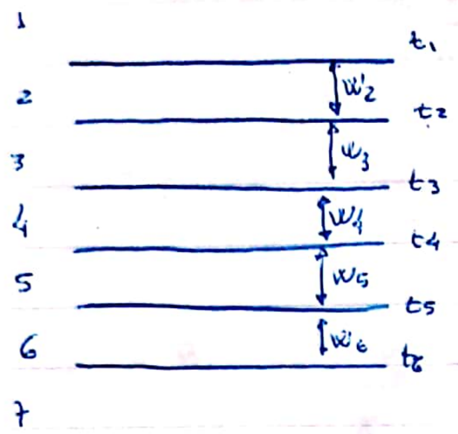



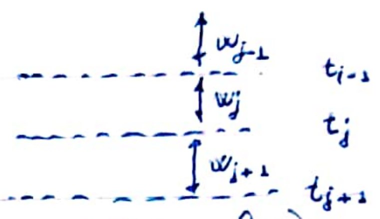


$$e^{j(\omega t - \beta z)}$$

$$\frac{\delta^2 H_y}{\delta x^2} - (\beta^2 - k_0^2 n^2) H_y = 0$$

$$\gamma_j = \sqrt{\beta^2 - k_0^2 n_j^2}$$

$$H_y^j(x) = A_j \exp[\gamma_j (x - t_j)] + B_j \exp[-\gamma_j (x - t_j)]$$

B. C. : $$H_j(t_{j-1}) = H_{j-1}(t_{j-1}) \quad (1)$$

$$\frac{1}{n_j^2} \frac{d}{dx} H_j(t_{j-1}) = \frac{1}{n_{j-1}^2} \frac{d}{dx} H_{j-1}(t_{j-1}) \quad (2)$$

$$A_j \exp[\gamma_j (t_{j-1} - t_j)] + B_j \exp[-\gamma_j (t_{j-1} - t_j)] =$$

$$A_{j-1} \exp[\gamma_{j-1} (t_{j-1} - t_{j-1})] + B_{j-1} \exp[-\gamma_{j-1} (t_{j-1} - t_{j-1})]$$

$$\theta_j = (t_j - t_{j-1}) \gamma_j = w_j \gamma_j$$

$$\boxed{A_{j-1} + B_{j-1} = A_j \exp(-\theta_j) + B_j \exp(\theta_j)} \quad (3)$$

$$\frac{\gamma_j}{n_j^2} A_j \exp[\gamma_j (t_{j-1} - t_j)] - \frac{\gamma_j}{n_j^2} B_j \exp[-\gamma_j (t_{j-1} - t_j)] =$$

$$\frac{\gamma_{j-1}}{n_{j-1}^2} A_{j-1} \exp[\gamma_{j-1} (t_{j-1} - t_{j-1})] - \frac{\gamma_{j-1}}{n_{j-1}^2} B_{j-1} \exp[-\gamma_{j-1} (t_{j-1} - t_{j-1})]$$

$$\frac{\gamma_j}{n_j^2} A_j \exp(-\theta_j) - \frac{\gamma_j}{n_j^2} B_j \exp(\theta_j) = \frac{\gamma_{j-1}}{n_{j-1}^2} A_{j-1} - \frac{\gamma_{j-1}}{n_{j-1}^2} B_{j-1}$$

multiplicar por : $\frac{n_{j-1}^2}{\gamma_{j-1}}$

$$A_{j-1} - B_{j-1} = \frac{\gamma_j}{\gamma_{j-1}} \cdot \frac{n_{j-1}^2}{n_j^2} A_j \exp(-\theta_j) - \frac{\gamma_j}{\gamma_{j-1}} \cdot \frac{n_{j-1}^2}{n_j^2} B_j \exp(\theta_j) \quad (4)$$

(3) + (4) :

$$2A_{j-1} = \left[1 + \frac{\gamma_j}{\gamma_{j-1}} \cdot \frac{n_{j-1}^2}{n_j^2}\right] A_j \exp(-\theta_j) + \left[1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] B_j \exp(\theta_j)$$

$$\boxed{A_{j-1} = \frac{1}{2}\left[1 + \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] A_j \exp(-\theta_j) + \frac{1}{2}\left[1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] B_j \exp(\theta_j)} \quad (5)$$

(5) → (3) :

$$B_{j-1} = A_j \exp(-\theta_j) - \frac{1}{2}\left[1 + \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] A_j \exp(-\theta_j) +$$

$$B_j \exp(\theta_j) - \frac{1}{2}\left[1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] B_j \exp(\theta_j)$$

$$\boxed{B_{j-1} = \frac{1}{2}\left[1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] A_j \exp(-\theta_j) + \frac{1}{2}\left[1 + \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right] B_j \exp(\theta_j)}$$

(6)

Arranjando (5) e (6) na forma matricial:



$$\begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \left(1 + \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right) \exp(-\theta_j) & \left(1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right) \exp(\theta_j) \\ \left(1 - \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right) \exp(-\theta_j) & \left(1 + \frac{\gamma_j}{\gamma_{j-1}} \frac{n_{j-1}^2}{n_j^2}\right) \exp(\theta_j) \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} = \begin{bmatrix} T_j \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = T_2 \cdot T_3 \cdot \ldots \cdot T_N \begin{bmatrix} A_N \\ B_N \end{bmatrix}$$

$$T_{WG} = \prod_{k=2}^{N} T_k = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} A_N \\ B_N \end{bmatrix}$$

$$x \to +\infty \Rightarrow A_N = 0$$
$$x \to -\infty \Rightarrow B_1 = 0$$

$$\begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$a = t_{22}\, b$$
$$0 = t_{22}\, b \Rightarrow \boxed{t_{22}(\beta) = 0}$$

Segunda abordagem:

$$H_j(x) = A_j \exp\left[\gamma_j (x - t_j)\right] + B_j \exp\left[-\gamma_j (x - t_j)\right]$$

$$H_{j+1}(x) = A_{j+1} \exp\left[\gamma_{j+1} (x - t_{j+1})\right] + B_{j+1} \exp\left[-\gamma_{j+1} (x - t_{j+1})\right]$$

B.C. na interface $x = t_j$:

$$A_j \exp\left[\gamma_j (t_j - t_j)\right] + B_j \exp\left[-\gamma_j (t_j - t_j)\right] =$$

$$A_{j+1} \exp\left[\gamma_{j+1} (t_j - t_{j+1})\right] + B_{j+1} \exp\left[-\gamma_{j+1} (t_j - t_{j+1})\right]$$

$$\theta_{j+1} = (t_{j+1} - t_j)\gamma_{j+1} = w_{j+1}\gamma_{j+1}$$

$$\boxed{A_j + B_j = A_{j+1} \exp(-\theta_{j+1}) + B_{j+1} \exp(\theta_{j+1})} \quad (8)$$

$$\frac{1}{n_j^2} \frac{d}{dx} H_j(t_j) = \frac{1}{n_{j+1}^2} \frac{d}{dx} H_{j+1}(t_j)$$

$$\frac{\gamma_j}{n_j^2} A_j - \frac{\gamma_j}{n_j^2} B_j = \frac{\gamma_{j+1}}{n_{j+1}^2} A_{j+1} \exp(-\theta_{j+1}) - \frac{\gamma_{j+1}}{n_{j+1}^2} B_{j+1} \exp(\theta_{j+1})$$

multiplicar por $\dfrac{n_j^2}{\gamma_j}$

$$\boxed{A_j - B_j = \frac{\gamma_{j+1}}{\gamma_j} \frac{n_j^2}{n_{j+1}^2} A_{j+1} \exp(-\theta_{j+1}) - \frac{\gamma_{j+1}}{\gamma_j} \frac{n_j^2}{n_{j+1}^2} B_{j+1} \exp(\theta_{j+1})} \quad (9)$$

(8) + (9):

$$2A_j = \left[1 + \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] A_{j+1}\exp(-\theta_{j+1}) +$$

$$\left[1 - \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] B_{j+1}\exp(+\theta_{j+1})$$

$$\boxed{A_j = \frac{1}{2}\left[1 + \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] A_{j+1}\exp(-\theta_{j+1}) + \frac{1}{2}\left[1 - \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] B_{j+1}\exp(\theta_{j+1})} \quad (10)$$

(10) $\rightarrow$ (8) :

$$B_j = \left[1 - \frac{1}{2} - \frac{1}{2}\frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] A_{j+1}\exp(-\theta_{j+1}) +$$

$$\left[1 - \frac{1}{2} + \frac{1}{2}\frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] B_{j+1}\exp(\theta_{j+1})$$

$$\boxed{B_j = \frac{1}{2}\left[1 - \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] A_{j+1}\exp(-\theta_{j+1}) + \frac{1}{2}\left[1 + \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right] B_{j+1}\exp(\theta_{j+1})} \quad (11)$$

Na forma matricial:

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \left(1 + \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right)\exp(-\theta_{j+1}) & \left(1 - \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right)\exp(\theta_{j+1}) \\ \left(1 - \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right)\exp(-\theta_{j+1}) & \left(1 + \frac{\gamma_{j+1}}{\gamma_j}\frac{n_j^2}{n_{j+1}^2}\right)\exp(\theta_{j+1}) \end{bmatrix}\begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = T_{j+1} \begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} A_N \\ B_N \end{bmatrix}$$

$$H(x) = A_j \exp[\gamma_j (x - t_j)] + B_j \exp[-\gamma_j (x - t_j)]$$

$$P = \int S_z \, dx = \frac{1}{2} \int_{-\infty}^{\infty} \bar{E} \times \bar{H}^* \, dx = 1 \ W/m$$

$$E_x \times H_y$$

$$E_x = \frac{\beta}{n^2 \omega \epsilon_0} H_y \qquad E_z = -\frac{i}{n^2 \omega \epsilon_0} \cdot \frac{\delta H_y}{\delta x}$$

$$\frac{1}{2} \cdot \frac{\beta}{\omega \epsilon_0} \int_{-\infty}^{\infty} \frac{1}{n^2} H_y \cdot H_y^* \, dx = 1$$

$$H_y \cdot H_y^* = \{ A_j \exp[\gamma_j (x - t_j)] + B_j \exp[-\gamma_j (x - t_j)] \cdot \}$$

$$\{ A_j^* \exp[\gamma_j^* (x - t_j)] + B_j^* \exp[-\gamma_j^* (x - t_j)] \}$$

$$= |A_j|^2 \exp[(\gamma_j + \gamma_j^*)(x - t_j)] + A_j B_j^* \exp[(\gamma_j - \gamma_j^*)(x - t_j)] +$$

$$A_j^* B_j \exp[-(\gamma_j - \gamma_j^*)(x - t_j)] + |B_j|^2 \exp[-(\gamma_j + \gamma_j^*)(x - t_j)]$$

$$\frac{1}{2} \frac{\beta}{\omega \epsilon_0} \cdot \frac{1}{n^2} \Bigg[ \frac{|A_j|^2}{(\gamma_j + \gamma_j^*)} \cdot \exp[(\gamma_j + \gamma_j^*)(x - t_j)] +$$

$$\frac{A_j B_j^*}{(\gamma_j - \gamma_j^*)} \cdot \exp[(\gamma_j - \gamma_j^*)(x - t_j)] -$$

$$\frac{A_j^* B_j}{(\gamma_j - \gamma_j^*)} \exp[-(\gamma_j - \gamma_j^*)(x - t_j)] -$$

$$\frac{|B_j|^2}{(\gamma_j + \gamma_j^*)} \exp[-(\gamma_j + \gamma_j^*)(x - t_j)] \Bigg]_a^b = 1$$