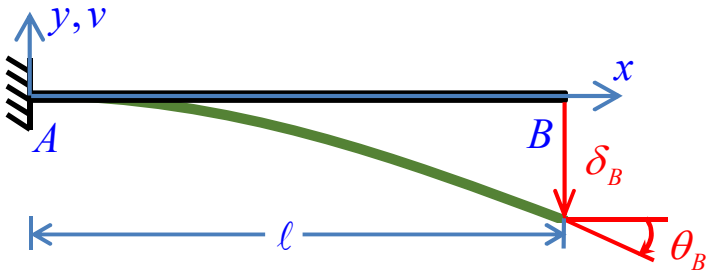


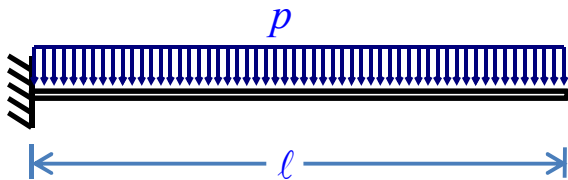
TABELAS DE DESLOCAMENTOS E ROTAÇÕES EM VIGAS

• Vigas engastadas



- Deslocamentos transversais: $v(x) = v$
- Rotações: $v'(x) = v'$
- Deslocamento transversal $\delta_B = |v_B|$ em B:
- Rotação em B: $\theta_B = |v'_B|$
- $EI = \text{constante}$

1.



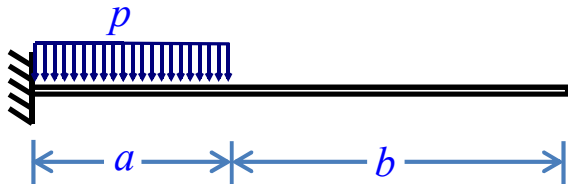
$$v = -\frac{px^2}{24EI} (6l^2 - 4lx + x^2)$$

$$\delta_B = \frac{p\ell^4}{8EI}$$

$$v' = -\frac{px}{6EI} (3l^2 - 3lx + x^2)$$

$$\theta_B = \frac{p\ell^3}{6EI}$$

2.



$$v = -\frac{px^2}{24EI} (6a^2 - 4ax + x^2) \quad (0 \leq x \leq a)$$

$$v' = -\frac{px}{6EI} (3a^2 - 3ax + x^2) \quad (0 \leq x \leq a)$$

$$v = -\frac{pa^3}{24EI} (4x - a) \quad (a \leq x \leq \ell)$$

Em $x = a$:

$$v = -\frac{pa^4}{8EI}$$

$$v' = -\frac{pa^3}{6EI}$$

$$v' = -\frac{pa^3}{6EI}$$

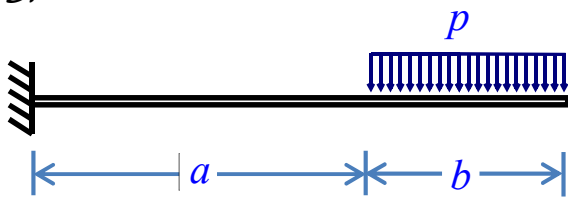
$$(a \leq x \leq \ell)$$

$$\delta_B = \frac{pa^3}{24EI} (4\ell - a)$$

$$\theta_B = \frac{pa^3}{6EI}$$



3.



$$v = -\frac{pbx^2}{12EI}(3\ell + 3a - 2x) \quad (0 \leq x \leq a)$$

$$v' = -\frac{pbx}{2EI}(\ell + a - x) \quad (0 \leq x \leq a)$$

$$v = -\frac{P}{24EI}(x^4 - 4\ell x^3 + 6\ell^2 x^2 - 4a^3 x + a^4) \quad (a \leq x \leq \ell)$$

$$v' = -\frac{P}{6EI}(x^3 - 3\ell x^2 + 3\ell^2 x - a^3) \quad (a \leq x \leq \ell)$$

$$\delta_B = \frac{P}{24EI}(3\ell^4 - 4a^3\ell + a^4)$$

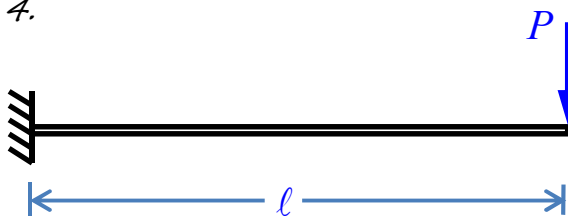
Em $x = a$:

$$v = -\frac{pa^2b}{12EI}(3\ell + a)$$

$$v' = -\frac{pabl}{2EI}$$

$$\theta_B = \frac{P}{6EI}(\ell^3 - a^3)$$

4.



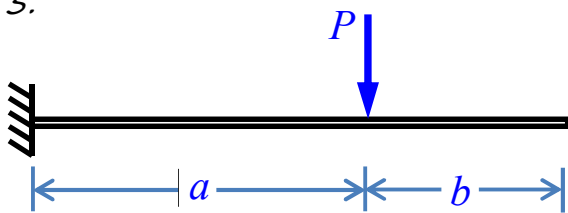
$$v = -\frac{Px^2}{6EI}(3\ell - x)$$

$$\delta_B = \frac{P\ell^3}{3EI}$$

$$v' = -\frac{Px}{2EI}(2\ell - x)$$

$$\theta_B = \frac{P\ell^2}{2EI}$$

5.



$$v = -\frac{Px^2}{6EI}(3a - x) \quad (0 \leq x \leq a)$$

$$v' = -\frac{Px}{2EI}(2a - x) \quad (0 \leq x \leq a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \quad (a \leq x \leq \ell)$$

Em $x = a$:

$$v = -\frac{Pa^3}{3EI}$$

$$v' = -\frac{Pa^2}{2EI}$$

$$v' = -\frac{Pa^2}{2EI}$$

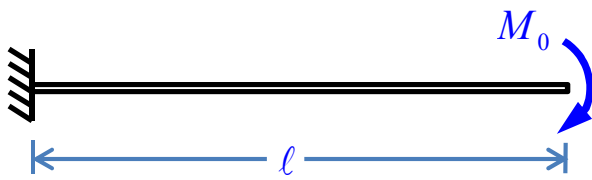
$$(a \leq x \leq \ell)$$

$$\delta_B = \frac{Pa^2}{6EI}(3\ell - a)$$

$$\theta_B = \frac{Pa^2}{2EI}$$



6.



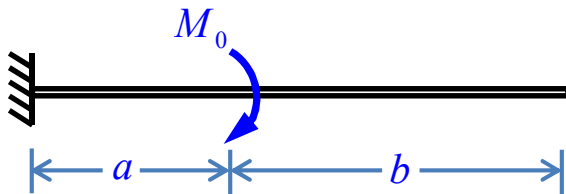
$$v = -\frac{M_0 x^2}{2EI}$$

$$v' = -\frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 \ell^2}{2EI}$$

$$\theta_B = \frac{M_0 \ell}{EI}$$

7.



$$v = -\frac{M_0 x^2}{2EI} \quad (0 \leq x \leq a)$$

$$v' = -\frac{M_0 x}{EI} \quad (0 \leq x \leq a)$$

$$v = -\frac{M_0 a}{2EI} (2x - a) \quad (a \leq x \leq \ell)$$

$$(a \leq x \leq \ell)$$

Em $x = a$:

$$v' = -\frac{M_0 a}{EI} \quad (a \leq x \leq \ell)$$

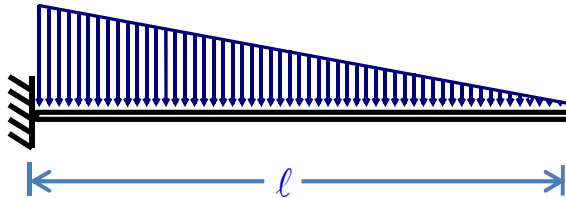
$$(a \leq x \leq \ell)$$

$$v = -\frac{M_0 a^2}{2EI}$$

$$v' = -\frac{M_0 a}{EI}$$

$$\delta_B = \frac{M_0 a}{2EI} (2\ell - a)$$

$$\theta_B = \frac{M_0 a}{EI}$$

8. p_0 

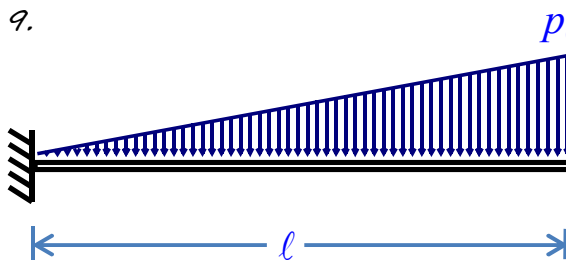
$$v = -\frac{p_0 x^2}{120\ell EI} (10\ell^3 - 10\ell^2 x + 5\ell x^2 - x^3)$$

$$v' = -\frac{p_0 x}{24\ell EI} (4\ell^3 - 6\ell^2 x + 4\ell x^2 - x^3)$$

$$\delta_B = \frac{p_0 \ell^4}{30EI}$$

$$\theta_B = \frac{p_0 \ell^3}{24EI}$$

9.



$$v = -\frac{p_0 x^2}{120\ell EI} (20\ell^3 - 10\ell^2 x + x^3)$$

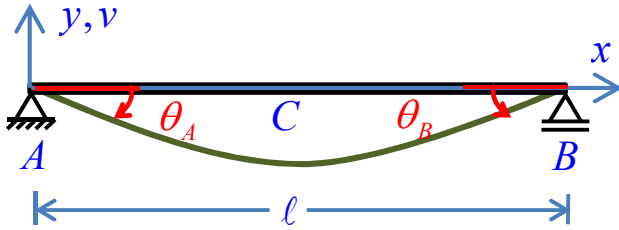
$$v' = -\frac{p_0 x}{24\ell EI} (8\ell^3 - 6\ell^2 x + x^3)$$

$$\delta_B = \frac{11p_0 \ell^4}{120EI}$$

$$\theta_B = \frac{p_0 \ell^3}{8EI}$$

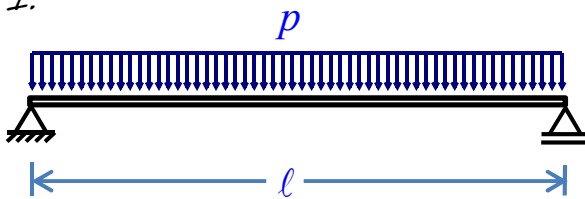


• Vigas simplesmente apoiadas



- Deslocamentos transversais: $v(x) = v$
- Rotações: $v'(x) = v'$
- Distância entre A e o ponto de $\delta_{\text{máx}}$: x_1
- Deslocamento transversal máximo: $\delta_{\text{máx}} = |v(x_1)|$
- Deslocamento transversal em C (ponto médio): $\delta_C = |v_C|$
- Rotação em A: $\theta_A = |v'_A|$
- Rotação em B: $\theta_B = |v'_B|$
- $EI = \text{constante}$

1.



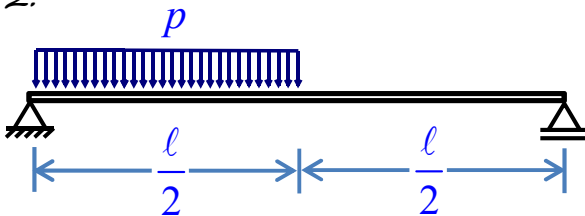
$$v = -\frac{px}{24EI}(\ell^3 - 2lx^2 + x^3)$$

$$v' = -\frac{p}{24EI}(\ell^3 - 6lx^2 + 4x^3)$$

$$\delta_C = \delta_{\text{máx}} = \frac{5p\ell^4}{384EI}$$

$$\theta_A = \theta_B = \frac{p\ell^3}{24EI}$$

2.



$$v = -\frac{px}{384EI}(9\ell^3 - 24lx^2 + 16x^3) \quad \left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$\left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$\delta_C = \frac{5pL^4}{768EI}$$

$$v' = -\frac{p}{384EI}(9\ell^3 - 72lx^2 + 64x^3) \quad \left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$\left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$\theta_A = \frac{3p\ell^3}{128EI}$$

$$v = -\frac{p\ell}{384EI}(8x^3 - 24lx^2 + 17\ell^2x - \ell^3) \quad \left(\frac{\ell}{2} \leq x \leq \ell\right)$$

$$\left(\frac{\ell}{2} \leq x \leq \ell\right)$$

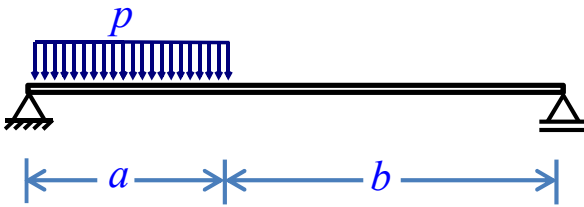
$$\theta_B = \frac{7p\ell^3}{384EI}$$

$$v' = -\frac{p\ell}{384EI}(24x^2 - 48lx + 17\ell^2) \quad \left(\frac{\ell}{2} \leq x \leq \ell\right)$$

$$\left(\frac{\ell}{2} \leq x \leq \ell\right)$$



3.



$$v = -\frac{px^2}{24\ell EI} (a^4 - 4a^3\ell + 4a^2\ell^2 + 2a^2x^2 - 4alx^2 + \ell x^3) \quad (0 \leq x \leq a)$$

$$v' = -\frac{P}{24\ell EI} (a^4 - 4a^3\ell + 4a^2\ell^2 + 6a^2x^2 - 12alx^2 + 4\ell x^3) \quad (0 \leq x \leq a)$$

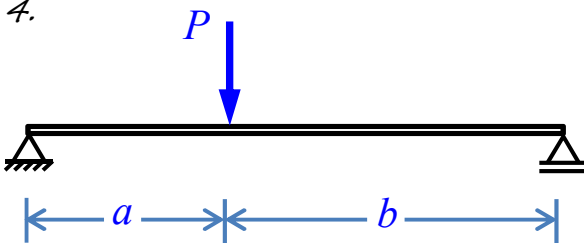
$$v = -\frac{pa^2}{24\ell EI} (-a^2\ell + 4\ell^2x + a^2x - 6\ell x^2 + 2x^3) \quad (a \leq x \leq \ell)$$

$$v' = -\frac{pa^2}{24\ell EI} (4\ell^2 + a^2 - 12\ell x + 6x^2) \quad (a \leq x \leq \ell)$$

$$\theta_A = \frac{pa^2}{24\ell EI} (2\ell - a)^2$$

$$\theta_B = \frac{pa^2}{24\ell EI} (2\ell^2 - a^2)$$

4.



$$v = -\frac{Pbx}{6\ell EI} (\ell^2 - b^2 - x^2) \quad (0 \leq x \leq a)$$

$$v' = -\frac{Pb}{6\ell EI} (\ell^2 - b^2 - 3x^2) \quad (0 \leq x \leq a)$$

$$\theta_A = \frac{Pab}{6\ell EI} (\ell + b)$$

$$\theta_B = \frac{Pab}{6\ell EI} (\ell + a)$$

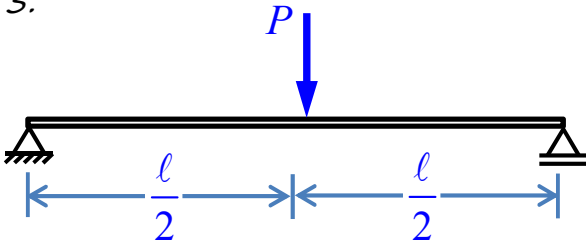
$$\text{Se } a \geq b: \quad \delta_C = \frac{Pb(3\ell^2 - 4b^2)}{48EI}$$

$$\text{Se } a \leq b: \quad \delta_C = \frac{Pa(3\ell^2 - 4a^2)}{48EI}$$

$$\text{Se } a \geq b: \quad x_1 = \sqrt{\frac{\ell^2 - b^2}{3}} \quad e \quad \delta_{\max} = \frac{Pb(\ell^2 - b^2)^{3/2}}{9\sqrt{3}\ell EI}$$



5.



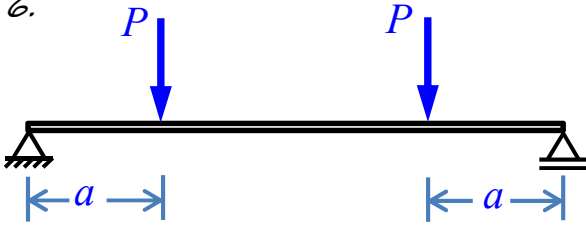
$$v = -\frac{Px}{48EI}(3\ell^2 - 4x^2) \quad \left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$v' = -\frac{P}{16EI}(\ell^2 - 4x^2) \quad \left(0 \leq x \leq \frac{\ell}{2}\right)$$

$$\delta_C = \delta_{\max} = \frac{P\ell^3}{48EI}$$

$$\theta_A = \theta_B = \frac{P\ell^2}{16EI}$$

6.



$$v = -\frac{Px}{6EI}(3a\ell - 3a^2 - x^2) \quad (0 \leq x \leq a)$$

$$v' = -\frac{P}{2EI}(a\ell - a^2 - x^2) \quad (0 \leq x \leq a)$$

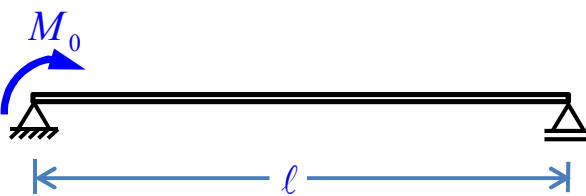
$$\theta_A = \theta_B = \frac{Pa}{2EI}(\ell - a)$$

$$v = -\frac{Pa}{6EI}(3\ell x - 3x^2 - a^2) \quad (a \leq x \leq \ell - a)$$

$$\delta_C = \delta_{\max} = \frac{Pa}{24EI}(3\ell^2 - 4a^2)$$

$$v' = -\frac{Pa}{2EI}(\ell - 2x) \quad (a \leq x \leq \ell - a)$$

7.



$$v = -\frac{M_0 x}{6\ell EI}(2\ell^2 - 3\ell x + x^2)$$

$$v' = -\frac{M_0}{6\ell EI}(2\ell^2 - 6\ell x + 3x^2)$$

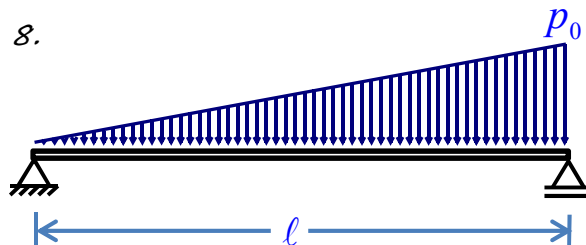
$$\delta_C = \frac{M_0 \ell^2}{16EI}$$

$$\theta_A = \frac{M_0 \ell}{3EI}$$

$$\theta_B = \frac{M_0 \ell}{6EI}$$

$$x_1 = \ell \left(1 - \frac{\sqrt{3}}{3}\right) \quad e \quad \delta_{\max} = \frac{M_0 \ell^2}{9\sqrt{3}EI}$$

8.



$$v = -\frac{p_0 x}{360\ell EI}(7\ell^4 - 10\ell^2 x^2 + 3x^4)$$

$$v' = -\frac{p_0}{360\ell EI}(7\ell^4 - 30\ell^2 x^2 + 15x^4)$$

$$\delta_C = \frac{5p_0 \ell^4}{768EI}$$

$$\theta_A = \frac{7p_0 \ell^3}{360EI}$$

$$\theta_B = \frac{p_0 \ell^3}{45EI}$$

$$x_1 = 0,5193\ell \quad e \quad \delta_{\max} = 0,00652 \frac{p_0 \ell^4}{EI}$$