$$\frac{d}{dz}\int \left[E^{(1)} \times H^{(2)} - E^{(2)} \times H^{(1)}\right] \cdot \hat{z}\, dxdy =$$

$$i\omega \int \left[\epsilon^{(2)} - \epsilon^{(1)}\right] E^{(1)} \cdot E^{(2)}\, dxdy \qquad (1)$$

Passo 1:

Estrutura $\tilde{\tilde{n}}$: Perturbada:

$$\epsilon^{(2)} = \epsilon(x,y) \qquad (2)$$

$$E^{(2)} = \left(E_t^T - \hat{z} E_z^T\right) e^{-i\beta_T z} \qquad (3)$$

$$H^{(2)} = \left(-H_t^T + \hat{z} H_z^T\right) e^{-i\beta_T z} \qquad (4)$$

Modos Estrutura (2) propagando em $-\hat{z}$

Eq. (4) obtida de (3) via Eq. de Maxwell

Estrutura Perturbada:

$$\epsilon^{(1)} = \epsilon(x,y) + \epsilon_0 \Delta n^2 f(z) \qquad (5)$$

$$E_t^{(1)} = a(z) E_t^T + b(z) E_t^B \qquad (6)$$

$$H_t^{(1)} = a(z) H_t^T + b(z) H_t^B \qquad (7)$$

$$f(z) = \frac{2}{\pi} \cos\left(\frac{2\pi}{\Lambda} z\right)$$

$$\Delta n^2 = n_j^2 - n_{j-1}^2 \qquad j = 1, 2, 3, \ldots N$$

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Os campos da Estrutura 1 propagam em $+\hat{z}$

Para obtermos as componentes longitudinais, utilizamos as Eq. de Maxwell.

$$\left.\begin{aligned} \nabla \times E &= \cancel{-}\,{+}\,i\omega\mu H \\ \nabla \times H &= -i\omega\epsilon E \end{aligned}\right\} \text{sinais}$$

$\therefore$

$$E_z = -\frac{1}{i\omega\epsilon} \nabla_t \times H_t \quad ✓ \qquad (8)$$

$$H_z = \frac{1}{i\omega\mu} \nabla_t \times E_t \quad ✓ \qquad (9)$$

substituindo (6) e (7) $\rightarrow$ (8) e (9)

porque tem q/ ser a estrutura ñ perturb.

$$E_z^{(1)} = -\frac{1}{i\omega\epsilon^{(1)}}\left[a(z)\,\underbrace{\nabla_t \times H_t^T}_{-i\omega\epsilon^{(2)}E_z^T} + b(z)\,\underbrace{\nabla_t \times H_t^B}_{-i\omega\epsilon^{(2)}E_z^B}\right]$$

$$H_z^{(1)} = \frac{1}{i\omega\mu}\left[a(z)\,\nabla_t \times E_t^T + b(z)\,\nabla_t \times E_t^B\right]$$

$$E_z^{(1)} = \frac{\epsilon^{(2)}}{\epsilon^{(1)}}\left[a(z)\,E_z^T + b(z)\,E_z^B\right] \qquad (10)$$

$$H_z^{(1)} = a(z)\,H_z^T + b(z)\,H_z^B \qquad (11)$$

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Os campos perturbados totais tornam-se:

$$E^{(1)} = a(z)\left(E_t^T + \hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^T\right) + b(z)\left(E_t^B + \hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^B\right) \quad (12)$$

$$H^{(1)} = a(z)\left(H_t^T + \hat{z}\,H_z^T\right) + b(z)\left(H_t^B + \hat{z}\,H_z^B\right) \quad (13)$$

Considere o lado esquerdo de (1):

$$LE = \frac{d}{dz}\int \Big[\underbrace{E^{(1)} \times H^{(2)}}_{1^{\circ}\text{ termo}} - \underbrace{E^{(2)} \times H^{(1)}}_{2^{\circ}\text{ termo}}\Big]\cdot \hat{z}\,dx\,dy \quad (14)$$

substituindo (3), (4), (12) e (13) → (14)

$$1^{\circ}\text{ termo} = E^{(1)} \times H^{(2)}$$

$$= \left\{a(z)\left(E_t^T + \hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^T\right) + b(z)\left(E_t^B + \hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^B\right)\right\} \times$$

$$\left(-H_t^T + \hat{z}\,H_z^T\right) e^{-i\beta_T z}$$

$$a(z)\Big[-E_t^T \times H_t^T + \overset{*0}{\cancel{E_t^T \times \hat{z}\,H_z^T}} + \overset{*0}{\cancel{\hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^T \times (-H_t^T)}} +$$

$$\underset{0}{\cancel{\hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^T \times \hat{z}\,H_z^T}}\Big]\,e^{-i\beta_T z} + \qquad \hat{z}\times\hat{z} = 0$$

* por causa do $\cdot\hat{z}$ em (14)

$$b(z)\Big[-E_t^B \times H_t^T + \overset{*0}{\cancel{E_t^B \times \hat{z}\,H_z^T}} + \overset{*0}{\cancel{\hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^B \times (-H_t^T)}} +$$

$$\underset{0}{\cancel{\hat{z}\,\frac{\epsilon^{(2)}}{\epsilon^{(1)}}\,E_z^B \times \hat{z}\,H_z^T}}\Big]\,e^{-i\beta_T z}$$

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$$E^{(1)} \times H^{(2)} = -a(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^T - b(z)\, e^{-i\beta_T z}\, E_t^B \times H_t^T$$

2^o termo $= E^{(2)} \times H^{(1)}$

$$= \left(E_t^T - \hat{z} E_z^T\right) e^{-i\beta_T z} \times \left\{ a(z)\left(H_t^T + \hat{z} H_z^T\right) + b(z)\left(H_t^B + \hat{z} H_z^B\right) \right\}$$

$$= a(z)\left[E_t^T \times H_t^T + \cancel{E_t^T \times \hat{z} H_z^T}^{\,0} - \cancel{\hat{z} E_z^T \times H_t^T}^{\,0} - \cancel{\hat{z} E_z^T \times \hat{z} H_z^T}^{\,0}\right] e^{-i\beta_T z} +$$

$$b(z)\left[E_t^T \times H_t^B + \cancel{E_t^T \times \hat{z} H_z^B}^{\,0} - \cancel{\hat{z} E_z^T \times H_t^B}^{\,0} - \cancel{\hat{z} E_z^T \times \hat{z} H_z^B}^{\,0}\right] e^{-i\beta_T z}$$

$$E^{(2)} \times H^{(1)} = a(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^T + b(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^B$$

Logo,

$$LE = \frac{d}{dz}\int \Big[-a(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^T - b(z)\, e^{-i\beta_T z}\, E_t^B \times H_t^T - a(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^T - b(z)\, e^{-i\beta_T z}\, E_t^T \times H_t^B\Big] \cdot \hat{z}\, dx\, dy$$

$$LE = -2\frac{d}{dz}\left(a(z)\, e^{-i\beta_T z}\right) \int E_t^T \times H_t^T \cdot \hat{z}\, dx\, dy - \frac{d}{dz}\left(b(z)\, e^{-i\beta_T z}\right) \int \left[E_t^B \times H_t^T + E_t^T \times H_t^B\right] \cdot \hat{z}\, dx\, dy \qquad (15)$$

Definindo: Integral de Overlapping

$$C_{mn} = \frac{1}{2}\int E_t^n \times H_t^m \cdot \hat{z}\, dx\, dy \qquad (16)$$

$$m, n = T, B$$

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Como estamos lidando com modos da estrutura inteira :

$$C_{TT} = C_{BB} = 1 \qquad C_{TB} = C_{BT} = 0$$

Portanto :

$$LE = -4 \frac{d}{dz}\left(a(z)\, e^{-i\beta_T z} \right)$$

$$LE = -4\left[\frac{da(z)}{dz} e^{-i\beta_T z} - i\beta_T a(z) e^{-i\beta_T z} \right] \qquad (17)$$

Para concluir, divida (17) por 4 :

$$\boxed{LE = \left[-\frac{da(z)}{dz} + i\beta_T a(z) \right] e^{-i\beta_T z}} \qquad (18)$$

Considere, agora, o lado direito de (1) :

$$LD = i\omega \int \left[\epsilon^{(2)} - \epsilon^{(1)} \right] E^{(1)} \cdot E^{(2)}\, dx\, dy \qquad (19)$$

Substituindo (2), (3), (5) e (12) $\rightarrow$ (19)

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$$LD = i\omega \int \left[\epsilon^{(2)} - \epsilon^{(1)} \right] \left\{ a(z) \left(E_t^T + \hat{z} \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^T \right) + b(z) \left(E_t^B + \hat{z} \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^B \right) \right\} \cdot$$

$$\left(E_t^T - \hat{z} E_z^T \right) e^{-i\beta_T z} \, dx dy$$

$$LD = i\omega \int \Delta\epsilon \left\{ a(z) \left(E_t^T \cdot E_t^T - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^T \cdot E_z^T \right) + b(z) \left(E_t^B \cdot E_t^T - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^B \cdot E_z^T \right) \right\}$$

$$e^{-i\beta_T z} \, dx dy \tag{20}$$

onde: $\Delta\epsilon = \epsilon^{(2)} - \epsilon^{(1)} = \epsilon(x,y) - \epsilon(x,y) - \epsilon_0 \Delta n^2 f(z) = -\epsilon_0 \Delta n^2 f(z)$

Como feito p/ (17), divida (20) por 4:

$$LD = \frac{i\omega}{4} \int \Delta\epsilon \left[a(z) \left(E_t^T \cdot E_t^T - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^T \cdot E_z^T \right) + b(z) \left(E_t^B \cdot E_t^T - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^B \cdot E_z^T \right) \right] e^{-i\beta_T z} \, dx dy \tag{21}$$

Podemos reduzir (21) definindo uma nova quantidade:

$$K_{mn} = \frac{\omega \epsilon_0 \Delta n^2}{4} \int \left[E_t^m \cdot E_t^n - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^m \cdot E_z^n \right] dx dy \tag{22}$$

$$m, n = T, B$$

Coeficiente de Acoplamento

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$$LD = -\left[i a(z) K_{TT} f(z) + i b(z) K_{BT} f(z) \right] e^{-i\beta_T z} \quad (23)$$

$$LE = LD$$

$$\left[-\frac{da(z)}{dz} + i\beta_T a(z) \right] \cancel{e^{-i\beta_T z}} = -\left[i a(z) K_{TT} f(z) + i b(z) K_{BT} f(z) \right] \cancel{e^{-i\beta_T z}}$$

$$\boxed{\frac{da(z)}{dz} = i\left[\beta_T + K_{TT} f(z) \right] a(z) + i K_{BT} f(z) b(z)} \quad (24)$$

Passo 2

Deixe q/ os campos $E^{(2)}$ e $H^{(2)}$ e a permissividade $\epsilon^{(2)}$ representem, novamente, a estrutura multimodo não-perturbada; mas suponha q/ os campos não-perturbados sejam representados pelos modos do guia interior:

$$\epsilon^{(2)} = \epsilon(x,y) \quad (25)$$

$$E^{(2)} = \left(E_t^B - \hat{z} E_z^B \right) e^{-i\beta_B z} \quad (26)$$

$$H^{(2)} = \left(-H_t^B + \hat{z} H_z^B \right) e^{i\beta_B z} \quad (27)$$

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Os modos da estrutura perturbada é como no passo anterior:

$$\epsilon^{(1)} = \epsilon(x,y) + \epsilon_0 \Delta n^2 f(z) \quad (28)$$

$$E^{(1)} = a(z)\left(E_t^T + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^T\right) + b(z)\left(E_t^B + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^B\right) \quad (29)$$

$$H^{(1)} = a(z)\left(H_t^T + \hat{z}H_z^T\right) + b(z)\left(H_t^B + \hat{z}H_z^B\right) \quad (30)$$

Substituindo (26), (27), (29) e (30) $\rightarrow$ (14):

$$E^{(1)} \times H^{(2)} = \left\{a(z)\left(E_t^T + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^T\right) + b(z)\left(E_t^B + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^B\right)\right\} \times \left(-H_t^B + \hat{z}H_z^B\right)e^{-i\beta_B z}$$

$$= a(z)e^{-i\beta_B z}\left[-E_t^T \times H_t^B + \cancel{E_t^T \times \hat{z}H_z^B}^{\,0} + \cancel{\hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^T \times (-H_t^B)}^{\,0} + \cancel{\hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^T \times \hat{z}H_z^B}^{\,0}\right] +$$

$$b(z)e^{-i\beta_B z}\left[-E_t^B \times H_t^B + \cancel{E_t^B \times \hat{z}H_z^B}^{\,0} + \cancel{\hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^B \times (-H_t^B)}^{\,0} + \cancel{\hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^B \times \hat{z}H_z^B}^{\,0}\right]$$

$$\boxed{E^{(1)} \times H^{(2)} = -a(z)e^{-i\beta_B z}E_t^T \times H_t^B - b(z)e^{-i\beta_B z}E_t^B \times H_t^B}$$

$$E^{(2)} \times H^{(1)} = \left(E_t^B - \hat{z}E_z^B\right)e^{-i\beta_B z} \times \left\{a(z)\left(H_t^T + \hat{z}H_z^T\right) + b(z)\left(H_t^B + \hat{z}H_z^B\right)\right\}$$

$$= a(z)e^{-i\beta_B z}\left[E_t^B \times H_t^T + \cancel{E_t^B \times \hat{z}H_z^T}^{\,0} - \cancel{\hat{z}E_z^B \times H_t^T}^{\,0} - \cancel{\hat{z}E_z^B \times \hat{z}H_z^T}^{\,0}\right] +$$

$$b(z)e^{-i\beta_B z}\left[E_t^B \times H_t^B + \cancel{E_t^B \times \hat{z}H_z^B}^{\,0} - \cancel{\hat{z}E_z^B \times H_t^B}^{\,0} - \cancel{\hat{z}E_z^B \times \hat{z}H_z^B}^{\,0}\right]$$

$$\boxed{E^{(2)} \times H^{(1)} = a(z)e^{-i\beta_B z}E_t^B \times H_t^T + b(z)e^{-i\beta_B z}E_t^B \times H_t^B}$$

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$$LE = \frac{d}{dz}\int\left\{\left[-a(z)\,e^{-i\beta_B z}\,E_t^T \times H_t^B - b(z)\,e^{-i\beta_B z}\,E_t^B \times H_t^B\right] e - \left[a(z)\,e^{-i\beta_B z}\,E_t^B \times H_t^T + b(z)\,e^{-i\beta_B z}\,E_t^B \times H_t^B\right]\right\} \cdot \hat{z}\, dx\, dy$$

$$LE = \frac{d}{dz}\int\left\{-a(z)\left[E_t^T \times H_t^B + E_t^B \times H_t^T\right] - b(z)\left[E_t^B \times H_t^B + E_t^B \times H_t^B\right]\right\} e^{-i\beta_B z} \cdot \hat{z}\, dx\, dy$$

$$LE = -\frac{d}{dz}\int\left\{a(z)\left[E_t^T \times H_t^B + E_t^B \times H_t^T\right] + 2b(z)\,E_t^B \times H_t^B\right\} e^{-i\beta_B z} \cdot \hat{z}\, dx\, dy$$

Utilizando (16):

$$LE = -4\frac{d}{dz}\left(b(z)\,e^{-i\beta_B z}\right) \qquad (31)$$

Dividindo (31) por 4 e fazendo a derivada:

$$LE = \left[-\frac{d\,b(z)}{dz} + i\beta_B\, b(z)\right] e^{-i\beta_B z} \qquad (32)$$

Considere, agora, o lado direito de (1) como dado em (19):

$$LD = i\omega\int\left[\epsilon^{(2)} - \epsilon^{(1)}\right]\left(E_t^B - \hat{z}\,E_z^B\right) e^{-i\beta_B z} \cdot \left\{a(z)\left(E_t^T + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^T\right) + b(z)\left(E_t^B + \hat{z}\frac{\epsilon^{(2)}}{\epsilon^{(1)}}E_z^B\right)\right\} dx\, dy$$

e^{-} $E^{(1)} \cdot E^{(2)}$ ou μ $E^{(2)} \cdot E^{(1)}$

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$$LD = i\omega \int \Delta\epsilon \left\{ a(z)\left(E_t^T \cdot E_t^B - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^T \cdot E_z^B \right) + b(z)\left(E_t^B \cdot E_t^B - \frac{\epsilon^{(2)}}{\epsilon^{(1)}} E_z^B \cdot E_z^B \right) \right\} e^{-i\beta_B z} dx dy$$ (33)

$$\Delta\epsilon = \epsilon^{(2)} - \epsilon^{(1)} = \epsilon(x,y) - \epsilon(x,y) - \epsilon_0 \Delta n^2 f(z) = -\epsilon_0 \Delta n^2 f(z)$$

Dividindo (33) por 4 e utilizando (32):

$$LD = -i\left[a(z) K_{TB} f(z) + b(z) K_{BB} f(z) \right] e^{-i\beta_B z}$$ (34)

$$LD = LE$$

$$\left[-\frac{d b(z)}{dz} + i\beta_B b(z) \right] \cancel{e^{-i\beta_B z}} = -i\left[a(z) K_{TB} f(z) + b(z) K_{BB} f(z) \right] \cancel{e^{-i\beta_B z}}$$

$$\boxed{\frac{d b(z)}{dz} = i K_{TB} f(z)\, a(z) + i\left[\beta_B + K_{BB} f(z) \right] b(z)}$$ (35)

As amplitudes complexas $a(z)$ e $b(z)$ são funções de z com variações rápidas. Estas variações podem ser removidas introduzindo amplitudes c/ variações lentas $A(z)$ e $B(z)$

$$a(z) = A(z)\, e^{i\beta_T z}$$ (36)

$$b(z) = B(z)\, e^{i\beta_B z}$$ (37)

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Substituindo (36) e (37) em (29) e (35)

$$\begin{cases} \dfrac{dA(z)}{dz} = i f(z) K_{TT} A(z) + i f(z) K_{BT} B(z) e^{i(\beta_B - \beta_T) z} & (38) \\[2em] \dfrac{dB(z)}{dz} = i f(z) K_{TB} A(z) e^{i(\beta_T - \beta_B) z} + i f(z) K_{BB} B(z) & (39) \end{cases}$$

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