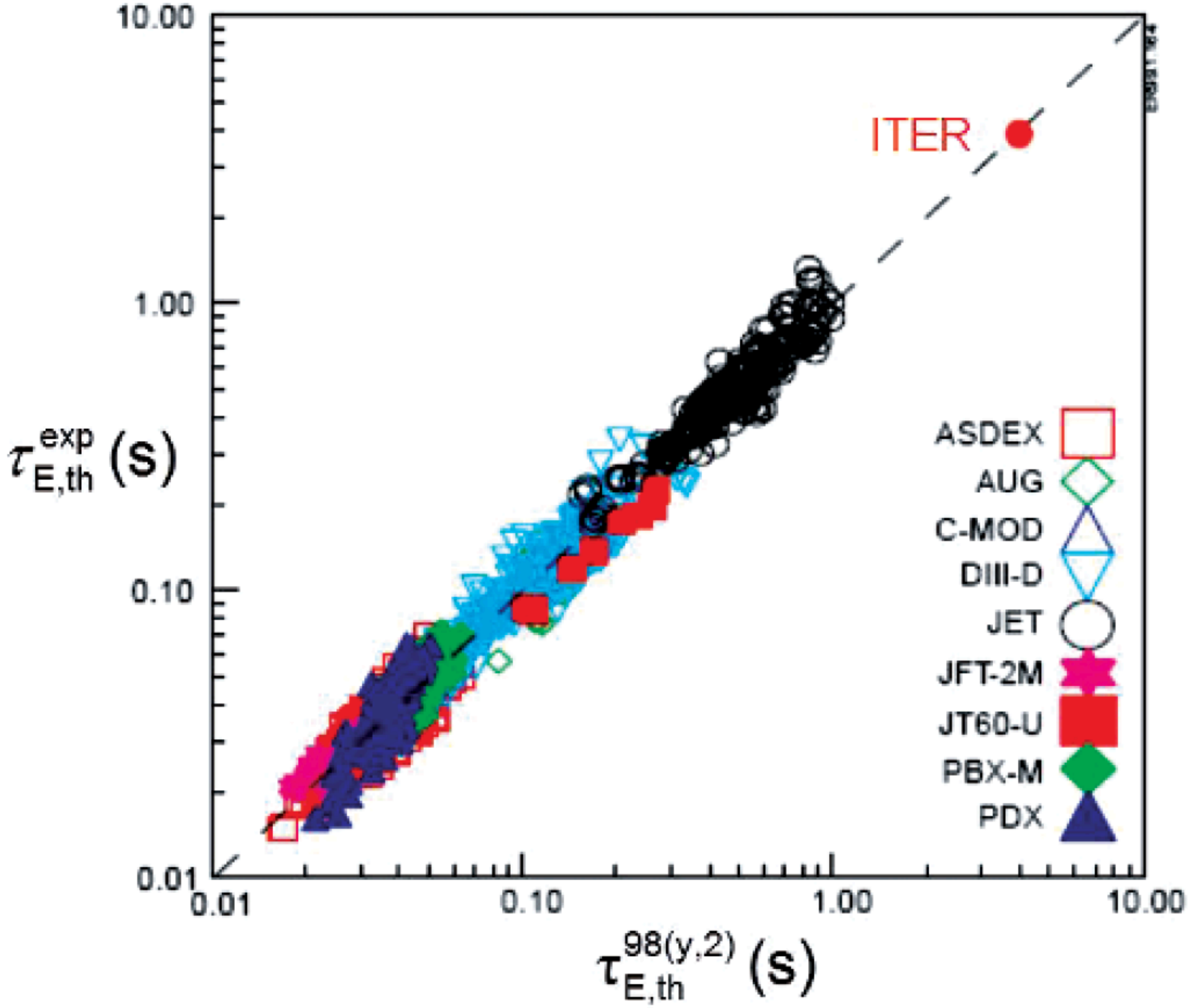
(Note that, in this simple case, there is only one free parameter left for the data fitting)

  - *This method does not give information about geometrical ratios, such as a/R₀*
  - *This method is of less value when atomic processes play a significant role*
  - *A more complex model, and, consequently, a more complete form for $\tau_{E,th}$, would lead to less constraints*

Scaling laws

- **The thermal energy confinement time scaling law (the IPB98(y,2) scaling) is**

$$\tau_{E,th} = 0.0562\, I_P^{0.93}\, B_0^{0.15}\, n_{19}^{0.41}\, P_H^{-0.69}\, R_0^{1.97}\, \epsilon^{0.58}\, \kappa_a^{0.78}\, M^{0.19}$$



References

- **Nuclear fusion and tokamak physics**
  - *Nuclear fusion reactions: Wesson, Ch. 1*
  - *Thermonuclear fusion: Wesson, Ch. 1*
  - *Breakeven and the Lawson criterion for ignition: Wesson, Ch. 1*
  - *Scaling laws: Wesson, Ch. 4, section 15*