Directional Couplers Made of Nonidentical Asymmetric Slabs. Part II: Grating-Assisted Couplers

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Abstract—**In this second part of the paper we discuss TE and TM mode directional couplers made of nonsynchronous, nonidentical asymmetric slab waveguides. Coupling is accomplished with the help of a diffraction grating that is placed on either of the two guides on the side facing the opposite slab. The grating couples the compound modes of the complete slab structure to each other. However, since the two slabs are nonsynchronous, the compound modes resemble closely the modes of each slab in isolation. We also provide approximate expressions that result from approximating the compound modes with the modes of the individual slabs. The accuracy of these approximations is estimated by comparison with the exact results.**

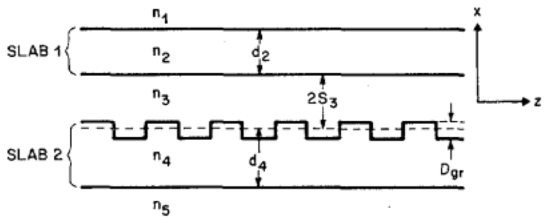

Fig. 1. Schematic of a grating-assisted slab directional coupler showing two slab cores (width d_2 and d_4) and a square shaped index grating on the lower slab.

INTRODUCTION

IN THE FIRST part of this paper we discussed directional couplers made of nonidentical slab waveguides [1]. Complete exchange of light power between the two guides occurs for those modes that have identical propagation constants. However, complete power exchange between the two slab waveguides is possible even if their modes have different propagation constants, provided a diffraction grating is etched into the face of at least one slab on the side facing the opposite slab [2], [3]. A grating-assisted directional coupler is schematically shown in Fig. 1. The grating may be located on either slab, but its effectiveness depends on its position. Complete power exchange between the two slabs can occur if the difference of the propagation constants of the modes that are to be coupled satisfies the relationship [4]

$$\beta_2 - \beta_1 = \frac{2\pi}{\Lambda} \tag{1}$$

with Λ indicating the length of one period of the diffraction grating.

The performance of a grating-assisted directional coupler can be described by conventional coupled mode theory [5]. However, this theory is based on the assumption that the interacting modes are mutually orthogonal. This requirement excludes the use of the modes of each individual slab since these modes are not orthogonal once the two slabs are placed in close proximity to each other.

In the first part of this paper we have made use of the compound modes of the combined structure of the two slabs to describe the directional coupler. These modes are mutually orthogonal. Their superposition at the input end of the directional coupler could be arranged so that most of the light power is concentrated over the input slab. Exchange of power occurs due to the difference in the propagation constants of the two compound modes. After the distance $L = 2\pi/(\beta_2 - \beta_1)$, the phase relationship among the modes is reversed. Thus, the superposition of the two modes now results in cancellation of the light power over the input slab and reinforcement over the opposite slab, which results in transfer of light from one slab to the other. The same process can approximately be described by a system of coupled differential equations for the amplitudes of the modes of the individual slabs. Both approaches yield results that are in good agreement. However, the coupled equations thus obtained are only approximately valid and do not follow directly from the conventional theory of coupled orthogonal modes.

The standard coupled mode theory for describing a grating-assisted directional coupler requires the use of a system of orthogonal modes [6]. The logical choice for such a mode system are the modes of the compound structure consisting of the two slabs. In the absence of the grating, these modes are not coupled with each other; coupling is provided by the grating. Even though compound modes of the combined slab structure are used for describing power exchange in synchronous directional couplers as well as in asynchronous grating-assisted couplers, their roles are very different in the two cases. In the asynchronous couplers, the modes of the compound structure—even though they overlap both slabs—carry power predominantly only in the region of one or the other slab.

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Thus, superposition of these two modes cannot lead to complete exchange of power among the two slab waveguides. But when the two compound modes are coupled by the diffraction grating, they do interchange their power with resultant power exchange among the slabs of the directional coupler.

Use of the compound modes of the two slabs complicates the mathematical description of the directional coupler, preventing us from obtaining closed-form analytical expressions for the coupling coefficients. Since the two compound modes resemble the modes of the individual slab waveguides, it is tempting to replace them with these more easily computed modes. This approximation leads to mathematical expressions in closed form for the coupling coefficients. These approximations often yield the correct order of magnitude of the coupling coefficients but they are not adequate for precise predictions of the behavior of the directional coupler. A comparison of the exact and approximate coupling coefficients is the principal contribution of this paper.

Coupled Wave Equations for the Grating Coupler

Since we are only interested in the interaction of two guided modes with mode amplitudes a_1 and a_2 and propagation constants β_1 and β_2, we use the following set of coupled equations to describe the directional coupler [6]

$$\frac{da_1}{dz} = -i\beta_1 a_1 + i\overline{K} a_2 \tag{2}$$

$$\frac{da_2}{dz} = -i\beta_2 a_2 + i\overline{K} a_1. \tag{3}$$

The coupling coefficient $\overline{K}$ is defined as

$$\overline{K} = 2\hat{K} f(z) \tag{4}$$

with [5]

$$\hat{K} = -\frac{\omega\epsilon_0}{8PD_{gr}} \cdot (n_4^2 - n_3^2) \int_{-(S_3 + D_{gr}/2)}^{-(S_3 + D_{gr}/2)} \vec{E}_1(x) \cdot \vec{E}_2(x)\, dx. \tag{5}$$

In these equations, $\vec{E}_1$ and $\vec{E}_2$ are the electric vectors of the mode fields that are coupled together. In the case of the directional coupler they are the modes of the compound structure in the absence of the grating; they were defined and discussed in [1]. If the grating is positioned as shown in Fig. 1, n_4 and n_3 are the refractive index values of the core of the lower slab and of the medium between the two slabs. P is the (unit) power carried by each mode field. This is just a constant normalizing the fields and is assumed the same for both modes. The light is assumed to be monochromatic with angular frequency ω; ϵ_0 is the electric permittivity of vacuum. The integral extends over the transverse dimension of the grating and the function $f(z)$ describes the deformation of the core boundary of the slab that constitutes the grating. According to Fig. 1, $f(z)$ is a step function. However, when the step function is expressed as a Fourier series of sinusoidal functions, only one of the Fourier components, usually the fundamental component, takes part in the interaction of the two waves [6]. Thus, we express $f(z)$ as the fundamental component of its Fourier transform

$$f(z) = \frac{2}{\pi} D_{gr} \cos\left(\frac{2\pi}{\Lambda} z\right) \tag{6}$$

with grating depth D_{gr}. The propagation constants β_1 and β_2 appearing in the coupled wave equations may have opposite signs permitting mode coupling in opposite directions if the grating period is sufficiently short.

The wave amplitudes a_1 and a_2 are rapidly varying functions of z. This rapid z-variation can be removed by introducing slowly varying amplitudes A_1 and A_2

$$a_j = A_j e^{-i\beta_j z}, \quad j = 1, 2. \tag{7}$$

Substitution of (4), (6), and (7) into (2) and (3) results in

$$\frac{dA_1}{dz} = i\frac{2}{\pi} D_{gr}\hat{K} A_2 \left(e^{i(2\pi/\Lambda)z} + e^{-i(2\pi/\Lambda)z}\right) e^{-i(\beta_2 - \beta_1)z} \tag{8}$$

$$\frac{dA_2}{dz} = i\frac{2}{\pi} D_{gr}\hat{K} A_1 \left(e^{i(2\pi/\Lambda)z} + e^{-i(2\pi/\Lambda)z}\right) e^{i(\beta_2 - \beta_1)z}. \tag{9}$$

The product of the exponential function $\exp(\pm i(\beta_2 - \beta_1)z)$ with the two terms of the exponential expansion of the cosine function results in two different expressions. If (1) holds, the first expression becomes a constant (unity), while the second expression is a rapidly varying function that oscillates with a spatial frequency $2(2\pi/\Lambda)$. When integrating the differential equation, this second term makes a negligible contribution. Thus, we may write the system of coupled wave equations in the form

$$\frac{dA_1}{dz} = iKA_2 \tag{10}$$

$$\frac{dA_2}{dz} = iKA_1 \tag{11}$$

with

$$K = \frac{2}{\pi} D_{gr}\hat{K}. \tag{12}$$

Equations (5), and (10)–(12) provide the mathematical tools for the description of the grating-assisted directional coupler. The electric field components of the compound modes and the eigenvalue equation for computing the propagation constants β_1 and β_2 are given in [1]. Unfortunately, these equations are so cumbersome as to make it impractical to write down closed-form expressions for the coupling coefficient. Instead, it is simpler to compute numerical values for K and study its properties by computing values for specific examples.

Approximate Coupling Coefficient

We remarked in the introduction (and shall show later) that the compound modes of the structure consisting of the two slabs closely resemble the modes of each slab taken in isolation. This observation makes it attractive to approximate the mode fields $\vec{E}_1$ and $\vec{E}_2$ appearing in (5) by the modes of the isolated slabs. The mode field expressions are given in [6] and the eigenvalue equation in [1] and [6]. Straightforward calculations yield the following approximate expression for the coupling coefficient of TE modes

$$K = \frac{D_{gr}}{\pi} \frac{\kappa_2 \kappa_4 (n_4^2 - n_3^2)^{1/2} \, e^{-2\gamma_{23} S_3}}{\left[|\beta_2 \beta_4| (n_2^2 - n_3^2) \left(d_2 + \frac{1}{\gamma_{23}} + \frac{1}{\gamma_1} \right) \left(d_4 + \frac{1}{\gamma_{43}} + \frac{1}{\gamma_5} \right) \right]^{1/2}}. \tag{13}$$

As usual, the coupling coefficient for TM modes is much more complicated

$$K = \frac{D_{gr}}{\pi} \frac{\left(1 - \frac{n_4^2}{n_3^2} \frac{\gamma_{23}\gamma_{43}}{\beta_2 \beta_4}\right) n_2 n_3^2 (n_4^2 - n_3^2) \, \kappa_2 \kappa_4 \, |\beta_2 \beta_4|^{1/2} \, e^{-2\gamma_{23} S_3}}{n_4 \left[(n_3^4 \kappa_2^2 + n_2^4 \gamma_{23}^2)(n_3^4 \kappa_4^2 + n_4^4 \gamma_{43}^2) \, C_2 C_4 \right]^{1/2}} \tag{14}$$

with

$$C_2 = d_2 + \frac{n_2^2 n_3^2}{\gamma_{23}} \frac{\kappa_2^2 + \gamma_{23}^2}{n_3^4 \kappa_2^2 + n_2^4 \gamma_{23}^2} + \frac{n_1^2 n_2^2}{\gamma_1} \frac{\kappa_2^2 + \gamma_1^2}{n_1^4 \kappa_2^2 + n_2^4 \gamma_1^2} \tag{15}$$

and

$$C_4 = d_4 + \frac{n_4^2 n_5^2}{\gamma_5} \frac{\kappa_4^2 + \gamma_5^2}{n_5^4 \kappa_4^2 + n_4^4 \gamma_5^2} + \frac{n_4^2 n_3^2}{\gamma_{43}} \frac{\kappa_4^2 + \gamma_{43}^2}{n_3^4 \kappa_4^2 + n_4^4 \gamma_{43}^2}. \tag{16}$$

The symbols β_2 and β_4 indicate propagation constants of modes belonging to the slabs with core index n_2 and n_4, respectively. With the free space propagation constant $k = 2\pi / \lambda$ (λ = vacuum length) we define the parameters appearing in (13)–(15) as follows

$$\begin{aligned} \kappa_2^2 &= n_2^2 k^2 - \beta_2^2 & \kappa_4^2 &= n_4^2 k^2 - \beta_4^2 \\ \gamma_1^2 &= \beta_2^2 - n_1^2 k^2 & \gamma_5^2 &= \beta_4^2 - n_5^2 k^2 \\ \gamma_{23}^2 &= \beta_2^2 - n_3^2 k^2 & \gamma_{43}^2 &= \beta_4^2 - n_3^2 k^2. \end{aligned} \tag{17}$$

As written, the coupling coefficients belong to a grating on the lower slab, as shown in Fig. 1, with core index n_4. If, instead, the grating is placed on the upper slab, the following transformation of indices must be made

$$\begin{aligned} 1 &\to 5, \quad 2 \to 4, \quad 3 \to 3, \\ 4 &\to 2, \quad 5 \to 1. \end{aligned} \tag{18}$$

For asymmetric slabs, the value of the coupling coefficient depends strongly on whether the grating is placed on the upper or lower slab.

The approximate coupling coefficients (13) and (14) are useful for computing order-of-magnitude estimates, but we shall show that they are not very accurate.

Examples and Numerical Results

To illustrate the results of the theory we consider two examples. In the first case, we assume that the directional coupler consists of two dissimilar slabs. The upper slab (as defined in Fig. 1) has a core width $d_2 = 1$ μm and a core refractive index of $n_2 = 3.3$. The lower slab has a core width of $d_4 = 0.3$ μm and a core refractive index of $n_4 = 3.5$. The refractive index of the medium above the upper slab is assumed to be air with $n_1 = 1$. The refractive index of the medium between the two slabs is $n_3 = 3.2$. Finally, the index of the medium below the lower slab has the refractive index $n_5 = 3$. The vacuum wavelength is assumed to be $\lambda = 1.5$ μm.

Fig. 2(a)–(d) shows the E_y component of the electric field of the lowest order TE compound modes of the directional coupler as solid lines for several values of the spacing $2S_3$ between the two slabs. The corresponding pictures for the H_y component of compound TM modes look quite similar. The dotted lines indicate the corresponding mode fields of the isolated slabs that would exist in the absence of the opposite slab. The boundaries of the slab cores are indicated by the upwards pointing arrows. This figure shows clearly how closely the compound modes resemble the mode fields of the individual, isolated slabs for large slab spacing. As the slabs move closer together the mode fields depart more and more from the modes of the isolated slabs. This departure is particularly significant at the core boundaries of the slab opposite the field maximum because, for a grating coupler, it is the value of the product of the electric field strengths at the position of the grating that determines the coupling strength. Obviously, the field values of the compound and isolated slab modes at this critical point can be completely different.

The grating is located either on the lower slab, as shown in Fig. 1, or in a corresponding position on the upper slab. The length L required for total exchange of power between the slabs is inversely proportional to the coupling coefficient K appearing in the coupled wave equations (10)

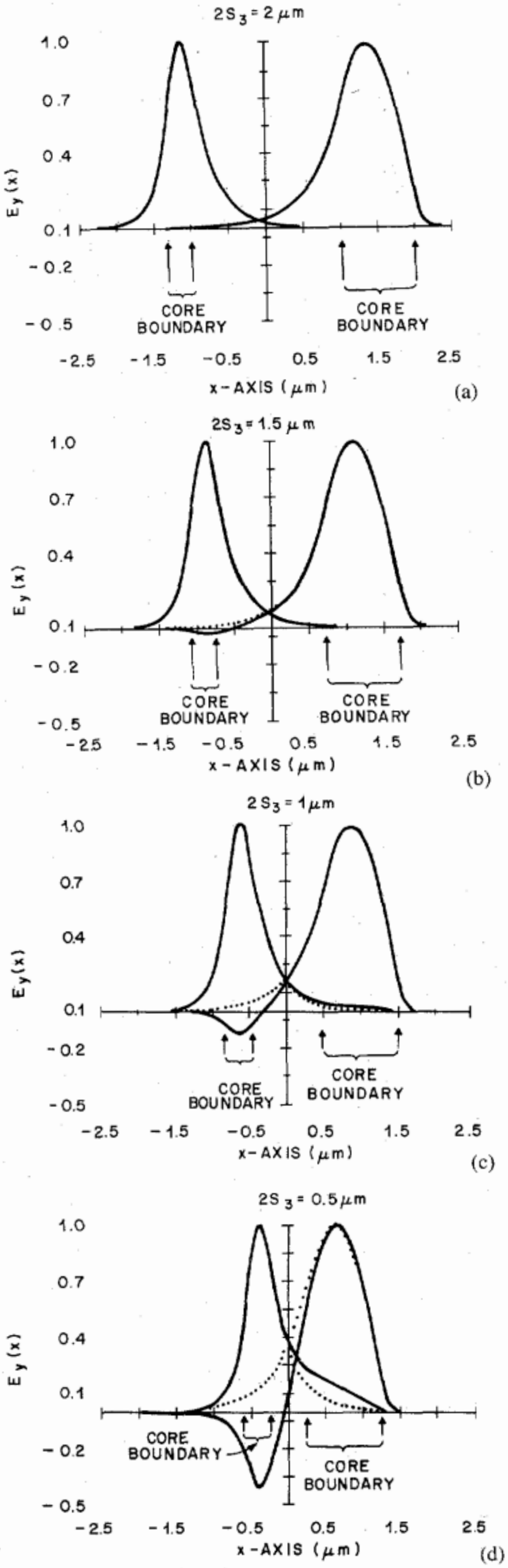

Fig. 2. (a)–(d) The electric E_y component of the two compound modes of lowest order for four different slab core spacings. The physical parameters used for these calculations are spelled out in the text.

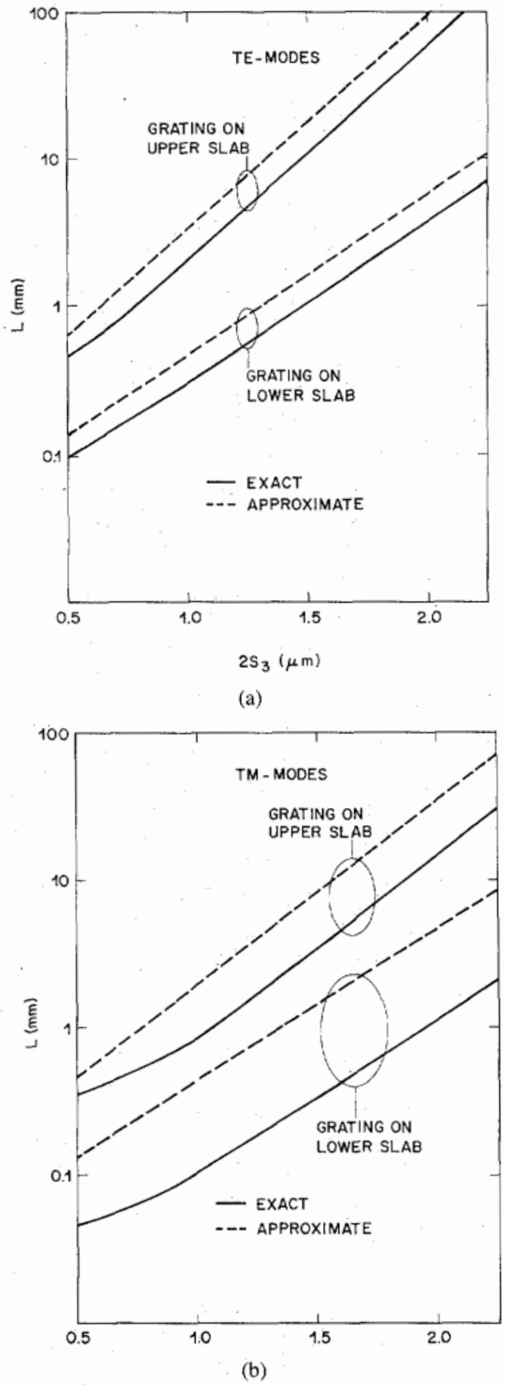

Fig. 3. Power exchange length L in millimeters as a function of slab core spacing for: (a) TE modes and (b) TM modes. The solid curves represent the exact results based on compound modes, the dotted curves are approximations obtained by replacing the compound modes with the modes of that individual slabs.

and (11)

$$L = \frac{\pi}{2|K|}. \tag{19}$$

Fig. 3(a) shows the power exchange lengths L for TE modes for a square grating of depth $D_{gr} = 0.1\ \mu m$ as functions of the spacing $2S_3$ between the slab cores. Fig. 3(b) shows the corresponding results for TM modes. The solid lines in both figures were computed by substituting the fields of the compound modes into (5). The dotted curves were obtained by using the approximate coupling coefficients (13) and (14). For TE modes (Fig. 3(a)) the approximate power exchange length is roughly 1.5 times as

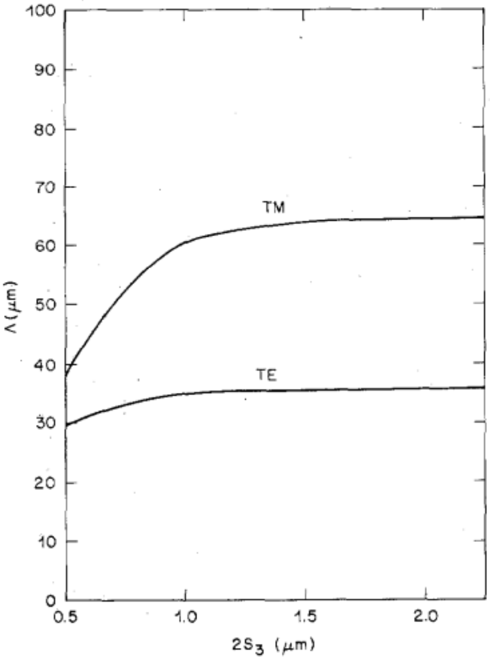

Fig. 4. The grating period Λ as a function of slab core spacing for TE and TM mode coupling.

large as the exact value. For TM modes (Fig. 3(b)) the discrepancy between the exact theory and the approximation is much larger. In spite of the large discrepancy, the approximate formulas for the coupling coefficients are useful in two ways. They often yield order-of-magnitude estimates and they give an indication how the coupling coefficient K depends on grating depth and slab core spacing.

If significant cross coupling is to occur, the grating condition (1) must be satisfied. Since this condition holds for the compound modes, it is a function of the spacing between the slab cores as shown in Fig. 4. For the approximate modes, (1) yields a value that is independent of the core spacing and is equal to the limits shown in Fig. 4 for large core spacing.

Looking at the compound mode fields in Fig. 2 we see that one of them crosses the horizontal axis near the core boundary of the opposite slab. If this zero crossing should occur right at the core boundary that is carrying the grating, the coupling efficiency would suffer significantly. To illustrate this point we look at another example. Now we assume that the refractive index distributions are more nearly symmetrical: $n_1 = n_3 = n_5 = 3.2$. The two slabs also have equal width $d_2 = d_4 = 1$ μm. However the refractive indices of the slab cores are different, $n_2 = 3.25$ and $n_4 = 3.23$. The wavelength is still $\lambda = 1.5$ μm.

Fig. 5(a)–(d) shows the E_y components of the compound TE modes as functions of the slab spacing $2S_3$. The most significant difference between this figure and Fig. 2(a)–(d) is the fact that the zero crossing of one of the compound modes now occurs very near one of the core boundaries. The consequence of this fact becomes dramatically apparent in Fig. 6(a) and (b) which corresponds to Fig. 3(a) and (b). The power exchange length for a grating-assisted coupler whose grating is located on the

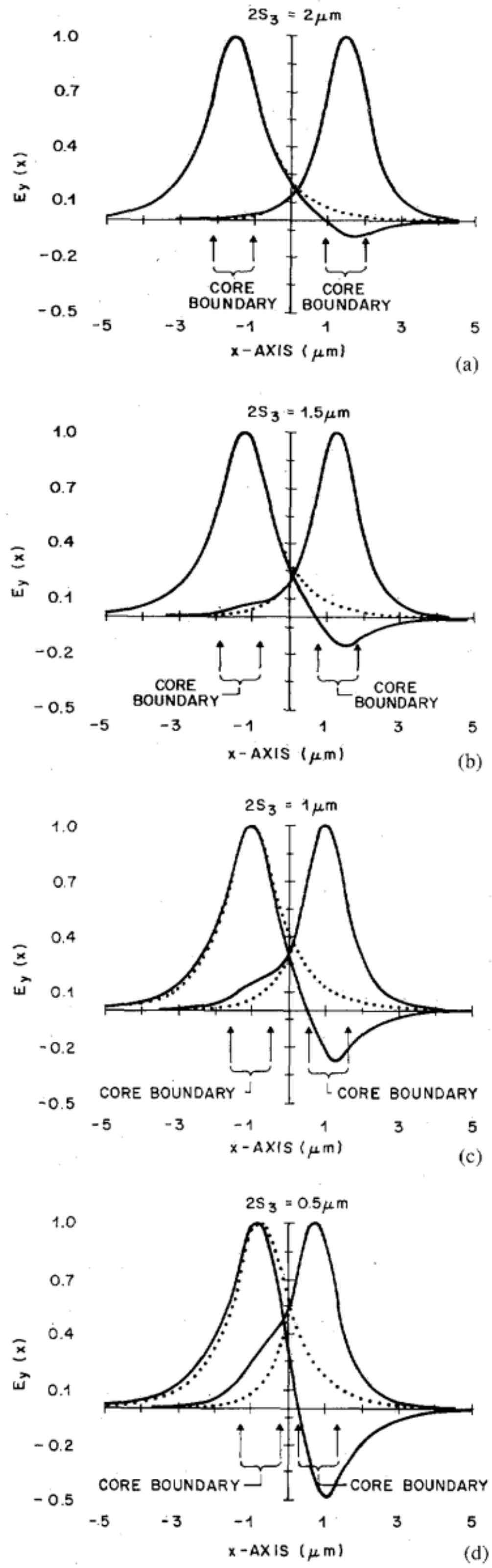

Fig. 5. (a)–(d) This figure is similar to Fig. 2 for different slab parameters.

boundary of the core of the upper slab (where the zero crossing of the field occurs) is very much larger in this case. In fact, at one point it reaches infinite values, because the zero crossing of the field at the grating is located such that the integral in (5) vanishes. In this case, the discrepancy between the exact and approximate theories

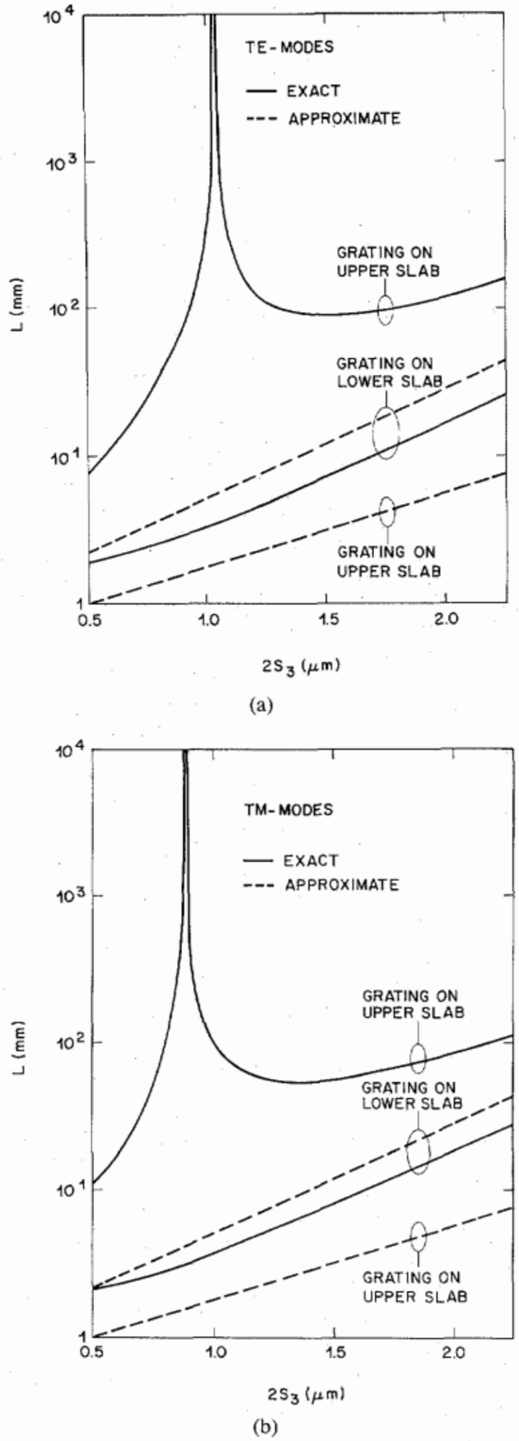

Fig. 6. (a), (b) This figure is similar to Fig. 3 for different slab parameters.

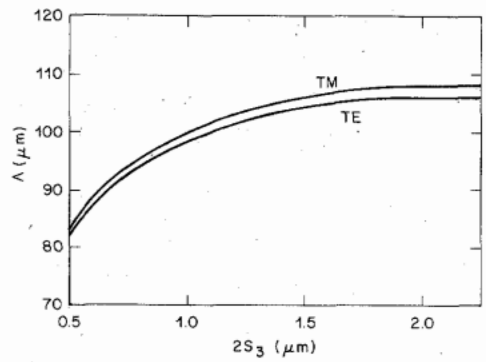

Fig. 7. This figure is similar for Fig. 4 for different slab parameters.

is enormous for gratings located on the upper slab core. For a grating on the lower slab nothing unusual happens and the discrepancy between exact and approximate theory is no worse than in Fig. 3(a). Fig. 6(b) shows that TM mode coupling displays very much the same behavior.

The grating period as a function of length for this second example is shown in Fig. 7. Since the refractive index differences are very much smaller in this second case ($n_1 = n_3$ in this case while $n_3 - n_1 = 2.2$ in the first example) the TM modes are very similar to the TE modes.

Conclusions

The description of a grating-assisted slab directional coupler requires the use of the compound modes of the combined slabs. Since this description does not lend itself to the derivation of simple formulas, we derived approximate analytical expressions for the coupling coefficients by approximating the compound modes by the modes of the individual, isolated slabs. Comparison of the exact and approximate results shows that the approximation yields only an order-of-magnitude estimate. In particular, it can happen that the compound modes have zeros at the position of the coupling grating. When that happens the grating coupler is ineffectual and the approximate theory yields completely misleading results. Thus it is advisable to look at the shape of the compound modes to judge if they resemble the approximate modes sufficiently closely to justify the use of the approximate formulas for the coupling coefficients.

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