# Understanding energy as a subtle concept: A model for teaching and learning energy

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A study of physics textbooks from the 1860s up to the present reveals shortcomings of our contemporary approach to teaching the concept of energy. In response, this paper offers a coordinated set of conceptual definitions of force, work, and energy, which can provide a somewhat more accessible grounding on which to develop the subject pedagogically. 

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## I. INTRODUCTION

This treatise is not about teaching methodology per se. Instead it deals primarily with three related concerns: first, it shows that the present widely held understanding of "energy" is inadequate for several reasons; second, it argues that potential energy is a highly useful bookkeeping device and not an empirical measurable quantity; and third, it provides a more modern way to appreciate what energy, in totality, is. No matter how effectively we teach outmoded formulations, we fail. No matter how thoroughly we bring to bear the cannons of research on students' ideas and pedagogical approaches, if the concepts we proffer are long out of date, we cannot meet our responsibilities. It is time to upgrade the simplistic 19th-century understanding of "energy," and then focus on the techniques of teaching it.[1]

In the real world, everything is in motion from atoms to galaxies. Nothing is truly static; bridges expand and contract; skyscrapers sway; planets revolve. When you step on the floor, the floor sags, it too changes. Only when the net force acting on a material object is zero, will there be no change in its state. In large measure, physics studies events; it studies change; change that has occurred, and change that is yet to occur. To begin to clarify and integrate our conceptions of energy, force, work, and mass, we will employ the even more fundamental ideas of matter, interaction, and change.

It might come as a surprise to some to learn that we physicists do not already have communally agreed-upon definitions of our basic concepts, energy being just one. Despite the obvious difficulties, the goal here is to formulate a working definition that addresses what energy *is*; a definition that can serve as the foundation of instructional discourse. The concern is not merely with writing equations like $KE = \frac{1}{2}mv^2$ or $PE = mgh$ (both only approximations) that appear to tell us how we might measure specific forms of energy, but beyond that, what is it that is being measured? What does a cannonball have when it has 1000 J of energy, either *KE* or *PE*?

Surely, if we can succeed in conceptualizing *energy* on a deeper level than has been done thus far in our contemporary textbooks, we will be better able to teach the subtleties associated with the concept in a more unified way. But first we have to recognize, and be willing to come to grips with, the considerable inadequacies of our familiar formulations. Clearly, we have been able to successfully do physics without being overly careful about defining the foundational basics, but teaching physics without conceptual rigor is a different matter.

We know that physics is an ever-evolving dynamic creation, and so a definition of any fundamental concept—frozen in time, as it must be—will have to be broad enough to allow for future discovery and evolution. Moreover, we should expect that what was once accepted usage will require updating after 160 years. Certainly, it would be naïve to think that "energy" could have been satisfactorily defined before Einstein (1907) gave us $E_0 = mc^2$, or before Noether (1915) devised her theorem.

Even a cursory study of physics textbooks from the 1860s and onward makes it apparent that they have been presenting *energy* pretty much the same way for all that time, despite everything that has been learned in the interim. The consequence of this was underscored by Nobel laureate Richard P. Feynman who pointed out (1963), "It is important to realize that in physics today, we have no knowledge of what energy is."[2] It is now more than 50 years later and we—the community of physicists—still have not resolved that dilemma. The idea of energy is central to all of physics, and yet one would be hard pressed to find a textbook which provides an effective definition that goes beyond the usual tautologies. Our most widely accepted definitions, the ones in terms of "work," are, as we shall see, all solidly mired in the 19th-century, and all are simplistically circular.

Nonetheless the inherent shortcomings of defining energy in terms of *work* are widely unappreciated, and that flawed approach is commonly proffered in modern textbooks, and hence in classrooms worldwide. Since the 1970s, there have been dozens of excellent papers published on the subjects of force, work, energy, and mass.[3] No doubt as a result, textbook writers, reasonably enough, have become more circumspect and adept at treating energy; alas, most often without ever actually addressing what energy is. Typically, they define both kinetic energy and potential energy in terms of work—which we will soon see is simplistic—and then they quietly jump to the conclusion that having defined *KE* and *PE* somehow defines energy itself. It does not; just as distinguishing between a chicken and a frog does not establish what life is.

The obvious bottom line is that we cannot properly teach the concept unless we can say, at the outset, what energy is. By providing a framework of a few interrelated conceptual definitions (of energy, force, work, matter, mass, etc.) it is hoped that we can at least begin to address the problem.

## II. A BRIEF HISTORY

To go beyond where we are now in our conceptualization of *energy*, it is imperative to know how we got here, essentially stuck in the 19th century. Accordingly, what follows is a brief historical sketch of the key related concepts of: (A) Force and work; (B) Matter and mass; (C) *Vis Viva* and energy; (D) Kinetic energy; and (E) Potential energy.

Teachers who have had little time to study the history of physics will hopefully find some of this material useful in the classroom. Beyond that, the reader will gain perspective on how the epistemological situation that now prevails, quite naturally came to be. In other words, why do we usually start teaching about *energy* with the far less basic notion of *work*?

### A. Force and work

Force—we'll come back to what it is later—is one of those primitive concepts that has been around since ancient times.[4] For centuries it was enough to define force as "pushes and pulls," which are of course forces. Nonetheless, that sufficed to appreciate the several so-called "simple machines" conceived in antiquity. An inclined plane allows one to push a load up to some height with less force than lifting it directly, but the distance over which that force must be exerted increases proportionately. The same is true with the lever. The early Greeks knew that for a pulley system, if a force of say, a quarter of the weight of an object could hold that object motionless, you would have to slowly draw the rope through four times the distance that the load would simultaneously rise. Force and the distance over which it acted were intriguingly linked. These nascent ideas were elaborated on across the Middle Ages by both Latin and Muslim philosophers. Their efforts in turn influenced the thinking of scholars in the 16th and 17th centuries.

It was the French hydraulics engineer Solomon of Caux in his book *Les Raisons des forces mouvantes avec diverses machines* (1615) who was probably the first to explicitly write about the concept of *work* in an almost modern form.[5] Albeit crudely put, work is force times the distance over which it acts.

The ever-popular notion—*energy is the ability to do work*—derives from the historical centrality of the concept of work, which preceded the technical idea of "energy" by centuries. That's probably why most textbooks, and hence many teachers, still introduce energy by way of work. Unlike energy, which is an exquisitely subtle fundamental idea relating to the very nature of space-time, work is a practical notion that was quantified, and measured, long ago. Just think of Guillaume Amontons (1663–1705) who built a heat engine (1699) and compared its efficacy with the "labour of a horse," which he defined as the "force exerted by the horse multiplied by the speed of the horse…"[6] Starting a discussion of energy with work is really putting the horse before the cart.

### B. Matter and mass

Early Greek scholars held that matter—we'll come back to what it is later—could neither be created nor destroyed; we would say it was *conserved*. And so, it was reasonable for the theologian and philosopher Giles of Rome (c. 1243–1316), aka *Aegidius Romanus*, to be troubled about the transubstantiation. Given that the body of Christ was transformed into the bread of the Eucharist, how could it be that weight and volume were so profoundly altered if matter was conserved? Giles concluded that there had to be a new more abiding measure, the *quantitas materi*æ, or "quantity-of-matter."[7] That was a purely theological concept, and there was no need to define it any further.

Aware of Aegidius's suggestion, Jean Buridan (c. 1295–c. 1358) recognized its physical significance in relation to his own Impetus Theory of motion. He asserted that speed alone could not be the true measure of motion; instead it had to be quantity-of-matter and speed "conjointly." Impetus Theory survived in one form or another across the 17th century and well beyond. The product of quantity-of-matter and speed ultimately came to be known as *momentum*, or *quantity-of-motion*—Newton favored the latter. We mention momentum here because the precursor of the rival idea of *energy* sprang from the mind of Newton's continental nemesis Gottfried Leibniz (1646–1716). One might even speculate that some portion of Leibniz's motivation was to distract from Newton's preeminence.

In the early 1600s, the term *quantitas materi*æ had powerful religious overtones and philosophers usually avoided it. Johannes Kepler (1571–1630) replaced quantity-of-matter with several equivalent terms such as *copia materiæ*, but he also used the Latin word *moles* meaning *mass*.[7] In the *Principia* (1687) Newton, perhaps to distinguish himself from the followers of Kepler, introduced an alternative Latin word *massa* for mass, though he preferred, and most frequently used, *quantitas materi*æ as a synonym for mass. Nonetheless, the idea of *mass* came into widespread use on the wings of the *Principia*.

Amazingly, today one can find physics textbooks that define mass as the "[q]uantity of matter in an object"[8] even though, as we will see, that's quite meaningless. That definition has pretty much been replaced in our more serious textbooks by a definition of mass in term of inertia, which as we will also learn, is not a great deal better.[9] Since mass is at the heart of any 21st-century understanding of energy, we have to sort all of this out.

### C. Vis Viva and energy

Billiards having become quite popular, in 1668 the Royal Society of London called for papers analyzing collisions. The request was responded to by, among others, Christiaan Huygens (1629–1695) who analyzed elastic collisions. At the time, quantity-of-motion, which was not yet widely called momentum ($mv$), was the premiere dynamical quantity. We do not know what Huygens was thinking, but there is something about momentum that is a little counterintuitive; the net momentum of two identical balls flying toward each other at great speed can nonetheless be zero. Perhaps Huygens squared the speeds to get rid of the directionality and so avoid that issue; in any event, he found that a quantity corresponding to $mv^2$ was conserved in elastic collisions.

Huygens's one-time student Leibniz recognized that there was something extraordinary about $mv^2$ and he gave it the Latin name *vis viva*, living force. In those days, the German word *Kraft* or force might mean strength, vigor, energy, or power and so "living force" was not actually force. There followed a several-decades-long dispute between those who embraced $mv$ (mostly Newtonians) and those who alternatively took $mv^2$ as the true measure of motion (mostly Leibnizians). Neither side won, but over the next hundred years or so, *vis viva* slowly transformed into our *kinetic energy*.

Historically, energy and momentum were two competing ideas, and we still teach them separately. And yet a hundred years ago Relativity Theory brought energy and momentum together; they are the two complementary pieces of a four-dimensional whole—the energy-momentum 4-vector. No matter on what level we teach, we should at least mention that these two concepts are intimately related.

### D. Kinetic energy

In 1735 Johann Bernoulli (1667–1748), a friend of Leibniz, used the word energy in a technical sense, but it was not until 1807 that *energy* was formally associated with $mv^2$. That was initiated by Dr. Thomas Young (1773–1829) who asserted that "labour expended in producing any motion, is proportional to the energy which is obtained." That observation would reverberate across the centuries. Indeed, its inverse, "energy expended is labour obtained" calls to mind, "energy is the ability to do work."

Like so many in the 1800s, Gaspard Coriolis (1792–1843) was interested in the technical aspects of steam engines, *vis viva*, force, and work. Sometime around 1829 he did a piece of mathematical analysis that established what Young had earlier proposed. By integrating force over distance, Coriolis showed that the work done in changing the speed ($v$) of a mass ($m$) was equal to the change in the quantity $\frac{1}{2}mv^2$. This was a significant step toward recasting *vis viva* ($mv^2$) as energy ($\frac{1}{2}mv^2$), using what Coriolis called the "principle of the transmission of work." With this bit of mathematics, Coriolis, almost two hundred years ago, set the path for a legion of textbooks to follow.

To better distinguish between the concepts of force and energy, in 1849 Lord Kelvin suggested the term *kinetic energy* for $\frac{1}{2}mv^2$ instead of *vis viva*, which slowly vanished from the vocabulary of physics.

### E. Potential energy

Remarkably, Leibniz, the genius who gave us *vis viva* was also responsible for the earliest theorizing that would, a hundred and fifty years later, evolve into *potential energy*. He proposed (1695) the idea that there could be inactive unmanifested *vis viva*; matter could harbor *vis mortua* (i.e., *dead force*). This was a static propensity-for-motion, a "solicitation to motion" without there being motion.[10] A system restrained by forces into motionless equilibrium, "has the power of acting" even though it may not act. A raised body, or a stretched elastic object possesses *vis mortua*. That unrealized capacity to act was the forerunner of the concept of potential energy, though Leibniz never explicitly linked it with *vis viva* to form a conserved gestalt.

Daniel Bernoulli (1700–1782), Johann's son, in his treatise *Hydrodynamics* (1736) made conservation of *vis viva* his central analytic principle—that's what Bernoulli's Equation is all about. In a letter to the mathematician Leonhard Euler (1743), Bernoulli spoke of "potential living force." At the turn of the century Lazare, Count Carnot (1753–1823), Sadi's father, called the product of weight and height, *latent living force*. In 1853, William Rankine (1820–1872) compared "actual or sensible energy" (i.e., *KE*) to "potential or latent" energy (i.e., *PE*). After his treatise of 1847 on conservation of energy, Hermann von Helmholtz (1821–1894), adopting Rankine's terminology, posited "the decrease in the potential energy is always equal to the increase of the kinetic energy."

## III. WHAT DO TEXTBOOKS SAY?

Most often we teach in association with, and guided by, textbooks, and so it's essential to see what those textbooks have had to say about energy. Already in the late 1860s authors were informing:

> *Every moving mass is said to have an actual or dynamical energy. Every mass so situated that it can be moved by the forces acting upon it is said to have possible or potential energy.*[11]

One famous text of the 1880s, having first defined work, then posited:

> *The fact that any agent is capable of doing work is usually expressed by saying that it possesses Energy, and the quantity of energy it possesses is measured by the amount of work it can do.*[12]

And an equally popular treatise of the time asserted:

> *The work that must be done upon a body to give it its actual motion, supposing it to have been initially at rest, is called the energy of motion or the kinetic energy of the body.*[13]

*A First Course in Physics* by Millikan and Gale (1906) set the standard for much of what followed in the 20th century. They gave us the unequivocal statement, "The *energy* of a body is defined as its *capacity for doing work*."[14] By the 1920s that definition had become commonplace in the literature. Thus, one popular (1928) text asserted:

> *Energy has been defined as stored work, or as the capacity for doing work.*[15]

Throughout much of the 20th century, schoolbooks maintained that energy was simply "the ability to do work" even though there were compelling thermodynamic reasons to suggest otherwise.[16] At the same time there was a growing more-cautious pedagogical approach of merely moving from *work* directly to *KE* and then on to *PE*, without ever troubling to attempt to define *energy* itself.[17] That tactic has persisted in textbooks right up to the present where it is by far the most commonly encountered.[18]

Serway and Jewett (*Physics for Scientists and Engineers*) recognized the problem and admitted that "energy cannot be easily defined." They even cautioned against accepting the proposition that energy is "the capacity to do work."[19] Commendably, a few modern-day textbook authors, aware of the conceptual shortcoming, have tried to address it, alas, with limited success. Debora Katz in her book (*Physics for Scientists and Engineers: Foundations and Connections*) told us, "the term energy describes the state of a particle, object, or system."[20] That's certainly true, but after offering a similar definition, Jearl Walker (*Fundamentals of Physics*) asserted that it was "too vague to be of much help" and he was right.[21] Randall Knight came close to the essence of energy when he lightheartedly wrote in his book (*Physics for Scientists and Engineers A Strategic Approach*), "energy is the ability to make things happen."[22]

## IV. A REEVALUATION

And so we arrive at the two related energy formulations, one or the other of which is taught in almost every introductory physics classroom in the world: succinctly put, (1) work is essentially force times distance; work produces a change in either or both $KE$ and $PE$; and these concepts taken together define energy. (2) Alternatively, energy is simply the ability to do work.[23] The problem with both of these approaches is that they, we will show next, rely on a superficial notion of what *work* is.

### A. How do we teach "work"?

One of the most widely used introductory physics textbooks defines work as "force $\times$ distance."[24] Overlooking that this is simplistically stated, it is a mathematical representation that can guide a measurement; it's not a conceptual definition of what work is. It doesn't tell a student what is happening (energy-wise) to the system when work is done on it. Nor does it say what, if anything, is happening to the thing doing the work. Work involves overcoming forces: friction, inertia, gravity, and so forth. It involves interactions, and that's how energy is transferred. We get little of that from "force $\times$ distance."

The formal modern textbook definition of work is the integral of the scalar product of force and the displacement over which that force acts. Strictly speaking, ***that definition only applies to structure-less point particles or perfectly rigid objects***, neither of which can internalize energy.[25] Except for a few fundamental particles (because no real composite object is perfectly rigid), the ubiquitous integral representation has little value as a rigorous definition of work—at least one possessing generality. Nonetheless, most contemporary textbooks define work via that integral. Careful textbook authors advise that that formulation only applies to point particles, but then they quietly go on to treat ordinary "objects" or "bodies."

The assertion, "The work done on an object [presumably by an external force] is thus equal to the *change* in its kinetic energy," is called the Work Energy Theorem, and it is seriously flawed.[26] We have already seen that the integral/force-displacement formulation of work only applies approximately to actual macroscopic objects. Beyond that, there are forces that act on bodies, and impart motion to them, and yet do no work on those bodies. Jump and the reaction of the floor pushes you up, but that externally applied force does only a minute amount of work on you; the point of application of the normal force is hardly displaced. Your acquired $KE$ comes not via the Work Energy Theorem, but from your lunch. Self-propelled bodies (people, dogs, cars, trains, planes, etc.) are accelerated by external forces that do little or no work on them; each provides its own source of energy internally. Clearly, using work to define the entirety of what encompasses energy is problematic. *Work done by the action of an external force on a system is but one modality for imparting KE to that system.*

If possible, the definition of a physical concept such as *work* should be conceptually framed in terms of more fundamental ideas. Energy is one of those more fundamental notions. That accords with James Clerk Maxwell's 1877 prescient assertion that "work, therefore, is a transfer of energy from one system to another."[27] Albert Einstein (1907) agreed: "energy" he wrote, "may be transferred to the system in the form of work or heat…"[28] And Enrico Fermi (1936) maintained that "besides mechanical work, other means of transfer of energy must be taken into account…"[29] Our modern literature abounds with similar statements:[30] ***work is the transfer of energy from one system to another via the action of force over distance.***

Work is a process; once done it no longer exists. It is something that cannot be stored; what is stored is energy. Positive work done on a system increases the energy content of that system, and negative work done on a system decrease its energy content. Work is just one of several mechanisms for changing the energy state of matter.

### B. What's wrong with our accepted definition of energy?

The notion that energy is "the capacity for performing work" was formalized in the mid 1800s, primarily through the independent efforts of William Rankine and William Thomson, Lord Kelvin. That inchoate definition has survived for well over a hundred years, and it lives on today in too many classrooms. Its most obvious failing follows from Maxwell's insight—given that work "is a transfer of energy from one system to another," and further that *energy is the ability to do work*, we can conclude that ***energy is the ability to transfer energy***. The thing is an obvious tautology; its circular nature cannot be lost on even the most unsophisticated student.

In the spirit of Rankine and Kelvin, we might assert that "energy is the ability to melt snow," or perhaps "energy is the ability to light up dark places." At this point anyone needing further convincing can find highly compelling thermodynamic and quantum mechanical reasons why *"energy is the ability to do work"* is unacceptable as a rigorous definition.[31] Just think of the Second Law of Thermodynamics, or the concept of zero-point energy.

Today our leading physics textbooks reasonably enough tend to be coy, made cautious by Feynman's famous admonition, and the dozens of modern journal articles. Most contemporary schoolbooks avoid defining energy altogether; instead they provide the mathematical/integral statement of work, and then wrongly use it to talk about the $KE$, and $PE$ of macroscopic objects. But since work is the transfer of energy, we again have a tautology.

If we transfer energy to a system it can supposedly be manifest as $KE$ or $PE$—that just tells us that energy, whatever it is, has different presentations. Alas, it's really only okay to posit that "kinetic energy is the energy of motion" provided you first have a usable definition of *energy*. Clearly, all those chapters in all those physics books are a bit of smoke and mirrors when it comes to answering the students' common question; What *is* energy?

## V. WORKING DEFINITIONS

The plan here is to attempt to create a consistent set of definitions for the basic concepts of matter, force, energy, and *mass*; definitions that can guide the discourse in and out of the classroom. Knowing beforehand that this can only be accomplished with limited success, it is nonetheless worth the effort, especially if it stimulates the readers of this essay to improve upon the offering.

Keep in mind that $KE \approx {}^1\!/_2 mv^2$ *is really only a low-speed approximation, and that too should caution against using it to define energy as a whole*. Of course, $KE \approx {}^1\!/_2 mv^2$ is not the energy of a photon, even though that energy is entirely kinetic. How then can we continue to base our definition of *energy* on that rather restricted expression?

## A. Matter

Perhaps the most defining characteristic of matter is that it interacts. On a very basic level, there is matter, interaction, and change. ***Matter is that which interacts and as a result manifests observable physical properties that are amenable to being measured.*** There is matter that has mass, and matter that is massless. *Ordinary everyday matter is comprised of clusters of quarks, and leptons, and possesses mass. Light* (*i.e., photons*) *are massless matter*. Here we are in accord with some of the most respected physicists of the 20th and 21st centuries: W. Pauli concluded, "even light has become matter now, due to Einstein's discoveries." L. de Broglie concurred, "Light is, in short, the most refined form of matter." E. Schrödinger maintained "even in the remotest voids of the universe there is always starlight—and *that* is matter."[32] More recently, Nobel laureate F. Wilczek asserted "light is another form of matter."[33] A clever student might well ask, "What then is *not* matter?" Time is not matter; energy, force, space, wavefunctions, electric fields, magnetic fields, gravity, sound, and heat are all *not* matter, though all are in some way associated with matter.

## B. Interaction and force

On a very basic level, there is matter, interaction, and change. Matter interacts, and it does so via equal and oppositely directed forces, so-called interaction pairs. If matter did not interact, it could not be known and could not be said to exist. *There is no such thing as a single force*. That of course is the modern reinterpretation of Newton's Third Law. All matter interacts in one way or another, either long range, or short range.

***Force is the agent of all observable physical change.*** Nothing determinable happens without the action of force. There are four fundamental forces: *gravitational*, *electromagnetic*, *strong*, and *weak*. Every physical phenomenon, every change, every transformation, that we have been able to study, is the result of one or more of these four interactions. Clearly, defining force as "[a]ny influence that tends to accelerate an object; a push or pull"[34] ignores an entire century of discovery.

## C. Energy

On a very basic level, there is matter, interaction, and change. Determinable physical change occurs as a result of interactions. ***Energy is a measure of the capacity of matter interacting with matter to effectuate physical change. It is the conserved scalar measure of the extent of change that has already occurred, and/or change that can yet occur.*** As Max Planck (1945) suggested "The energy of a system is, therefore, sometimes briefly denoted as the faculty to produce external effects."[35] And long before that, when discussing *vis viva*, Johann Bernoulli and Leonhard Euler talked about the "capacity of action" (*Facultas agendi*).[36]

A chunk of plastic explosive might have the capacity to liberate 1000 J of *KE* in various forms (sound, light, etc.); but that can only happen after the electromagnetic forces between its atoms are prompted to change the explosive's chemistry and thereby reduce its mass. Think of a bullet fired in a gun. Force changes the bullet's state of being; the *KE* acquired as it leaves the muzzle measures much of the extent of that change. If matter did not interact, energy in all its manifestations would vanish; the concept would be meaningless.

A stretched spring has a quantity of energy stored (via internal interactions); work has been done on it, and hence it has been changed—an applied force was the agent of that change. The spring in its additionally energized state has the capacity in future to generate thermal energy, to do work, and to radiate photons; it can effectuate a variety of changes. The amount of energy stored is a measure of the ability of the spring to produce change (internally and externally) via the action of the forces it can exert.

On the other hand, momentum is a vector quantity and has a spatial aspect. It can pass, in whole or in part, from one entity in space to another; it can be altered by forces, but it cannot change form; it has only one manifestation [$\mathbf{p} = \gamma m\mathbf{v}$ where $\gamma = (1 - v^2/c^2)^{-1/2}$]. By contrast, energy is a scalar quantity, it can be manifested in time in several different forms.

Energy and time are conjugate concepts, as witnessed by both the Heisenberg Uncertainty Principle and Noether's Theorem. Force is that which changes the state of a system; energy is a measure of the extent of that change, whether it has already happened or is yet to happen. Energy is associated with all the "stuff" that exists, from immense whirling galaxies to miniscule particles. Energy is a measure of the change resulting from matter interacting with matter; it is not an entity in and of itself, independent of matter. "Dark energy" aside, there is no such thing as "pure" energy wafting around, energy free of interacting matter; just as there is no such thing as "pure" momentum.

As a result of the symmetrical nature of interactions, a change in one entity/system is accompanied by an energy-equivalent change in the entity/system with which it interacts. Accordingly, we have the principle of *Conservation of Energy—the total energy associated with any system can be accounted for at all times*.

## D. Energy of motion and energy of rest

There are two forms of energy ($E$), energy of motion ($KE$) and energy of rest ($E_0$). The energy of any moving entity (whether possessing mass or not) is known as kinetic energy. It is the capacity of any moving entity to effectuate change due to its relative motion. Kinetic energy can be thought of as a measure of either the change that occurred in imparting relative motion to any entity, or as a measure of the change that can yet occur when that entity interacts and subsequently loses all or part of its relative motion.

The total energy of any particle, or collection of particles—a photon, an electron, an atom, or even a frog—is the sum of its *rest energy* ($E_0 = mc^2$), and its *kinetic energy*. We will discuss mass ($m$) presently, for the moment suffice it to say, mass is Lorentz invariant (i.e., speed independent). We know from the Special Theory of Relativity that the total energy of a physical system moving at speed $v$ can be expressed as $E = E_0 + KE = \gamma mc^2$. The $E_0$ term is the *internal energy* associated with the system at rest ($\mathbf{p} = 0$). The $KE$ term is the *organized* kinetic energy of the system moving as a whole ($KE = \gamma mc^2 - mc^2$). At low speeds of course, $KE \approx \frac{1}{2}mv^2$. Using the customary definitions, if energy is the ability to do work, and mass is the resistance to changes in motion, then $E_0 = mc^2$ makes no sense at all.

### E. Potential energy

Traditionally, **potential energy ($PE$) is the energy said to be stored as a result of a configurational change in a system of interacting parts.** As we have seen that notion was created in order to support the conceit of conservation of energy in situations where $KE$ vanishes, wholly or partially. As is discussed elsewhere in the literature, $PE$ is an excellent highly useful theoretical concept.[37] It survives because energy is conserved. Even so, $PE$ is an idea, not an empirical quantity; it is not itself amenable to measurement.[38]

Briefly put, only changes in $PE$ are defined, and these are defined as the work done on a system by conservative forces. Such work is measurable as it is being done. However, once work is done it no longer exists and is no longer measurable; if $\Delta PE$ is *only* defined by work done (e.g., $mgh$, or $\frac{1}{2}kx^2$, or $\frac{1}{2}CV^2$) it cannot be measured in stasis while it supposedly exists.

The fact that the zero of $PE$ floats, underscores the fact that $PE$ is not a real physically measurable quantity. Moreover, there is no universal equation for $PE$. You cannot measure the $PE$ of a stretched spring, or a stick of dynamite, or anything else—at least not without releasing it. What is only defined is the change in $PE$, and when the system experiences, say a change of 100 J of $PE$ it actually experiences a mass change of $(100\,\mathrm{J})/c^2$.

Potential energy is a tremendously useful bookkeeping device; it is not, however, a quantity possessing measurable physicality—it's a purely theoretical concept much like the quantum mechanical wavefunction. As further evidence of this, consider that fact that $PE$ is not in accord with the Special Theory of Relativity. "You have to recognize," Robert Mills (of quantum field theory fame) cautioned, "that the idea of potential energy is not truly fundamental and that it breaks down in the relativistic world…"[39] Considering a "system of particles," Landau and Lifshitz explained, "[t]he fact that the potential energy depends only on the positions of the particles at a given instant [and is not expressed as a function of time] shows that a change in the position of any particle instantaneously affects all the other particles. We may say that the interactions are instantaneously propagated.… If the propagation of interactions were not instantaneous… [t]he laws of motion for interacting bodies would then be different in different inertial frames, a result which would contradict the relativity principle."[40] And of course, instantaneous propagation violates Special Relativity.

For many of the great 19th-century scientists, people like W. Rankine, Lord Kelvin, James Joule, John Herschel, J. J. Thomson, Ernst Mach, and James Clerk Maxwell, there was "actual" energy ($KE$), and "possible" energy ($PE$). As Maxwell put it in 1877,

> *Rankine introduced the term potential energy, a very felicitous term, since it not only signifies the energy which the system has not in actual possession, but only has the power to acquire.*[37]

At the time, all actual energy was assumed to be kinetic. $PE$ was the energy a system could acquire; it was not energy already possessed by the system and somehow stored. Hence $PE$ would not be there to be measured with the system in stasis. As Einstein (1946) pointed out, "so long as none of the [internal] energy is given off externally, it cannot be observed."[41] Internal energy cannot be observed; it cannot be measured as it presumably exists.

*A composite system (at rest) possesses energy when it can undergo spontaneous change*. A hot cup of tea, or a human being, or an atomic nucleus possesses some kind of energy if interacting internally it can undergo observable change all by itself. A stretched spring is said to contain elastic energy; when released it can experience self-initiated change. If a physical property of a system of interacting parts (temperature, volume, etc.) undergoes a change, that property can be associated with a resulting difference in energy (e.g., $PdV$).

Because the interaction energy—which is called $PE$—is associated with the system's internal energy, it is conceptually subsumed into the rest energy, $E_0$. And since $E_0 = mc^2$, whenever the conceptual quantity known as the $PE$ of a system increases or decreases, the mass of that system actually, measurably, increases or decreases. The mass of U-235 in an atomic bomb is a measure of energy, rest energy, a portion of which waits to be liberated.

Physics textbooks tend to be very traditional; they uncritically accept $\Delta PE$ as real because it's a reliable bookkeeping device, albeit an unmeasurable anachronistic idea.[42] A more realistic alternative is to think in terms of the corresponding $\Delta m = \Delta PE/c^2$ of the system, which is at least in principle, if not always in practice, measurable. What happens when any nucleus is formed is that its mass is measurably less than the sum of the separate masses of the constituent nucleons—that's what is called the *mass defect*. What happens when we stretch a spring is that its mass actually increases. To raise the temperature of 1.00 kg of water 1.00 °C we add 4.186 kJ of thermal energy. That increases $E_0$ by 4.186 kJ and hence increases the mass of the water by $4.66 \times 10^{-14}$ kg.

Imagine that you raise a cannonball of mass $m$ into the air a height $h$. You push up on the ball, and down on the Earth. You do work ($W \approx mgh$), you transfer (i.e., lose) energy, your chemical energy, hence your mass ($M$) decreases ($\Delta M \approx mgh/c^2$). The mass of the interacting ball-Earth system increases by exactly that same amount. The mass of the isolated you-Earth-ball system remains unchanged. Surely, there is no additional potential energy floating around somewhere in that system.

$\Delta PE$ is a convenient way to keep track of the energy stored via interactions within a system, but it is actually stored as mass: $\Delta m = \Delta PE/c^2$. When the concept of $PE$ was invented most everyone knew it was an accounting device needed to rationalize conservation of energy. It took Einstein to discover that energy can be stored as mass.

### F. Mass

On a very basic level, there is matter, interaction, and change. There is matter that has mass and matter that is massless. Matter that manifests mass can interact, experience change, and continue to exist. Matter that has no mass, individual photons, only exist "on the wing" at speed $c$. As for a definition of mass, today in books and classrooms, there are three common contenders, all faulty: (1) Mass is a measure of the quantity-of-matter. (2) Mass is that which manifests inertia, and is the measure thereof. (3) Mass is that which produces gravity, and is the measure thereof.

We know now that mass is an extremely subtle concept. Teachers, especially early in an introductory course, tend to avoid the complexities and just assert that "the mass of an object is a measure of its quantity-of-matter." Although never formally defined, *quantity-of-matter* is simply assumed to be synonymous with the "amount of stuff," a measure presumably having to do with the type and number of particles constituting the material object.

That *quantity-of-matter* definition stopped being meaningful in 1905 with Einstein's second paper on Relativity. A rubber band has more mass stretched than flaccid—the same amount of rubber, the same number of atoms, but more mass. A cannonball when hot has more mass than when cold. Apparently, mass is not additive, as it would be if it were simply a measure of quantity-of-matter. Any ordinary bound system, from a tiny nucleus to the Earth itself, has less mass than the sum of the masses of its separated constituents. The self-gravity of the Earth as it came together reduced the mass of what was being accreted by a multiplicative factor of roughly $4.2 \times 10^{-10}$. Just think of the fact that you would have to add energy to any common bound system to pull it apart and that energy would reappear as mass.

Much of the mass of ordinary matter in our Universe is associated with neutrons and protons. As far as we can tell, a proton is a composite of three nearly massless interacting quarks—two "up" (each around $2.3\,\mathrm{MeV}/c^2$) and one "down" (around $4.8\,\mathrm{MeV}/c^2$). That's a tiny amount compared to the proton mass of $938.27\,\mathrm{MeV}/c^2$. Most of the mass of nucleons comes, not directly from the quarks, but from the interactions that confine those quarks within those nucleons. If the physics is right, you and I are mostly ($\approx$99.8%) quark confinement energy—we're supposedly made of very little actual "stuff." Sorry Newton; would seem equating mass with quantity-of-matter was not such a great idea.

Today's more rigorous physics texts define mass in terms of inertia. Unfortunately, that approach has serious drawbacks as well. We are given to believe that since a force is required to change the motion of a material body, which is the essence of "inertia," we can use that phenomenon to define mass. Hence, it is all too often stated that, *the mass of an object is a measure of, and gives rise to, its resistance to changes in motion, its inertia.* That's at best only partially true.

The problem with that approach goes back to the statement of Newton's Second Law, which is better given as $\mathbf{F} = d\mathbf{p}/dt$, rather than $\mathbf{F} = m\mathbf{a}$. The latter is only the low speed approximation and so should not be used to define anything. Using $\mathbf{p} = \gamma m\mathbf{v}$ and taking its derivative, produces all sorts of interesting results. Generally, $\mathbf{F}$ and $\mathbf{a}$ are not even parallel.[43] The relationship between $\mathbf{F}$ and $\mathbf{a}$ depends on the orientation of $\mathbf{F}$ with respect to $\mathbf{v}$; the velocity vector fixes a special direction in space. When $\mathbf{F}$ and $\mathbf{v}$ are perpendicular, $\mathbf{F}_\perp = \gamma m\mathbf{a}_\perp$, and when $\mathbf{F}$ and $\mathbf{v}$ are parallel, $\mathbf{F}_{||} = \gamma^3 m\mathbf{a}_{||}$. Clearly, $m$ is not a scalar proportionality constant between $\mathbf{F}$ and $\mathbf{a}$.

Often, when there is a causative agent in one direction, and a system response in an entirely different (non-perpendicular) direction, we can expect that the two are related by a tensor, not just a scalar. Thus, if you wish to define "inertia" (call it $m_\mathrm{I}$) via $\mathbf{F} = m_\mathrm{I}\mathbf{a}$, then $m_\mathrm{I}$ must be a tensor. Mass and inertia are connected concepts, but not in any simple way. Only when $v \ll c$ and $\gamma \approx 1$ will $m \approx F/a$.

To see how all of this comes together, imagine a spaceship firing its constant-thrust ion motors. The ship will accelerate in the direction of $\mathbf{v}$. As its speed increases, $\gamma$ increases, and, even though the thrust $\mathbf{F}$, and $m$ are essentially constant, with $\mathbf{F} = \gamma^3 m\mathbf{a}$, $\mathbf{a}$ must continually decrease such that $v$ never reaches $c$ (if $v = c$, $\gamma = \infty$). On the other hand, if $\mathbf{a}$ is to be kept constant, $\mathbf{F}$ must increase toward infinity, as $v$ increases toward $c$. It becomes harder and harder to sustain the acceleration; consequently, the object's "inertia" increases even though its mass is essentially constant.

Change the mass of a system and you change its inertia, but the inertia of a system also depends on its speed. Another way to see that, is via $KE = \gamma mc^2 - mc^2$. As $v$ goes to $c$, $\gamma$ approaches infinity and the $KE$ of the object possessing mass approaches infinity—hence, it would take the input of an infinite amount of energy for such an object to be propelled up to $c$. As Einstein and Infield pointed out, "If two bodies have the same rest mass, the one with the greater kinetic energy resists the action of an external force more strongly."[44] Apparently, ***energy is the determinant of inertia***. The definition, "mass is that which manifests inertia, and is the measure thereof," was okay 110 years ago, but is now no longer so.

There remains one last semi-popular definition to contend with: *Mass is that which produces gravity, and is the measure thereof.* That, of course, comes from what we now call Newton's Universal Law of Gravitation: $F_\mathrm{G} = GmM/r^2$. Since gravity is the domain of the General Theory of Relativity, we will have to look to it for guidance. Accordingly, let's turn to one of the premier descriptors of all forms of matter, the *mass-stress-energy-momentum-density tensor* (aka the energy-momentum tensor, or just the *energy tensor*). That conceptual device represents the flow and density of all forms of energy associated with a matter continuum. As Einstein put it, "The Special Theory of Relativity has led to the conclusion that inert mass is nothing more or less than energy, which finds its complete mathematical expression in a symmetrical tensor of the second rank, the energy tensor."[45]

The energy tensor is a $4 \times 4$ array, a 16-term compilation of the various sources of the gravitational field. It embodies the configuration, state, and flow of matter at each event, each point in four-dimensional space-time. General Relativity equates gravity with the curvature of space-time. Indeed, gravitation as we have conceived of it for centuries, is actually the expression of the curvature of space-time, a curvature impressed upon it by distributions of mass and energy. The Keplerian/Newtonian idea of mass as the exclusive source of gravitation is replaced by the more encompassing energy tensor.[46]

So then what is mass? It would seem that energy is a more fundamental concept than mass; after all, everything that has mass always has energy, but not everything that has energy always has mass. Think of a single photon, or a collimated beam of photons. Considering $E_0 = mc^2$, we have learned that the *mass of a composite system is a property of its matter, the internal motions thereof, and the interactions therein.* Mass subsumes $PE$ and so, as Nobel laureate (1914) Max von Laue pointed out, "we can determine the total amount of energy in a body from its mass. We thereby get rid of the arbitrariness of the zero point of [potential] energy…"[47]

It would seem that the mass of any composite entity represents a kind of story, an account of the entity's history from the beginning of the Universe: ***The mass of any object/system at rest (i.e., net*** $\mathbf{p} = 0$***) is a measure of the amount of energy that went into creating that object as it exists at that moment.*** If an entity/system cannot exist at rest (such that its net momentum is zero), that system has no mass.[48]

## VI. CONCLUSIONS

As a guide to how one might introduce the concept of energy in the classroom, consider the following: there is matter, it interacts, and that interaction produces observable change. Interactions (forces) effectuate all change, all measurable occurrences. As a result of the action of one or more of the four fundamental interactions, a system (composed of quarks and/or leptons and/or photons) possesses a quantity of energy. Energy is the capacity of such a system, acted upon by forces, to experience a specific amount of change; change that has already been imparted to it, or change that it can impart to itself and/or to its environment in future. The measure of energy in joules is the measure of the extent of that change.

Work, heat, and electromagnetic radiation correspond to energy in transit and are means of transferring energy from one material system to another. With the vocabulary established we can carefully introduce a mathematical expression for *work*, spelling out its limitations. Once that is in place, we can discuss *KE* and conservation of energy. That opens the way for what is traditionally known as *PE*, which is a convenient way of keeping track of stored energy.

At that juncture, it is appropriate to point out that Special Relativity maintains that there is energy of rest, $E_0 = mc^2$, and energy of motion, *KE*. The highly useful theoretical concept called *PE* is then subsumed into the mass of any system of interacting parts.[49]

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[1]See, for example, B. A. Lindsey, P. R. L. Heron, and P. S. Shaffer, "Student understanding of energy: Difficulties related to systems," Am. J. Phys. **80**, 154–163 (2012); "Student ability to apply the concepts of work and energy to extended systems," **77**(11), 999–1009 (2009); J. W. Jewett, "Energy and the confused student. Part I–V," Phys. Teach. **46**, 38–43 (2008), **46**, 81–86 (2008), **46**, 149–153 (2008), **46**, 210–217 (2008), **46**, 269–274 (2008); C. Singh and D. Rosengrant, "Multiple-choice test of energy and momentum concepts," Am. J. Phys. **71**(6), 607–617 (2003); H. Goldring and J. Osborne, "Students' difficulties with energy and related concepts," Phys. Educ. **29**(1), 26–32 (1994); R. A. Lawson and L. C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," Am. J. Phys. **55**(9), 811–817 (1987).

[2]Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics* (Addison-Wesley Pub. Co., Reading, Mass., 1963), pp. 4–2.

[3]Herman Erlichson, "Work and kinetic energy for an automobile coming to a stop," Am. J. Phys. **45**(8), 769 (1977); Claude Penchina, "Pseudowork and real work," *ibid.* **46**(3), 295–296 (1978); Bruce A. Sherwood, "Pseudowork and real work," *ibid.* **51**(7), 597–602 (1983); Hanna Goldring, "Energy revisited," Phys. Teach. **22**(5), 318 (1984); A. John Mallinckrodt and Harvey Leff, "All about work," Am. J. Phys. **60**(4), 356–365 (1992); Robert Bauman, "Physics that textbook writers usually get wrong," Phys. Teach. **30**(5), 264–269 (1992); Kenneth Mendelson, "Physical and colloquial meaning of the term 'work'," Am. J. Phys. **71**(3), 279–281 (2003).

[4]René Dugas, *A History of Mechanics* (Dover Pub., Inc., New York, 1988) <https://archive.org/details/historyofmachani000518mbp/page/n7>.

[5]Reference 4, p. 128.

[6]Jennifer Coopersmith, *Energy the Subtle Concept* (Oxford U.P., Oxford, 2010), p. 61.

[7]Eugene Hecht, "Kepler and the origins of pre-Newtonian mass," Am. J. Phys. **85**(2), 115–123 (2017).

[8]Paul G. Hewitt, *Conceptual Physics*, 12th ed. (Addison-Wesley, Reading, MA, 2015), p. G-10.

[9]Eugene Hecht, "There is no really good definition of mass," Phys. Teach. **44**(1), 40–45 (2006).

[10]Reference 6, p. 102.

[11]W. J. Rolfe and J. A. Gillet, *Handbook of Natural Philosophy* (Potter, Ainsworth, and Co., NY, 1869), p. 275.

[12]E. Atkinson, *Elementary Treatise on Physics* (William Wood and Co., NY, 1886), p. 50.

[13]A. Privat Deschanel, *Elementary Treatise on Natural Philosophy* (D. Appleton and Co., NY, 1887), p. 73.

[14]Robert Millikan and Henry Gale, *A First Course in Physics* (Ginn & Co., Boston, MA, 1906), p. 162.

[15]Ervin S. Ferry, *General Physics* (John Wiley & Sons, Reading, NY, 1928), p. 104.

[16]For example, see, Erich Hausmann and Edgar Slack, *Physics* (D. Van Nostrand Co., NY, 1935), p. 130; F. Bueche, *Principles of Physics* (McGraw-Hill, NY, 1972), p. 90; Arthur Beiser, *Physics* (The Benjamin Cummings Pub. Co., Menlo Park, CA, 1978), p. 842; Edwin Jones and Richard Childers, *Contemporary College Physics* (McGraw-Hill, Boston, MA, 1999), p.181.

[17]For example, see, Robert B. Lindsay, *General Physics* (John Wiley & Sons, London, 1940), pp. 88–109; Francis Sears and Mark Zemansky, *University Physics* (Addison-Wesley Pub. Co., Reading, MA, 1st ed. 1940), p. 117 (2nd ed., 1955); Marcelo Alonso and Edward Finn, *Fundamental University Physics* (Addison-Wesley Pub. Co., Reading, MA, 1967), pp. 195–212; Jerry Wilson and Anthony Buffa, *College Physics* (Prentice Hall, NJ, 1997), pp. 133–141; Douglas Giancoli, *Physics*, 5th ed. (Prentice Hall, NJ, 1998), pp. 145–167; Richard Wolfson and Jay Pasachoff, *Physics* (Addison-Wesley Pub. Co., Reading, MA, 1999), pp. 163–187; John Cutnell and Kenneth Johnson, *Physics*, 5th ed. (John Wiley & Sons, NY, 2001), pp. 151–179.

[18]For example, see, Hugh Young and Roger Freedman, *University Physics* (Pearson, San Francisco, CA, 2004), p. 207; Raymond Serway and Cris Vuille, *College Physics* (Cengage Learning, Australia, 2015), pp. 127–138; Nicholas Giordano, *College Physics* (Cengage Learning, Australia, 2010), pp. 165–176.

[19]Raymond Serway and John Jewett, *Physics for Scientists and Engineers* (Cengage Learning, Australia, 2014), pp. 177–202.

[20]Debora Katz, *Physics for Scientists and Engineers* (Cengage Learning, Australia, 2016), p. 215.

[21]Jearl Walker, *Fundamentals of Physics* (John Wiley & Sons, NY, 2014), p. 149.

[22]Randall Knight, *Physics for Scientists and Engineers: A Strategic Approach* (Pearson Education, Boston, MA, 2017), p. 207.

[23]Art Hobson, "Energy and work," Letter to the Editor, Phys. Teach. **42**, 260 (2004).

[24]Reference 8, p. G-17.

[25]See, for example, Ref. 3, A. John Mallinckrodt and Harvey Leff. Also see, Carl Mungan, "A primer on work-energy relationships for introductory physics," Phys. Teach. **43**(1), 10–15 (2005); see also, A. B. Arons, "Developing the energy concepts in introductory physics," *ibid.* **27**(10), 506–517 (1989).

[26]Reference 18, Giordano, p. 171; or Serway and Vuille, p. 133.

[27]James C. Maxwell, *Matter and Motion* (Dover Pub. Inc., Mineola, NY, 1991), p. 56.

[28]Albert Einstein, "On the relativity principle and the conclusions drawn from it," Jahrbuch Radioaktivität Elektronik **4**, 411–462 (1907).

[29]Enrico Fermi, *Thermodynamics* (Dover Pub. Inc., Mineola, NY, 1936), pp. 12–15.

[30]Clifford Swartz in "Why use the work-energy theorem?" Letter to the Editor, Am. J. Phys. **72**, 1145 (2004), agreed that "Work is just one way of transferring energy," Carl Mungan in "Work and energy," Letter to the Editor, *ibid.* **73**, 197 (2005) concurred: "it is correct to define work as an energy transfer."; See H. B. Callen, *Thermodynamics* (John Wiley & Sons, New York, 1962), p. 7.

[31]Nancy Hicks, "Energy is the capacity to do work—or is it?," Phys. Teach. **21**(11), 529–530 (1983); Robert Lehman, "Energy is not the ability to do work," *ibid.* **11**(1), 15–18 (1973).

[32]Eugene Hecht, "Energy and change," Phys. Teach. **45**(2), 88–92 (2007).

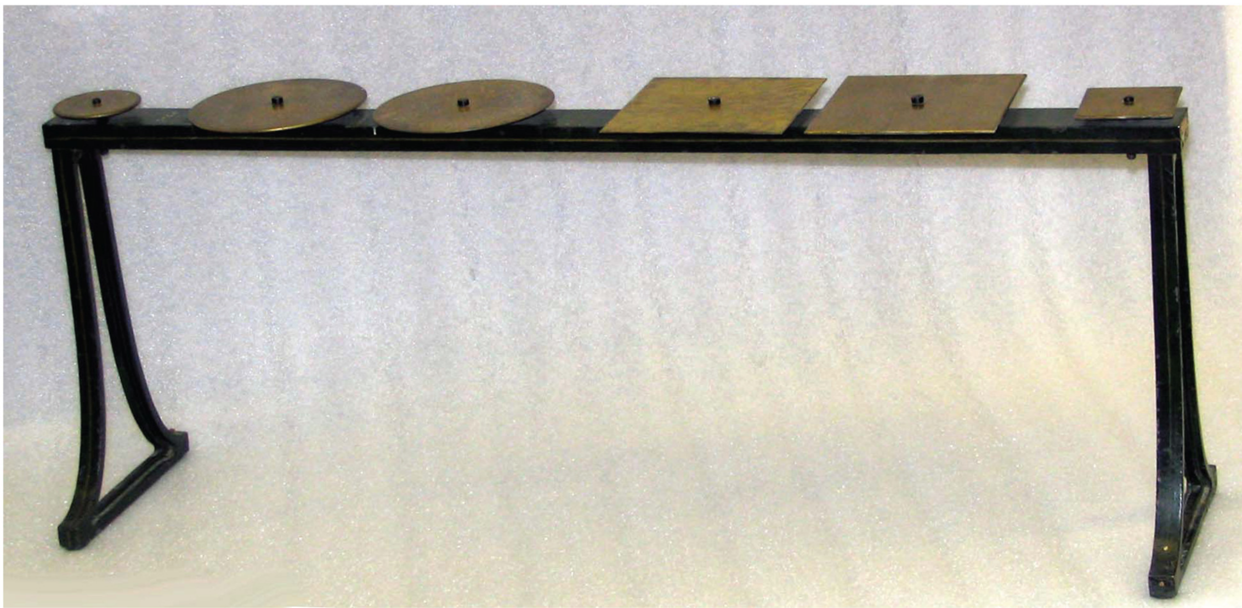
[33]Frank Wilczek, *The Lightness of Being* (Basic Books, New York, NY, 1963), p. 24.

[34]Compare Reference 8, 8th ed. p. 716 and 12th ed. p. G-6.

[35]Max Planck, *Treatise on Thermodynamics*, 3rd English ed. (Dover, NY, 1945), p. 41. The idea that energy is "the capacity for producing physical change" goes back at least to the 1860s; see Ref. 12, Atkinson, p. 51.

[36]Max von Laue, "Inertia and energy," in *Albert Einstein Philosopher–Scientist*, edited by P. Schilpp (Harper Torchbooks, NY, 1951), Vol. II, pp. 503–533.

[37]Eugene Hecht, "Relativity potential energy, and mass," Eur. J. Phys. **37**, 065804 (2016); "An historic-critical account of potential energy: Is *PE* really real?," Phys. Teach. **41**(11), 486–493 (2003).
[38]Eugene Hecht, "Reply to a comment on 'Relativity potential energy, and mass'," Eur. J. Phys. **40**, 028002 (2019).
[39]Robert Mills, *Space, Time and Quanta* (W. H. Freeman, NY, 1994), p. 151.
[40]L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon Press, Oxford, 1960), p. 7.
[41]Albert Einstein, "$E = Mc^2$," in *Ideas and Opinions* (Dell, New York, 1976), p. 330.
[42]John Roche, "What is potential energy?" Eur. J. Phys. **24**(2), 185–196 (2003).
[43]Lev B. Okun, "The concept of mass (mass, energy, relativity)," Sov. Phys. Usp. **32**, 629–638 (1989); "The concept of mass," Phys. Today **42**(6), 31–36 (1989); Gordon J. Aubrecht II, "Comment on 'Apparatus to measure relativistic mass increase'," Am. J. Phys. **72**(7), 970–971 (2004).
[44]Albert Einstein and Leopold Infeld, *The Evolution of Physics* (Simon and Schuster, NY, 1938), p. 207. See also Carl G. Adler, "Does mass really depend on velocity, Dad?," Am. J. Phys. **55**, 739–743 (1987).
[45]Albert Einstein, "The foundation of the general theory of relativity," in *The Principle of Relativity*, translated by W. Perrett, and G. B. Jeffery (Dover Pub. Inc, NY, 1923), p. 148.
[46]Dennis Lehmkuhl, "Mass-energy-momentum in general relativity. only there because of spacetime," Br. J. Philos. Sci. **62**(3), 453–488 (2011).
[47]Reference 36, p. 529.
[48]Eugene Hecht, "On defining mass," Phys. Teach. **49**(1), 40–44 (2011).
[49]Eugene Hecht, *Physics In Perspective* (Addison Wesley Pub. Co., Reading, Mass., 1980), p. 158; *Physics: Algebra/Trig*, 3rd ed. (Brooks Cole Pub. Co., Pacific Grove, CA, 2003), p. 171 (1st ed. 1994, p. 278). For a philosophical discussion of energy as change see *Scientific Realism, Selected Essays of Mario Bunge*, edited by Martin Mahner (Prometheus Books, Amherst, N.Y., 2001), p. 51.



**Chladni Plates**

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