PGF5112 - Plasma Physics I

By

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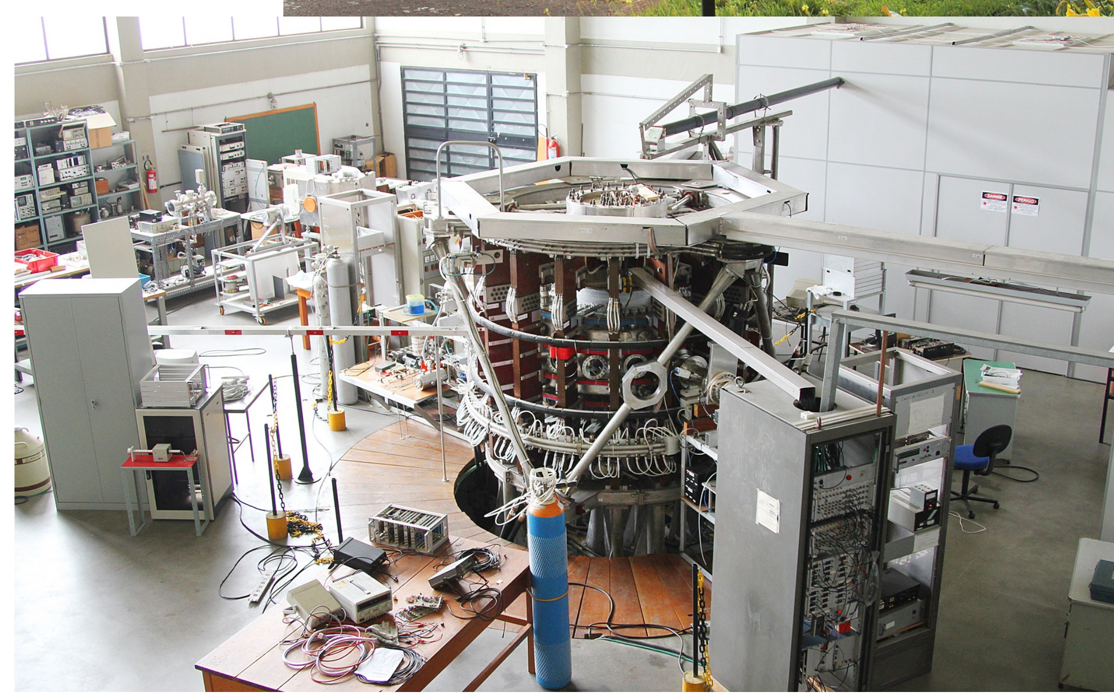

Graduate course ministered at the

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São Paulo - SP, 23 March 2023

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

- **Macroscopic features of plasmas**
  - *Quasi-neutrality*
  - *Debye shielding*
  - *Plasma oscillations*
  - *The plasma definition criteria*

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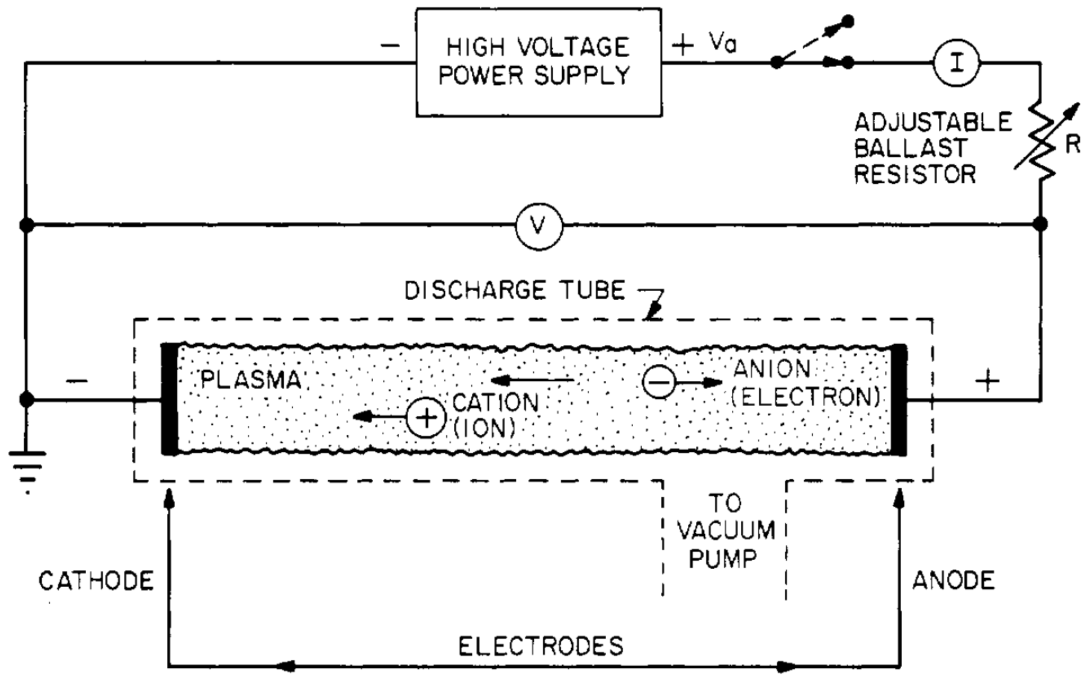
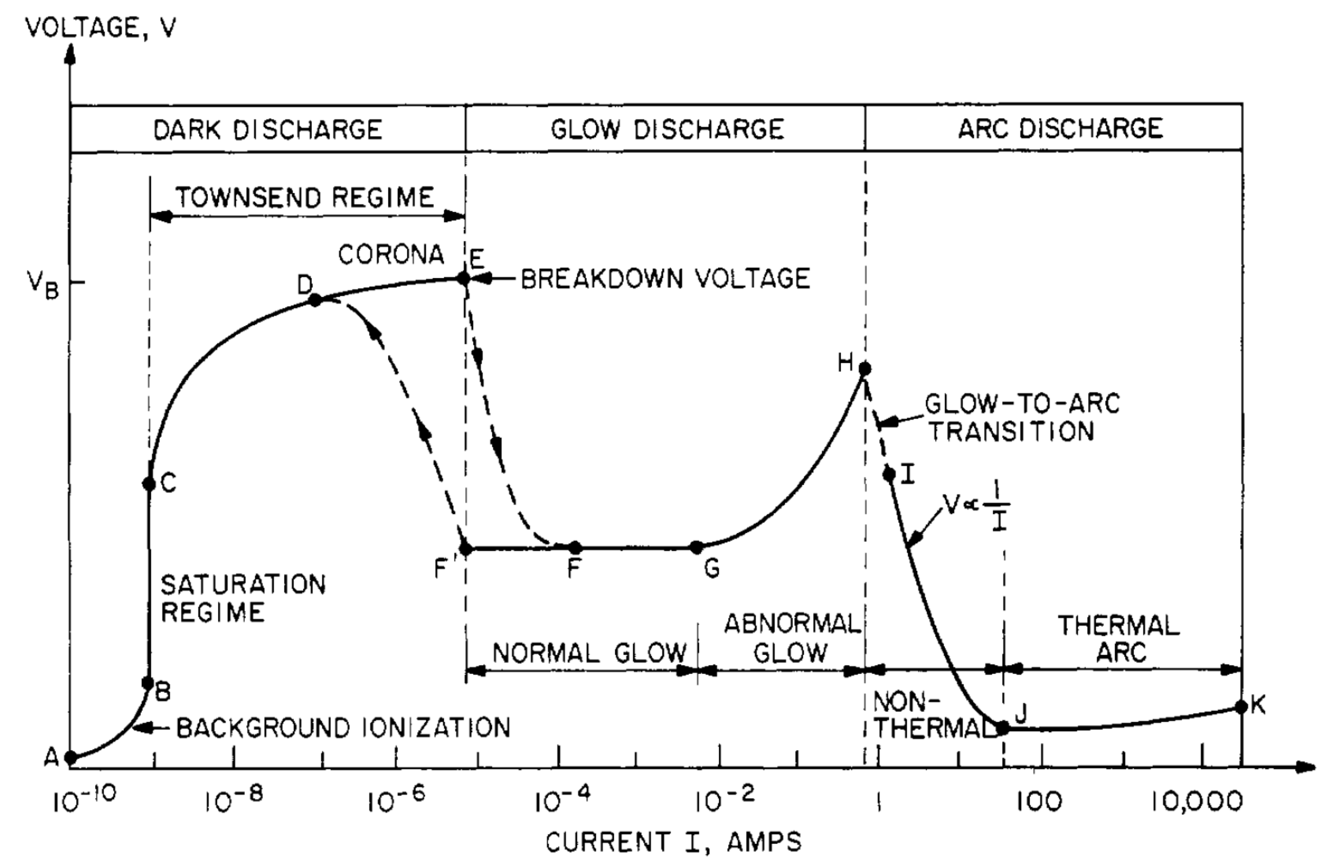
Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - + *Corona*
    - + *Breakdown*
- **Glow discharge**
  - Normal glow
  - Abnormal glow
- **Arc discharge**
  - Non-thermal
  - Thermal





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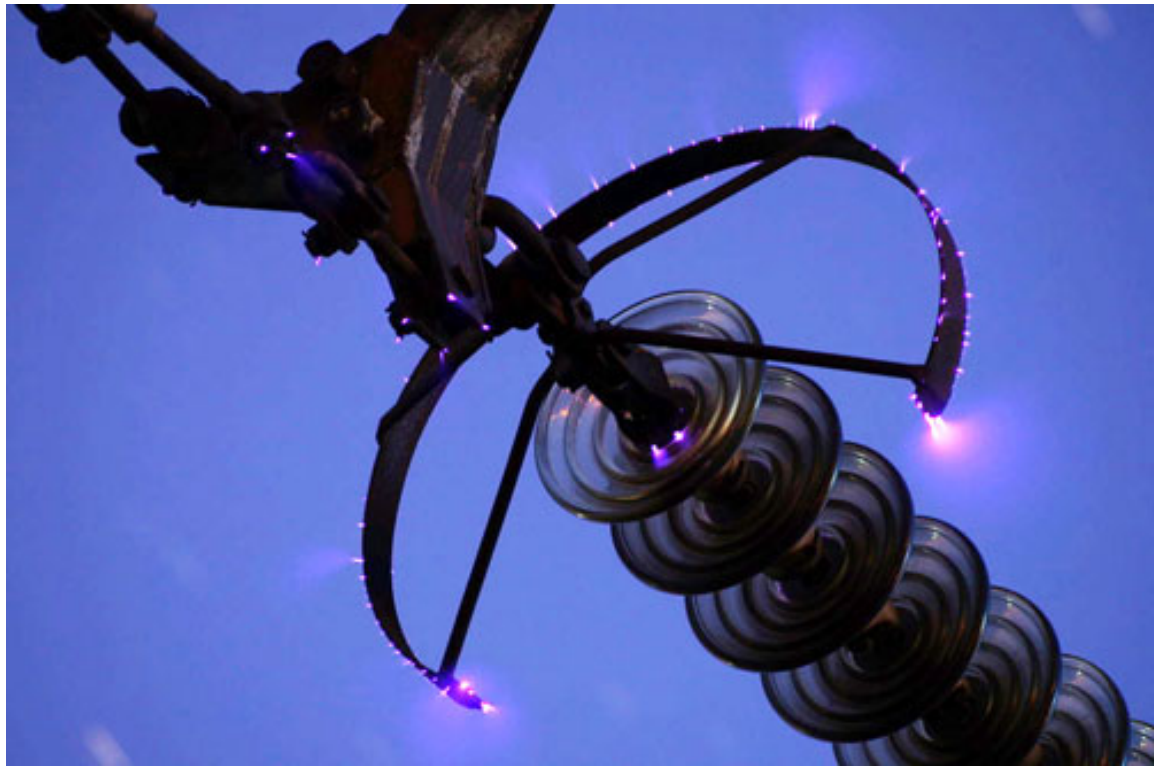
This regime is called Dark because the plasma does not emit enough light to be seem by human eye

This regime ends with the corona effect and finally the breakdown of the gas

Corona due to high voltage



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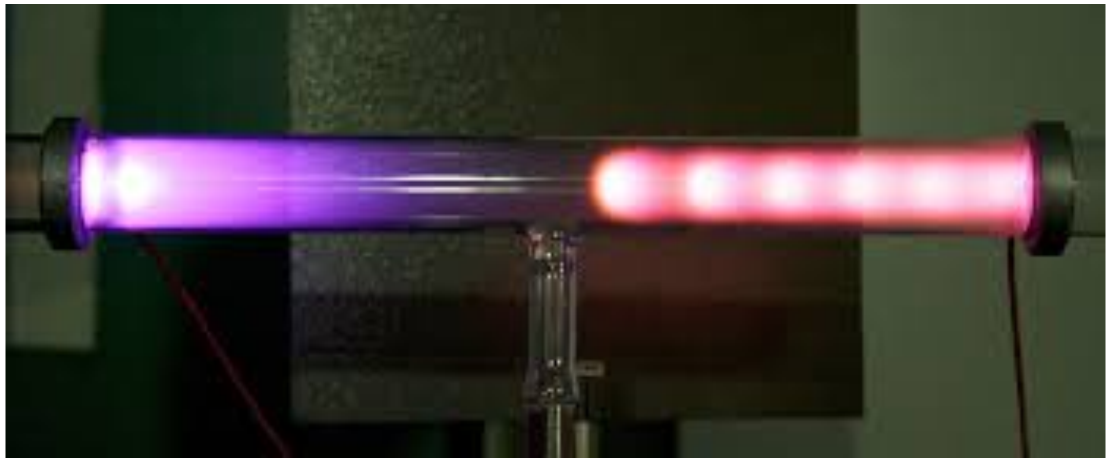
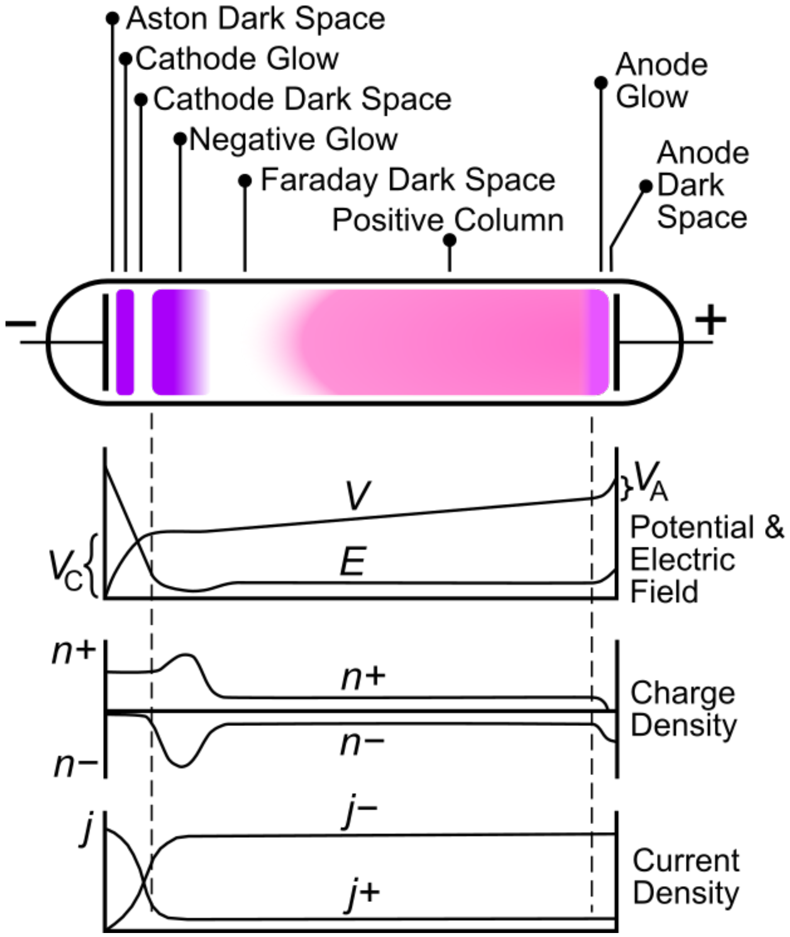


Very interesting phenomena appear in tube discharges (outside the scope of this course)

For those who are interested, see Ch. 9 of Industrial Plasma Engineering, Vol. 1



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Important applications (outside the scope of this course)

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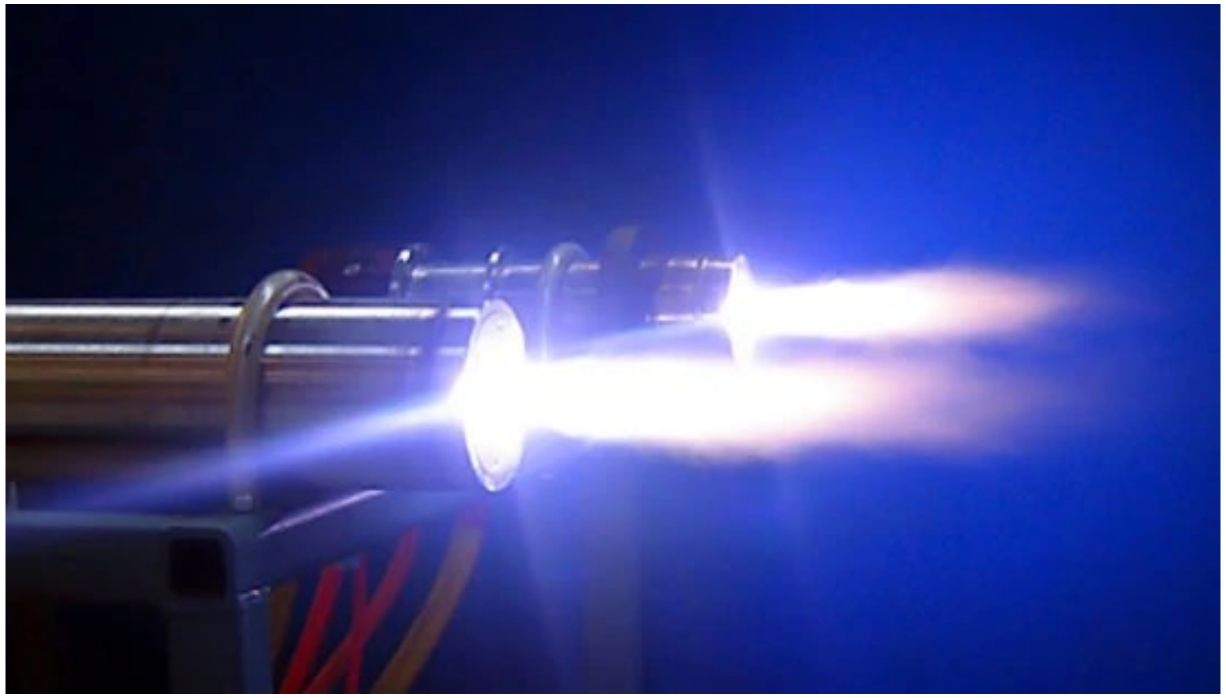
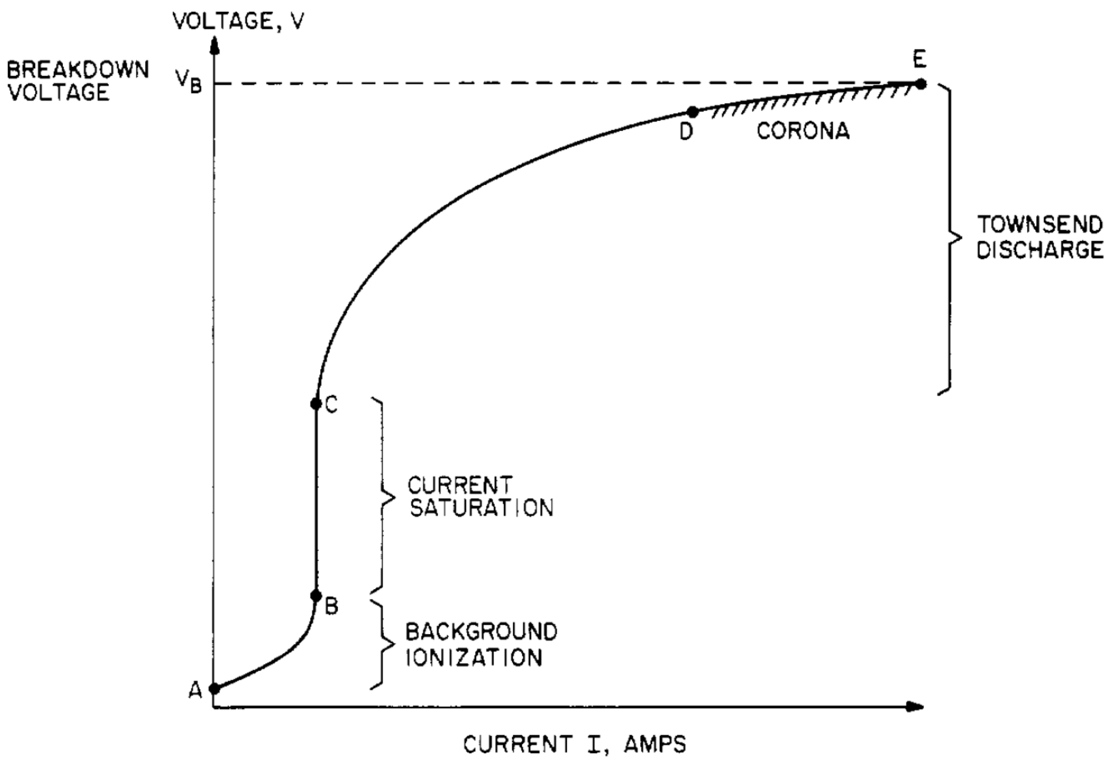


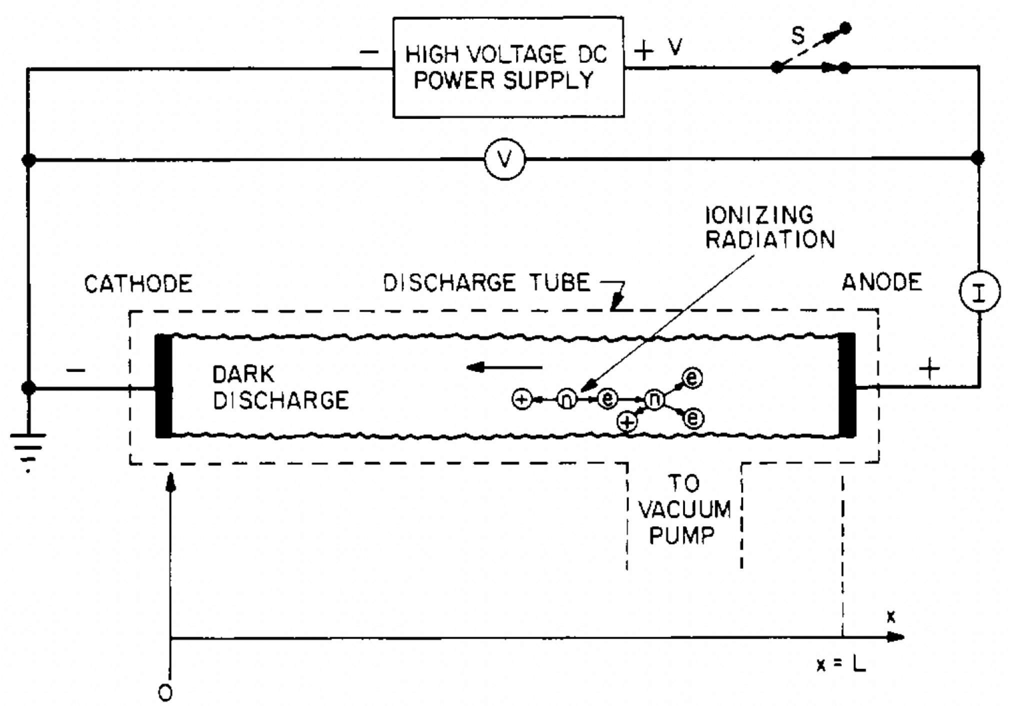
Important applications (outside the scope of this course)

For those who are interested, see Ch. 10 of Industrial Plasma Engineering, Vol. 1

Modeling of the dark discharge regime, specially the Townsend regime, is important for plasma breakdown

- **Background ionization**
  - Ions and electrons created by radiation, cosmic rays, radioactive mineral, etc.
- **Saturation current**
  - Eventually, all charged particles are contributing to the current
  - If $S_e = S_i = dn/dt$, the saturation current is $I_{\text{sat}} = dq/dt = d(enAL)/dt = eSAL$ (pA to nA)
- **Townsend discharge**
  - The current increases due to an avalanche process

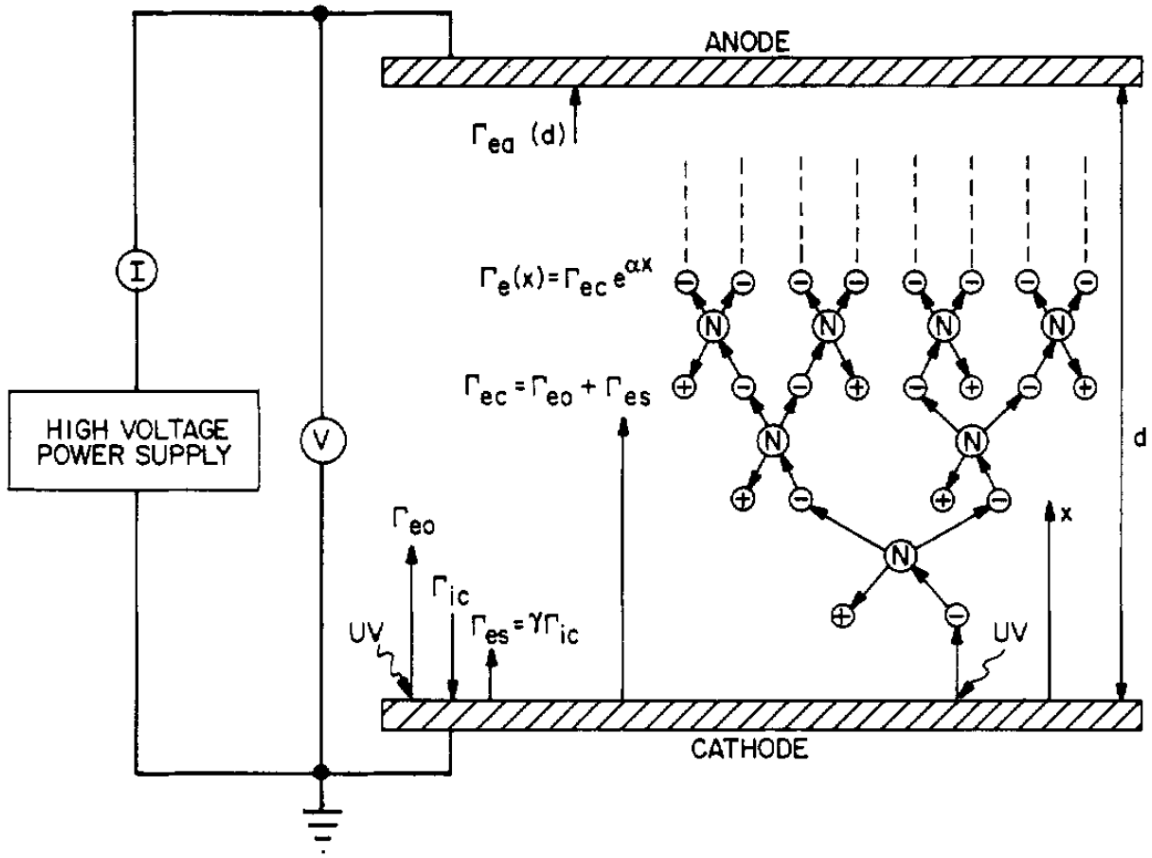




Modeling of the dark discharge regime, specially the Townsend regime, is important for plasma breakdown

- **Electron-ion pair created by background ionization are accelerated by the applied electric field (E = V/d)**

- **At sufficiently high voltages, these electrons can eventually ionize the neutral gas**
  - α is the Townsend's first ionization coefficient: # of ionizations per meter



$$d\Gamma_e = \alpha\Gamma_e dx \qquad \int_{\Gamma_{e0}}^{\Gamma_e} \frac{d\Gamma_e}{\Gamma_e} = \int_0^x \alpha dx \qquad \Gamma_e(x) = \Gamma_{e0}e^{\alpha x}$$

- **If there exists a volumetric source of electrons, e.g. due to cosmic rays,** $S_e = \dfrac{dn}{dt}$

$$d\Gamma_e = \alpha\Gamma_e dx + S_e dx \qquad \int_0^{\Gamma_e} \frac{d\Gamma_e}{\alpha\Gamma_e + S_e} = \int_0^x dx = \frac{\ln(\alpha\Gamma_e + S_e)}{\alpha}\Bigg|_0^{\Gamma_e} \qquad \Gamma_e(x) = \frac{S_e}{\alpha}\left(e^{\alpha x} - 1\right)$$

The Townsend first ionization coefficient

- **The probability that an electron has not ionized a neutral after traveling a distance x is (λ_i is the m.f.p. for ionization)**

$$\frac{n_e(x)}{n_{e0}} = e^{-\frac{x}{\lambda_i}}$$

- **The Townsend's first coefficient can be estimated as**

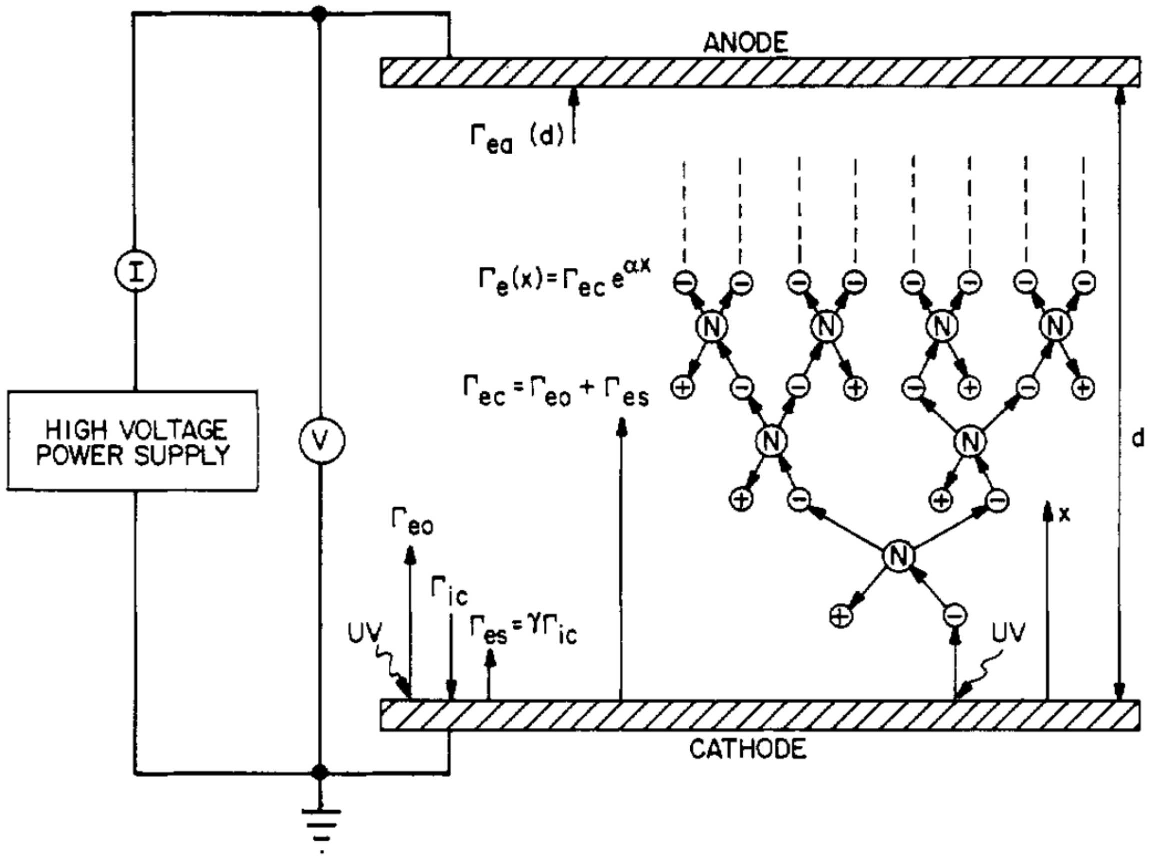
$$\alpha = \frac{1}{\lambda_i}\frac{n_e(x)}{n_{e0}} = \frac{e^{-\frac{x}{\lambda_i}}}{\lambda_i}$$

- **The mean free path for ionization is**

$$\frac{1}{\lambda_i} = \frac{n_0\langle\sigma_{\text{ion}} v_e\rangle}{\langle v_e\rangle} \approx n_0\langle\sigma_{\text{ion}}\rangle = Ap$$

- **If one sets $x = x_i = V_{\text{i}}/E$ as the distance an electron have to travel in the electric field E to gain enough energy to ionize an neutral, one has that**

$$\alpha = Ape^{-\frac{C}{E/p}}$$



A and C are constants that depends of the type of gas

V_{i} is the ionization potencial

Exercise: the Stoletow point

- **What is the pressure that maximizes the current in the plasma for a fixed electric field?**
  - Hint: It is the pressure that maximizes the Townsend first ionization coefficient

  - Answer: $p_{\max} = \dfrac{E}{C}$ (Eq. 8.33 of Industrial Plasma Engineering, Vol. 1)

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The effect of secondary electrons

- **The flux of electrons at the Anode is**

$$\Gamma_{ea} = \Gamma_{ec} e^{\alpha d}$$

- **The emission of secondary electrons is proportional to the ion flux at the cathode**
  - *γ is the secondary electron emission coefficient*

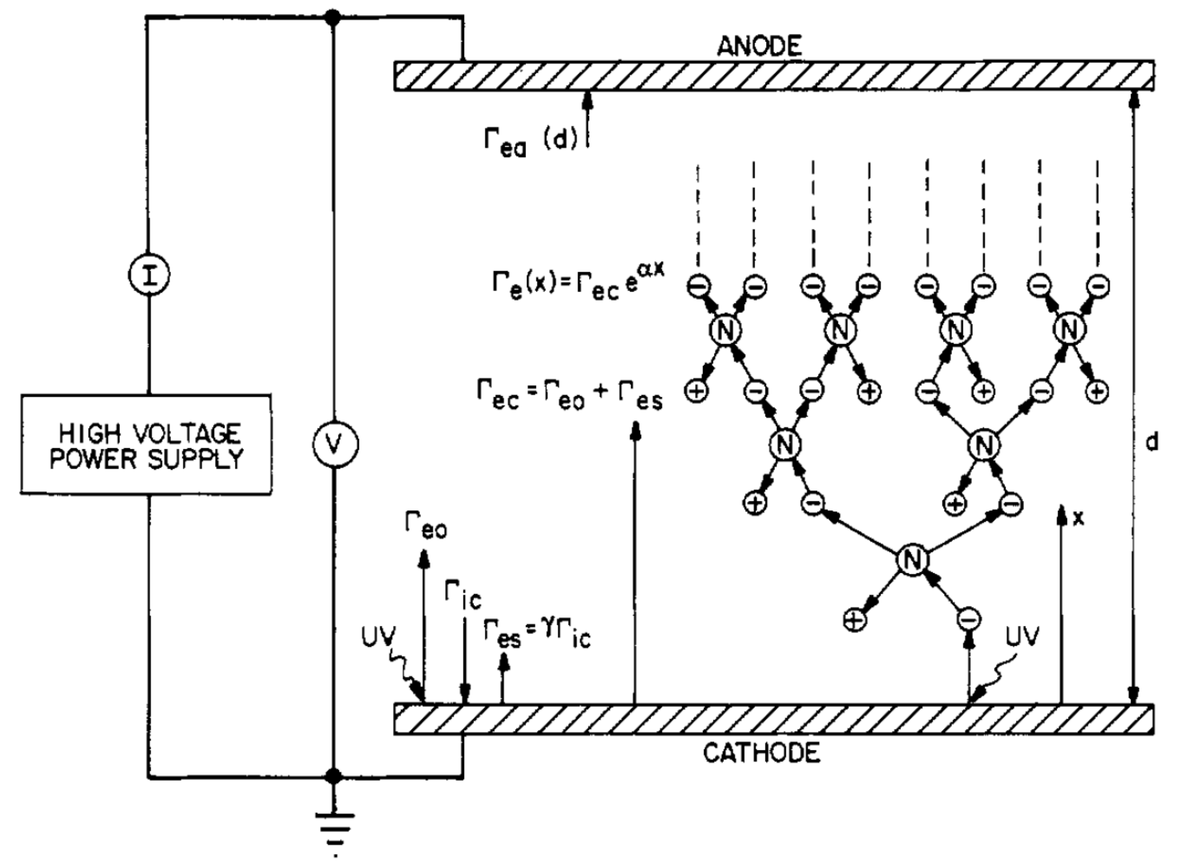
$$\Gamma_{ec} = \Gamma_{e0} + \Gamma_{es} = \Gamma_{e0} + \gamma \Gamma_{ic}$$

- **In the steady state, one must have**

$$\Gamma_{ea} - \Gamma_{ec} = \Gamma_{ic}$$

- **Therefore, after some algebra:**

$$\Gamma_{ea} = \Gamma_{e0} \frac{e^{\alpha d}}{1 - \gamma \left(e^{\alpha d} - 1 \right)}$$



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The Townsend criterion for breakdown

- **The Townsend criterion for breakdown is obtained from the condition for which the current between the two plates increases without limit**

$$J_{ea} = = e\Gamma_{ea} = e\Gamma_{e0}\frac{e^{\alpha d}}{1-\gamma\left(e^{\alpha d}-1\right)} \longrightarrow 1-\gamma\left(e^{\alpha d}-1\right)=0 \longrightarrow \gamma e^{\alpha d} = \gamma + 1$$

- **Defining the breakdown voltage as $V_b = E_b d$ and using the expression for α**

$$\ln\left(1+\frac{1}{\gamma}\right) = \alpha d = Apd\exp\left(-\frac{Cpd}{V_b}\right)$$

Gas	A ion pairs/m-Torr	C V/m-Torr
A	1200	20 000
Air	1220	36 500
CO_2	2000*	46 600
H_2	1060	35 000
HCl	2500*	38 000
He	182	5 000
Hg	2000	37 000
H_20	1290*	28 900
Kr	1450	22 000
N_2	1060	34 200
Ne	400	10 000
Xe	2220	31 000

- **Manipulating this equation leads to the Paschen law**

$$V_b = \frac{Cpd}{\ln\left[Apd/\ln\left(1+\frac{1}{\gamma}\right)\right]} = f(pd)$$

The Townsend criterion for breakdown

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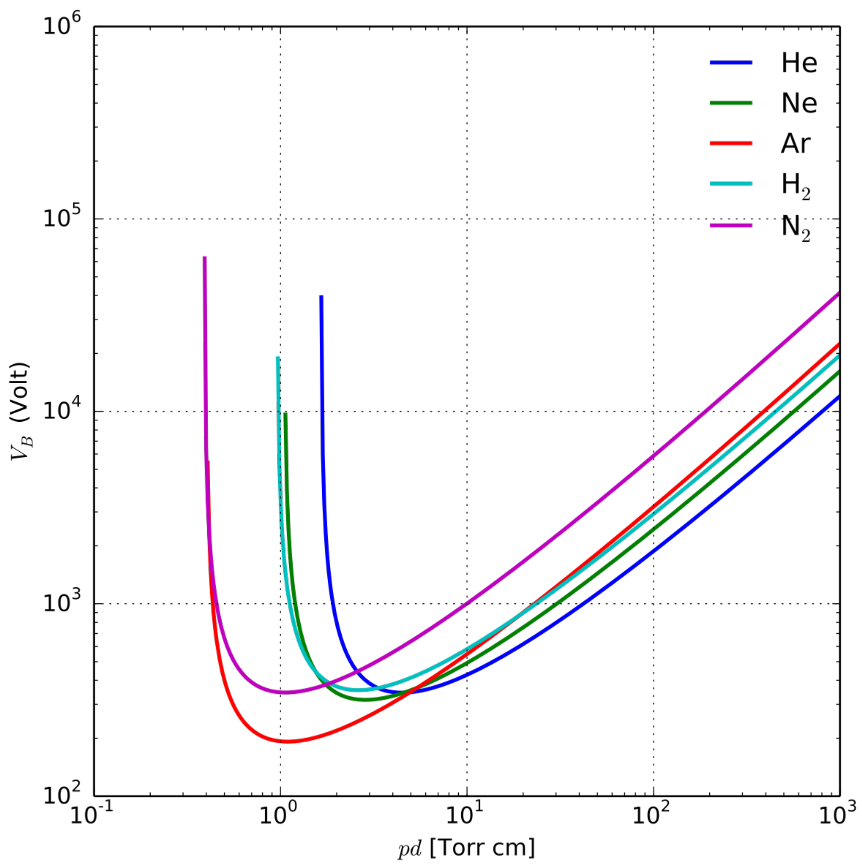


The minimum breakdown voltage

- **Differentiating the expression of V_b with respect to pd yield**

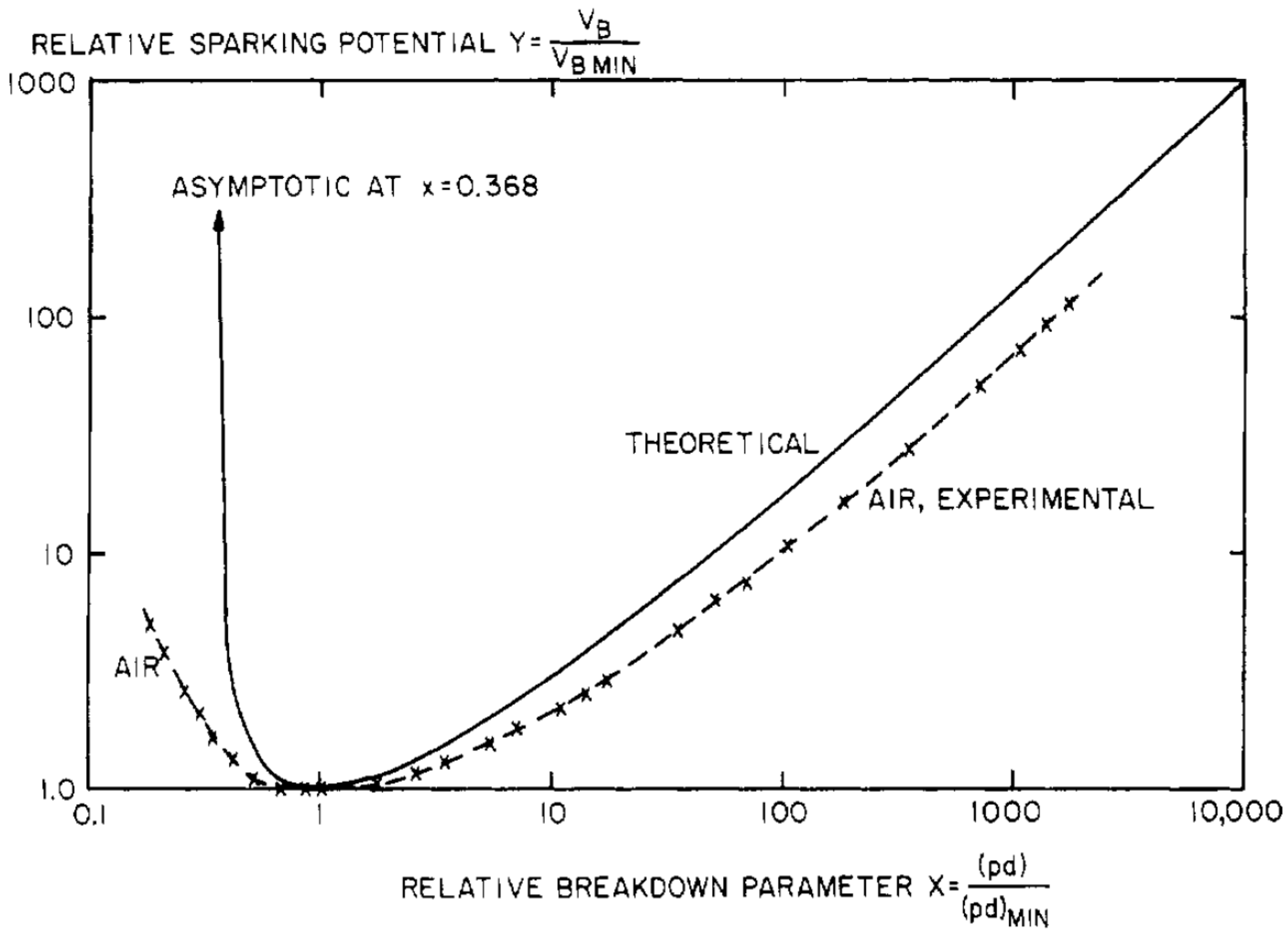
$$(pd)_{\min} = \frac{e^1}{A} \ln\left(1 + \frac{1}{\gamma}\right)$$

- **Let's define the parameters**

$$X = \frac{pd}{(pd)_{\min}} \qquad Y = \frac{V_\mathrm{b}}{V_\mathrm{b,min}}$$

- **The universal Paschen curve**

$$Y = \frac{X}{1 + \ln X}$$



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Macroscopic features of plasmas: *quasi*-neutrality

- **Plasmas have a natural tendency of remaining electrically neutral**

$$\rho = \sum_{\text{j (all species)}} q_j n_j = 0 \quad \text{or} \quad \sum_{\text{i (ions)}} Z_i n_i = n_e \quad \text{(Charge neutrality condition)}$$

- **If there exists any charge imbalance within a small spherical region in a plasma:**
  - An electric field is established
  - Electrons are accelerated in the direction of the positive charges
  - In a short time interval, *quasi*-neutrality is restored

Net electric charge due to charge imbalance: $Q = \frac{4}{3}\pi a^3 (n_i - n_e) e$

Electric potential at the surface of the spherical region: $\phi = \frac{1}{4\pi\epsilon_0}\frac{Q}{a} = \frac{e(n_i - n_e)}{3\epsilon_0}a^2$

Macroscopic features of plasmas: *quasi*-neutrality

- **For a plasma with $n_{i0} = n_{e0} = 1\times10^{20}$ m^{-3} and a charge imbalance $\delta n_i/n_{i0} = (n_i - n_{i0})/n_{i0} = 1\,\%$ within a spherical region of 1 mm of radius:**

$$Q = \frac{4}{3}\pi a^3(n_i - n_e)e = \frac{4}{3}\pi a^3(n_{i0} + \delta n_i - n_{e0})e = \frac{4\pi e n_{i0}}{3}\frac{\delta n_i}{n_{i0}}a^3 = 6.7\times10^{-10}\ C$$

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{Q}{a} = \frac{e(n_i - n_e)}{3\epsilon_0}a^2 = 6.0\ kV \qquad E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{a^2} = \frac{e(n_i - n_e)}{3\epsilon_0}a = 6.0\ MV/m$$

$$|\mathbf{F}| = |e\mathbf{E}| = 1.6\times10^{-19}\times6.0\times10^{6} \approx 9.6\times10^{-13}\ N$$ **(Force doesn't seem so strong)**

$$|\mathbf{a}| = \frac{|\mathbf{F}|}{m_e} = \frac{|e\mathbf{E}|}{m_e} = \frac{1.6\times10^{-19}\times6.0\times10^{6}}{9.11\times10^{-31}} \approx 1\times10^{18}\ m/s^2$$

(Acceleration is huge!!!)

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Macroscopic features of plasmas: Debye shielding

- **Charge imbalance occurs naturally in plasmas only in a small region whose typical size is of the order of the so-called Debye length (see Bittencourt ch. 11, section 2)**

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_0 e^2}}$$

- **The Debye length is the typical distance at which the particle electric potencial energy balances its thermal energy**
  - The electric potential energy tend to restore *quasi*-neutrality
  - The thermal energy of the particles tend to break *quasi*-neutrality

Macroscopic features of plasmas: Debye shielding

- **Let's isolate one single charge from a (globally neutral) plasma and see how this charge interacts with all the other particles**

- **From Poisson's equation:**

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} = -\frac{\left[Z_i\, e\, n_i(\mathbf{r}) - e\, n_e(\mathbf{r})\right]}{\epsilon_0} - \frac{q_t}{\epsilon_0}\delta(\mathbf{r})$$

- **From Boltzmann's equation (see Bittencourt ch. 7, section 5)**

$$n_j(\mathbf{r}) = n_{j0}\exp\left[-\frac{U(\mathbf{r})}{k_B\, T_j}\right] = n_{j0}\exp\left[-\frac{q_j\,\phi(\mathbf{r})}{k_B\, T_j}\right]$$

- **Suppose that** $n_e(\mathbf{r}\to\infty) = n_i(\mathbf{r}\to\infty) = n_0$

$$\nabla^2\phi + \frac{en_0}{\epsilon_0}\left[Z_i\exp\left(-\frac{Z_i e\phi}{k_B T_i}\right) - \exp\left(\frac{e\phi}{k_B T_e}\right)\right] = -\frac{q_t}{\epsilon_0}\delta(\mathbf{r})$$

Macroscopic features of plasmas: Debye shielding

- **Let's suppose that the perturbing electrostatic potential due to the test charge is weak so that the electrostatic potential energy of the particles is much smaller than their mean thermal energy, i.e.** $q_j\phi(r) \ll k_B T_j$ **:**

$$\exp\left(-\frac{q_j\phi}{k_B T_j}\right) \approx 1 - \frac{q_j\phi}{k_B T_j}$$

- **Therefore,** $\nabla^2\phi - \frac{\phi}{\lambda_D^2} = -\frac{q_t}{\epsilon_0}\delta(\mathbf{r})$**, with** $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_{\text{eff}}}{n_0 e^2}}$ **or** $\lambda_D = 7433\sqrt{\frac{T_{\text{eff}}}{n_0}}$

(Debye length)

$T_{\text{eff}} = \frac{T_e T_i}{(Z_i T_e + T_i)}$ **in eV**

n_0 **in m⁻³**

- **Debye length for some particular cases**
  - Tokamaks ($n_0 = 1\times10^{20}\ m^{-3};\ T_e = T_i = 1\times10^{8}\ K$): $\lambda_D = 5\times10^{-5}\ m$
  - Solar corona ($n_0 = 1\times10^{12}\ m^{-3};\ T_e = T_i = 1\times10^{6}\ K$): $\lambda_D = 0.05\ m$

Macroscopic features of plasmas: Debye shielding

- **In spherical coordinates, one has that**

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\phi}{dr}\right]-\frac{\phi}{\lambda_D^2}=-\frac{q_t}{\epsilon_0}\delta(\mathbf{r})$$

- **Let's try to solve this equation with the ansatz** $\phi(r)=\phi_c(r)f(r)=\frac{q_t}{4\pi\epsilon_0}\frac{f(r)}{r}$ **, where** $f(r\rightarrow 0)=1$ **and** $\phi(r\rightarrow\infty)=0$**.**

- **For** $\mathbf{r}\neq 0$**, one has that:** $\frac{d^2f}{dr^2}=\frac{f}{\lambda_D^2}\ \rightarrow\ f(r)=A\exp\left(\frac{r}{\lambda_D}\right)+B\exp\left(-\frac{r}{\lambda_D}\right)$

- **The condition** $\phi(r\rightarrow\infty)=0$ **implies that** $A=0$**, while the condition** $f(r\rightarrow 0)=1$ **implies that** $B=1$**. Therefore,**

$$\phi(r)=\phi_c(r)\ \exp\left(-\frac{r}{\lambda_D}\right)=\frac{1}{4\pi\epsilon_0}\frac{q_t}{r}\exp\left(-\frac{r}{\lambda_D}\right)$$

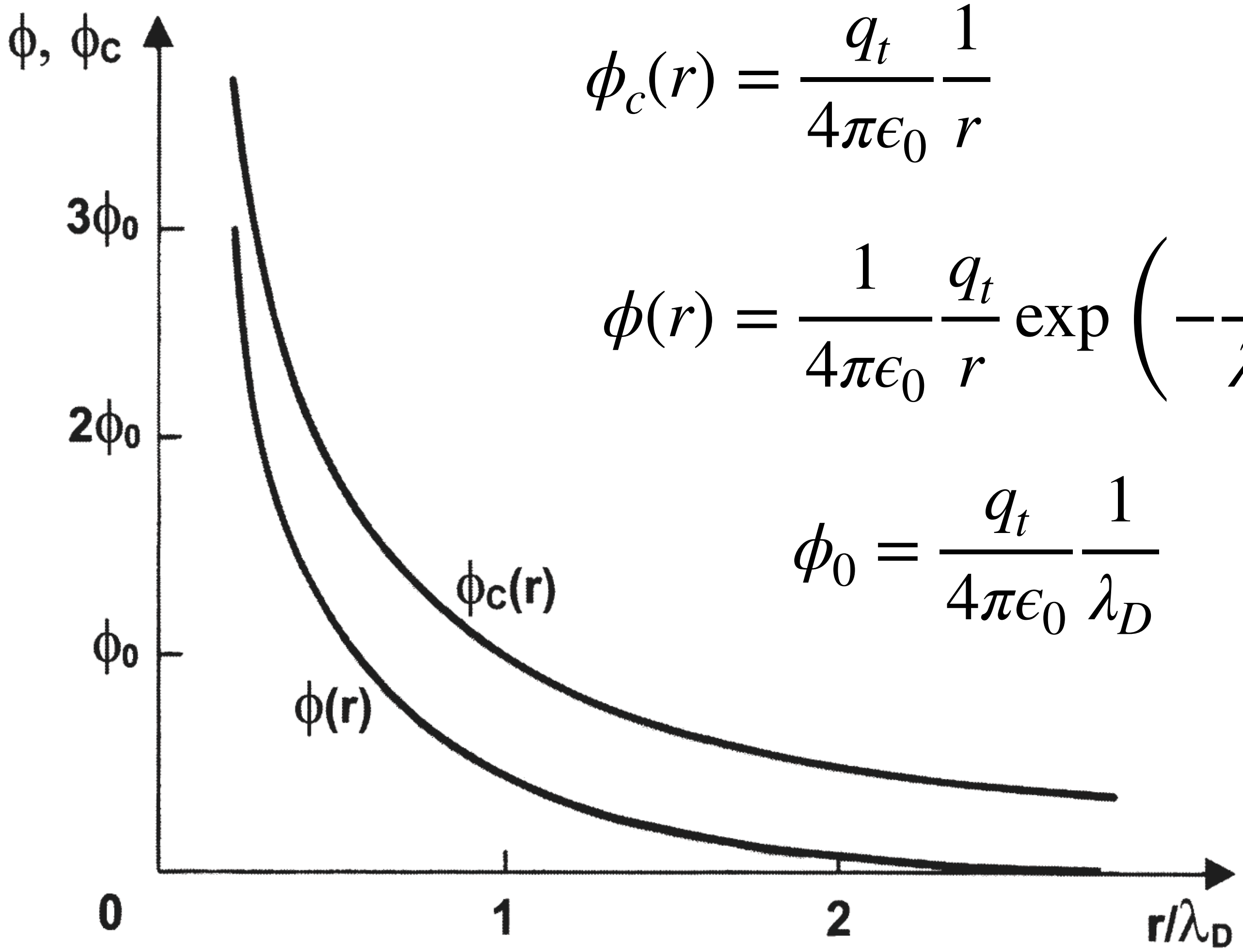
Macroscopic features of plasmas: Debye shielding

- Near the test particle ($r \ll \lambda_D$), the electric potential created by the test charge is given by the Coulomb potential

- Far from the test particle ($r \gg \lambda_D$), the electric potential is significantly smaller that the Coulomb potential of the test charge

- The number of particles that interact collectively with the test charge is about the number of charges within the Debye sphere

$$N_D = \frac{4\pi}{3}\lambda_D^3 n_0$$



$$\phi_c(r) = \frac{q_t}{4\pi\epsilon_0}\frac{1}{r}$$

$$\phi(r) = \frac{1}{4\pi\epsilon_0}\frac{q_t}{r}\exp\left(-\frac{r}{\lambda_D}\right)$$

$$\phi_0 = \frac{q_t}{4\pi\epsilon_0}\frac{1}{\lambda_D}$$

Macroscopic features of plasmas: Debye shielding

- **Close to the test charge, the assumption $q_j\phi(r) \ll k_B T_j$ is unlikely to be fulfilled**

- **Taking $q_t = e$, let's calculate the parameter**

$$\frac{e\phi}{k_B T} = \frac{e^2}{4\pi\epsilon_0 k_B T}\frac{\exp(-r/\lambda_D)}{r}$$

- **If one sets $r = n_0^{-1/3}$ (average distance between charged particles) one has that**

$$\left.\frac{e\phi}{k_B T}\right|_{r=n_0^{-1/3}} = \frac{e^2}{4\pi\epsilon_0 k_B T}\frac{\exp(-n_0^{-1/3}/\lambda_D)}{n_0^{-1/3}}$$

- **Defining the so-called Plasma Parameter $g = \dfrac{1}{n_0\lambda_D^3}$**

$$\left.\frac{e\phi}{k_B T}\right|_{r=n_0^{-1/3}} = \frac{g^{3/2}\exp(-g^3)}{4\pi}$$

- **Therefore, a necessary condition for Debye shielding to occur is $g \ll 1$**

Exercises

- **(1) What is the total charge of the plasma mentioned in slide 24? Is the plasma globally neutral?**

$$Q = \int \rho \, dV \qquad \textbf{(Total charge)}$$

- **(2) If $T_e \gg T_i$, which particle species determines the Debye length? Why?**

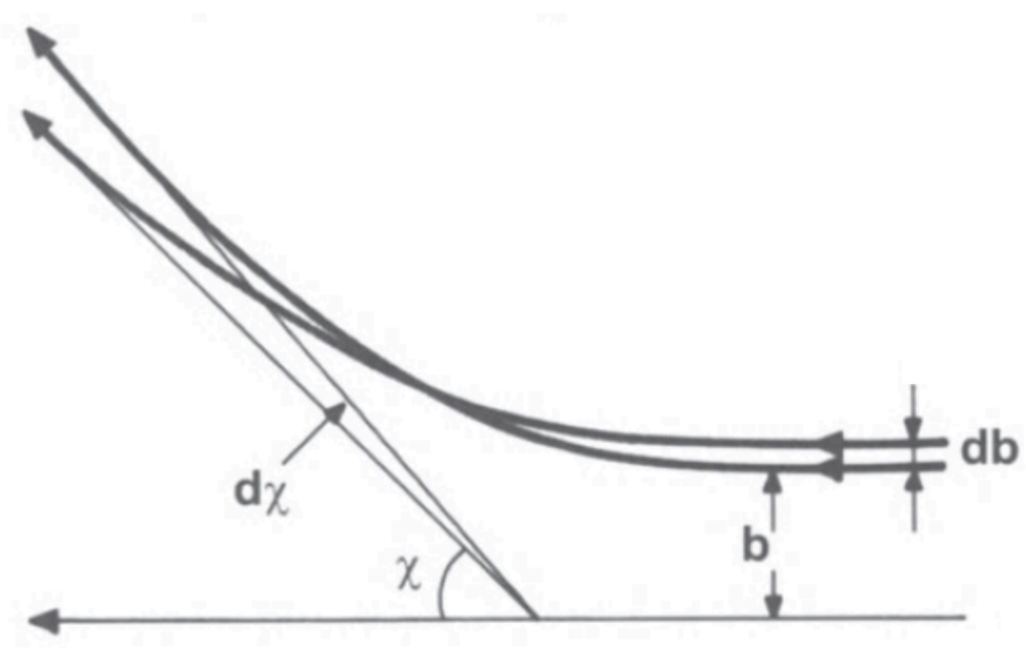
Screening of the Coulomb potential

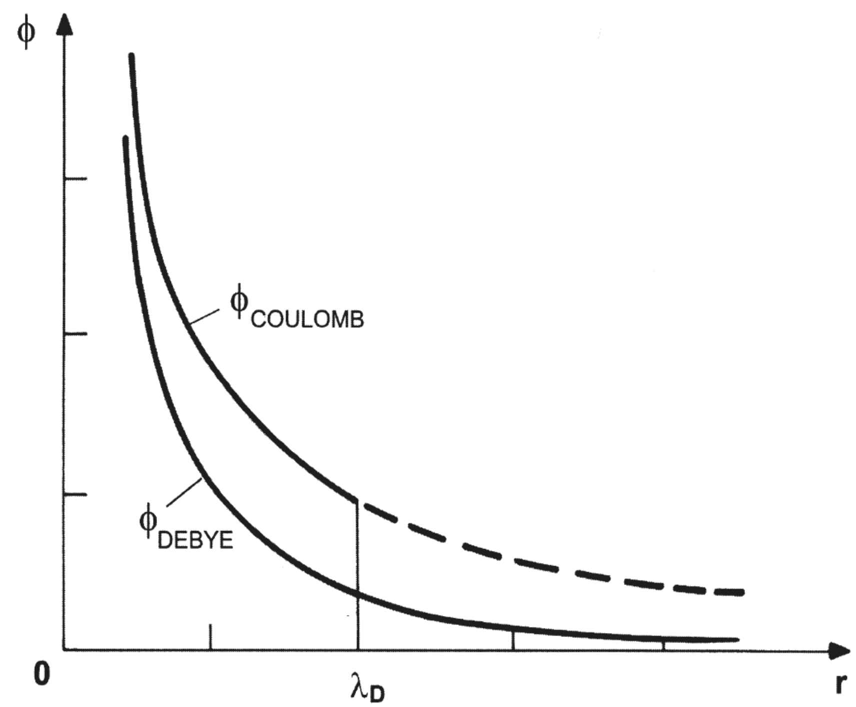
- **Total cross section (considering $\sigma(\chi,\epsilon)d\Omega = b\,db\,d\epsilon$ independent of ϵ)**

$$\sigma_t = \int \sigma(\chi,\epsilon)d\Omega = \int_{b_{\min}}^{b_c}\int_0^{2\pi} b\,db\,d\epsilon = 2\pi\int_{b_{\min}}^{b_c} b\,db = \pi(b_c^2 - b_{\min}^2)$$

- **Deflections with $0 < b < b_0$, which yield scattering angles $\pi/2 < \chi < \pi$ are usually called** *large-angle deflections* **or** *close encounters*

$$\sigma_{t,large} = \pi b_0^2$$





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- **Deflections with $0 < b < b_0$, which yield scattering angles $\pi/2 < \chi < \pi$ are usually called** *large-angle deflections* **or** *close encounters*

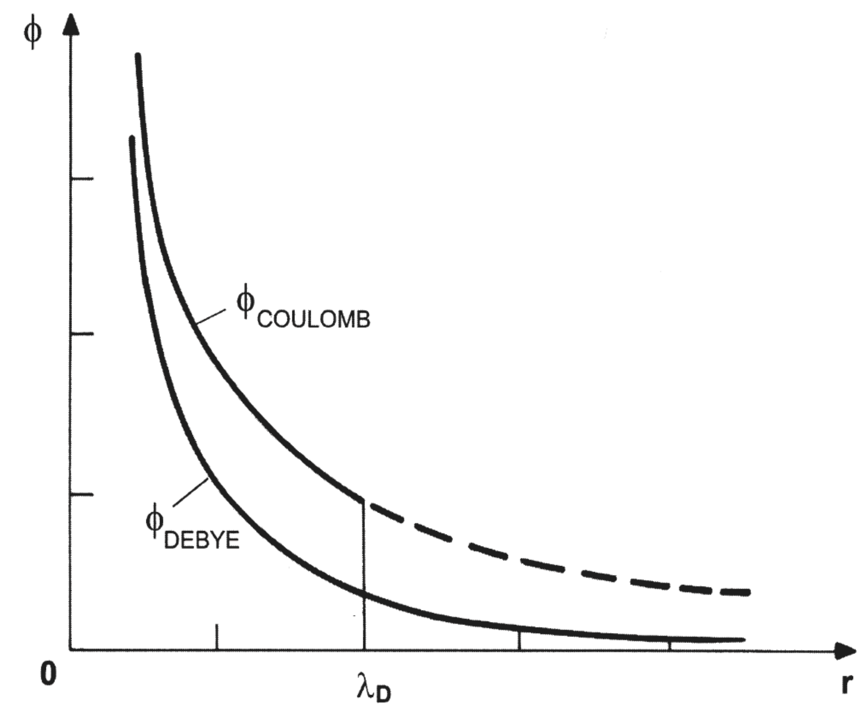
$$\sigma_{t,large} = \pi b_0^2$$

- **Deflections with $b_0 < b < \lambda_D$, which yield scattering angles $\chi < \pi/2$, are usually called** *small-angle deflections*

$$\sigma_{t,small} = \pi(\lambda_D^2 - b_0^2) \approx \pi\lambda_D^2 \qquad (\lambda_D \gg b_0)$$

- **Small-angle deflections are much more frequent than large-angle deflections**

$$\frac{\sigma_{t,small}}{\sigma_{t,large}} = \frac{\lambda_D^2}{b_0^2} - 1 \approx \frac{\lambda_D^2}{b_0^2} \gg 1$$



Momentum transfer cross section due to the Coulomb potential

- **Momentum transfer cross section (see Bittencourt ch. 20, section 8)**

$$\sigma_m = \int (1 - \cos\chi)\sigma(\chi, \epsilon) d\Omega$$

- **For the special case of an isotropic interaction potential**

$$\sigma_m = 2\pi \int_0^{\pi} (1 - \cos\chi)\sigma(\chi)\sin\chi\, d\chi$$

- **Using the result,** $\sigma(\chi) = b_0^2/(1 - \cos\chi)^2$ **one has that**

$$\sigma_m = 4\pi b_0^2 \ln\left[\frac{1}{\sin(\chi_{\min}/2)}\right]$$

- **Using that** $\sin(\chi_c/2) = \left(1 + b_c^2/b_0^2\right)^{-1/2}$ **one obtains** $\sigma_m = 4\pi b_0^2 \ln\Lambda$ **, where** $\Lambda = \dfrac{\lambda_D}{b_0}$
- **It is interesting to note that the parameter** $\Lambda = \dfrac{12\pi\epsilon_0 k_B T}{e^2}\lambda_D = 12\pi n_0 \lambda_D^3 = \dfrac{12\pi}{g} = 9N_D$

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

- **Macroscopic features of plasmas**
  - *Quasi-neutrality*
  - *Debye shielding*
  - *Plasma oscillations*
  - *The plasma definition criteria*

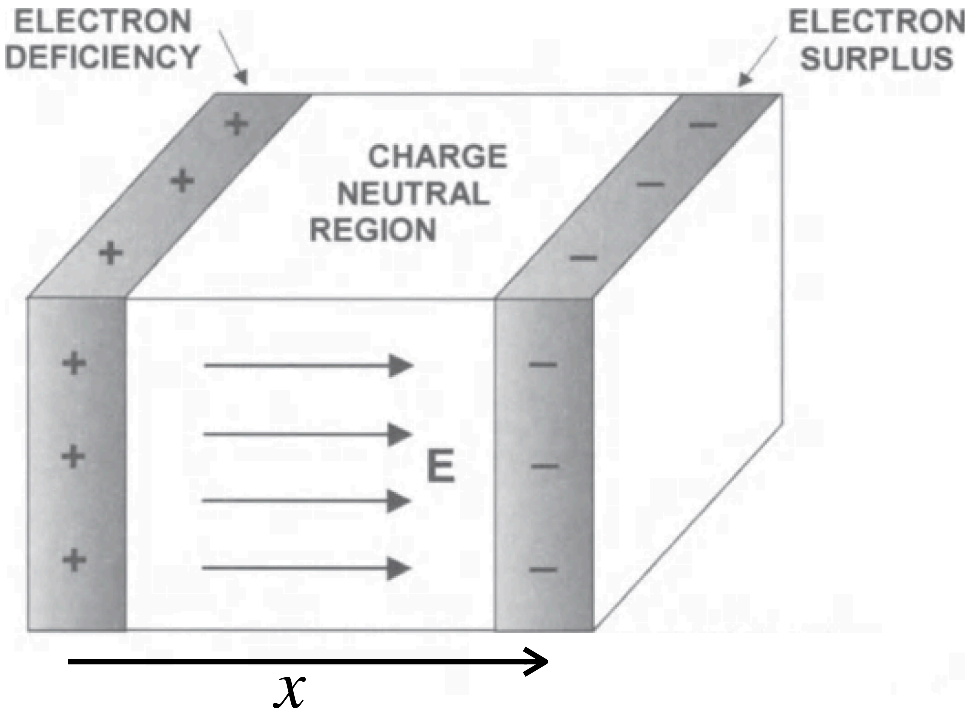
Macroscopic features of plasmas: plasma oscillations

- **The Debye shielding is a dynamic effect**
- **When charge imbalance is created in the plasma, e.g. by thermal fluctuations**
  - A strong electric field arises
  - Electrons accelerate towards positive charges
  - Most of the electrons have small-angle deflections
  - Electrons oscillate around the test particle

From Gauss law: $E_x = \left(\dfrac{n_0 e}{\epsilon_0}\right) x$

From the motion equation of an electron:

$$\frac{d^2x}{dt^2} = -\frac{eE_x}{m_e} = -\omega_{pe}^2 x \qquad \omega_{pe} = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}}$$



Electron plasma frequency (ω_{pe})

Macroscopic features of plasmas: plasma oscillations

- **The average time an electron takes to complete an oscillation depends on its average thermal speed** $v_{th,e} = \sqrt{k_B T_e / m_e}$

- **Note that there is a relation between** ω_{pe}, λ_{De} **and** $v_{th,e}$

$$\lambda_{De} = \sqrt{\frac{\epsilon_0 k_B T_e}{n_0 e^2}} = \sqrt{\frac{\epsilon_0 m_e}{n_0 e^2}\frac{k_B T_e}{m_e}} = \frac{v_{th,e}}{\omega_{pe}} \quad \rightarrow \quad v_{th,e} = \lambda_{De}\omega_{pe}$$

- **Electron plasma frequency**

$$\omega_{pe} = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}} \qquad f_{pe} = \frac{\omega_{pe}}{2\pi} = 9.0\sqrt{n_0} \qquad \textbf{(}n_0 \textbf{ in m}^{-3}\textbf{)}$$

- **Electron plasma frequency for some particular cases**
  - Tokamaks ($n_0 = 1 \times 10^{20}\ m^{-3}$): $f_{pe} = 90\ GHz$
  - Solar corona ($n_0 = 1 \times 10^{12}\ m^{-3}$): $f_{pe} = 9\ MHz$

Macroscopic features of plasmas: plasma oscillations

- **The collision between electrons and neutral gas particles might prevent electron plasma oscillations to be established thus preventing the Debye shielding**
  - Typical time between collision between electrons and neutral gas

$$\tau_{en} = \frac{1}{\nu_{en}} = \frac{1}{n_n \sigma_{en} v_{th,e}}$$

  - Taking $\sigma_{en} \approx \pi a_0^2$ (with a_0 being the Bohr radius - Hydrogen) and $v_{th,e} = \sqrt{\frac{k_B T}{m_e}}$

$$\tau_{en} \approx \frac{1 \times 10^{17}}{n_n \sqrt{T}}$$ **(given n_i, n_n can be calculated from the Saha equation)**

  - Tokamaks ($n_i = 1 \times 10^{20}\ m^{-3};\ T = 1 \times 10^{8}\ K$): $\tau_{en} = 2.4 \times 10^{6}\ s$
  - Solar corona ($n_i = 1 \times 10^{12}\ m^{-3};\ T = 1 \times 10^{6}\ K$): $\tau_{en} = 2 \times 10^{20}\ s$

- **For the cases above, the criterion $\tau_{en} \gg 2\pi/\omega_{pe}$ is well satisfied**

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Macroscopic features of plasmas: the plasma definition criteria

- **The main difference between ionized gases and plasmas is the presence of collective effects, which are present if**
  - (1) The plasma dimensions be much larger than the Debye length

$$L \gg \lambda_D$$

  - (2) The # of electrons within a Debye sphere be much larger than unity

$$N_D = \frac{4}{3}\pi\lambda_D^3 n_0 \gg 1 \qquad\qquad g = \frac{1}{n_0\lambda_D^3} \ll 1$$

(Plasma Parameter)

  - (3) The average time between collision of electrons and neutral particles be much larger than the time for one electron plasma oscillation to take place

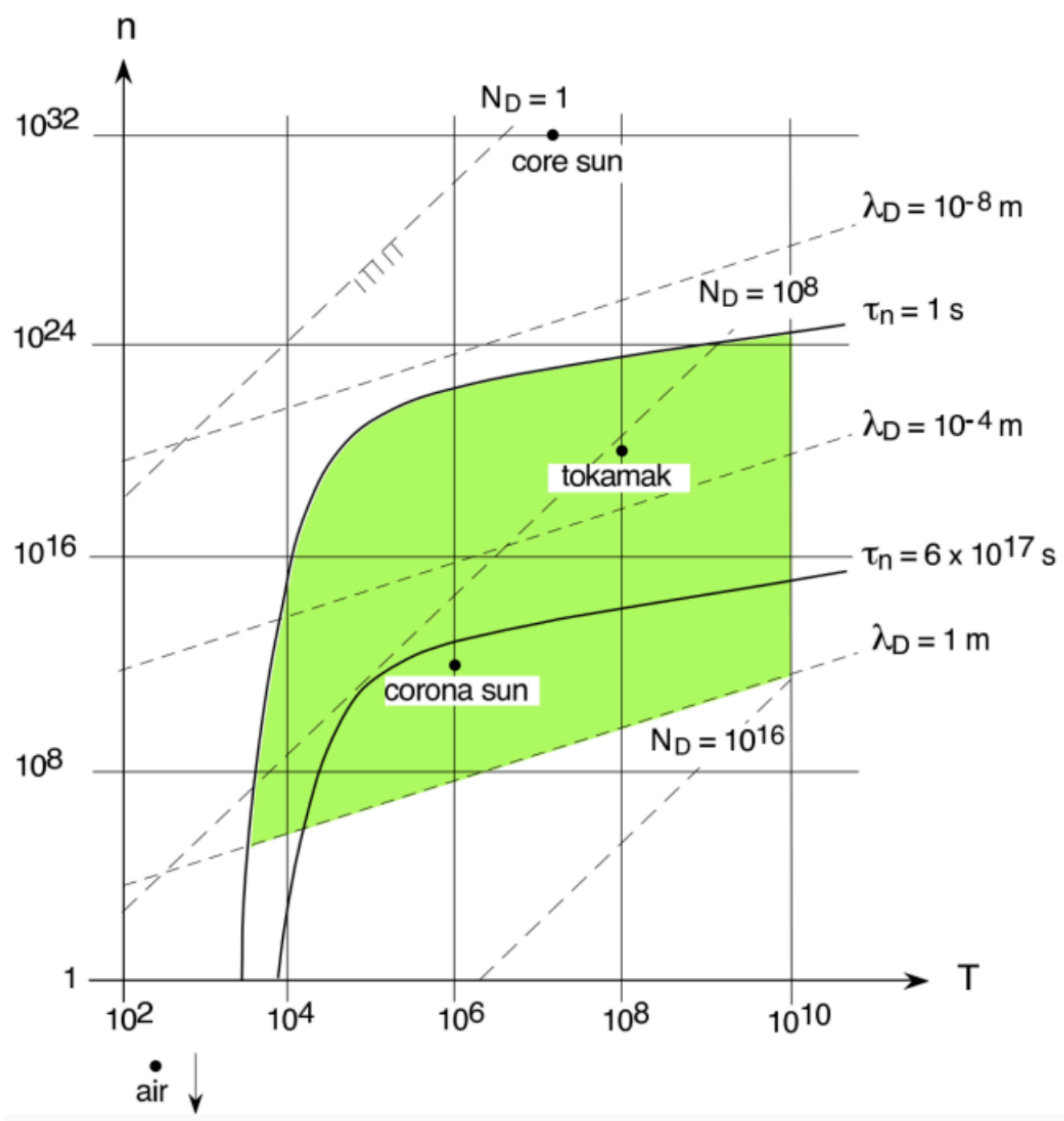
$$\tau_{en} \gg \tau_{pe} = \frac{1}{f_{pe}} = \frac{2\pi}{\omega_{pe}}$$

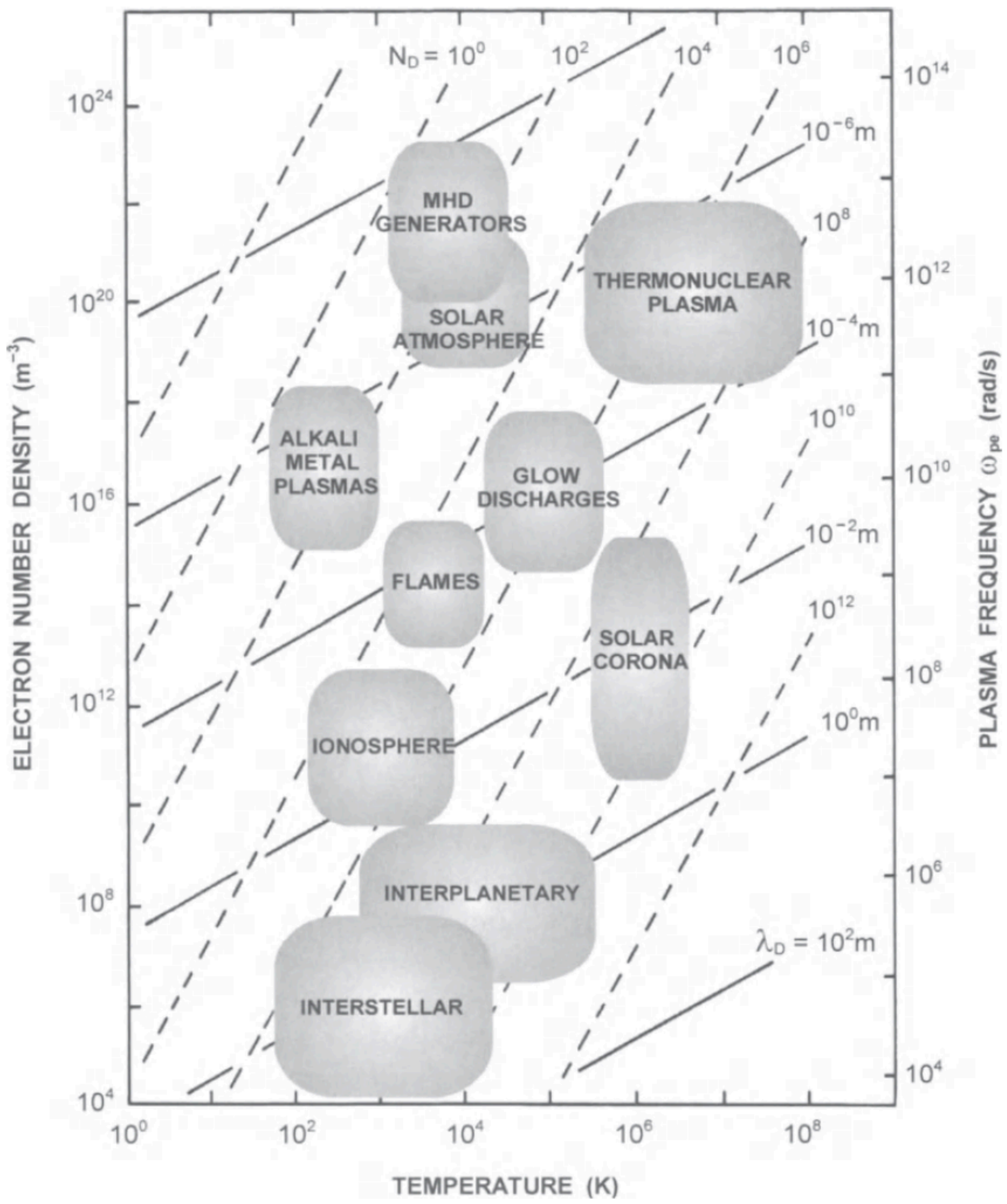
- **Note that the first criterion already implies macroscopic charge neutrality**
  - Sometimes, charge neutrality is considered a 4th criterion, even though it is not an independent one

$$\sum_j Z_j n_j = n_e$$

Macroscopic features of plasmas: the plasma definition criteria

- **Conditions for collective plasma behavior are satisfied within the green region**





References

- **Low pressure electrical discharges**
  - *Industrial Plasma Engineering, Vol. 1: Ch. 8 and Ch. 4 (Section 9)*
  - *Breakdown in low pressure gases: part I*
  - *Breakdown in low pressure gases: part II*

- **Quasi-neutrality**
  - *Bittencourt: Ch. 1, section 2*

- **Debye shielding**
  - *Bittencourt: Ch. 11, section 2*
  - *Bittencourt: Ch. 7, section 5*
  - *Bittencourt: Ch. 20, section 8*

- **Plasma oscillations**
  - *Bittencourt: Ch. 1, section 2*