PGF5112 - Plasma Physics I

By

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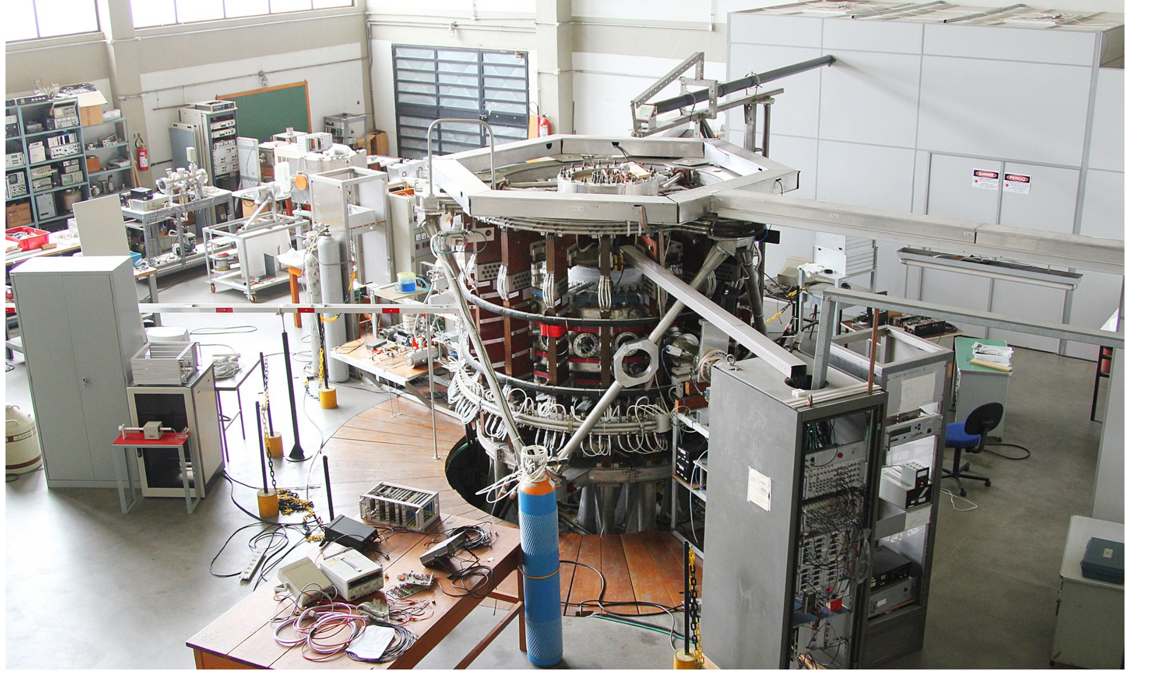

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PGF5112 - Plasma Physics I

- **Introduction**
- **Theoretical descriptions of plasma phenomena**
- **Review of basic concepts in kinetic theory of gases**
- **Particle interactions in plasmas**
  - *Collision cross section*
  - *The Rutherford cross section*
  - *Collision parameters*
  - *Collisional processes*
- **Particle detailed balance**

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This course aims at providing a broad view about the various phenomena occurring in plasmas

- **What are plasmas?**
  - Plasmas are ionized gases whose atoms have been dissociated (not necessarily all of them) into ions and electrons

- **All ionized gases are considered plasmas?**
  - No, plasmas are ionized gases that exhibit collective effects

- **How plasmas are produced and maintained?**
  - Plasmas are produced by the ionization of atoms, which can happens through a variety of collisional processes
  - To maintain a steady state plasma, particles and/or energy must be supplied constantly

This course aims at providing a broad view about the various phenomena occurring in plasmas

- **How can we describe/model the behavior of plasmas?**
  - From first principles: following the trajectory of each individual particle
  - From a statistical approach: kinetic theory
  - Assuming the plasma is a continuous medium: fluid model

- **Why is it important to study plasma physics?**
  - Plasmas are used in an enormous number of technological applications
  - Important astrophysical phenomena for human life: solar thunderstorms
  - Energy production through thermonuclear fusion: tokamaks

- **How stable are plasmas in tokamaks?**
  - Sometimes, plasmas can find a path towards a lower energy state
  - Plasma instabilities are the result of plasmas accessing a lower energy state

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The particle orbit theory: a first principles plasma model

- **Motion equation of a charged particle of especies α in an electromagnetic field:**

$$\frac{d\mathbf{r}_\alpha}{dt} = \mathbf{v}_\alpha$$

$$m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = \mathbf{F}_\alpha = q_\alpha \left(\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}\right)$$

- **Maxwell's equations:**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- **Constitutive relations:**

$$\rho = \rho_{ext} + \rho_{plasma} = \rho_{ext} + \sum_\alpha q_\alpha \delta[\mathbf{r} - \mathbf{r}_\alpha(t)] \qquad \mathbf{E} = \mathbf{E}_{\mathbf{ext}} + \mathbf{E}_{\mathbf{plasma}}$$

$$\mathbf{J} = \mathbf{J}_{\mathbf{ext}} + \mathbf{J}_{\mathbf{plasma}} = \mathbf{J}_{\mathbf{ext}} + \sum_\alpha q_\alpha \mathbf{v}_\alpha(t) \delta[\mathbf{r} - \mathbf{r}_\alpha(t)] \qquad \mathbf{B} = \mathbf{B}_{\mathbf{ext}} + \mathbf{B}_{\mathbf{plasma}}$$

The particle orbit theory: a first principles plasma model

- **The particle orbit theory provides a well-defined self-consistent model to describe plasmas, however, this model has limitations in practice**
  - The large number of particles ($\sim 10^{20}$ m^{-3}) makes this model prohibitive
  - The amount of information contained in this model is unnecessarily large: (# of particles) x (3 positions) x (3 velocities) x (# of temporal steps)

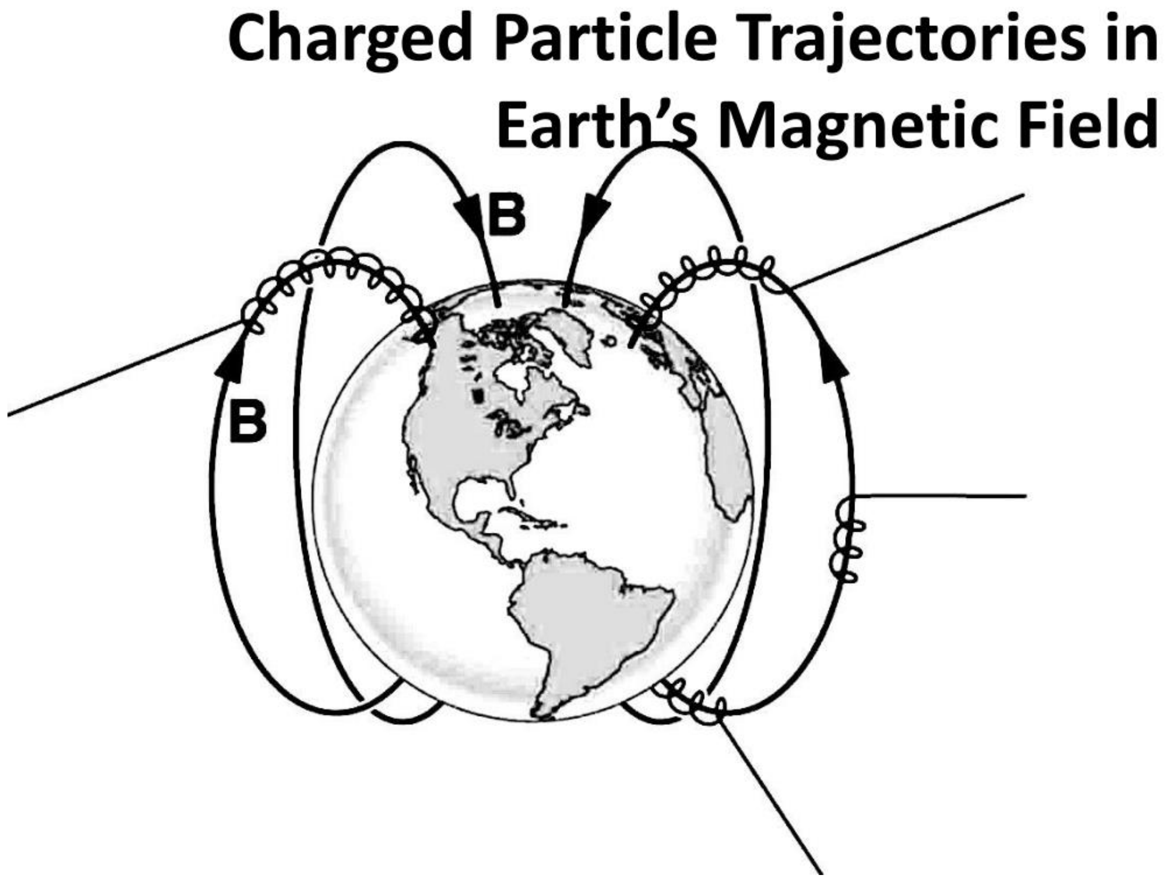
- **To simplify the model, the response/reaction of charged particles to EM fields from other charged particles is neglected ($\mathbf{E}_{\mathbf{ext}} >> \mathbf{E}_{\mathbf{plasma}}$ and $\mathbf{B}_{\mathbf{ext}} >> \mathbf{B}_{\mathbf{plasma}}$)**
  - The charged particle trajectory is, therefore, determined by ONLY the externally applied EM fields
  - This model neglects collective effects (not well suited for plasmas)

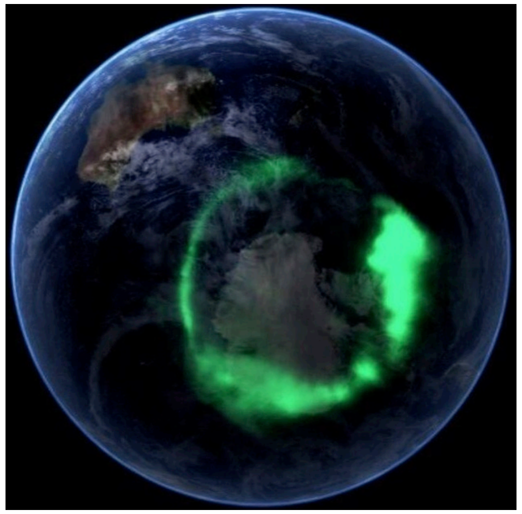
$$\mathbf{E} = \mathbf{E}_{\mathbf{ext}} + \cancel{\mathbf{E}_{\mathbf{plasma}}} \qquad \frac{d\mathbf{r}_\alpha}{dt} = \mathbf{v}_\alpha$$

$$\mathbf{B} = \mathbf{B}_{\mathbf{ext}} + \cancel{\mathbf{B}_{\mathbf{plasma}}} \qquad m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = q_\alpha \left(\mathbf{E}_{\mathbf{ext}} + \mathbf{v}_\alpha \times \mathbf{B}_{\mathbf{ext}}\right)$$

The particle orbit theory is well suited to study the trajectory of charged particles entering Earth's atmosphere

- **Charged particles arriving to Earth's atmosphere are deflected towards the poles by the terrestrial magnetic field**
  - The collision of these charged particles with the air molecules in the atmosphere gives rise to the so-called auroras





The particle orbit theory: the particle-in-cell simulations

- **To make simulations possible, so-called *super-particles* are used**
  - A super-particle (or *macroparticle*) is a computational particle that represents many real particles; it may be millions of electrons or ions

- **For plasma applications, the leapfrog method takes the following form:**

$$\mathbf{x_{k+1}} = \mathbf{x_k} + \Delta t\, \mathbf{v_{k+1/2}}$$
$$\mathbf{v_{k+1/2}} = \mathbf{v_{k-1/2}} + \frac{q}{m}\left(\mathbf{E_k} + \frac{\mathbf{v_{k+1/2}} + \mathbf{v_{k-1/2}}}{2} \times \mathbf{B_k}\right)$$

$\mathbf{E_k}$ and $\mathbf{B_k}$ $\rightarrow$ come from a field solver : finite element method

The particle orbit theory: the particle-in-cell simulations

- **To make simulations possible, so-called *super-particles* are used**
  - A super-particle (or *macroparticle*) is a computational particle that represents many real particles; it may be millions of electrons or ions

- **There is also the Boris scheme:**

$$\mathbf{x_{k+1}} = \mathbf{x_k} + \Delta t\, \mathbf{v_{k+1/2}}$$

$$\mathbf{v_{k+1/2}} = \mathbf{u}' + q'\mathbf{E_k}$$

$\mathbf{E_k}$ and $\mathbf{B_k}$ $\rightarrow$ come from a field solver : finite element method

  - onde $\mathbf{u}' = \mathbf{u} + \left[\mathbf{u} + (\mathbf{u} \times \mathbf{h})\right] \times \mathbf{s}$, $\mathbf{u} = \mathbf{v_{k-1/2}} + q'\mathbf{E_k}$, $\mathbf{h} = q'\mathbf{B_k}$,

    $\mathbf{s} = 2\mathbf{h}/\left(1 + h^2\right)$ and $q' = \Delta t\, q/(2m)$

  - Because of its excellent long term accuracy, the Boris algorithm is *de facto* the standard scheme used for advancing charged particles
  - The excellent long term accuracy of the Boris algorithm is due to the fact it conserves phase space volume, even though it is not symplectic

Due to its large number of particles, plasmas can also be described by means of a statistical approach: the kinetic theory of plasmas

- **In kinetic theory, all the information of the system is contained in the distribution function, $f_\alpha = f_\alpha(\mathbf{r}, \mathbf{v}, t)$, which is defined for each particle species**

- **The evolution of the system is given by the so-called Boltzmann equation:**

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_\alpha = \sum_\beta C_{\text{coll}}[f_\alpha, f_\beta]$$

- **Maxwell's equations:**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- **Constitutive relations:**

$$\rho = \rho_{ext} + \rho_{plasma} = \rho_{ext} + \sum_\alpha q_\alpha \int_{\mathbf{v}} f_\alpha(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$\mathbf{J} = \mathbf{J}_{\mathbf{ext}} + \mathbf{J}_{\mathbf{plasma}} = \mathbf{J}_{\mathbf{ext}} + \sum_\alpha q_\alpha \int_{\mathbf{v}} \mathbf{v} f_\alpha(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

Particle density

$$n_\alpha(\mathbf{r}, t) = \int_{\mathbf{v}} f_\alpha(\mathbf{r}, \mathbf{v}, t)\; d\mathbf{v}$$

Mean velocity

$$\mathbf{u}_\alpha(\mathbf{r}, t) = \frac{1}{n_\alpha(\mathbf{r}, t)} \int_{\mathbf{v}} \mathbf{v}\, f_\alpha(\mathbf{r}, \mathbf{v}, t)\; d\mathbf{v}$$

Depending of the conditions, each particle species in a plasma can be treated as a separate continuous medium: the multi-fluid model

- **Plasma fluid transport equations can be derived for each species by taking the moments of the Boltzmann equation**

Mass conservation

$$\frac{\partial \rho_{m\alpha}}{\partial t} + \nabla \cdot \left(\rho_{m\alpha}\mathbf{u}_\alpha\right) = S_\alpha$$

Sources of EM fields

$$\mathbf{J} = \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha$$

$$\rho = \sum_\alpha n_\alpha q_\alpha$$

Momentum conservation

$$\rho_{m\alpha}\left[\frac{\partial \mathbf{u}_\alpha}{\partial t} + \left(\mathbf{u}_\alpha \cdot \nabla\right)\mathbf{u}_\alpha\right] = n_\alpha q_\alpha \left(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_\alpha + \mathbf{A}_\alpha - \mathbf{u}_\alpha S_\alpha$$

Energy conservation

$$\frac{3}{2}\left[\frac{\partial p_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla p_\alpha\right] + \frac{3\,p_\alpha}{2}\left(\nabla \cdot \mathbf{u}_\alpha\right) + \left(\mathbf{P}_\alpha \cdot \nabla\right) \cdot \mathbf{u}_\alpha + \nabla \cdot \mathbf{q}_\alpha = M_\alpha - \mathbf{u}_\alpha \cdot \mathbf{A}_\alpha + \frac{1}{2}u_\alpha^2 S_\alpha$$

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Depending of the conditions, the whole plasma can be treated as one continuous medium: the single-fluid model

- **The behavior of the plasma as a whole can be determined by adding the contributions of the various particle species in the plasma**

Mass conservation

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \left(\rho_m \mathbf{u}\right) = 0$$

Momentum conservation

$$\rho_m \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u}\right] = \rho \mathbf{E} + \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{P}$$

Energy conservation

$$\frac{3}{2}\left[\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p\right] + \frac{3\,p}{2}(\nabla \cdot \mathbf{u}) + (\mathbf{P} \cdot \nabla) \cdot \mathbf{u} + \nabla \cdot \mathbf{q} = \mathbf{J} \cdot \mathbf{E} - \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) - \rho \mathbf{u} \cdot \mathbf{E}$$

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Generalized Ohm's law

$$\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{J}' + \mathbf{J}\mathbf{u}) - \frac{e}{m_e}\nabla \cdot \mathbf{P_e} = \frac{ne^2}{m_e}(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \frac{e}{m_e}\mathbf{J} \times \mathbf{B} - \nu_{ei}\mathbf{J}$$

The magnetohydrodynamic model

- **In the single-fluid approach ($\rho = 0$), the magnetohydrodynamic (MHD) model focuses on large (plasma size) scale and (relatively) low frequency phenomena**
  - In the MHD model, the information about the plasma is contained in ρ_m, $\mathbf{u}$ e p

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \left(\rho_m \mathbf{u}\right) = 0$$ (Mass conservation)

$$\rho_m \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u}\right] = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}$$ (Momentum conservation)

$$p = \left(\frac{k_B T}{m_i}\right) \rho_m$$ (Energy conservation)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ (Faraday’s law)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$ (Ampère’s law)

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J}$$ (Ohm’s law)

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Review of basic concepts in the kinetic theory of gases

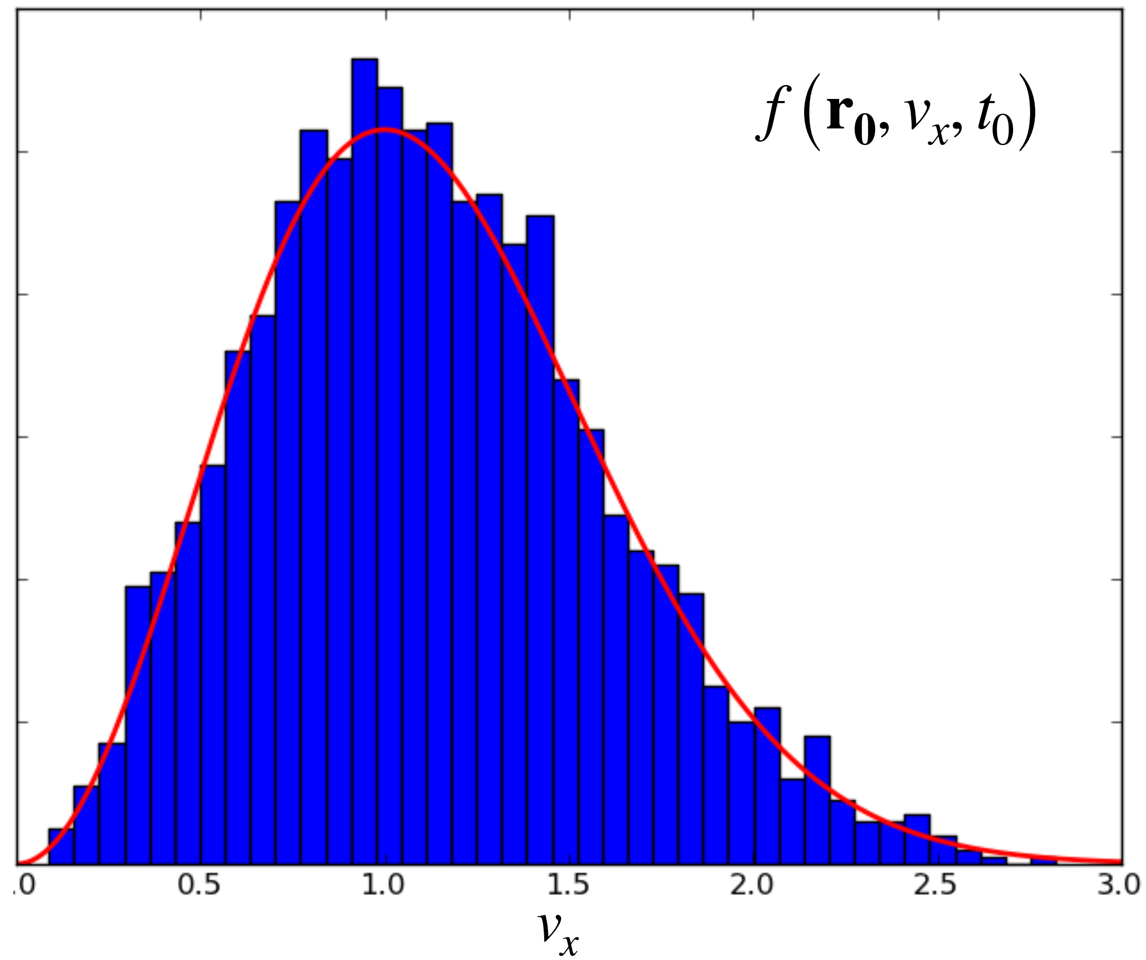
- **Distribution function**
  - Number of particles per unit of volume dx^3 with velocity between $\mathbf{v}$ and $\mathbf{v}+d\mathbf{v}$
  - The $f(\mathbf{r},\mathbf{v},t)$ has units of s^3/m^6

$$d^6N = f(\mathbf{r},\mathbf{v},t)\, d^3v\, d^3x$$

$$d^6N = f(\mathbf{r},\mathbf{v},t)\, d\mathbf{r}\, d\mathbf{v}$$

$$N(t) = \int_{\mathbf{r}}\int_{\mathbf{v}} d^6N\ d\mathbf{r}\, d\mathbf{v}$$

$$N(t) = \int_{\mathbf{r}}\int_{\mathbf{v}} f(\mathbf{r},\mathbf{v},t)\, d\mathbf{r}\, d\mathbf{v}$$



Review of basic concepts in the kinetic theory of gases

- **Particle density**
  - Number of particles per unit of volume (independent of their velocity) within a volume d^3x around $\mathbf{r}$

$$n(\mathbf{r},t) = \iiint f(\mathbf{r},\mathbf{v},t)\,d\mathbf{v}$$

- **Mean velocity**
  - Average velocity of the particles within a volume d^3x around $\mathbf{r}$

$$\mathbf{u}(\mathbf{r},t) = \frac{\iiint \mathbf{v}\, f(\mathbf{r},\mathbf{v},t)\,d\mathbf{v}}{n(\mathbf{r},t)}$$

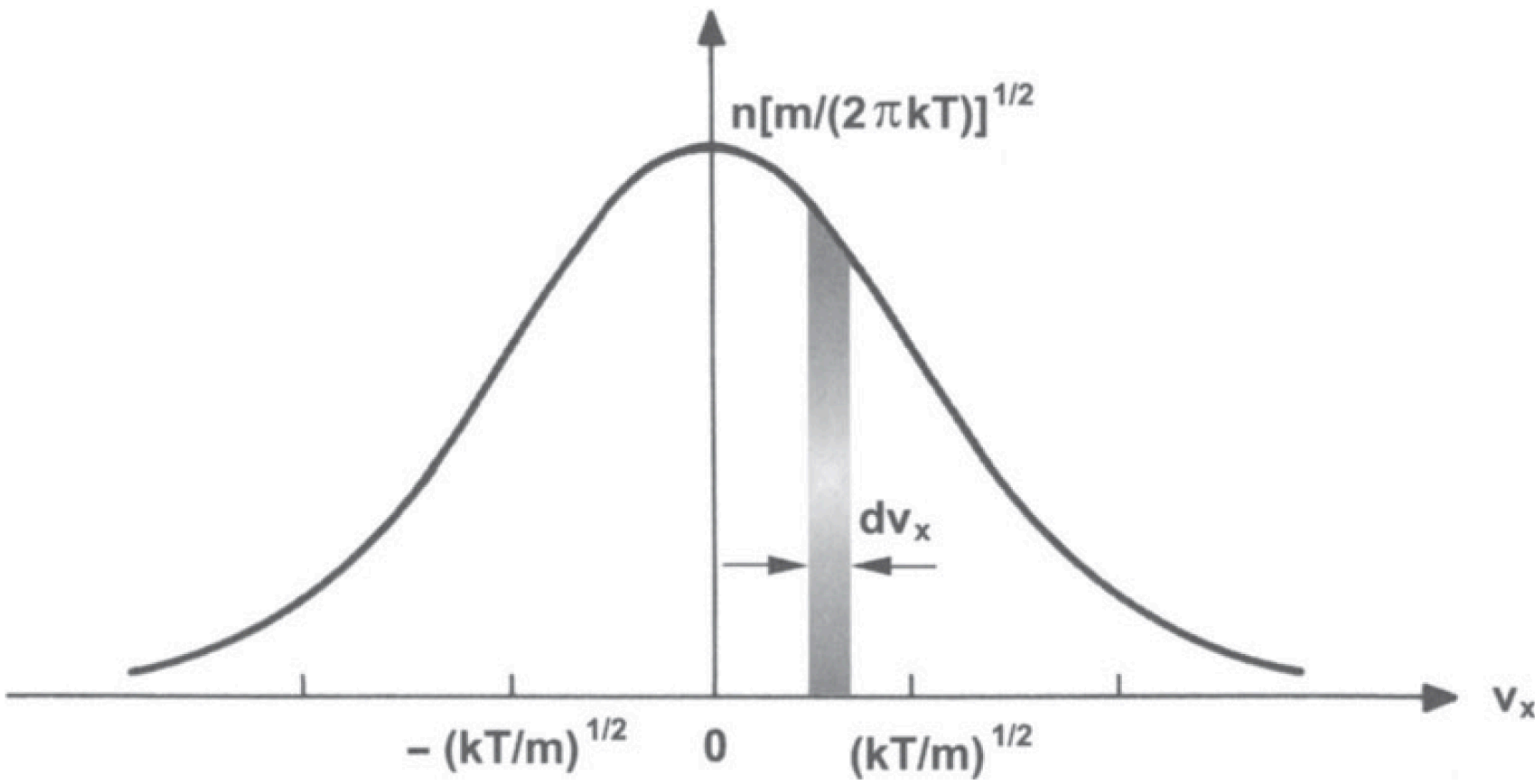
- **Mean energy**
  - Average energy of the particles within a volume d^3x around $\mathbf{r}$

$$K(\mathbf{r},t) = \frac{\iiint \frac{1}{2}mv^2\, f(\mathbf{r},\mathbf{v},t)\,d\mathbf{v}}{n(\mathbf{r},t)}$$

Review of basic concepts in the kinetic theory of gases

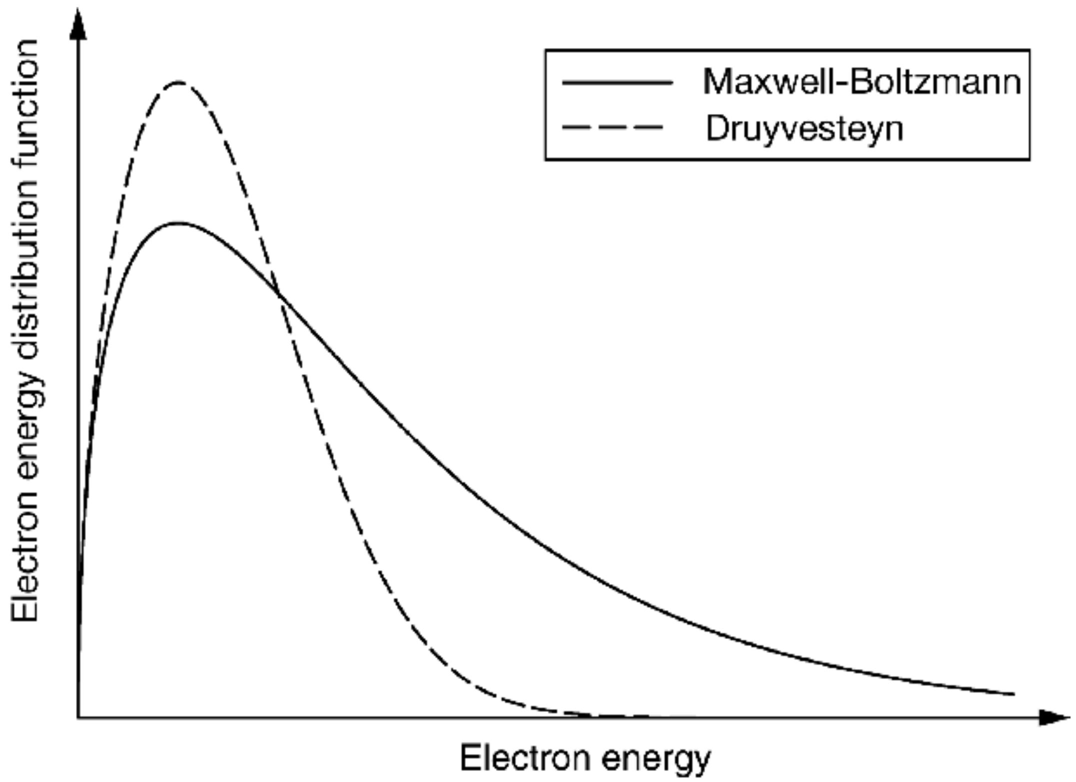
- **In the kinetic theory of gases, one can show (H-theorem) that the velocity distribution function of the particles tends to the Maxwell-Boltzmann distribution when the gas reaches thermodynamic equilibrium through collisions**

$$f_M(v) = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) \qquad v^2 = v_x^2 + v_y^2 + v_z^2$$



Review of basic concepts in the kinetic theory of gases

- **In plasmas, however, the general form of the H-theorem does not hold**
  - There are frequent situations in what the electron distribution function does not evolves towards the Maxwell-Boltzmann distribution (for example, it can evolve to a Druyvesteyn distribution when an electric field is applied)



  - The shape of the distribution function has an impact on the reaction rates, in which electrons usually play the significant role

Review of basic concepts in the kinetic theory of gases

- **Mean velocity in a Maxwell-Boltzmann distribution**

$$\langle v_x \rangle = \frac{1}{n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v_x \, n \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp \left[- \frac{m \left(v_x^2 + v_y^2 + v_z^2 \right)}{2 k_B T} \right] dv_x \, dv_y \, dv_z$$

$$\langle v_x \rangle = \left(\frac{m}{2\pi k_B T} \right)^{3/2} \underbrace{\left[\int_{-\infty}^{\infty} v_x \exp \left(- \frac{m v_x^2}{2 k_B T} \right) dv_x \right]}_{0} \underbrace{\left[\int_{-\infty}^{\infty} \exp \left(- \frac{m v_y^2}{2 k_B T} \right) dv_y \right]}_{\sqrt{\frac{2\pi k_B T}{m}}} \underbrace{\left[\int_{-\infty}^{\infty} \exp \left(- \frac{m v_z^2}{2 k_B T} \right) dv_z \right]}_{\sqrt{\frac{2\pi k_B T}{m}}}$$

- **In a Maxwell-Boltzmann distribution, the mean velocities** $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$

Review of basic concepts in the kinetic theory of gases

- **Mean energy in a Maxwell-Boltzmann distribution:** $\langle E \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$

$$\langle v_x^2 \rangle = \frac{1}{n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v_x^2 \, n \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp \left[- \frac{m \left(v_x^2 + v_y^2 + v_z^2 \right)}{2 k_B T} \right] dv_x \, dv_y \, dv_z$$

$$\langle v_x^2 \rangle = \left(\frac{m}{2\pi k_B T} \right)^{3/2} \underbrace{\left[\int_{-\infty}^{\infty} v_x^2 \exp \left(- \frac{m v_x^2}{2 k_B T} \right) dv_x \right]}_{\frac{\sqrt{\pi}}{2} \left(\frac{2 k_B T}{m} \right)^{3/2}} \underbrace{\left[\int_{-\infty}^{\infty} \exp \left(- \frac{m v_y^2}{2 k_B T} \right) dv_y \right]}_{\sqrt{\frac{2\pi k_B T}{m}}} \underbrace{\left[\int_{-\infty}^{\infty} \exp \left(- \frac{m v_z^2}{2 k_B T} \right) dv_z \right]}_{\sqrt{\frac{2\pi k_B T}{m}}}$$

- **In a Maxwell-Boltzmann distribution, the mean velocities** $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{k_B T}{m}$

- **This result is consistent with the theorem of equipartition of energy:**

$$\frac{1}{2} m \langle v_j^2 \rangle = \frac{1}{2} k_B T \qquad\qquad \langle E \rangle = \frac{3}{2} k_B T$$

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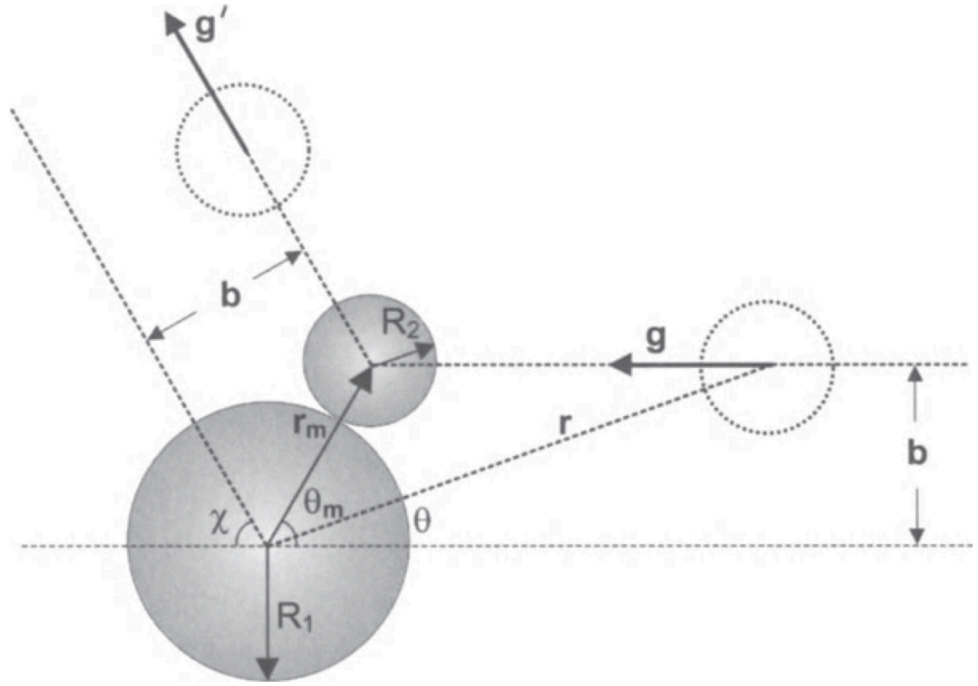
The most basic concept of cross section

- **The cross section of a certain collision process corresponds to the effective area of the target particle, assuming that the projectile is a point particle**

$$\sigma_t = \pi b_0^2$$

- **As an example, consider the collision between two neutrals (or between a neutral and a charged particle)**

$$\sigma_t = \pi \left(R_1 + R_2\right)^2$$



- **Considering** $R_1 = R_2 = a_0 = 0.98 \times 10^{-10}$ m
(Argon)

$$\sigma_t = 0.96 \times 10^{-20}\ \text{m}^2$$

The most basic concept of cross section

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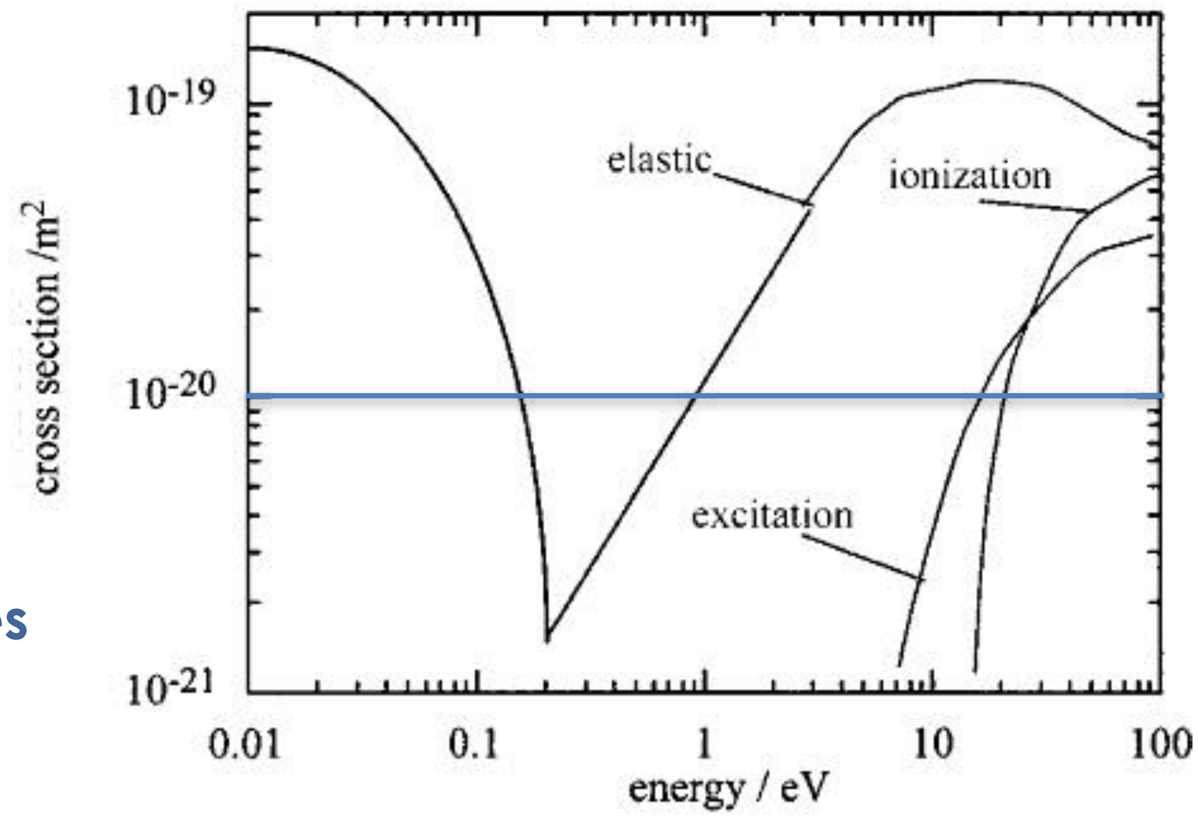
$$\sigma_t = 0.96 \times 10^{-20} \text{ m}^2$$

Argon Plasma



For energies lower than 1 eV, a resonant effect causes a decrease in the elastic collision cross section (The Ramsauer effect)

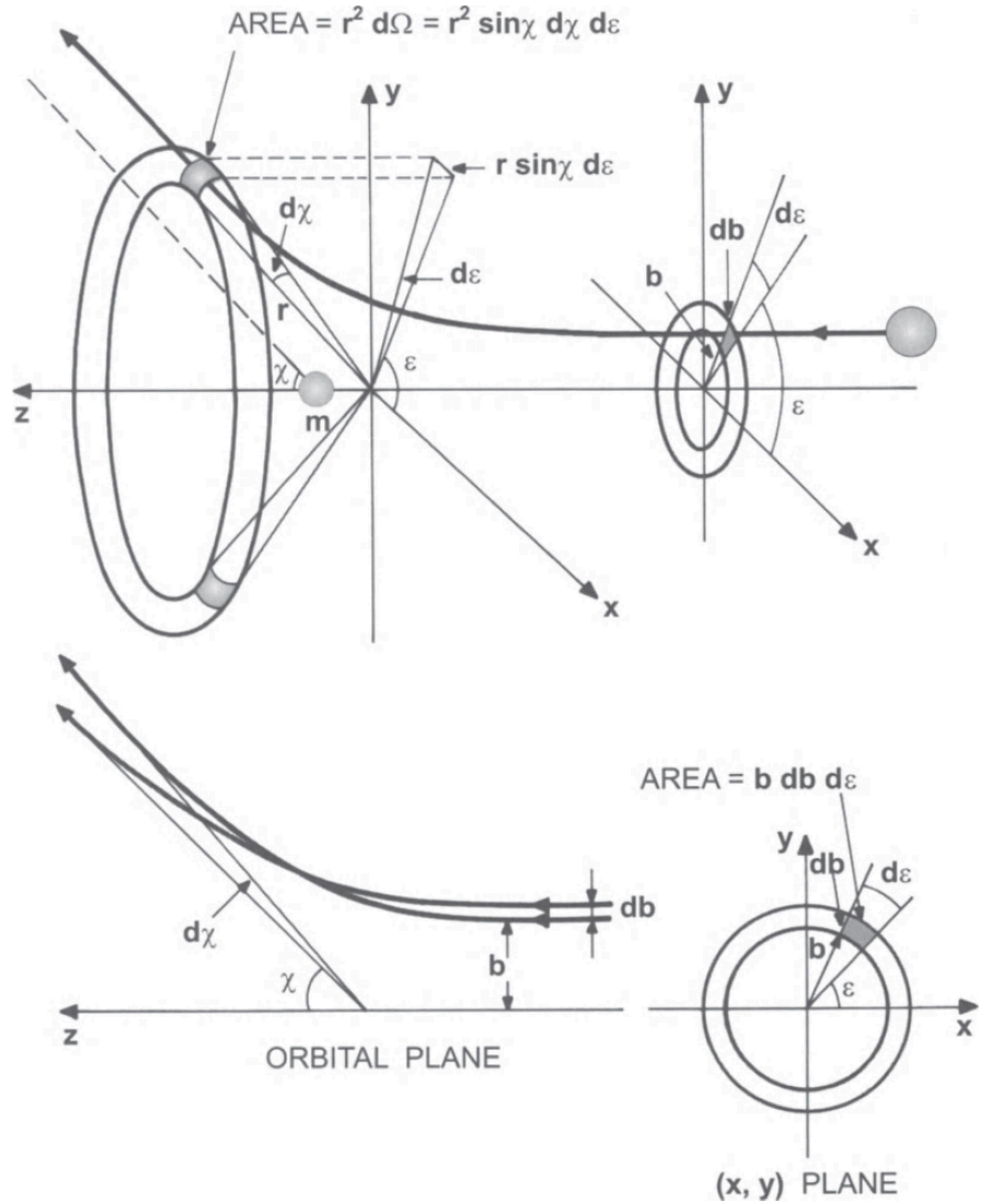
The precise concept of cross section

- **The precise definition of a cross section accounts for the scattering of an incoming particle as follows**
  - Γ is the particle flux (#/s/m²)
  - $\sigma(\chi,\epsilon)$ is the differential cross section
  - b is the impact parameter
  - $\dot{N}$ is the # of particles scattered, per unit time, into $d\Omega$

$$\frac{dN}{dt} = \Gamma\, d\sigma_t = \Gamma\, \sigma(\chi,\epsilon)\, d\Omega = \Gamma\, b\, d\epsilon\, db$$

- **Since $d\Omega = \sin(\chi)\, d\chi\, d\epsilon$, one finds that**

$$\sigma(\chi,\epsilon) = \frac{b}{\sin(\chi)}\left|\frac{db}{d\chi}\right|$$



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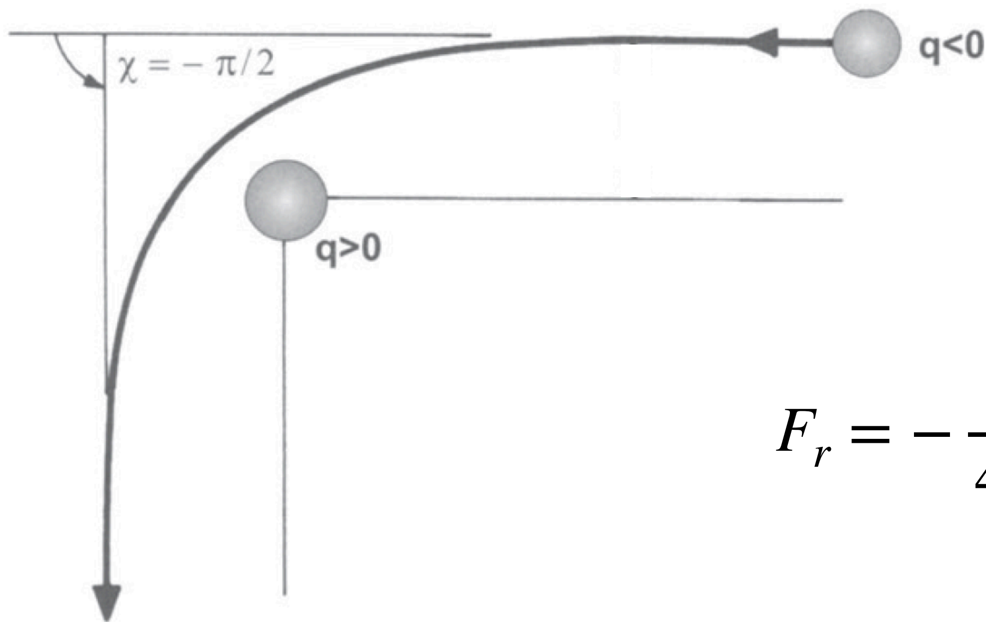
The Rutherford cross section corresponds to the elastic scattering of charged particles due to the Coulomb interaction

- **When an electron (q=-e) approaches a positive ion (q=Ze) by a distance b, it experiences an attractive force (the Coulomb force)**



$$F_r = -\frac{Ze^2}{4\pi\epsilon_0 b^2}$$

- **To have a substantial change in the trajectory of the electron, the energy of the interaction must be of the same order of the electron's kinetic energy**

$$\frac{Ze^2}{4\pi\epsilon_0 b} \approx \frac{1}{2}mv^2 \qquad b \approx \frac{2Ze^2}{4\pi\epsilon_0 v^2} \qquad \sigma \approx \pi b^2 \approx \frac{4\pi Z^2 e^4}{(4\pi\epsilon_0)^2 v^4} \propto \frac{1}{v^4}$$

This dependence has important consequences on plasma resistivity and diffusion:

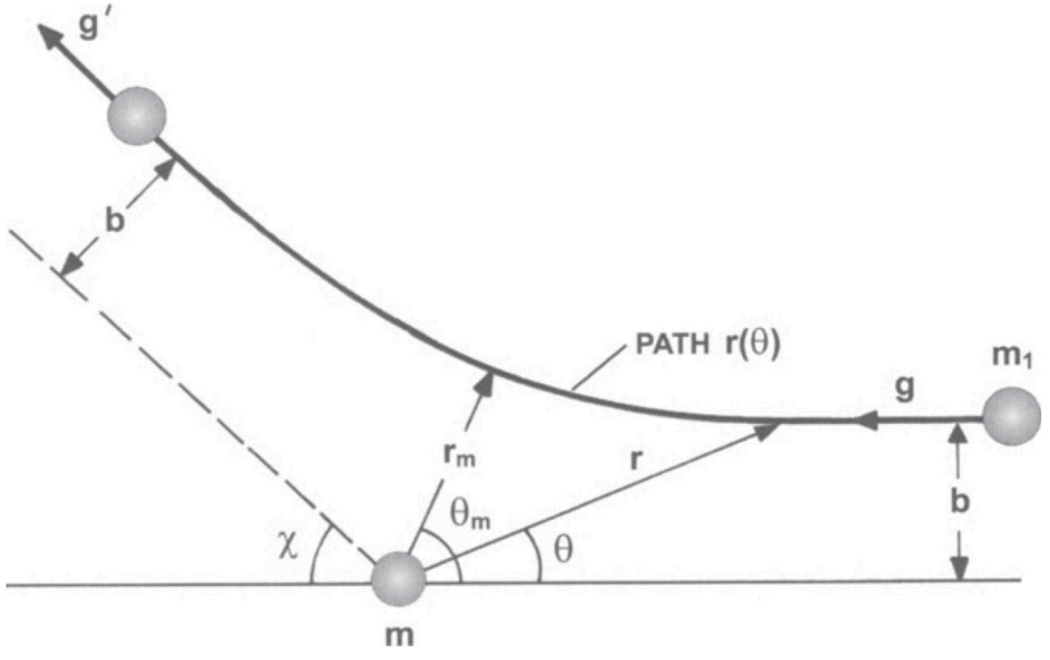
Collisions in plasmas become less frequent at higher velocities/temperatures

Exercise

- **Using polar coordinates, centered on an ion scattering center, and using energy and momentum conservation, show that**

$$\tan\left(\frac{\chi}{2}\right) = \frac{q\,q_2}{4\pi\epsilon_0\mu g^2 b}$$

(Eq. 20.4.13 from Bittencourt's book)



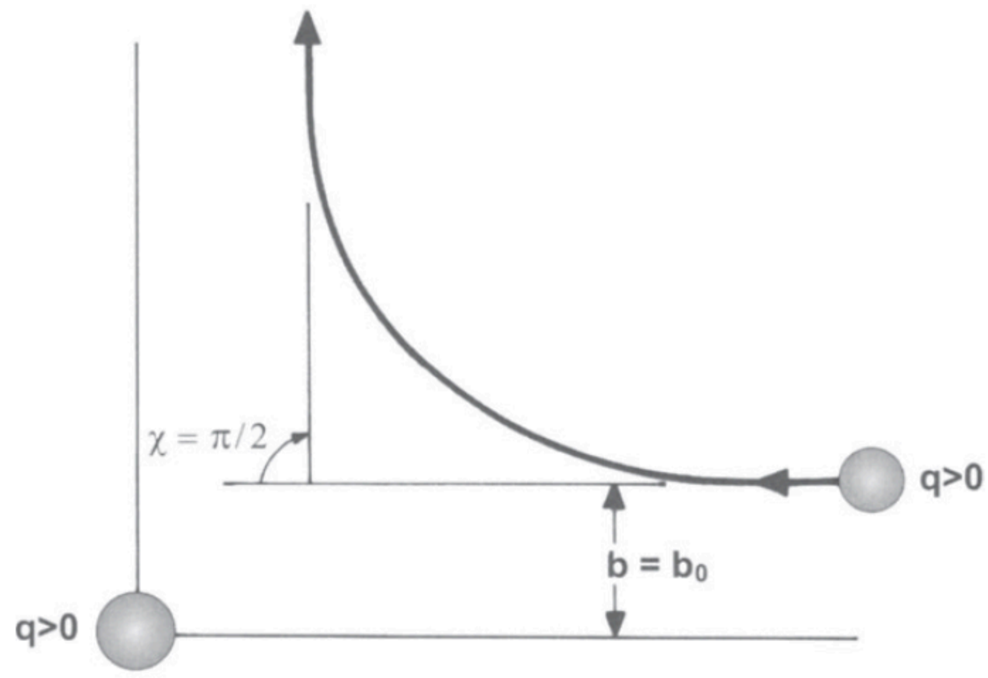
– $\mu = \dfrac{m\,m_1}{m + m_1}$ is the reduced mass

– $\mathbf{g} = \mathbf{v_1} - \mathbf{v}$ is the relative velocity

The scattering of particles via the Coulomb interaction

- **When a pair of particles interact via the Coulomb force, the scattering angle can be determined from (given as exercise):**

$$\tan\left(\frac{\chi}{2}\right) = \frac{b_0}{b}, \text{ with } b_0 = \frac{q\, q_1}{4\pi\epsilon_0 \mu g^2}$$



- **Differentiating this equation yields**

$$\left|\frac{db}{d\chi}\right| = \frac{b^2}{2b_0\cos^2(\chi/2)}$$

- **Therefore, the differential cross section for the Coulomb potential is**

$$\sigma(\chi) = \frac{b}{\sin(\chi)}\left|\frac{db}{d\chi}\right| = \frac{b_0^2}{4\sin^4(\chi/2)} = \frac{b_0^2}{(1-\cos\chi)^2}$$

The Rutherford scattering

The total cross section due to the Coulomb interaction diverges

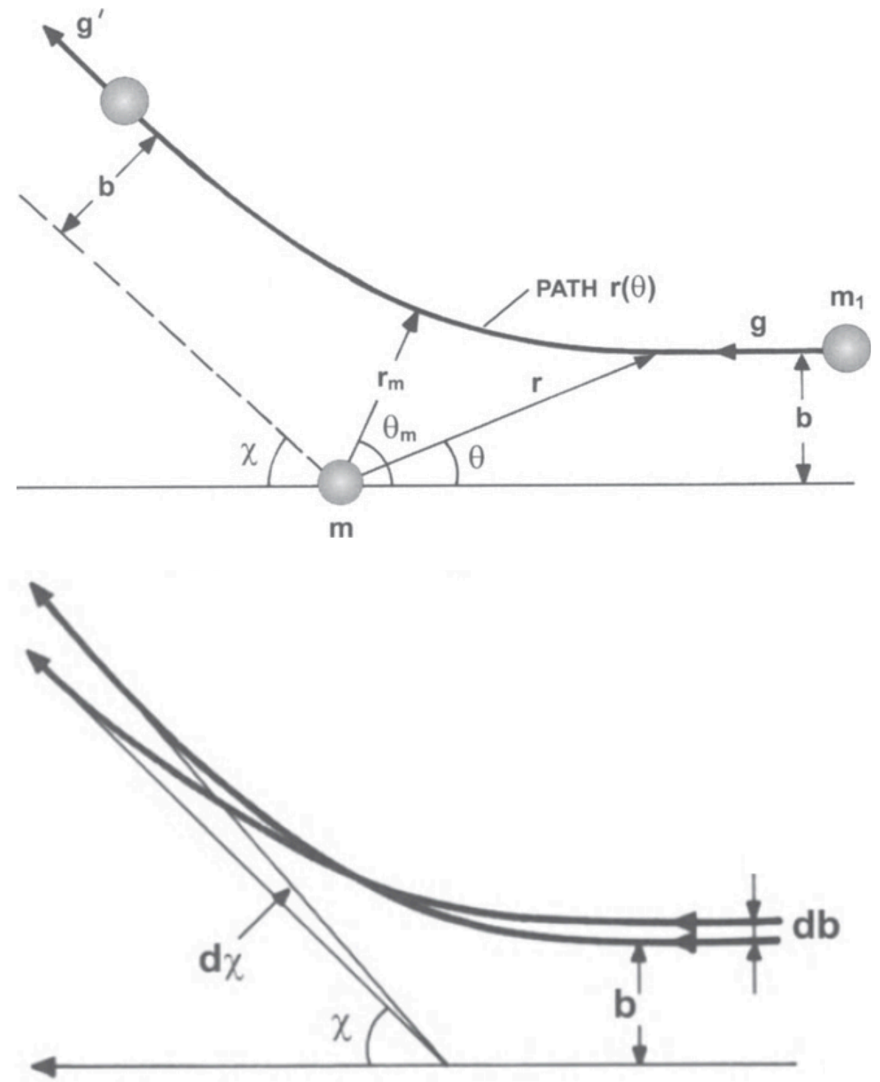
- **The total cross section can be found as**

$$\sigma_t = \int \sigma(\chi,\epsilon)\, d\Omega = \int_0^{2\pi}\int_{\chi_{\min}}^{\pi} \sigma(\chi,\epsilon)\, \sin\chi\, d\chi\, d\epsilon$$

$$\sigma_t = 2\pi \int_{\chi_{\min}}^{\pi} \sigma(\chi)\, \sin\chi\, d\chi$$

$$\sigma_t = 2\pi\, b_0^2 \int_{\chi_{\min}}^{\pi} \frac{\sin\chi}{(1-\cos\chi)^2}\, d\chi$$

$$\sigma_t = 2\pi\, b_0^2 \left[\frac{1}{\sin^2(\chi_{\min}/2)} - 1\right]$$



- **The total cross section becomes infinite for** $\chi_{\min} = 0$
  - Particles with very small deflection angles (very large value of b) contribute to make σ_t infinite ($1/r^2$ forces are of infinite range!)
  - In plasmas, there exists an upper limit for b (the so-called Debye length)

PGF5112 - Plasma Physics I

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  - *Collision parameters*
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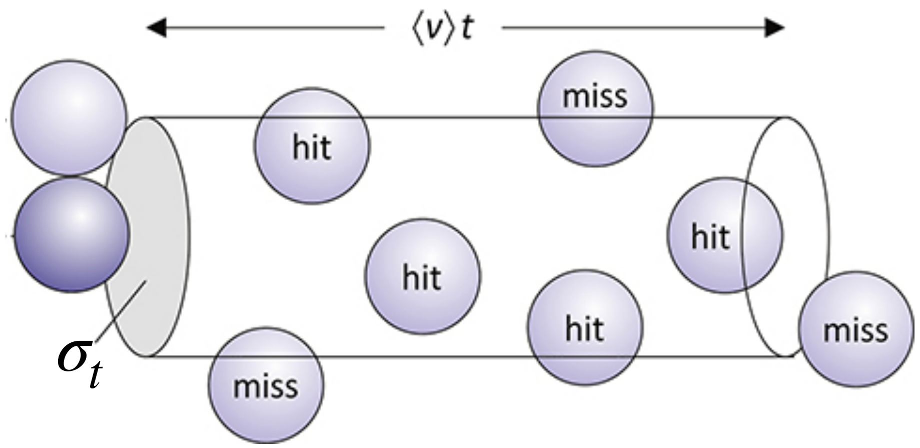
Collision frequency

- **The collision frequency corresponds to the number of collision per unit time**

- **Let's suppose that in a certain region of space (volume V) there exists N scattering centers: density of scattering centers n = N/V**



- **In a time interval Δt, a test particle travels a distance $d = v\,\Delta t$, where $v = |\mathbf{v_1} - \mathbf{v_2}|$ is the relative particle velocity. Therefore, the "collisional volume" associated to this particle will be $V_{\text{coll}} = \sigma\, v\, \Delta t$, with a number of encounters/ collisions $N_{\text{coll}} = nV_{\text{coll}} = n\,\sigma\, v\,\Delta t$**

- **Therefore, the collision frequency will be** $\nu_{\text{coll}}(v) = \dfrac{N_{\text{coll}}}{\Delta t} = \dfrac{n\,\sigma\, v\,\Delta t}{\Delta t} = n\,\sigma(v)\, v$

Collision frequency

- **In plasmas, the velocities of the particles are not all the same**

- **One must take the average over the velocity distribution function**

$$\langle \nu_{\mathrm{coll}} \rangle = n \langle \sigma v \rangle$$

$$\langle \sigma v \rangle = \frac{1}{n} \int \sigma(v)\, v\, f(v)\, dv \quad \textbf{with} \quad v = |\mathbf{v_1} - \mathbf{v_2}|$$

- **Therefore, the average collision frequency is given by**

$$\langle \nu_{\mathrm{coll}} \rangle = \int \sigma(v)\, v\, f(v)\, dv$$

Reaction rate

- **Let's suppose that there are two species in the plasma, with densities n_1 and n_2**

- **Take the particles of type 2 as scattering centers (targets) and one single particle of type 1 as the scattered particle (projectile)**

- **A more precise definition of the average collision frequency for a particle of type 1 is**

$$\nu_{12}(\mathbf{r},t) = n_2\langle\sigma v\rangle_{12} = \frac{1}{n_1}\int\int f_1(\mathbf{r},\mathbf{v},t) f_2(\mathbf{r},\mathbf{v},t)\sigma(|\mathbf{v_1}-\mathbf{v_2}|)\, d^3v_1\, d^3v_2$$

- **The reaction rate is defined as the # of collisions per unit volume, per unit time:**

$$R_{12}(\mathbf{r},t) = n_1\nu_{12}(\mathbf{r},t) = n_1 n_2\langle\sigma v\rangle_{12} = \int\int f_1(\mathbf{r},\mathbf{v},t) f_2(\mathbf{r},\mathbf{v},t)\sigma(|\mathbf{v_1}-\mathbf{v_2}|)\, d^3v_1\, d^3v_2$$

The concept of reaction rate will be important when we calculate the power produced in fusion plasmas

Mean free path

- **The mean free path is defined as the average distance a particle travels in between collisions**

$$\lambda = \frac{d}{N_{\text{coll}}}$$

- **Therefore,**

$$\lambda = \frac{\langle v \rangle \Delta t}{n \langle \sigma v \rangle \Delta t} = \frac{\langle v \rangle}{n \langle \sigma v \rangle}$$

$$\lambda = \frac{1}{n} \frac{\int v\, f(v)\, dv}{\int \sigma(v)\, v\, f(v)\, dv}$$

$$\lambda \approx \frac{1}{n \langle \sigma \rangle}$$

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Elastic collisions between electrons and neutral atoms

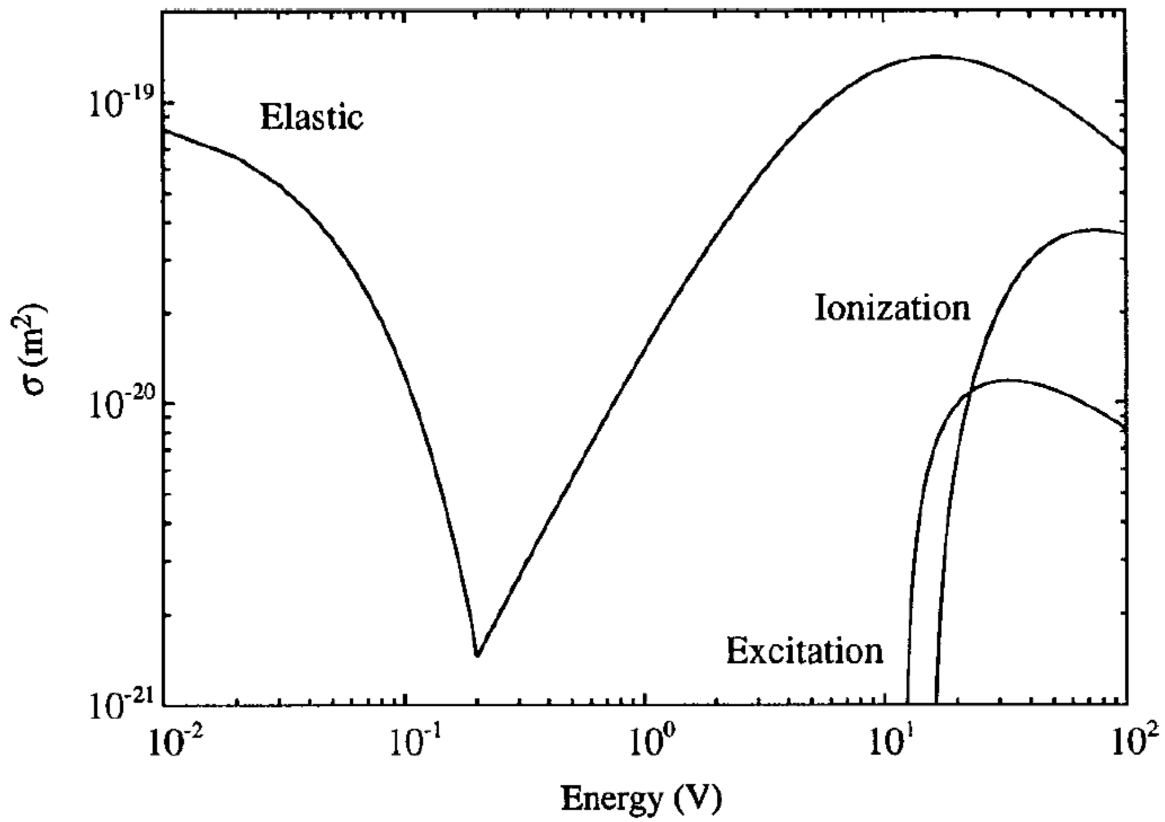
- **Due to the large mass difference between electrons and atoms, (almost) no energy is exchanged during an elastic electron-atom collision**

$$\Delta E = \frac{2m_e m_p}{(m_e + m_p)^2} \approx \frac{2m_e}{m_p} = 0.1\,\%$$

- **The cross section for elastic scattering of electrons in gases, in general, are significantly larger than in non-elastic collisions**

- **Elastic collisions between electrons and atoms/molecules is the main mechanism responsible for momentum transport**
  - Main effect responsible for the electric conductivity in weakly ionized plasmas

Argon Plasma



For E < 1 eV, the Ramsauer effect causes a decrease in the cross section. This effect does not occur in collision with molecules

Inelastic collisions: A large # of inelastic processes can occur in plasmas

- **Electrons (A: atoms or molecules)**
  - $e + A \rightarrow A^+ + 2e$ (Ionization by electronic impact)**
  - $e + A \rightarrow e + A^* \rightarrow e + A + h\nu$ (Excitation)
  - $e + A^* \rightarrow A^+ + 2e$ (Penning ionization)
  - $e + AB \rightarrow A + B + e$ (Molecular dissociation)*
  - $e + AB \rightarrow A^+ + B + 2e$ (Dissociative ionization)*
  - $e + AB \rightarrow A^- + B$ (Dissociative attachment)

***Most important processes in technological plasmas**

- **Ions (A and B: atoms or molecules)**
  - $A^+ + B \rightarrow A + B^+$ (Charge exchange)*
  - $A^+ + B \rightarrow A^+ + B^+ + e$ (Ionization)
  - $A^+ + B \rightarrow A^+ + B^* \rightarrow A^+ + B + h\nu$ (Excitation)
  - $A^+ + e + B \rightarrow A + B$ (Three-body recombination)**
  - $A^+ + BC \rightarrow A^+ + B + C$ (Dissociation)
  - $A + BC \rightarrow C + AB$ (Chemical reaction)

**** Dominat mechanism**

PGF5112 - Plasma Physics I

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The particle detailed balance

- The temporal evolution of electron density in a plasma depends on the reaction rate of each collisional processes that can happen in that plasma

$$\frac{dn_e}{dt} = \sum_j R^+_{e,j} - \sum_k R^-_{e,k}$$

- Suppose that the dominant processes occurring in a plasma are the ionization by electronic impact (σ_{ion}) and recombination (σ_{rec}). Therefore

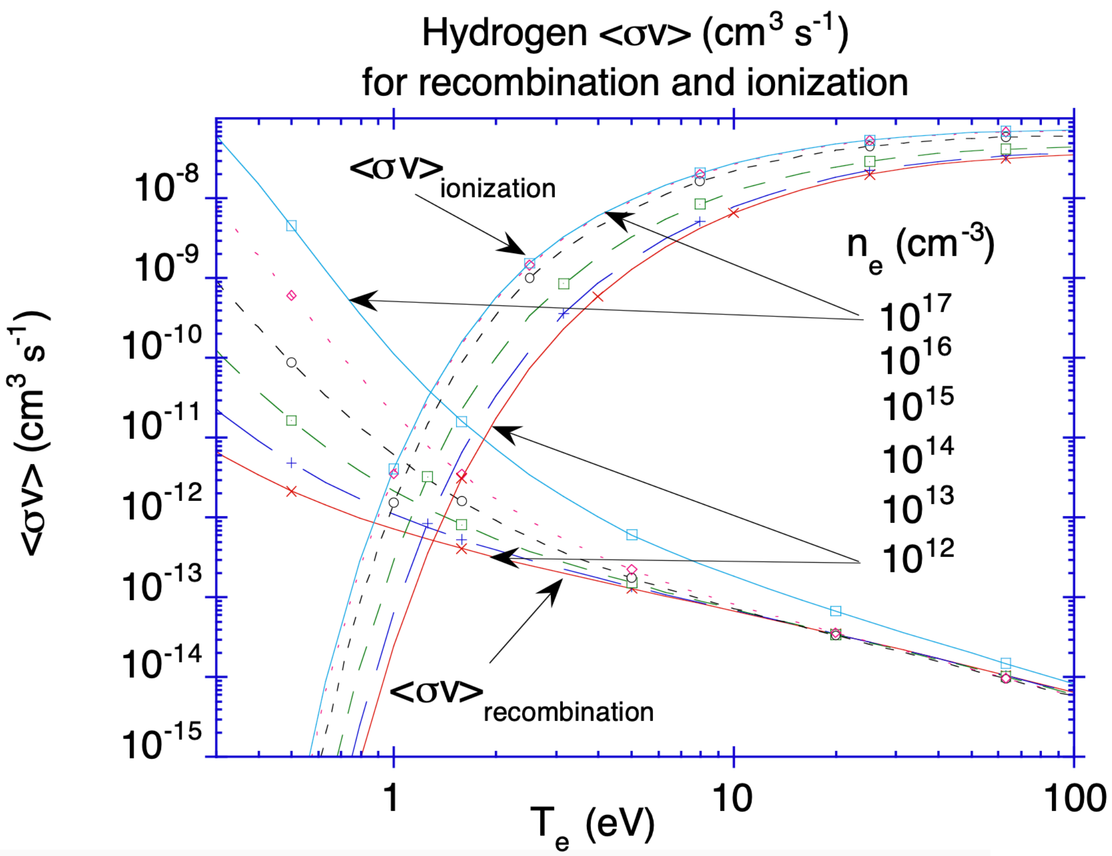
$$\frac{dn_e}{dt} = n_e n_0 \langle \sigma_{\text{ion}} v \rangle_e - n_e n_i \langle \sigma_{\text{rec}} v \rangle_e$$

$$\langle \sigma_{\text{ion}} v \rangle_e = \frac{1.0 \times 10^{-11} \sqrt{T_e}}{E_\infty^2 \left(6.0 + T_e/E_\infty\right)} \exp\left(-\frac{E_\infty}{T_e}\right) \left(\frac{m^3}{s}\right)$$

$$\langle \sigma_{\text{rec}} v \rangle_e = \frac{2.7 \times 10^{-19}}{T_e^{1/2}} \quad \left(\frac{m^3}{s}\right)$$

E_∞ = 13.6 eV is the hydrogen ionization energy

T_e (in eV) is the electron temperature



D.E. Post *et al.*, A Review of Recent Developments in Atomic Processes for Divertors and Edge Plasmas, Journal of Nuclear Materials 220-222 (1995) 143-157

The particle detailed balance

- **Plasma equilibrium is achieved when**

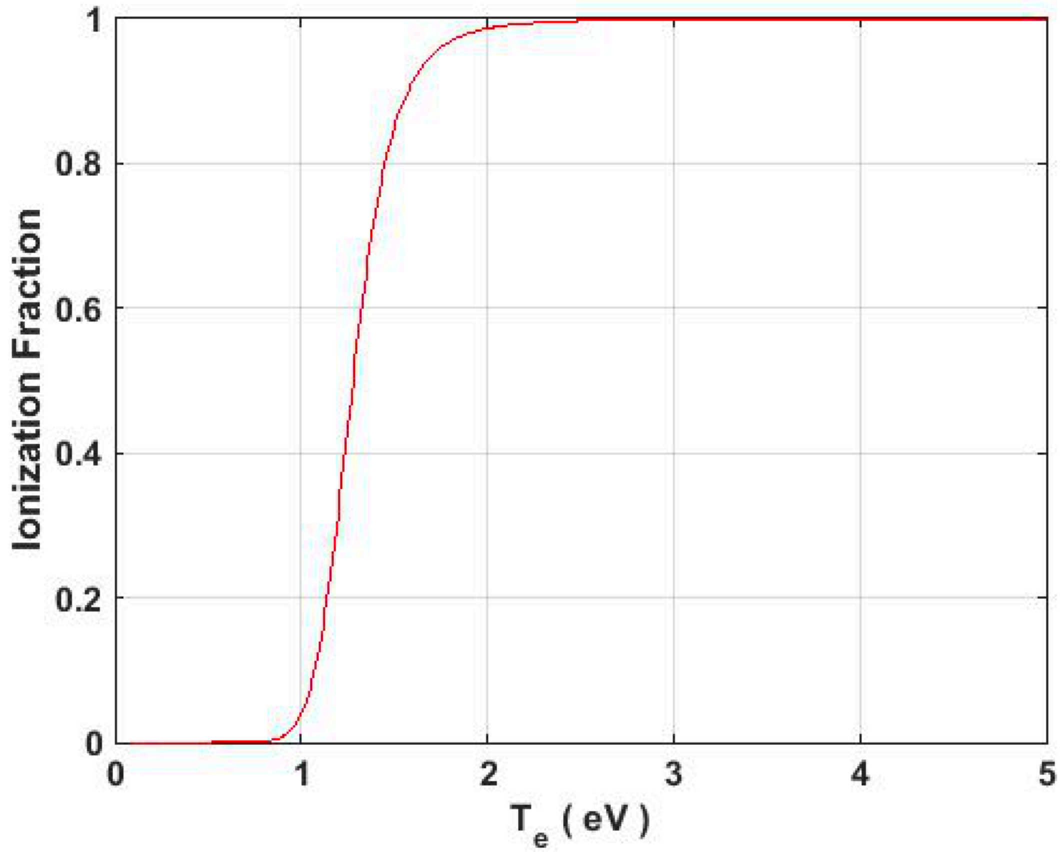
$$\frac{dn_e}{dt} = \sum_j R^+_{e,j} - \sum_k R^-_{e,k} = 0$$

- **For the case in which the dominant processes occurring in a plasma are: ionization by electronic impact and recombination, one obtain that**

$$n_e n_0 \langle \sigma_{\text{ion}} v \rangle_e = n_e n_i \langle \sigma_{\text{rec}} v \rangle_e$$

$$\frac{n_e}{n_0} = \frac{2.0 \times 10^5 \; T_e \; e^{-\frac{E_\infty}{T_e}}}{6.0 + T_e / E_\infty}$$

$$\alpha = \frac{n_e}{n_0 + n_e} = \frac{n_e/n_0}{1 + n_e/n_0}$$



Note that plasma transport was neglected in this very crude approximation

References

- **Theoretical descriptions of plasma phenomena**
  - *Bittencourt: Chap. 1 - Section 5*
- **Review of basic concepts in kinetic theory of gases**
  - *Bittencourt: Chap. 7*
- **Boltzmann's H theorem**
  - *Bittencourt: Chap. 21 - Section 3*
- **Particle interactions in plasmas**
  - *Bittencourt: Chap. 20*
- **Collisional processes**
  - *M.A. Lieberman: Ch. 3*