MAT2455 - Cálculo Diferencial e Integral III para Engenharia
Lista de Exercícios - Revisão de integrais

Calcule as integrais indefinidas abaixo (de 1 a 73):

1. $\int \frac{x^7 + x^2 + 1}{x^2}\, dx$
2. $\int e^{2x}\, dx$
3. $\int \cos\ 7x\ dx$
4. $\int \mathrm{tg}^2\ x\ dx$
5. $\int \frac{7}{x-2}\, dx$
6. $\int \mathrm{tg}^3\, x\, \sec^2 x\ dx$
7. $\int \frac{\mathrm{sen}^3\, x}{\sqrt{\cos\ x}}\, dx$
8. $\int \mathrm{tg}\ x\ dx$
9. $\int \mathrm{tg}^3\, x\ dx$
10. $\int \frac{x}{1+x^2}\, dx$
11. $\int \frac{x}{1+x^4}\, dx$
12. $\int \frac{x^2}{1+x^2}\, dx$
13. $\int x\sqrt{1-x^2}\, dx$
14. $\int \sec x\ dx$
15. $\int \frac{1}{x\sqrt{1+\ln x}}\, dx$
16. $\int x^2 \sqrt[5]{x^3+1}\, dx$
17. $\int \frac{4x+8}{2x^2+8x+20}\, dx$
18. $\int \frac{\sqrt{\ln x}}{x}\, dx$
19. $\int \frac{dx}{(\mathrm{arcsen}\, x)\ \sqrt{1-x^2}}$
20. $\int \frac{e^x}{1+e^x}\, dx$
21. $\int \frac{\mathrm{sen}\, 2x}{1+\cos^2 x}\, dx$
22. $\int e^{x^3} x^2\, dx$
23. $\int e^x\ \sqrt[3]{1+e^x}\, dx$
24. $\int \frac{\mathrm{sen}\sqrt{x}}{\sqrt{x}}\, dx$
25. $\int \frac{e^{\mathrm{arctg}x}}{1+x^2}\, dx$
26. $\int 2x(x+1)^{2006}\, dx$
27. $\int x\ \mathrm{sen}\, x\ dx$
28. $\int e^x\ \cos x\ dx$
29. $\int x^r\ \ln x\ dx, r \in \mathbb{R}$
30. $\int (\ln x)^2\ dx$
31. $\int xe^{-x}\, dx$
32. $\int x\, \mathrm{arctg}\ x\ dx$
33. $\int \mathrm{arcsen}\, x\ dx$
34. $\int \sec^3 x\ dx$
35. $\int \cos^2 x\, dx$
36. $\int \mathrm{sen}^2\, x\ \cos^3 x\ dx$
37. $\int \mathrm{sen}^2\, x\ \cos^2 x\ dx$
38. $\int \frac{1-\mathrm{sen}\, x}{\cos x}\, dx$
39. $\int \frac{3x^2+4x+5}{(x-1)(x-2)(x-3)}\, dx$
40. $\int \frac{1}{2x^2+8x+20}\, dx$
41. $\int \frac{3x^2+4x+5}{(x-1)^2(x-2)}\, dx$
42. $\int \frac{x^5+x+1}{x^3-8}\, dx$
43. $\int \frac{x^2}{\sqrt{1-x^2}}\, dx$
44. $\int x^2\sqrt{1-x^2}\, dx$
45. $\int e^{\sqrt{x}}\, dx$
46. $\int \ln(x+\ \sqrt{1+x^2})\, dx$
47. $\int \frac{dx}{\sqrt{5-2x+x^2}}$
48. $\int \sqrt{x}\ \ln x\ dx$
49. $\int \mathrm{sen}(\ln x)\, dx$
50. $\int \frac{x}{x^2-4}\, dx$
51. $\int \frac{3x^2+5x+4}{x^3+x^2+x-3}\, dx$
52. $\int \sqrt{a^2+b^2x^2}\, dx$
53. $\int \frac{1}{\sqrt{a^2+b^2x^2}}\, dx$
54. $\int \sqrt{x^2-2x+2}\, dx$
55. $\int \sqrt{3-2x-x^2}\, dx$
56. $\int \frac{1}{(1+x^2)\sqrt{1-x^2}}\, dx$
57. $\int \cos^3 x\, dx$
58. $\int \mathrm{sen}^5\, x\ dx$
59. $\int \frac{\cos^5 x}{\mathrm{sen}^3\, x}\, dx$
60. $\int \mathrm{sen}^3\left(\frac{x}{2}\right) \cos^5\left(\frac{x}{2}\right)\ dx$

61. $\int \frac{1}{\operatorname{sen}^5 x \cos^3 x}\,dx$

62. $\int \operatorname{sen}^4 x\,dx$

63. $\int \operatorname{sen}^2 x \cos^5 x\,dx$

64. $\int \operatorname{sen}^2 x \cos^4 x\,dx$

65. $\int \cos^6(3x)\,dx$

66. $\int \frac{\cos^2 x}{\operatorname{sen}^6 x}\,dx$

67. $\int \frac{1}{\operatorname{sen}^2 x \cos^4 x}\,dx$

68. $\int \sqrt{\frac{1-x}{1+x}}dx$

69. $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}dx$
(**Sugestão:** Faça $u = \sqrt[6]{x}$)

70. $\int \frac{x+1}{x^2(x^2+4)^2}dx$

71. $\int \frac{\operatorname{arctg}\ x}{x^2}dx$

72. $\int \frac{x^2 dx}{\sqrt{2x-x^2}}dx$

73. $\int \frac{4x^2-3x+3}{(x^2-2+2)(x+1)}dx$

74. Determine condições sobre $a, b, c, d \in \mathbb{R}$ para que as primitivas de $f(x) = \frac{(x-a)(x-b)}{(x-c)^2(x-d)^2}$ sejam funções racionais. (Resp.: $d = c$ ou $(a+b)(c+d) = 2(ab+cd)$)

75. Calcule $\int \frac{x^2}{(\cos x + x \operatorname{sen} x)^2}dx$. **Sugestão:** Calcule a derivada de $v(x) = \frac{-1}{\cos x + x \operatorname{sen} x}$. Use integração por partes. (Resp.: $\frac{\operatorname{sen} x - x\cos x}{\cos x + x \operatorname{sen} x} + C$)

RESPOSTAS

1) $\frac{x^6}{6} + x - \frac{1}{x} + k$

2) $\frac{e^{2x}}{2} + k$

3) $\frac{\operatorname{sen} 7x}{7} + k$

4) $\operatorname{tg} x - x + k$

5) $7\ln|x-2| + k$

6) $\frac{\operatorname{tg}^4 x}{4} + k$

7) $2\sqrt{\cos x}\left(\frac{\cos^2 x}{5} - 1\right) + k$

8) $-\ln|\cos x| + k$

9) $\frac{\operatorname{tg}^2 x}{2} + \ln|\cos\ x| + k$

10) $\frac{1}{2}\ln(1+x^2) + k$

11) $\frac{1}{2}\operatorname{arctg}\ x^2 + k$

12) $x - \operatorname{arctg}\ x + k$

13) $-\frac{1}{3}\sqrt{(1-x^2)^3} + k$

14) $\ln|\sec x + \operatorname{tg} x| + k$

15) $2\sqrt{1+\ln x} + k$

16) $\frac{5}{18}\sqrt[5]{(x^3+1)^6} + k$

17) $\ln(2x^2+8x+20) + k$

18) $\frac{2}{3}\sqrt{(\ln x)^3} + k$

19) $\ln|\operatorname{arcsen} x| + k$

20) $\ln(1+e^x) + k$

21) $-\ln(1+\cos^2 x) + k$

22) $\frac{1}{3}e^{x^3} + k$

23) $\frac{3}{4}\sqrt[3]{(1+e^x)^4} + k$

24) $-2\cos\sqrt{x} + k$

25) $e^{\operatorname{arctg} x} + k$

26) $2(x+1)^{2005}\left(\frac{x+1}{2006} - \frac{1}{2005}\right) + k$

27) $-x\cos x + \operatorname{sen} x + k$

28) $\frac{1}{2}e^x(\operatorname{sen} x + \cos x) + k$

29) $\begin{cases} \frac{x^{r+1}}{r+1}\ln x - \frac{x^{r+1}}{(r+1)^2} + k \text{ se } r \neq -1 \\ \frac{1}{2}(\ln x)^2 + k \text{ se } r = -1 \end{cases}$

30) $x(\ln x)^2 - 2(x\ln x - x) + k$

31) $(-x-1)e^{-x} + k$

32) $\frac{x^2}{2}\operatorname{arctg}\ x - \frac{x}{2} + \frac{1}{2}\operatorname{arctg}\ x + k$

33) $x\ \operatorname{arcsen}\ x + \sqrt{1-x^2} + k$

34) $\frac{1}{2}\sec x \operatorname{tg} x + \frac{1}{2}\ln|\sec x + \operatorname{tg} x| + k$

35) $\frac{1}{2}(x + \operatorname{sen} x \cos x) + k$

36) $\frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + k$

37) $\frac{1}{8}\left(x - \frac{\operatorname{sen} 4x}{4}\right) + k$

38) $\ln|1 + \operatorname{sen} x| + k$

39) $6\ln|x-1| - 25\ln|x-2| + 22\ln|x-3| + k$

40) $\dfrac{\sqrt{6}}{12}\operatorname{arctg}\left(\dfrac{x+2}{\sqrt{6}}\right) + k$

41) $-22\ln|x-1| + \dfrac{12}{x-1} + 25\ln|x-2| + k$

42) $\dfrac{x^3}{3} + \dfrac{35}{12}\ln|x-2| + \dfrac{61}{24}\ln\left[1 + \left(\dfrac{x+1}{\sqrt{3}}\right)^2\right] + \dfrac{\sqrt{3}}{12}\operatorname{arctg}\left(\dfrac{x+1}{\sqrt{3}}\right) + k$

43) $\dfrac{1}{2}\operatorname{arcsen} x - \dfrac{1}{2}x\sqrt{1-x^2} + k$

44) $\dfrac{x}{8}(2x^2-1)\sqrt{1-x^2} + \dfrac{1}{8}\operatorname{arcsen} x + k$

45) $2(\sqrt{x}-1)e^{\sqrt{x}} + k$

46) $x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + k$

47) $\ln|\sqrt{5-2x+x^2} + x - 1| + k$

48) $\dfrac{2}{3}x\sqrt{x}\left(\ln x - \dfrac{2}{3}\right) + k$

49) $\dfrac{x}{2}\left[\operatorname{sen}(\ln x) - \cos(\ln x)\right] + k$

50) $\dfrac{1}{2}\ln|x^2-4| + k$

51) $2\ln|x-1| + \dfrac{1}{2}\ln(x^2+2x+3) + \dfrac{1}{\sqrt{2}}\operatorname{arctg}\dfrac{x+1}{\sqrt{2}} + k$

52) $x\sqrt{a^2+b^2x^2} + \dfrac{a^2}{2b}\ln\left[\dfrac{bx}{a} + \dfrac{\sqrt{a^2+b^2x^2}}{a}\right] + k$

53) $\dfrac{1}{b}\ln\left[\dfrac{bx}{a} + \dfrac{\sqrt{a^2+b^2x^2}}{a}\right] + k$

54) $\dfrac{x-1}{2}\sqrt{x^2-2x+2} + \dfrac{1}{2}\ln(x-1+\sqrt{x^2-2x+2})$ +k

55) $\left(\dfrac{x+1}{2}\right)\sqrt{3-2x-x^2} + 2\operatorname{arcsen}\left(\dfrac{x+1}{2}\right) + k$

56) $\dfrac{1}{\sqrt{2}}\operatorname{arctg}\left(\dfrac{x\sqrt{2}}{\sqrt{1-x^2}}\right) + k$

57) $\operatorname{sen} x - \dfrac{1}{3}\operatorname{sen}^3 x + k$

58) $-\cos x + \dfrac{2}{3}\cos^3 x - \dfrac{1}{5}\cos^5 x + k$

59) $\dfrac{\operatorname{sen}^2 x}{2} - \dfrac{1}{2\operatorname{sen}^2 x} - 2\ln|\operatorname{sen} x| + k$

60) $\dfrac{1}{4}\cos^8\left(\dfrac{x}{2}\right) - \dfrac{1}{3}\cos^6\left(\dfrac{x}{2}\right) + k$

61) $\dfrac{\operatorname{tg}^2 x}{2} + 3\ln|\operatorname{tg} x| - \dfrac{3}{2\operatorname{tg}^2 x} - \dfrac{1}{4\operatorname{tg}^4 x} + k$

62) $\dfrac{3x}{8} - \dfrac{\operatorname{sen}(2x)}{4} + \dfrac{\operatorname{sen}(4x)}{32} + k$

63) $\dfrac{\operatorname{sen}^3 x}{3} - 2\dfrac{\operatorname{sen}^5 x}{5} + \dfrac{\operatorname{sen}^7 x}{7} + k$

64) $\dfrac{x}{16} - \dfrac{\operatorname{sen}(4x)}{64} + \dfrac{\operatorname{sen}^3(2x)}{48} + k$

65) $\dfrac{5}{16}x + \dfrac{1}{12}\operatorname{sen}(6x) + \dfrac{1}{64}\operatorname{sen}(12x) - \dfrac{\operatorname{sen}^3(6x)}{144} + k$

66) $-\dfrac{\operatorname{cotg}^3 x}{3} - \dfrac{\operatorname{cotg}^5 x}{5} + k$

67) $\operatorname{tg} x + \dfrac{\operatorname{tg}^3 x}{3} - 2\operatorname{cotg}(2x) + k$

68) $\operatorname{arcsen} x + \sqrt{1-x^2} + k$

69) $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + k$

70) $\dfrac{1}{4}\ln|x| - \dfrac{1}{4x} - \dfrac{1}{4}\left[\dfrac{1}{2}\ln(x^2+4) + \dfrac{1}{2}\operatorname{arctg}\left(\dfrac{x}{2}\right)\right] + k$

71) $\dfrac{-\operatorname{arctg} x}{x} + \ln|x| - \ln\sqrt{1+x^2} + k$

72) $\dfrac{3}{2}\operatorname{arcsen}(x-1) - \left(\dfrac{x+3}{2}\right)\sqrt{2x-x^2} + k$

73) $2\ln|x+1| + \ln(x^2-2x+2) + 3\operatorname{arctg}(x-1) + k.$