Coupled Mode Formulation for Directional Couplers with Longitudinal Perturbation

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***Abstract*—A grating-assisted directional coupler is investigated using an improved coupled-mode formulation for multiwaveguide systems with longitudinal perturbation. This approach is capable of handling three-dimensional structures as well as complex index of refraction and nonisotropic medias. The case of two coupled channel waveguides is closely examined, and numerical analysis is carried out for several cases of slab structures.**

I. Introduction

ENERGY coupling between parallel channels has been discussed extensively in the past [1] and reformulated lately by several groups under the title of "improved coupled mode theory" [2]–[7]. All the mentioned approaches are based on the underlying assumption that transverse field distribution of several parallel channels can be approximated, at any point along the propagation direction (z axis), by a linear combination of the separate transverse field solution of each channel, which is obtained by "erasing" the neighboring channels from the transverse index profile of the whole structure. For the case of two parallel channels, shown in Fig. 1, it can be formulated as follows:

$$E_{t(x,y,z)} \cong a(z)\, E_t^{(a)}(x, y) + b(z)\, E_t^{(b)}(x, y)$$
$$H_{t(x,y,z)} \cong a(z)\, H_t^{(a)}(x, y) + b(z)\, H_t^{(b)}(x, y) \qquad (1)$$

where the superscripts (a) and (b) denote the separate waveguiding channels, and $a(z)$ and $b(z)$ are the local amplitudes of each field along the direction of propagation. The main motivation behind this approach is that it is much easier to solve, sometimes analytically, the modal field and propagation constants of the separate waveguides, and then calculate the total field and propagation constants of the whole structure in terms of these results, rather than to analyze numerically the whole structure.

It was shown, under the validity of the mentioned approximation, that in any structure of directional parallel channels there is a certain amount of energy coupling, herein referred to as "natural coupling," which is expressed mathematically as a gradual increase of one of the channel field amplitudes, $a(z)$ or $b(z)$, on the account of the other along the direction of propagation. The efficiency of this natural coupling is strongly dependent on the geometry of the discussed structure. If the channel indexes of refraction are identical and the whole structure is symmetrical, then a complete exchange of energy between the two channels is possible. However, when there is a slight difference between the separate channel parameters, namely the dimensions or the refractive indexes, so that the propagation constants of the separate waveguide modes are not identical, then a complete exchange of energy is no longer possible and the efficiency might decrease significantly.

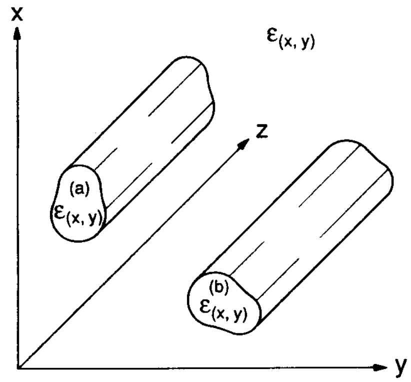

Fig. 1. A general structure of two parallel channels.

One way of improving the mode coupling efficiency in a nonidentical channel structure is to form a periodic perturbation along the direction of propagation in such a way that the wave number of that perturbation $K = 2\pi/\Lambda$ (Λ being the length of one period) is equal to the difference between the separate modes propagation constants, namely,

$$\beta^{(b)} - \beta^{(a)} = \frac{2\pi}{\Lambda}. \qquad (2)$$

The idea of creating a longitudinal grating in order to couple energy between modes that otherwise would not be coupled or would be coupled insufficiently was used before by Kogelnik [8] and by Stoll and Yariv [9] in order to couple energy between orthogonal modes of one structure. Some aspects of the approach for coupling between nonidentical channels were discussed by März and Nolting [10] who based their derivation on the formulation of Yariv. An interesting practical use for the grating-assisted directional coupler was presented recently by Alferness *et*

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al. [11] who reported the fabrication of a codirectional slab coupler made of InGaAsP–InP–InP. It turns out that the coupling efficiency in these kinds of structures has a strong wavelength selectivity; therefore, they can serve as optical filters in lightwave networks. Moreover, it was shown that this codirectional coupler has an improved tuning range with respect to that of the contradirectional type [12] by a factor equivalent to the ratio of the grating periods of the two structures (a calculated realistic value of the center wavelength shift of ~ 100 Å compared to ~ 6 Å). These kinds of devices can be easily monolithically integrated with other semiconductor optoelectric devices, and therefore they are the focus of increased attention.

An approximate formalism by which the coupling coefficients and the interaction length can be calculated was suggested by Marcuse [13]. This approach deals with asymmetric slab double-waveguide structures and is based on the fact that in strongly asymmetric index profile, each of the compound modes of a combined structure of two slabs, which are mutually orthogonal, carries power mainly in the region of one of the slabs; therefore, a grating which is constructed to couple energy between the compound modes can be used to couple energy between the two slabs. In order to simplify the calculation, and based on the fact that in strongly asymmetric structures the compound mode fields resemble the mode fields of the individual waveguides, the latter were used to calculate the coupling coefficients. It was found that by using this approximation, an order of magnitude value of the coupling coefficients and interaction length can be obtained, but it cannot provide an exact analysis of the directional coupler.

An improved approach was presented recently by Huang and Haus [14] (HH) who used the nonorthogonal coupled mode theory to analyze a grating-assisted slab coupler in a lossless system. Their approach was shown to be in good agreement with the compound mode approach suggested by Marcuse. However, in order to calculate the grating period and coupling length, the two most important parameters for designing such device, HH have carried a linear transformation from the nonorthogonal-mode formulation to the orthogonal (compound) mode one. That is, the power exchange process was assumed to take place between the two compound modes, with a grating period calculated for phase matching of their propagating constants. This approach fails to yield good results when the two waveguides are close to synchronism and/or are strongly coupled (and an assisting grating is no longer necessary). In many practical devices, the state of affairs is that in order to decrease the coupling length, they are indeed strongly coupled [11]. Therefore, a different approach is required.

The approach presented in this paper is based on the improved coupled mode theory [2]–[6] and, under the limitation of this technique, can be used to obtain highly accurate results. The presented formulation can be used to analyze multiwaveguide systems and three-dimensional directional couplers (such as fiber–fiber coupling), and it can be used to solve structures which have gain and/or loss or nonisotropic media [15]. The formulation is currently being used for the design of grating-assisted semiconductor tunable filters; it has also been used to analyze the spectral properties of such devices, yielding excellent agreement with experimental results [16].

In the following section, the coupled mode equations are presented. The resultant set of equations include terms which are obtained due to the natural coupling and terms which appear because of the assisted grating. In Section III, the case of two waveguide systems is examined more closely. In Section IV, the conditions for power conservation in the exact and the approximate formulation are discussed. In Section V, some application examples for slab structures, for which analytical expressions for the coupling coefficients can be obtained, are presented. These coefficients are calculated for several particular cases and are used for the calculations of the grating factor and the length of the interaction. Comparisons between the presented approach and former techniques are also given. Finally, the influence of the grating location and its magnitude is analyzed.

II. Formulation

The approach presented here deals with structures whose dielectric dependence in space $\epsilon(x, y, z)$ can be given by

$$\epsilon(x, y, z) = \epsilon(x, y) + \Delta\epsilon(x, y, z). \tag{3}$$

Here, $\epsilon(x, y)$ is a general medium that is translation invariant in the z-direction and is comprised of N waveguides such as shown in Fig. 2(a). Each of these waveguides can be described separately by $\epsilon^{(i)}(x, y)$ [Fig. 2(b)] such that

$$\epsilon(x, y) = \epsilon^{(i)}(x, y) + \Delta\epsilon^{(i)}(x, y). \tag{4}$$

The term $\Delta\epsilon(x, y, z)$ in (3) is a small perturbation of the medium so that for any value of z the inequality

$$\Delta\epsilon(x, y, z)/\epsilon(x, y) \ll 1 \tag{5}$$

holds.

We shall restrict the discussion to perturbations which can be expressed in the form

$$\Delta\epsilon(x, y, z) = \epsilon_0 f(x, y)\, \Delta n^2(z). \tag{6}$$

For the sake of convenience, we shall hereon omit the x, y dependency from the structure parameters ϵ, $\Delta\epsilon(z)$, $\Delta\epsilon^{(i)}$, and f, as well as from the various field amplitudes.

The derivation of the coupled mode equations is quite lengthy and tedious. Therefore, it appears in complete form in Appendix A. It is shown there, following some substantial manipulations, that the coupled mode differential equations for the described structure are given in a matrix form by

$$\frac{d}{dz} \boldsymbol{U}(z) = i(\boldsymbol{B} + \boldsymbol{C}^{-1}\tilde{\boldsymbol{K}} + \Delta n^2(z)\boldsymbol{C}^{-1}\hat{\boldsymbol{K}}^T)\, \boldsymbol{U}(z) \tag{7}$$

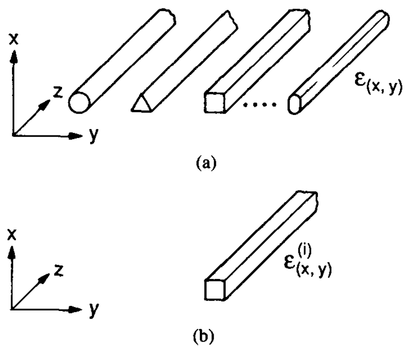

Fig. 2. (a) A multiwaveguide system $\epsilon(x, y)$ consists of N parallel channels. (b) A single waveguide $\epsilon^{(i)}_{(x,y)}$ isolated from the multiwaveguide system.

where $\boldsymbol{U}(z)$ is a vector that consists of the amplitudes of the individual waveguides modes, which are linearly superpositioned to approximate the total propagating electromagnetic field of the nonperturbed medium $\epsilon(x, y)$. $\boldsymbol{B}$ is a diagonal matrix, $\boldsymbol{B} = \text{diag}\,[\beta_1, \beta_2, \cdots, \beta_N]$, with β_i being the unperturbed propagation constants of the guided modes of the isolated waveguides. The matrix $\boldsymbol{C}$ is the overlapping matrix whose elements are defined by

$$C_{pq} = C_{qp} = \frac{c_{pq} + c_{qp}}{2} \tag{8}$$

where

$$c_{pq} = \frac{1}{2} \iint_{-\infty}^{\infty} [\boldsymbol{E}_t^{(p)} \times \boldsymbol{H}_t^{(q)}]_z \, dx\, dy. \tag{9}$$

The elements of the matrix $\tilde{\boldsymbol{K}}$ are defined by

$$\tilde{K}_{pq} = \frac{\omega}{4} \iint_{-\infty}^{\infty} \Delta\epsilon^{(q)} \left[\boldsymbol{E}_t^{(p)} \cdot \boldsymbol{E}_t^{(q)} - \frac{\epsilon^{(p)}}{\epsilon} \boldsymbol{E}_z^{(p)} \boldsymbol{E}_z^{(q)} \right] dx\, dy \tag{10}$$

and $\hat{\boldsymbol{K}}^T$ is the transposed matrix of $\hat{\boldsymbol{K}}$, whose elements are defined by

$$\hat{K}_{pq} = \frac{\omega}{4} \epsilon_0 \iint_{-\infty}^{\infty} f \left[\boldsymbol{E}_t^{(p)} \cdot \boldsymbol{E}_t^{(q)} - \frac{\epsilon^{(p)}}{\epsilon} \boldsymbol{E}_z^{(p)} \boldsymbol{E}_z^{(q)} \right] dx\, dy. \tag{11}$$

It should be noted that if the dielectric profile functions of the isolated waveguides $\epsilon^{(p)}(x, y)$ are chosen such that they all have the same value in the lateral grating region (that is, $f(x, y)\epsilon^{(p)} = f(x, y)\epsilon^{(q)}$), then $\hat{K}_{pq} = \hat{K}_{qp}$ for $p \neq q$. It is shown later in Section IV that such a selection is a necessary condition for satisfying the power conservation relation. Such a selection can be made easily by choosing the "DC term" of the grating as the dielectric constant value of the isolated waveguides in the lateral location of the grating as shown in Fig. 3(b). Another possibility is simply to neglect the influence of the grating "DC term" on the isolated waveguides solution and to assume that the value of the dielectric constant there is the same as the background (interguides) material [Fig. 3(c)].

Examination of the coupled-mode equation (7) for the longitudinal perturbed structure reveals that when the perturbation magnitude is being taken to the zero limit, $\Delta n^2(z) \to 0$, (7) is left with the first two terms on the right-hand side, which is the form derived earlier by the coupled-mode formalism for unperturbed structures (see, for example, [4, (19)]. Therefore, the present formulation agrees with and is actually extending the prior theory by adding the third term of (7).

III. Two Waveguide Systems

Since most of the grating-assisted coupler studies so far consist of two asymmetric waveguides [10]–[14], we shall examine more closely the results of the applying suggested formulation to these kinds of structures. Equation (7) can also be written in the form

$$\frac{d}{dz} \boldsymbol{U}(z) = i[\boldsymbol{M} + \Delta n^2(z) N]\, \boldsymbol{U}(z). \tag{12}$$

For the case of $N = 2$, the vector $\boldsymbol{U}(z) = \text{col}[a(z), b(z)]$ consists of the amplitudes of the isolated waveguide modes. The matrix $\boldsymbol{M}$ is defined by

$$\boldsymbol{M} = \begin{bmatrix} \gamma_a & K_{ab} \\ K_{ba} & \gamma_b \end{bmatrix} \tag{13}$$

with the matrix elements γ_a, γ_b, K_{ab}, and K_{ba} given by

$$\gamma_a = \beta_a + (\tilde{K}_{aa} - c\tilde{K}_{ba})/(1 - c^2) \tag{14a}$$

$$\gamma_b = \beta_b + (\tilde{K}_{bb} - c\tilde{K}_{ab})/(1 - c^2) \tag{14b}$$

$$K_{ab} = (\tilde{K}_{ab} - c\tilde{K}_{bb})/(1 - c^2) \tag{14c}$$

$$K_{ba} = (\tilde{K}_{ba} - c\tilde{K}_{aa})/(1 - c^2) \tag{14d}$$

and the matrix N is defined by

$$N = \begin{bmatrix} k_{aa} & k_{ab} \\ k_{ba} & k_{bb} \end{bmatrix} \tag{15}$$

with matrix elements

$$k_{aa} = (\hat{K}_{aa} - c\hat{K}_{ab})/(1 - c^2) \tag{16a}$$

$$k_{bb} = (\hat{K}_{bb} - c\hat{K}_{ba})/(1 - c^2) \tag{16b}$$

$$k_{ab} = (\hat{K}_{ba} - c\hat{K}_{bb})/(1 - c^2) \tag{16c}$$

$$k_{ba} = (\hat{K}_{ab} - c\hat{K}_{aa})/(1 - c^2). \tag{16d}$$

The constants γ_a and γ_b are corrected forms of the propagation constants of the modes in each of the individual waveguides, K_{ab} and K_{ba} are the natural coupling coefficients, while k_{ab} and k_{ba} are the coefficients for energy coupling due to the existence of the longitudinal perturbation. The elements k_{aa} and k_{bb} are the self-coupling factors, and it is shown in Appendix B that these terms can be neglected if the perturbation is small enough.

$\tilde{K}_{p,q}$ and $\hat{K}_{p,q}$ are defined by (10) and (11), respectively, where we have substituted $(p, q) = (a, b)$, and $c \equiv C_{ab} = C_{ba}$ is given by (8).

Assume a z-dependent periodic grating of the form

$$\Delta n^2(z) = 2d \cos\left(\frac{2\pi}{\Lambda} z\right) = d(e^{iK_g z} + e^{-iK_g z}) \quad (17)$$

where the grating constant $K_g \equiv 2\pi/\Lambda$ is chosen to phase match between the modes of the isolated waveguides, i.e.,

$$\frac{2\pi}{\Lambda} \simeq \gamma_a - \gamma_b \equiv 2\Delta, \quad (18)$$

and define a detuning factor δ, which measures the deviation of the grating constant from exact matching

$$2\delta \equiv 2\Delta - \frac{2\pi}{\Lambda} \quad (19)$$

as well as removing the rapid z-variation of $a(z)$ and $b(z)$ by introducing two slowly varying amplitudes $A(z)$ and $B(z)$, namely

$$a(z) = A(z)e^{i\gamma_a z} \quad (20a)$$

$$b(z) = B(z)e^{i\gamma_b z}. \quad (20b)$$

It can be shown that by substituting (17)–(20) into (12), a new form of coupled-mode equations (B1) is obtained which, upon neglecting asynchronous terms (Appendix B), is reduced to the form

$$\frac{d}{dz} A(z) = i(K_{ab}e^{-i2\Delta z} + dk_{ab}e^{-i2\delta z})\, B(z) \quad (21a)$$

$$\frac{d}{dz} B(z) = i(K_{ba}e^{i2\Delta z} + dk_{ba}e^{i2\delta z})\, A(z). \quad (21b)$$

All of the prior theories that dealt with the problem of energy coupling assisted by a longitudinal grating considered energy coupling between modes of a compound structure, for which no natural coupling exists. Even in the approach presented by Marcuse [13], which dealt with nonidentical asymmetric slabs, the set of coupled-mode equations and the formula for calculating the coupling coefficients are those of the compound structure, except that the mode fields of each isolated slab are used as an approximation of the compound modes. However, it is quite clear from observing the more accurate coupled mode equations, (B1) or (21), that a "natural" coupling exists between the isolated waveguide modes (represented by the first term at the right-hand side) and, in general, the two mechanisms of coupling should be taken into consideration by solving this set of equations numerically in order to obtain exact results.

Nevertheless, if the analyzed structure is strongly asymmetric, that is, if the parameter Δ is large enough, and the perturbation is strong enough to govern the coupling process, then the first term on the right-hand side of both equations (B1) or (21) can be discarded. In Appendix C we derive an exact formulation for this situation.

IV. Power Conservation

The power conservation relations are formally derived here as in [6]. If the analyzed structure bears no gain or loss, then the total guided power is given by

$$P(z) = \text{Re}\left[\frac{1}{2}\iint_{-\infty}^{+\infty} (\boldsymbol{E}_t \times \boldsymbol{H}_t) \cdot \hat{z}\, dx\, dy\right] \quad (22)$$

providing that $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$ are chosen to be real functions. If $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$ are given by (1), then

$$P(z) = aa^* + c(ab^* + ba^*) + bb^* \quad (23)$$

where c is a real quantity which was defined by (8). In order for the presented formalism to conserve power, we demand $(d/dz)P(z) = 0$. By using (12) for the z-derivatives of the modes' amplitudes and their complex conjugates, we find that

$$\frac{d}{dz} P(z) = i(ab^*\{[c\, 2\Delta - (K_{ab} - K_{ba})] + [c(k_{aa} - k_{bb}) - (k_{ab} - k_{ba})]\, \Delta n^2(z)\} - \text{c.c.}) \quad (24)$$

where c.c. denotes the complex conjugate of the first bracketed term. It is clear that for satisfying the power conservation condition, both of the square bracketed expressions should be equal to zero, that is,

$$K_{ab} - K_{ba} = c(\gamma_a - \gamma_b) \quad (25)$$

and

$$k_{ab} - k_{ba} = c(k_{aa} - k_{bb}). \quad (26)$$

The first relation (25) is known as the reciprocity condition which was derived before for lossless unperturbed systems. For the case of longitudinally perturbed systems, we have added now the second condition, given by (26) which, upon choosing $\epsilon^{(a)}$ and $\epsilon^{(b)}$ properly as discussed at the end of Section II, is found to be fulfilled by a straightforward substitution of the coupling parameters which were introduced by (16a–d).

The demand for power conservation might raise a problem when the approximated coupled-mode equations are used in cases where k_{ab} differs significantly from k_{ba}. In such an event, the expression for power exchange obtained from the approximated coupled-mode equations might be wrong, and as a result an erroneous coupling length [(C15)] is calculated. In that case, one should either solve numerically the exact form of the coupled-mode equations (B1), or carry a linear transformation from the nonorthogonal coupled-mode formulation to the orthogonal-mode one, such as done by Huang and Hause [14], and then obtain an approximate solution by neglecting asynchronous terms. In this case, the cross-coupling coefficients will be equal, and therefore the power is conserved.

It can be shown [17] that the approximated formulation solved in this paper is accurate as long as the analyzed structure obeys

$$c\left(\frac{E_g^a}{E_g^b} - \frac{E_g^b}{E_g^a}\right) \ll 1 \qquad (27)$$

where $E_g^{a,b}$ is the value of the isolated waveguide mode fields at the location of the grating. Since for most practical cases the demand for an efficient coupling leads to $E_g^a \simeq E_g^b$, the condition given by (27) is fulfilled and the approximation is valid.

V. Application Examples

A. A General Slab Structure

We shall illustrate the utility and improvement achieved by the presented theory by considering first the same structure given in [13]. This structure consists of a grating-assisted directional coupler made of two dissimilar slabs as shown schematically by Fig. 3(a). Although the presented formalism is capable of handling three-dimensional structures, we choose this example since, as in [13], it is easy to obtain and use analytical expression for the parameters given by (14) and (16). We emphasize again that the calculation is based on a separate modal analysis of each of the waveguides of which the whole structure is made; therefore, no further mathematical complication or computation ability is required. Two possibilities for selecting the isolated waveguide profiles, following the power conservation conditions derived in Section IV, are shown in Fig. 3(b) and (c).

We shall examine two different sets of parameters for this structure; the first, as in [13], consists of two dissimilar slabs. The upper one has a core width of $d_2 = 1\ \mu$m and a refractive index of $n_2 = 3.3$. The lower one has a width of $d_4 = 0.3\ \mu$m and a refractive index of $n_4 = 3.5$. The refractive index of the medium between the two slabs is $n_3 = 3.2$. The substrate and the superstrate have refractive indexes of $n_5 = 3$ and $n_1 = 1$, respectively, and the light wavelength in vacuum here, as well as in all the following examples, is assumed to be $\lambda = 1.5\ \mu$m.

We start by calculating the needed grating constant Λ for the structure as a function of the spacing $2s$ between the two waveguides. The exact grating condition, after substituting γ_a and γ_b from (14a) and (14b) into (18), is given by

$$\Lambda = \frac{\lambda}{n_{\text{eff}_a} - n_{\text{eff}_b} + [\tilde{K}_{aa} - \tilde{K}_{bb} + c(\tilde{K}_{ab} - \tilde{K}_{ba})]/[k_0(1 - c^2)]} \qquad (28)$$

where n_{eff_a} and n_{eff_b} are the effective indexes of the modes which propagate in each of the isolated waveguides (a) and (b). The factor $k_0 = 2\pi/\lambda_0$ is the free-space propagation constant, with λ_0 being the vacuum wavelength. Using the well-known equations for modal analysis of simple three-layer antisymmetric waveguides [8], analytic equations were developed for the parameters $\tilde{K}_{aa}$, $\tilde{K}_{ba}$, $\tilde{K}_{ba}$, and c. We have considered the case of TE modes only. The results are presented by the solid line in

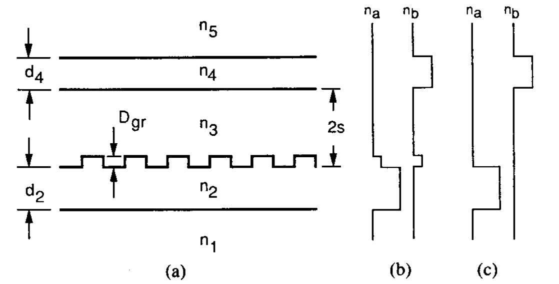

Fig. 3. (a) A grating-assisted directional coupler made of two dissimilar slabs. At the RHS are two possibilities of constructing the isolated waveguide profiles: (b) with and (c) without the average refractive index of the grating region.

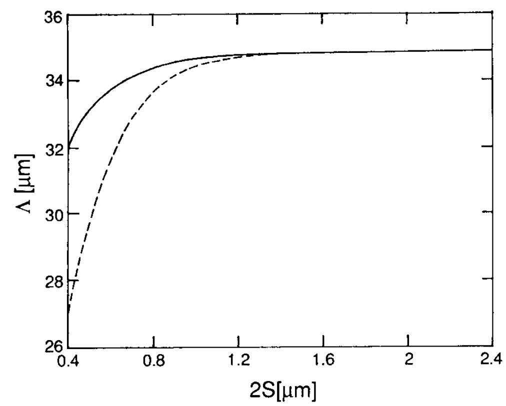

Fig. 4. The grating constant Λ as a function of the spacing $2s$ between the two waveguides: for coupling between the TE modes according to the presented approach (solid line) and for coupling between the compound modes (dashed lines).

Fig. 4. It is seen here that as the two waveguides are brought closer to each other, the difference between γ_a and γ_b is increased, and therefore a shorter grating period is needed. It was shown before [2] that the propagation constants of the compound modes of the whole structure are given by

$$\sigma_{1,2} = \bar{\gamma} \pm \sqrt{\Delta^2 + K_{ab}K_{ba}} \qquad (29)$$

where $\bar{\gamma} \equiv (\gamma_a + \gamma_b)/2$.

For the sake of comparison, we used this equation to calculate the grating factor which is needed in order to couple between the compound modes of the whole structure. The result appears as the dashed line in Fig. 4, and fits exactly the results obtained for the compound modes by solving the eigenvalue equation as was done in [13] and [14]. We can see that at a small separation, the difference between our result and that of the compound mode approach becomes significant. The reason for that is the presence of the coupling term $K_{ab}K_{ba}$ in the equation for calculating the propagating constants of the compound modes (29) which increases as the waveguides are getting closer to each other. In [13], an approximate procedure was suggested, according to which the propagation constant was calculated for each waveguide separately, without any correction for the existence of the second one. Thus, the grating constant calculated in this way is independent of the waveguide spacing, and is given by a horizontal line, which is the asymptote of (28).

Another interesting parameter is the interaction length for full power coupling from one channel to the other. If we assume that the natural coupling is much weaker than the grating induced one, that is, the condition (C17) is fulfilled, then the interaction length is given by (C15). At exact Bragg condition, the grating mismatching factor δ equals 0 and the interaction length becomes

$$l = \frac{\pi}{2d|k_{ab}k_{ba}|^{1/2}}. \tag{30}$$

k_{ab} and k_{ba} were calculated analytically using (16c, d). The grating that we chose was as shown by Fig. 3(a) with a width $D_{gr} = 0.1$ μm, which defines $f(x, y) = f(x)$ in (11). The grating amplitude is found using (17) and is $d = (n_2^2 - n_3^2)/\pi$. We have repeated the calculation for both grating location on the upper slab boundary and on the lower slab boundary. Fig. 5 shows the calculated interaction length l as a function of the spacing $2s$ between the two waveguides (solid line). It was found that, in this example, when the influence of each waveguide on the calculation of the other waveguide's propagation constant is ignored (by letting $c = 0$), the resultant k_{ab} and k_{ba} are both analytically equal to the coupling factor K in [13]. Using this approximation, the value of l was calculated and is shown as the dashed line in Fig. 5. For the sake of comparison, we have repeated the calculation of the interaction length between the two compound modes, which is being referred to as the "exact approach" in [13] and [14]. The results are given by the dotted line in Fig. 5. It is seen clearly that the results differ significantly at small waveguide separation, that is, when the waveguides are strongly coupled. It is consistent with the analysis given in [14], according to which the power exchange ratio calculated by the compound mode approach drops from unity as the waveguide separation becomes smaller. It is clear that this phenomenon has no basis in reality, since the purpose of the grating is to synchronize the coupler so that the coupling efficiency should be unity no matter what the separation is.

It is interesting to note that, as in the calculation for l which is carried in [13] for the compound modes, there is a certain situation in which l goes to infinity, that is, the grating of the certain configuration has no assisting influence on the power coupling. This phenomenon is explained there by the fact that the coupling coefficient is calculated by integrating the multiplication of the two compound modes, between which the grating is supposed to couple power, and the index perturbation amplitude, across the lateral perturbation location. If the region of integration happens to be around the crossing point of the asymmetrical compound mode, then since the rest of the

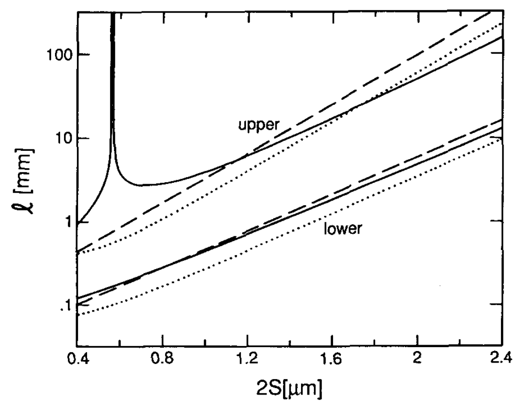

Fig. 5. The interaction length l, as a function of the spacing $2s$ between the two waveguides, for an assisted grating located once on the upper slab boundary and once on the lower slab boundary. An exact calculation is shown by the solid lines, and the weak coupling approximation ($c = 0$) is shown by the dashed lines; the results using the compound mode approach are shown by the dotted lines.

multiplicands are positive, the integrand is odd, and the results can become zero, which means infinite length for full power exchange. Unlike [13], this phenomenon occurs here in a different geometry since instead of dealing with coupling between the compound modes, we couple between the isolated waveguide modes [(11) and (16)]. These modes can be expanded by a superposition of the compound modes, and therefore a different geometry leads to the nulling of the coupling coefficients. However, this phenomenon, as discussed in the previous section, leads to an inefficient coupling as well as a possible error in the calculated coupling length, and therefore should be avoided in designing practical devices.

B. Passing Through the Symmetrical Case

In order to check the usefulness of the suggested approach in delicate situations that could not be handled before, we have picked a near-symmetrical case in which $n_1 = n_3 = n_6 = 3.2$, $n_2 = n_4 = 3.5$, $d_2 = 1$ μm, $2s = 0.5$ μm and the width of the lower waveguide was changed between 0.8 μm and 1.2 μm. As is well known [9] in the symmetrical case ($d_4 = 1$ μm), a full-power coupling is obtained without the necessity of using an assisting grating. One may expect this phenomenon to manifest itself when the calculation of the grating period is carried out.

Fig. 6 shows the calculated grating period for this case. The calculation was carried out first using the approach of this paper (solid line) and then again for coupling between the compound modes of the whole structure (dashed line). It is seen clearly that a full agreement is obtained using the presented procedure, since as the structure is getting closer to the symmetrical geometry, the calculated grating period goes to infinity; while by using the compound-mode approach, a finite value for the grating period is obtained. It can also be seen that the compound mode approach is good for calculating the grating factor

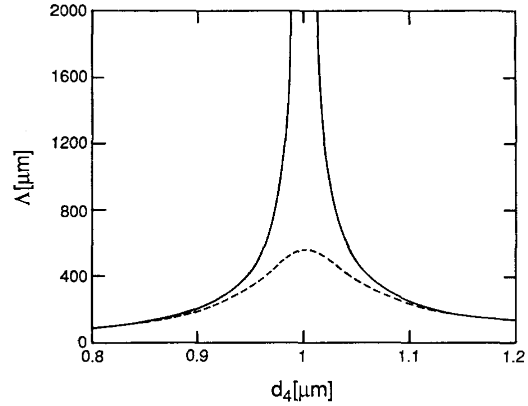

Fig. 6. The grating period Λ as a function of d_4 around the symmetrical case ($d_4 = 1$ μm) for the approach presented in this paper (solid line) and for coupling between the compound modes (dashed line).

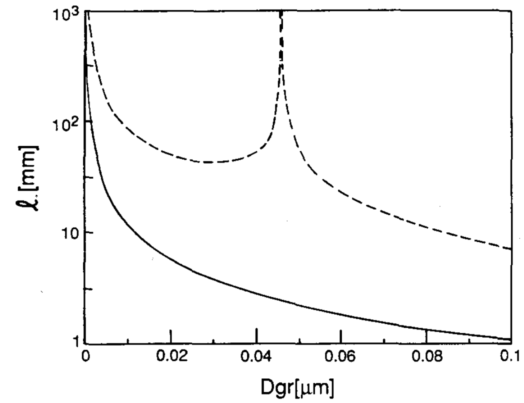

Fig. 7. The interaction length l, as a function of the grating width D_{gr}, for a grating located on the lower slab boundary (solid line) and for a grating located on the upper slab boundary (dashed line).

at strongly asymmetric structures since for differences of about 10% between d_2 and d_4, the curves tend to coincide with each other.

C. The Influence of the Grating Location and Depth

A useful application of the described analysis is demonstrated by examining the influence of changing the grating width D_{gr} on the interaction length. We have used the structure parameters $n_1 = n_3 = n_5 = 3.0$, $n_2 = 3.05$, and $n_4 = 3.1$, the waveguides widths were $d_2 = 1$ μm and $d_4 = 0.9$ μm, the spacing between the slabs was taken as $2s = 0.6$ μm, and as before the optical wavelength was $\lambda = 1.5$ μm. We have changed the grating width from zero to 0.1 μm and, using (30), checked its influence on the interaction length—once for a grating located on the lower slab boundary (solid line), and once for the upper slab boundary (dashed line).

The results are shown in Fig. 7. The dramatic influence of creating and increasing the grating height on the interaction length is clearly seen, but it is also shown that most of the influence is achieved at a grating depth of no more than 0.05 μm; therefore, a deeper grating would not be necessary. Another important result is the fact (for this example) that the grating has more influence when it is located on the lower slab than on the upper one. As in the first example, there is a certain delicate geometry where the interaction length goes to infinity and the grating has no assisting influence. The ability to predict these results in a simple, straightforward analysis procedure is the main benefit of the suggested theory.

VI. Conclusions

We have presented a formalism by which the parameters and the influence of longitudinal perturbations, and particularly a periodical grating, on any kind of directional coupler can be calculated. As in the improved coupled-mode formalism, the procedure is based on separate analysis of each of the waveguides and uses a proper linear combination of the results obtained to give approximate, yet highly accurate, results for the whole structure. This formulation can serve as a design tool for almost any geometry, including two-dimensional channels, and it can be applied on complex-index and nonisotropic mediums. By using this formulation, one can obtain design criteria for the grating parameters and location as well as structure parameters such as channel-layer dimensions and materials composition. Unlike prior formulations, this approach is not limited to weakly coupled and strongly unsynchronized structures.

Appendix A
Derivation of the Coupled Mode Equation

By using a simple, straightforward derivation, one can show that any two electromagnetic fields $\boldsymbol{E}^{(1)}$, $\boldsymbol{H}^{(2)}$ which satisfy the Maxwell equations as well as the corresponding boundary condition for any two related mediums $\epsilon^{(1)}(x, y, z)$ and $\epsilon^{(2)}(x, y, z)$, respectively, fulfill the relation

$$\frac{\partial}{\partial z} \iint_{-\infty}^{\infty} (\boldsymbol{E}^{(1)} \times \boldsymbol{H}^{(2)} - \boldsymbol{E}^{(2)} \times \boldsymbol{H}^{(1)}) \cdot \hat{z}\, dx\, dy = i\omega \iint_{-\infty}^{\infty} [\epsilon^{(2)}(x, y, z) - \epsilon^{(1)}(x, y, z)]\, \boldsymbol{E}^{(1)} \cdot \boldsymbol{E}^{(2)}\, dx\, dy \tag{A1}$$

which is essentially the integral form of the Lorenz reciprocity, obtained by applying it to cylindrical geometry with an infinitesimal z-change. In the derivation, as well as in the rest of this paper, we used the time convention $\exp(-i\omega t)$.

The derivation of the coupled-mode equation for the medium $\epsilon(x, y, z)$ is based upon proper selection and substitution of the mediums $\epsilon^{(1)}(x, y, z)$ and $\epsilon^{(2)}(x, y, z)$, as well as the related fields in (A1). For $\epsilon^{(2)}(x, y, z)$, we substitute the function that described the structure of waveguide (p) itself, namely,

$$\epsilon^{(2)}(x, y, z) = \epsilon^{(p)}(x, y). \quad \text{(A2)}$$

The related-field will, of course, be a guided mode solution of this single waveguide structure, and we choose to use a mode which propagates to the negative direction of the z axis, namely,

$$\begin{aligned} \boldsymbol{E}^{(2)} &= \boldsymbol{E}^{(p)^-} e^{-i\beta_p^z} \\ &= (\boldsymbol{E}_t^{(p)^-} + \hat{z} E_z^{(p)^-}) e^{-i\beta_p^z} \\ &= (\boldsymbol{E}_t^{(p)} - \hat{z} E_z^{(p)}) e^{-i\beta_p^z} \end{aligned} \quad \text{(A3-1)}$$

and

$$\begin{aligned} \boldsymbol{H}^{(2)} &= \boldsymbol{H}^{(p)^-}(x, y) e^{-i\beta_p^z} \\ &= (\boldsymbol{H}_t^{(p)^-} + \hat{z} H_z^{(p)}) e^{-i\beta_p^z} \\ &= (\boldsymbol{H}_t^{(p)} + \hat{z} H_z^{(p)}) e^{-i\beta_p^z}. \end{aligned} \quad \text{(A3-2)}$$

For $\epsilon^{(1)}(x, y, z)$, we choose the entire structure $\epsilon(x, y, z)$, that is,

$$\epsilon^{(1)}(x, y, z) = \epsilon(x, y) + \epsilon_0 f(x, y)\, \Delta n^2(z) \quad \text{(A4)}$$

and combine linearly the transverse parts of the field solutions of the separate waveguides $\epsilon^{(i)}(x, y)$ to approximate the transverse part of the entire structure field solution

$$\boldsymbol{E}_t^{(1)}(x, y, z) \cong \sum_{q=1}^{N} a_q(z) \boldsymbol{E}_{t(x,y)}^{(q)} \quad \text{(A5-1)}$$

$$\boldsymbol{H}_t^{(1)}(x, y, z) \cong \sum_{q=1}^{N} a_q(z) \boldsymbol{H}_t^{(q)}(x, y). \quad \text{(A5-2)}$$

It can be shown that the z components of the field of the entire structure are given by

$$E_z^{(1)}(x, y, z) \cong \sum_{q=1}^{N} \frac{\epsilon^{(q)}(x, y)}{\epsilon(x, y, z)} a_q(z)\, \boldsymbol{E}_z^{(q)}(x, y) \quad \text{(A5-3)}$$

$$H_z^{(1)}(x, y, z) \cong \sum_{q=1}^{N} a_q(z)\, \boldsymbol{H}_z^{(q)}(x, y). \quad \text{(A5-4)}$$

By substituting (A2)–(A5) into (A1), one gets

$$\sum_{q=1}^{N} C_{pq} \frac{d}{dz} a_q(z) = \sum_{q=1}^{N} i[\beta_p C_{pq} + \tilde{K}_{qp} + \Delta n^2(z) \hat{K}_{qp}] a_{q(z)} \quad \text{(A6)}$$

where β_p are the propagation constants of the isolated waveguides and C_{pq}, $\tilde{K}_{qp}$, and $\hat{K}_{qp}$ are defined by (8), (10), and (11). Noting that (A6) can be written in a matrix form

$$\boldsymbol{C} \frac{d}{dz} \boldsymbol{U}(z) = i[\boldsymbol{BC} + \tilde{\boldsymbol{K}} + \Delta n^2(z) \hat{\boldsymbol{K}}^T]\, \boldsymbol{U}(z), \quad \text{(A7)}$$

where $\boldsymbol{U}(z) = \text{col}\,[a_1(z), a_2(z) \cdots a_N(z)]$, and the superscript T denotes the matrix transpose, and using the identity [4]

$$\boldsymbol{BC} - \boldsymbol{CB} = \tilde{\boldsymbol{K}} - \tilde{\boldsymbol{K}}^T \quad \text{(A8)}$$

in (A7), one obtains finally the matrix form of the coupled-mode equation as given by (7).

Appendix B
A Limitation on the Magnitude of the Perturbation

Substituting (17)–(20) into (12) leads to the coupled-mode equations

$$\frac{d}{dz} A(z) = i(K_{ab} e^{-2\Delta z} + dk_{ab} e^{-i2\delta z} + dk_{ab} e^{-i(4\Delta - 2\delta)z})\, B(z) + ik_{aa} d(e^{i(2\pi/\Lambda)z} + e^{-i(2\pi/\Lambda)z})\, A(z)$$

$$\frac{d}{dz} B(z) = i(K_{ba} e^{i2\Delta z} + dk_{ba} e^{i2\delta z} + dk_{ba} e^{i(4\Delta - 2\delta)z})\, A(z) + ik_{bb} d(e^{i(2\pi/\Lambda)z} + e^{-i(2\pi/\Lambda)z})\, B(z). \quad \text{(B1)}$$

The third term in each of these equations can clearly be neglected due to its rapid z oscillations and therefore negligible contribution to the power exchange process. Although it is quite tempting to similarly get rid of the last term in each of these two equations, some care must be taken. From (18) we see that the first and third exponential terms in (B1) bear the same periodicity; therefore, one cannot claim that by integrating the differential equations (B1) the last term makes an oscillatory contribution negligible relative to the others. The solution to this problem is that a prior assumption that $\Delta\epsilon(x, y, z)$ is a "small" perturbation of the medium was made. This perturbation should contribute to energy coupling between the modes but should not change their basic properties. The last term of (B1) violates this assumption because it imposes self-dependent z variation on each amplitude. Therefore, in order to obtain the coupled mode equations form given by (21), we demand

$$k_{aa} d,\ k_{bb} d \ll K_{ab},\ K_{ba}. \quad \text{(B2)}$$

Appendix C
A Criterion for Efficient Perturbation

Let us examine first the set of equations for the case of nonperturbed structure, that is,

$$\frac{d}{dz} A(z) = iK_{ab} e^{-i2\Delta z} B(z)$$

$$\frac{d}{dz} B(z) = iK_{ba} e^{i2\Delta z} A(z). \quad \text{(C1)}$$

The solution of this set of equations imposed by the boundary condition $A(0) = A_0$, $B(0) = 0$ (waveguide (a) being used as an input port which starts at $z < 0$) is found to be

$$A(z) = A_0 \left(\cos Pz + i \frac{\Delta}{P} \sin Pz \right) e^{-i\Delta z}$$

$$B(z) = A_0 i \frac{K_{ba}}{P} \sin Pz\, e^{i\Delta z} \quad \text{(C2)}$$

where P is defined by

$$P^2 \equiv K_{ba} K_{ab} + \Delta^2. \quad \text{(C3)}$$

By expanding the output power in waveguide (*b*) in terms of the individual waveguide modes, it can be shown [15] that the coupling efficiency, defined by the ratio between the power in waveguide (*b*) and the power in the entrance to the system, is given by

$$\eta(z) = \frac{P_b(z)}{A_0^2} = c_{ab}c_{ba} + \frac{1 - c_{ab}c_{ba}}{1 + \xi^2} \cdot \sin^2\left[\sqrt{K_{ab}K_{ba}}(1 + \xi^2)^{1/2} z\right] \tag{C4}$$

where ξ, the coupler asynchronism factor [3], is defined as

$$\xi \equiv \frac{\gamma_a - \gamma_b}{2\sqrt{K_{ab}K_{ba}}} = \frac{\Delta}{\sqrt{K_{ab}K_{ba}}}. \tag{C5}$$

The maximum efficiency is obtained at a distance L given by

$$L = \frac{\pi}{2\sqrt{k_{ab}k_{ba}}(1 + \xi^2)^{1/2}} \tag{C6}$$

and its value is

$$\eta_{\max} = \frac{1 + c_{ab}c_{ba}\xi^2}{1 + \xi^2}. \tag{C7}$$

It is clear that for two identical waveguides, $\xi = 0$ and the maximum efficiency becomes unity. We assume, however, that this is not our case, and to improve this efficiency, we create the grating perturbation.

Let us assume now that by imposing the perturbation, we indeed increased the coupling efficiency so that the whole process is mainly governed by the second term of the RHS of (21). In that case, the solution, constrained by the same boundary conditions, is given by

$$A(z) = A_0\left(\cos pz + i\frac{\delta}{p}\sin pz\right)e^{-i\delta z} \tag{C8}$$

$$B(z) = A_0 i d \frac{k_{ba}}{p}\sin pz\, e^{i\delta z}. \tag{C9}$$

Here p is defined by

$$p^2 \equiv d^2 k_{ab}k_{ba} + \delta^2. \tag{C10}$$

If we again calculated the efficiency as before, we get

$$\eta_\epsilon(z) = c_{ab}c_{ba} + \frac{1 - c_{ab}c_{ba}}{1 + \zeta^2} \cdot \sin^2\left[d\sqrt{k_{ab}k_{ba}}(1 + \zeta^2)^{1/2} z\right] \tag{C11}$$

where ζ, the asynchronism factor of the grating-assisted structure, is defined as

$$\zeta \equiv \frac{\delta}{d\sqrt{k_{ab}k_{ba}}} = \frac{\Delta - \dfrac{\pi}{\Lambda}}{d\sqrt{k_{ab}k_{ba}}}. \tag{C12}$$

It should be noted that (C11) is slightly approximated due to the disregarding of the small contributing terms in the complete-form couple mode equations (B1). The approximation was made by setting the value "1" in the second term numerator instead of

$$\frac{dk_{ba} + 2c\delta}{dk_{ab}} \tag{C13}$$

where $c = c_{ab} + c_{ba}/2$, and the interaction length is assumed to contain an integer number of grating periods. For the case of $\delta = 0$, this expression becomes

$$\frac{\hat{K} - c\hat{K}_{aa}}{\hat{K} - c\hat{K}_{bb}} \cong 1 - c\frac{\hat{K}_{aa} + \hat{K}_{bb}}{\hat{K}} + c^2\frac{\hat{K}_{aa}\hat{K}_{bb}}{\hat{K}^2} \tag{C14}$$

where $\hat{K} \equiv \hat{K}_{ab} = \hat{K}_{ba}$. As we have shown in Section IV, the exact formulation conserves power. Therefore, the approximation is valid for the cases in which the last two terms of (C14) are negligible compared to 1, and the small amount of power gain or loss is strictly due to the perturbation approach, which leads to the disregarding of the asynchronous participants of the power exchange process.

The maximum efficiency is achieved now at the distance

$$l = \frac{\pi}{2d\sqrt{k_{ab}k_{ba}}(1 + \zeta^2)^{1/2}} \tag{C15}$$

and is given by

$$\eta_{\epsilon\max} \cong \frac{1 + c_{ab}c_{ba}\zeta^2}{1 + \zeta^2}. \tag{C16}$$

We demand

$$\eta_{\epsilon\max} \gg \eta_{\max}. \tag{C17}$$

Thus, by combining (C7), (C16), and (C17), and assuming $c_{ab}c_{ba} \ll 1$, we obtain

$$1 - \frac{K_g}{2\Delta} \ll \frac{d\sqrt{k_{ab}k_{ba}}}{\sqrt{K_{ab}K_{ba}}}. \tag{C18}$$

When this condition is fulfilled, the assisting grating governs the coupling process which is described by the second term in the coupled-mode equations (21).

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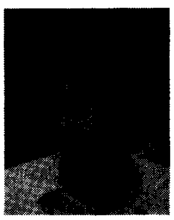

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