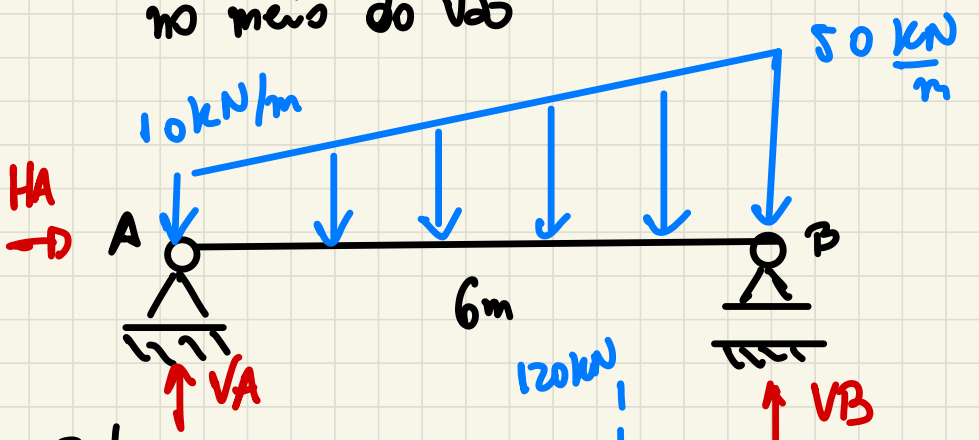
PEF3208

Aula 3

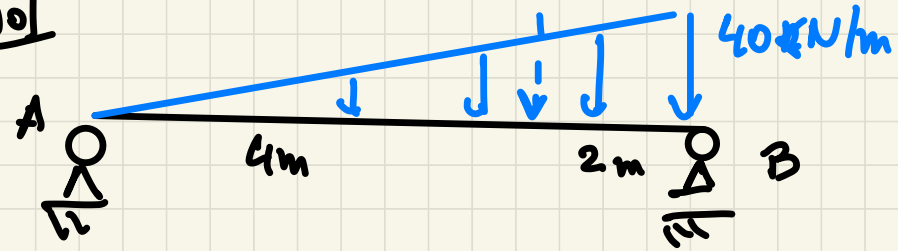
Turma 1

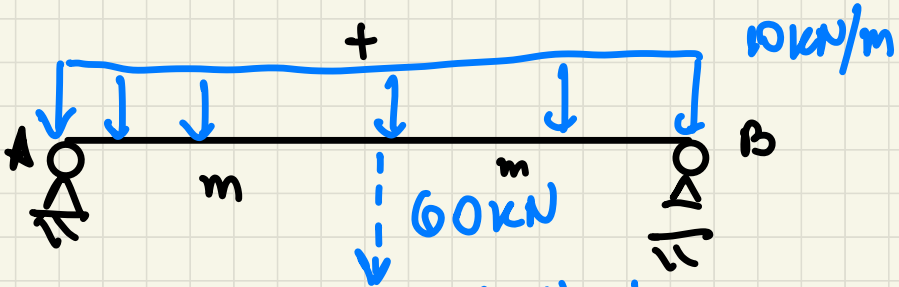
27/04/21

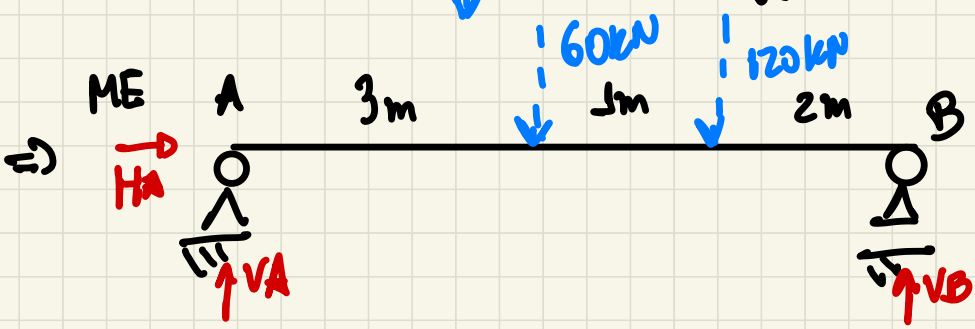
Ex: Achar os esforços solicitantes no meio do vão



Sol







i] Reações apoio

$\sum F_x = 0 \Rightarrow \boxed{H_A = 0\ kN}$

$\sum F_y = 0 \Rightarrow V_A - 60 - 120 + V_B = 0$

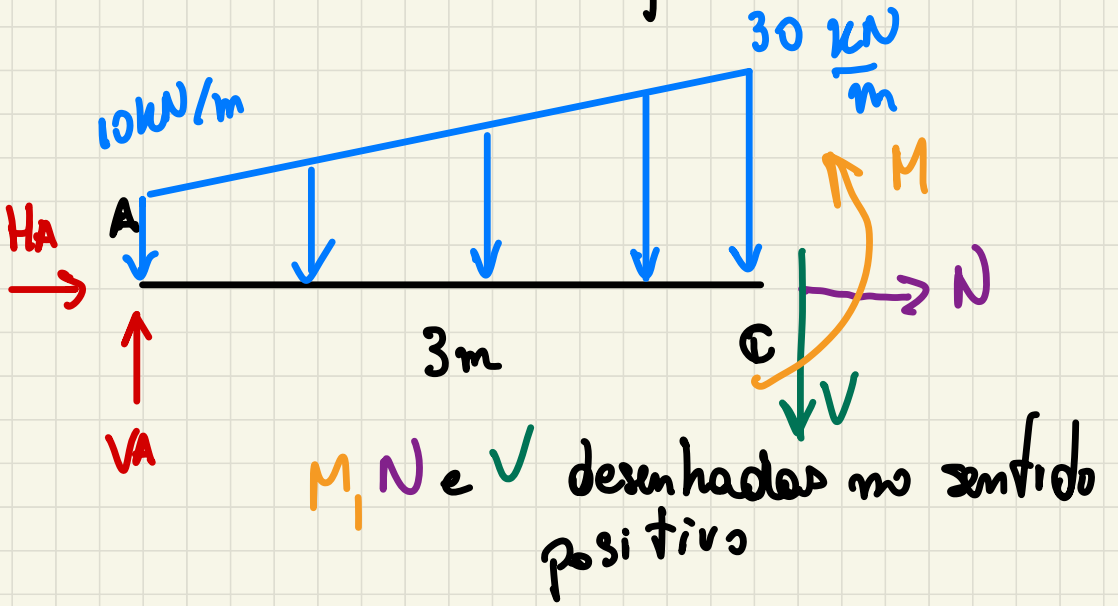
$\Rightarrow V_A + V_B = 180 \quad (a)$

$\sum M_A = 0 \Rightarrow -3.60 - 4.120 + 6V_B = 0$

$\boxed{V_B = 110\ kN} \quad (b)$

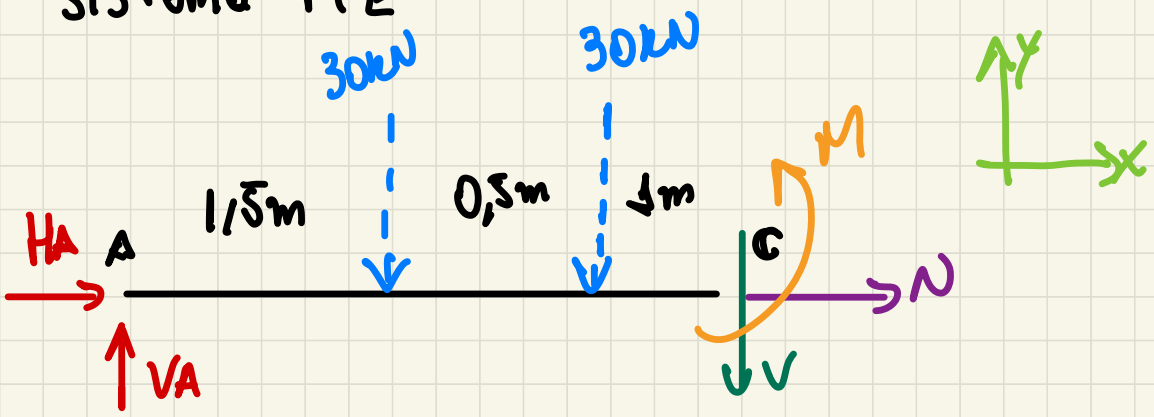
De (b) em (a) $\Rightarrow \boxed{V_A = 70\ kN}$

ii] Corte no meio do vão e desenho DCL da sub-est. à esquerda do corte C



M, N e V desenhados no sentido positivo

Equilíbrio sub-estrutura à esquerda de C. Lidarei c/ o carregamento distribuído de forma análoga ao feito p/ cálculo de reações de apoio. Já considerando sistema MF



Equilíbrio do trecho AC

$\sum F_x = 0 \Rightarrow H_A + N = 0 \Rightarrow \boxed{N = 0}$

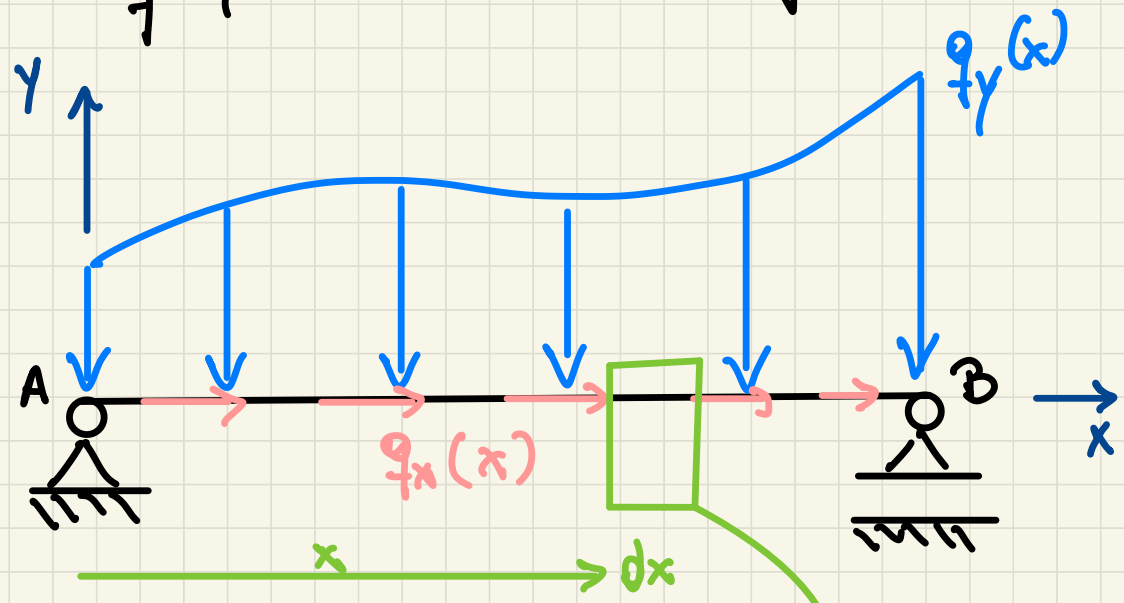
$\sum F_y = 0 \Rightarrow V_A - 30 - 30 - V = 0$

$\Rightarrow \boxed{V = V_A - 60 = 70 - 60 = 10\,kN}$

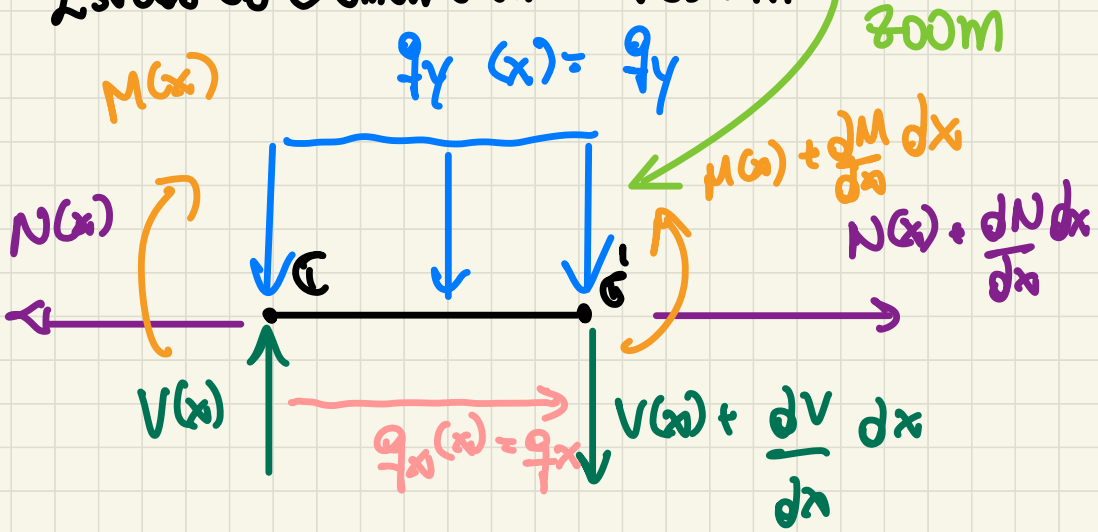
$\sum M_C = 0 \Rightarrow -3V_A + 30.1,5 + 30.1 + M = 0$

$\Rightarrow \boxed{M = 135\,kNm}$

Equações Diferenciais de Equilíbrio



Estudo do elemento infinitesimal



Equilíbrio do elemento infinitesimal

$$\sum F_x = 0 \Rightarrow -N(x) + q_x \cdot dx + N(x) + \frac{dN}{dx} dx = 0$$

$$\Rightarrow \boxed{\frac{dN}{dx} = -q_x}$$

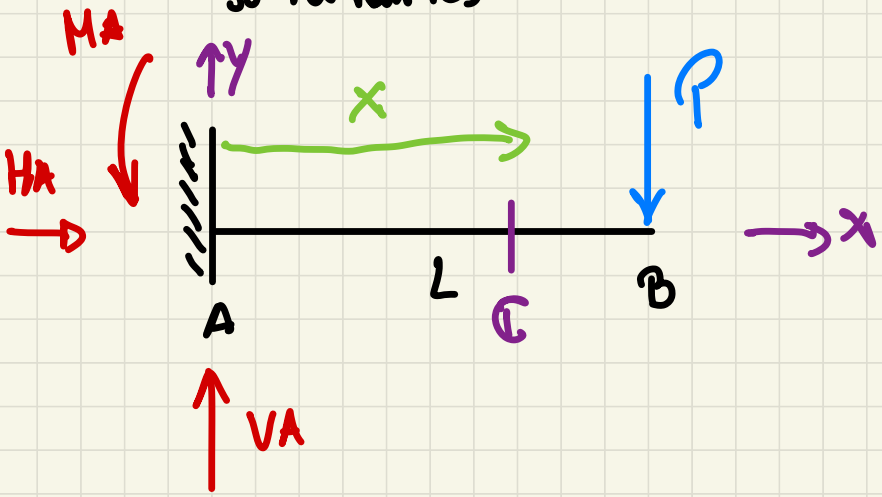
$$\sum F_y = 0 \Rightarrow V(x) - q_y\, dx - V(x) - \frac{dV}{dx} dx = 0$$

$$\Rightarrow \boxed{\frac{dV}{dx} = -q_y}$$

$$\sum M_{C'} \Rightarrow -V(x)\, dx - M(x) + M(x) + \frac{dM}{dx} dx = 0$$

$$\Rightarrow \boxed{\frac{dM}{dx} = V(x)}$$

Ex Traçar os diagramas de esforços solicitantes



Sol: Cálculo das reações de apoio

$$\Sigma F_x = 0 \Rightarrow H_A = 0$$

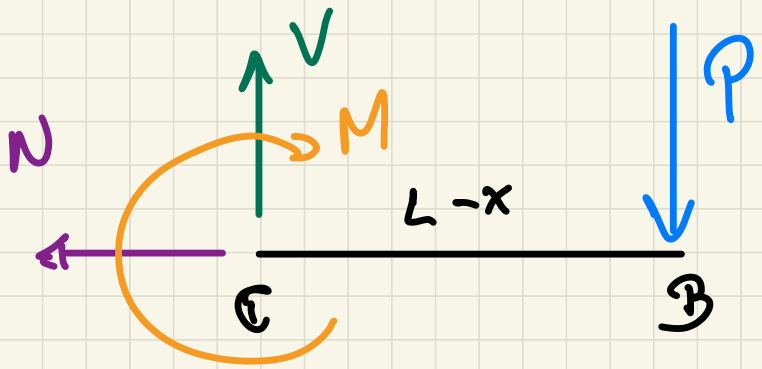
$$\Sigma F_y = 0 \Rightarrow V_A = P$$

$$\circlearrowleft + \Sigma M_A = 0 \Rightarrow M_A - PL = 0$$

$$M_A = PL$$

Corte a uma distância x do ponto A.

Equilíbrio da sub-estrutura à direita de A



Equilíbrio sub-estrutura à direita de C

$$\sum F_x = 0 \Rightarrow \boxed{N = 0}$$

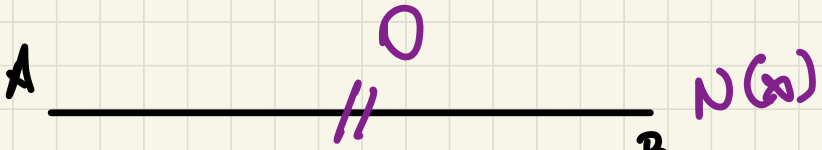
$$\sum F_y = 0 \Rightarrow \boxed{V = P}$$

$$\sum M_C = 0 \Rightarrow -M - P(L-x) = 0$$

$$\Rightarrow \boxed{M = P(x-L)}$$

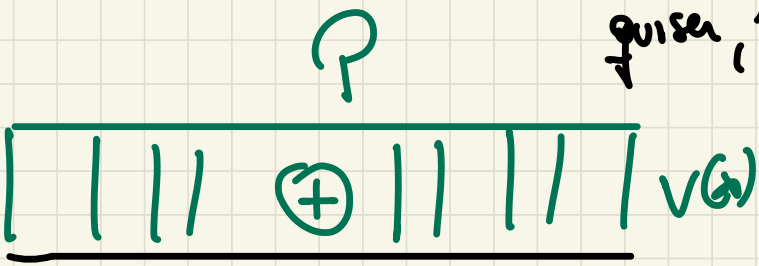
Veja que $\frac{dM}{dx} = P$

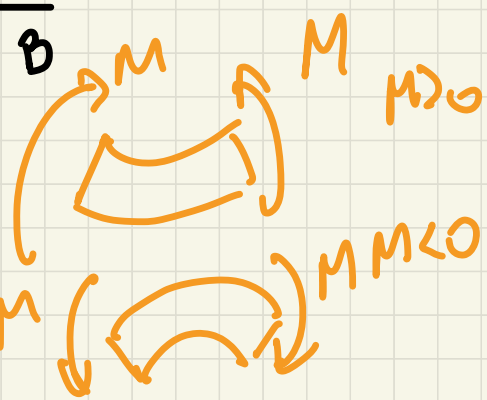
Traçando os diagramas

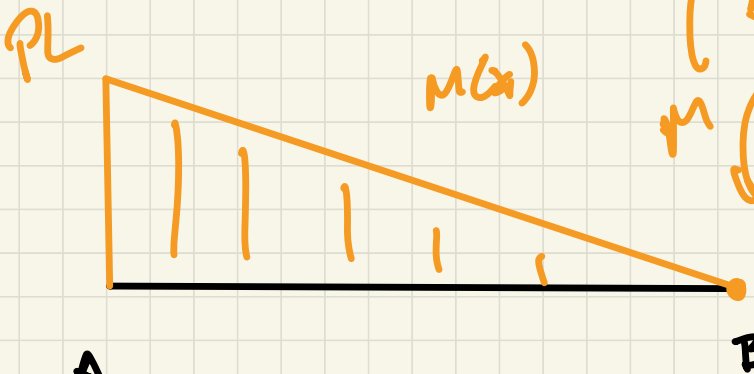


N(x) e V(x)

desenho onde quiser, mas indico sinal

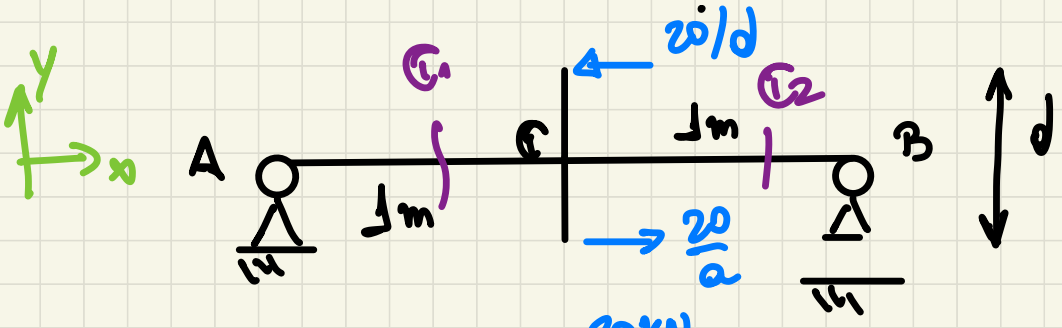




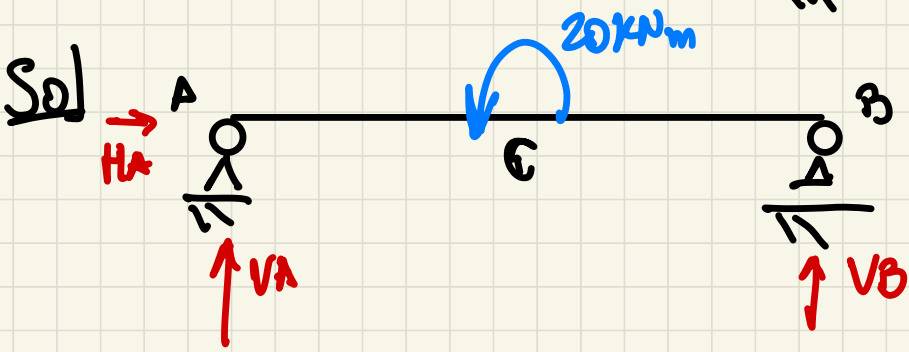


Desenho M(x) sem sinal, mas do lado das fibras tracionadas

Ex Traçar os diagramas de esforços solicitantes no trecho AB



Sol



Reações de apoio

$\sum F_x = 0 \Rightarrow H_A = 0$

$\sum F_y = 0 \Rightarrow V_A + V_B = 0$

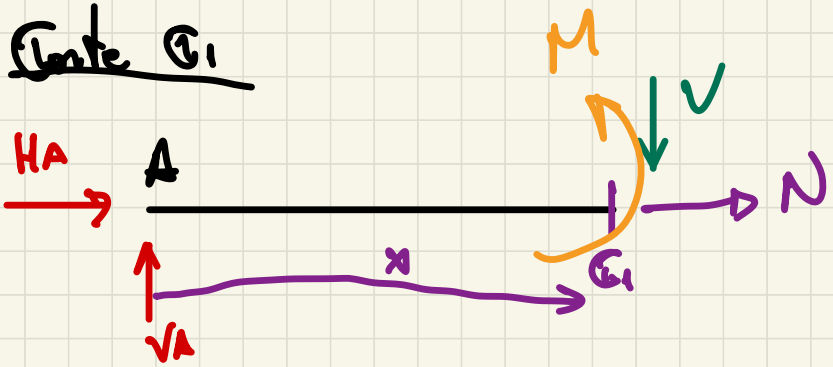
$\sum M_A = 0 \Rightarrow 20 + 2 V_B = 0$

$V_B = -10\ KN$

$V_A = 10\ KN$

Preciso de dois cortes C_1 e C_2

Corte C_1



Equilíbrio: $\Sigma F_x = 0 \Rightarrow N = 0$

$\Sigma F_y = 0 \Rightarrow V_A - V = 0$

$V = +V_A = +10kN$

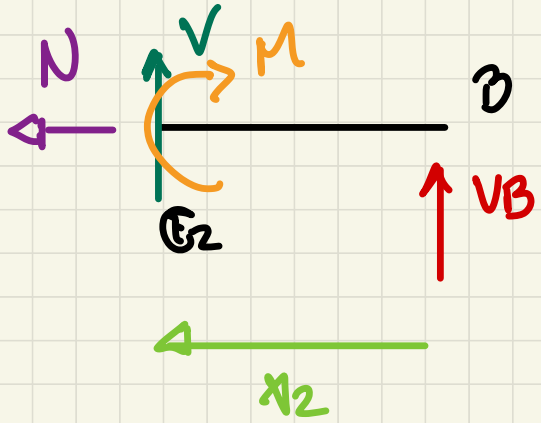
$\circlearrowleft + \Sigma M_{C_1} = 0 \Rightarrow -V_A x + M = 0$

$\Rightarrow M = +10x$

As expressões acima valem p/ $0 < x < 1m$

" A C_1 "

Corte C_2 e equilíbrio trecho BC2



Equilíbrio BC2

$$\sum F_x = 0 \Rightarrow \boxed{N = 0}$$

$$\sum F_y = 0 \Rightarrow V + V_B = 0$$

$$V - 10 = 0$$

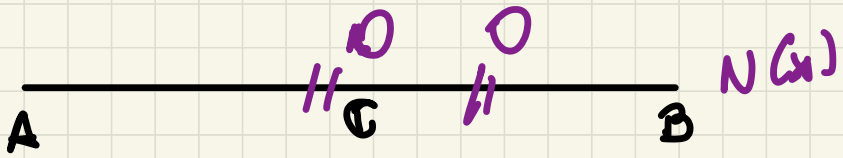
$$\boxed{V = 10\ kN}$$

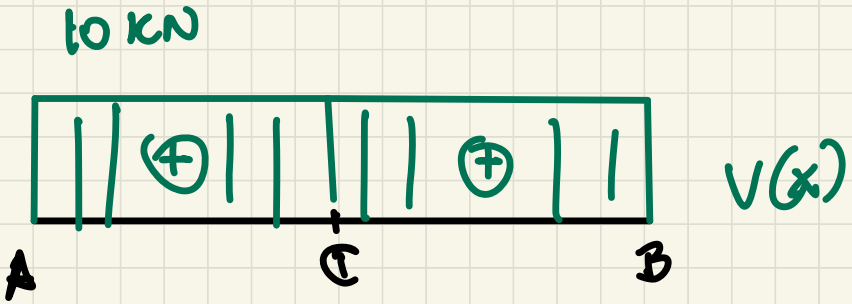
$$\circlearrowleft \sum M_{C2} = 0 \Rightarrow -M + V_B x_2 = 0$$

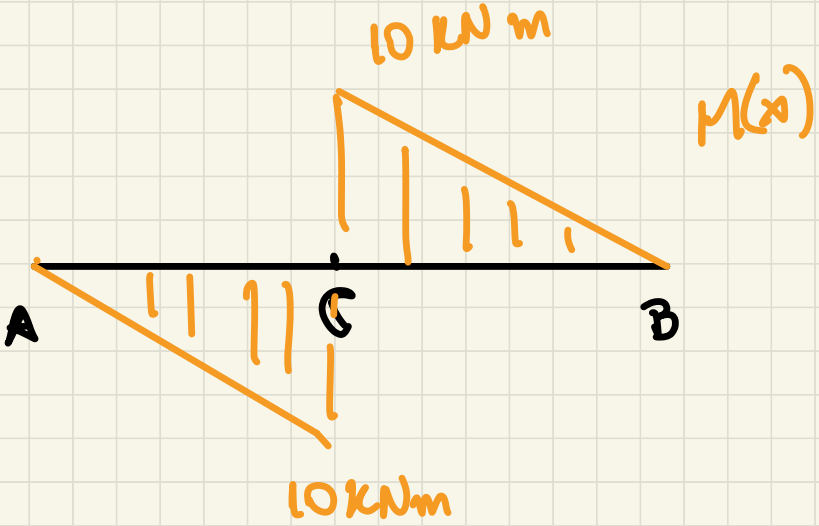
$$\boxed{M = -10 x_2}$$

Essas expressões valem no trecho BC2

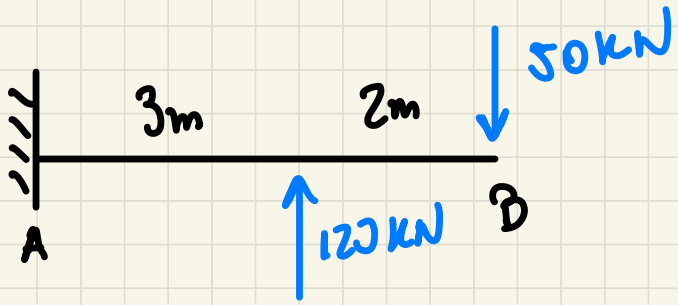
" $0 < x_2 < 1$"


A
C
B
N(x)


10 KN
A
C
B
V(x)


10 KN m
M(x)
A
C
B
10 KNm

Ex: Traçar os diagramas de esforços solicitantes



Exercício p/ presença