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How to select and how to rank projects: The PROMETHEE method

J.P. BRANS, Ph. VINCKE and B. MARESCHAL
Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium

Abstract: In this paper, we present the PROMETHEE methods, a new class of outranking methods in multicriteria analysis. Their main features are simplicity, clearness and stability. The notion of generalized criterion is used to construct a valued outranking relation. All the parameters to be defined have an economic signification, so that the decision maker can easily fix them. Two ways of treatment are proposed: It is possible to obtain either a partial preorder (PROMETHEE I) or a complete one (PROMETHEE II), both on a finite set of feasible actions. A comparison is made with the ELECTRE III method. The stability of the results given by the two methods is analysed. Numerical applications are given in order to illustrate the properties of the new methods and some further problems are discussed.

Keywords: Decision, multiple criteria programming

1. Introduction

Let us consider the multicriteria problem

$$\operatorname{Max}\{ f_1(a),\ldots,f_k(a) \mid a\in K \}, \tag{1.1}$$

where K is a finite set of actions and f_i, $i=1,\ldots,k$, are k criteria to be maximized. Each criterion is an application from K to $\mathbb{R}$ or any other ordered set.

The PROMETHEE methods (Preference Ranking Organization METHod for Enrichment Evaluations) belong to the family of the outranking methods, introduced by B. Roy, and include two phases:

- the construction of an outranking relation on K,
- the exploitation of this relation in order to give an answer to (1.1).

In the first phase, a valued outranking relation based on a generalization of the notion of criterion is considered: a preference index is defined and a valued outranking graph, representing the preferences of the decision maker, is obtained.

The exploitation of the outranking relation is realized by considering for each action a leaving and an entering flow in the valued outranking graph: a partial preorder (PROMETHEE I) or a complete preorder (PROMETHEE II) on the set of possible actions can be proposed to the decision maker in order to achieve the decision problem.

We point out the fact that only a few parameters are to be fixed in these methods and that they all have an economic signification so that the decision maker is able to determine their values easily. Furthermore, as we will briefly show at the end of this paper, some small deviations in the determination of these values do not often induce important modifications of the obtained rankings.

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2. The PROMETHEE valued outranking relation

a. Generalized criterion

Let f be a real-valued criterion:

$$f\colon K\to\mathbb{R}, \tag{2.1}$$

and suppose it has to be maximized (this is not restrictive). In this paper we consider $\mathbb{R}$, but the theory can easily be extended to the case of any other ordered set.

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For each action $a \in K$, $f(a)$ is an evaluation of this action.

When we compare two actions a, $b \in K$, we must be able to express the result of this comparison in terms of preference. We therefore consider a preference function P:

$$P\colon K \times K \to (0, 1), \tag{2.2}$$

representing the intensity of preference of action a with regard to action b and such that

- $P(a, b) = 0$ means an indifference between a and b, or no preference of a over b;
- $P(a, b) \sim 0$ means weak preference of a over b;
- $P(a, b) \sim 1$ means strong preference of a over b;
- $P(a, b) = 1$ means strict preference of a over b.

In practice, this preference function will often be a function of the difference between the two evaluations, so that we can write

$$P(a, b) = \mathscr{P}(f(a) - f(b)). \tag{2.3}$$

The graph of such a function is given by Figure 1 (2.4). It has to be a non-decreasing function, equal to zero for negative values of $d = f(a) - f(b)$.

For each criterion f we consider a *generalized criterion* defined by f and a corresponding preference function P.

b. Recommended generalized criteria for applications

We consider here six types of generalized criteria. This is of course not exhaustive but we think that they are sufficient in most of the practical cases.

In order to give a better view of the indifference

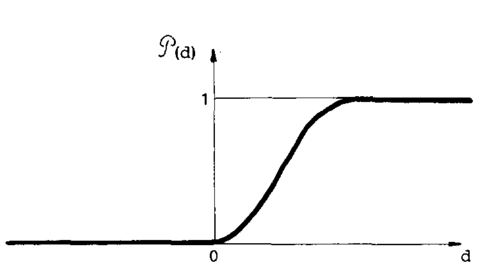


Figure 1. Preference function $\mathscr{P}(d)$

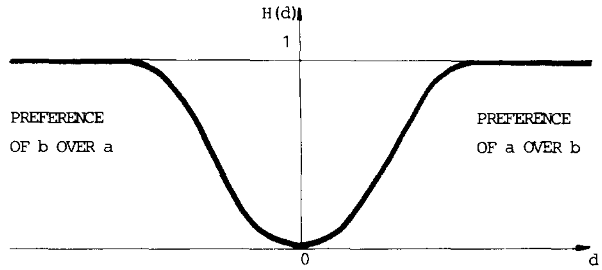


Figure 2. Funtion $H(d)$

area, we may consider a function $H(d)$ which is directly related to the preference function P:

$$H(d) = \begin{cases} P(a, b), & d \geqslant 0, \\ P(b, a), & d \leqslant 0. \end{cases} \tag{2.4}$$

This function is then represented by Figure 2.

(1) *Usual criterion*

$$H(d) = \begin{cases} 0 & \text{if } d = 0, \\ 1 & \text{if } d \neq 0. \end{cases} \tag{2.5}$$

In this case, there is indifference between a and b if and only if $f(a) = f(b)$; as soon as the two evaluations are different, the decision maker has a strict preference for the action having the greatest evaluation. The H function is represented by Figure 3.

In this case, no parameter has to be defined. This generalized criterion corresponds to the usual meaning of criterion (it is the true-criterion of B. Roy).

(2) *Quasi-criterion*

$$H(d) = \begin{cases} 0 & \text{if } -q \leqslant d \leqslant q, \\ 1 & \text{if } d < -q \text{ or } d > q. \end{cases} \tag{2.6}$$

As it can be seen in Figure 4, the two actions are indifferent to the decision maker as long as the difference between their evaluations, i.e. d, does

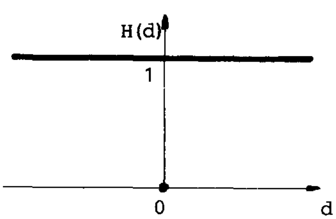


Figure 3. Usual criterion

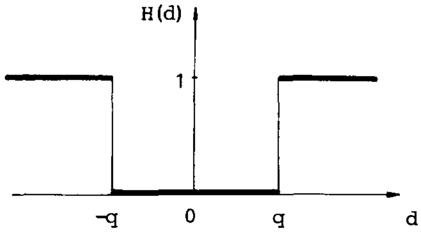


Figure 4. Quasi criterion

not exceed the indifference threshold q; if this is not the case, there is strict preference.

If the decision maker wishes to use a quasi-criterion, he has only to determine the value of q, and this has a clear economic signification: It is the greatest value of the difference between two evaluations, below which the decision maker considers the corresponding actions as indifferent.

(3) *Criterion with linear preference*

$$H(d)=\begin{cases} d/p & \text{if } -p \leqslant d \leqslant p, \\ 1 & \text{if } d<-p \text{ or } d>p. \end{cases} \tag{2.7}$$

As long as d is lower than p, the preference of the decision maker increases linearly with d. If d becomes greater than p, we have a strict preference situation. We then have the kind of function shown in Figure 5.

When the decision maker identifies some criterion to be of that type, he has to determine the value of the preference threshold up: This is the lowest value of d above which he considers that there is strict preference of one of the corresponding actions.

Until now we have described two types of thresholds:

- an indifference threshold q: the greatest value of d below which there is indifference,

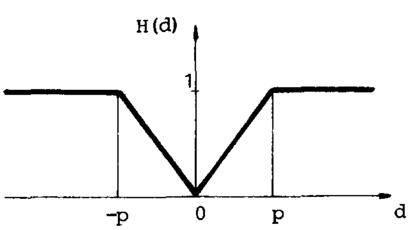


Figure 5. Criterion with linear preference

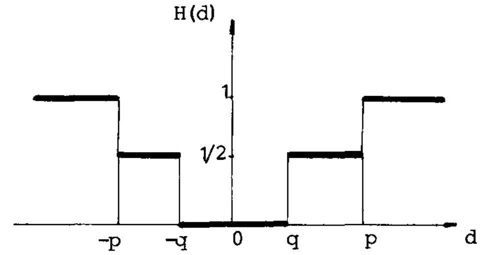


Figure 6. Level criterion

- a preference threshold p: the lowest value of d above which there is strict preference.

In practice these two thresholds are not necessarily equal. We therefore consider the two following types of criteria.

(4) *Level criterion*

$$H(d)=\begin{cases} 0 & \text{if } \quad |d| \leqslant q, \\ 1/2 & \text{if } \quad q<|d| \leqslant p, \\ 1 & \text{if } \quad p<|d|. \end{cases} \tag{2.8}$$

In this case, an indifference threshold q and a preference threshold p are simultaneously defined. If d lies between q and p, there is a weak preference situation ($H(d)=1/2$). The function is represented by Figure 6 and the decision-maker has this time two thresholds to define

(5) *Criterion with linear preference and indifference area*

$$H(d)=\begin{cases} 0 & \text{if } |d| \leqslant q, \\ (|d|-q)/(p-q) & \text{if } q<|d| \leqslant p, \\ 1 & \text{if } p<|d|. \end{cases} \tag{2.9}$$

In this case, the decision maker considers that his preference increases linearly from indifference

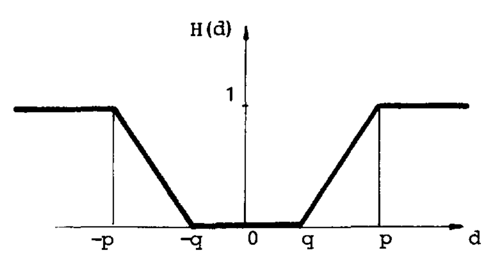


Figure 7. Linear preference and indifference area

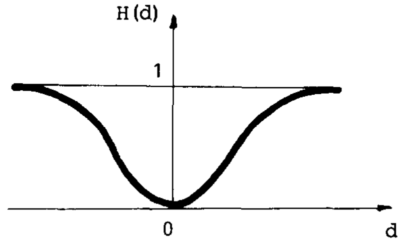


Figure 8. Gaussian criterion

to strict preference in the area between the two thresholds q and p. Two parameters are to be defined. The function H is then of the type shown in Figure 7.

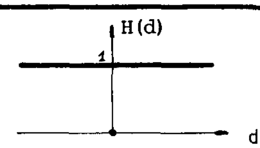
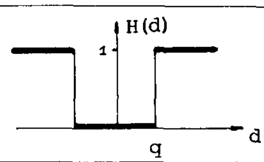
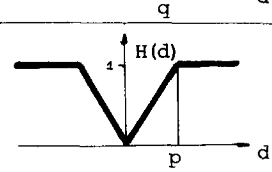
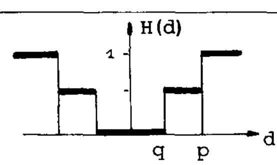
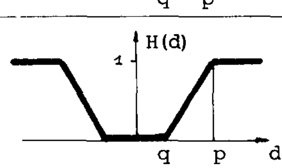
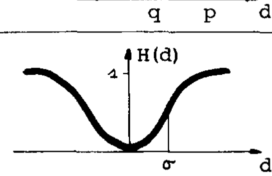
(6) *Gaussian criterion*

$$H(d) = 1 - \exp\{-d^2/2\sigma^2\}. \tag{2.10}$$

This function is represented in Figure 8. It only requires the determination of σ, which is made easily according to the experience obtained with the Normal distribution in statistics. This function having no discontinuities is interesting to guarantee stability of the results (cf. Section 5).

Table 1 summarizes the six types of generalized criteria among which the decision maker can choose, and the parameters which have to be fixed. We think that the presentation of this table could help the decision maker to easily choose the function $H(d)$ corresponding to his preferences and

Table 1
The six types of generalized criteria

Types of generalized criteria		Parameters
I. Usual criterion		-
II. Quasi-criterion		q
III. Criterion with linear preference		p
IV. Level criterion		q, p
V. Criterion with linear preference and indifference area		q, p
VI. Gaussian criterion		σ

this, for each point of view. The scientist has then to put the adequate questions in view to assess the associated parameters, which all have an economic signification.

c. Multicriteria preference index

Let us suppose that the decision maker has specified a preference function P_i and weight π_i for each criterion f_i $(i = 1, \ldots, k)$ of problem (1.1). The weight π_i is a measure of the relative importance of criterion f_i; if all the criteria have the same importance for the decision maker, all weights can be taken equal. We are fully aware of the very difficult problem of fixing these weights, but we do not wish to approach this question here.

The multicriteria preference index Π is then defined as the weighted average of the preference functions P_i:

$$\Pi(a, b) = \frac{\sum_{i=1}^{k} \pi_i P_i(a, b)}{\sum_{i=1}^{k} \pi_i}. \tag{2.11}$$

$\Pi(a, b)$ represents the intensity of preference of the decision maker of action a over action b, when considering simultaneously all the criteria. It is a figure between 0 and 1 and:
- $\Pi(a, b) \approx 0$ denotes a weak preference of a over b for all the criteria,
- $\Pi(a, b) \approx 1$ denotes a strong preference of a over b for all the criteria.

This preference index determines a valued outranking relation on the set K of actions. This relation can be represented as a *valued outranking graph*, the nodes of which are the actions of K. Between two nodes (actions), a and b, there are two arcs having values $\Pi(a, b)$ and $\Pi(b, a)$ (there is no particular relation between $\Pi(a, b)$ and $\Pi(b, a)$). See Figure 9.

3. The PROMETHEE rankings

a. Flows in the valued outranking graph

For each node a in the valued outranking graph,

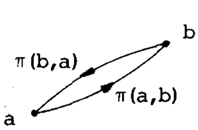


Figure 9

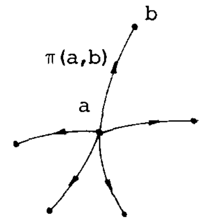


Figure 10

let us define the *leaving flow* by

$$\phi^+(a) = \sum_{b \in K} \Pi(a, b). \tag{3.1}$$

The leaving flow is the sum of the values of the arcs leaving node a and therefore provides a measure of the *outranking character* of a (cf. Figure 10).

Symmetrically, we define the *entering flow* by

$$\phi^-(a) = \sum_{b \in K} \Pi(b, a). \tag{3.2}$$

The entering flow measures the *outranked character* of a (cf. Figure 11).

We also consider the following *net flow*:

$$\phi(a) = \phi^+(a) - \phi^-(a). \tag{3.3}$$

b. PROMETHEE I

The higher the leaving flow and the lower the entering flow, the better the action.

The leaving and entering flows induce respectively the following preorders (3.4) and (3.5):

$$\begin{cases} aP^+b & \text{iff} \quad \phi^+(a) > \phi^+(b), \\ aI^+b & \text{iff} \quad \phi^+(a) = \phi^+(b); \end{cases} \tag{3.4}$$

$$\begin{cases} aP^-b & \text{iff} \quad \phi^-(a) < \phi^-(b), \\ aI^-b & \text{iff} \quad \phi^-(a) = \phi^-(b). \end{cases} \tag{3.5}$$

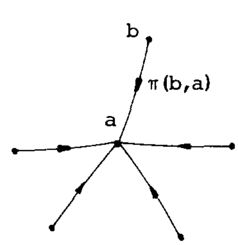


Figure 11

The PROMETHEE I partial preorder (P_I, I_I, R) is then obtained by considering the intersection of these two preorders:

$$\begin{cases} aP_Ib & (a \text{ outranks } b) & \text{if } aP^+b \text{ and } aP^-b, \\ & & \text{or } aP^+b \text{ and } aI^-b, \\ & & \text{or } aI^+b \text{ and } aP^-b; \\ aI_Ib & (a \text{ is indifferent} & \\ & \text{to } b) & \text{iff } aI^+b \text{ and } aI^-b; \\ aRb & (a \text{ and } b \text{ are} & \\ & \text{incomparable}) & \text{otherwise.} \end{cases} \tag{3.6}$$

This partial preorder is then proposed to the decision maker in order to achieve his decision problem. By using the PROMETHEE I method, some actions are remaining incomparable: Only confirmed outrankings are given by the partial preorder.

c. PROMETHEE II

In case a complete preorder on K is requested, avoiding any incomparabilities, the PROMETHEE II complete preorder (P_{II}, I_{II}) given in (3.7) and induced by the net flow (3.3) can be considered:

$$\begin{cases} aP_{II}b & (a \text{ outranks } b) & \text{iff } \phi(a) > \phi(b), \\ aI_{II}b & (a \text{ is indifferent} & \\ & \text{to } b) & \text{iff } \phi(a) = \phi(b). \end{cases} \tag{3.7}$$

Although it is easier for the decision maker to achieve the decision problem by using the complete preorder, the partial preorder contains more realistic information. This information, especially with regard to incomparabilities, can often be useful for the decision making.

Table 2

Crit.	Min or Max	Actions x_1	x_2	x_3	x_4	x_5	x_6	Type of crit.	Parameters	
f_1	Min	80	65	83	40	52	94	II	$q=10$	
f_2	Max	90	58	60	80	72	96	III		$p=30$
f_3	Min	6	2	4	10	6	7	V	$q=0.5$	$p=5$
f_4	Min	5.4	9.7	7.2	7.5	2.0	3.6	IV	$q=1$	$p=6$
f_5	Min	8	1	4	7	3	5	I	–	
f_6	Max	5	1	7	10	8	6	VI	$\sigma=5$	

4. Numerical applications

a. A location problem

Let us consider the following multicriteria problem: Six criteria are considered as relevant by the decision-maker to rank six hydroelectric powerstation projects ($x_1, \ldots, x_6$).

These criteria are:

f_1: manpower,
f_2: power (MW),
f_3: construction cost (10^9 \$),
f_4: maintenance cost (10^6 \$),
f_5: number of villages to evacuate,
f_6: security level.

The second and the last criterion have to be maximized, the others to be minimized.

Table 2 gives, for each criterion, the evaluations of the six actions, the type of generalized criterion specified by the decision maker, and the corresponding parameters.

Using the preference functions as given by Table 2, it is easy to compute the preference index. The six criteria are considered as having the same importance for the decision maker, so all the weights of (2.11) are equal.

The preference index is represented in Table 3.

Table 3

Π	x_1	x_2	x_3	x_4	x_5	x_6	$\phi^+(x)$
x_1		0.296	0.250	0.268	0.100	0.185	1.099
x_2	0.462		0.389	0.333	0.296	0.500	1.980
x_3	0.236	0.180		0.333	0.056	0.429	1.234
x_4	0.399	0.505	0.305		0.223	0.212	1.644
x_5	0.444	0.515	0.487	0.380		0.448	2.274
x_6	0.286	0.399	0.250	0.432	0.133		1.500
$\phi^-(x)$	1.827	1.895	1.681	1.746	0.808	1.774	

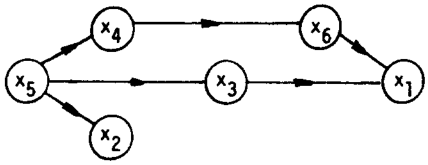


Figure 12.

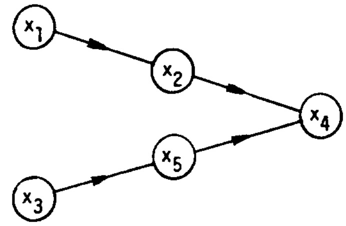


Figure 14.

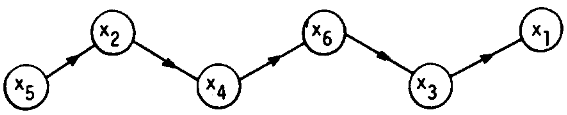


Figure 13.

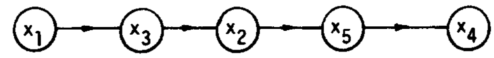


Figure 15.

The leaving flows are computed directly by adding the figures of each row of the table and the entering flows by adding the figures of each column.

The net flows are then easily obtained, using (3.3), resulting in Table 4.

(1) *PROMETHEE I*

The partial preorder is represented in Figure 12

We notice that incomparabilities appear in this ranking: For example, we see that x_1 is a large power-station and is incomparable to x_2 which has a very different shape.

(2) *PROMETHEE II*

The complete preorder induced by Table 4 is represented in Figure 13.

b. The advantage of a partial preorder

The following problem shows how the consideration of a complete preorder can lead to misinterpretations.

Five actions are evaluated on four criteria, all the information is resumed in Table 5.

The final results (which can easily be verified by the reader) are the following:

– PROMETHEE I partial preorder shown in Figure 14.

– PROMETHEE II complete preorder, shown in Figure 15.

It is clear that we lose a great deal of information when looking at Figure 15: The partial preorder shows that actions x_1 and x_2 are incomparable with actions x_3 and x_5.

5. Stability

Every outranking method involves the determination of some parameters (thresholds,...). It is interesting to know the influence they have on the rankings when small deviations in their values

Table 4

	x_1	x_2	x_3	x_4	x_5	x_6
$\phi(x)$	−0.728	0.085	−0.447	−0.102	1.466	−0.274

Table 5

Crit.	Min or Max	Actions x_1	x_2	x_3	x_4	x_5	Type of crit.	Parameters	
f_1	Min	16	11	15	7	13	V	$q=2$	$p=10$
f_2	Min	6	14	11	19	12	I	–	
f_3	Max	22	19	16	10	16	VI	$\sigma=7$	
f_4	Max	7	12	10	8	11	III		$p=6$

Table 6

PROMETHEE		ELECTRE III		
ρ_S	p	s	ρ_S	p
0.9836	0.0243	0.10	0.9351	0.0780
		0.15	0.9438	0.0696
		0.20	0.9506	0.0603

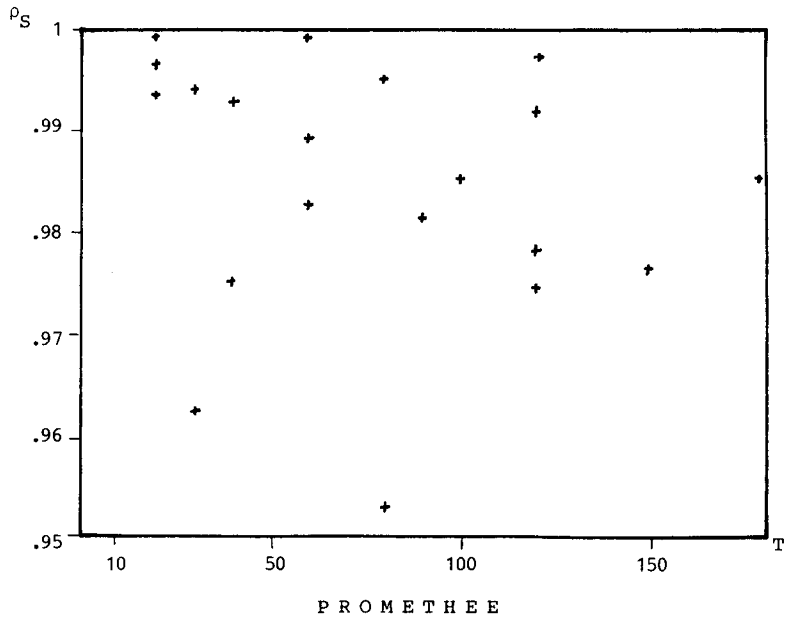
Table 7

PROMETHEE		ELECTRE III		
ρ_S	p	s	ρ_S	p
0.9448	0.0640	0.10	0.9102	0.0949

are introduced. As the decision maker usually cannot fix correctly their exact values, this stability problem is of a major importance. It looks as a necessary condition for a good outranking method.

On the other hand large deviations in the values of these parameters must imply modifications in the rankings, otherwise the method would not take into account the particularities of each problem.

It is also clear that when the parameters to be fixed have an economic signification, the decision



(5.3)

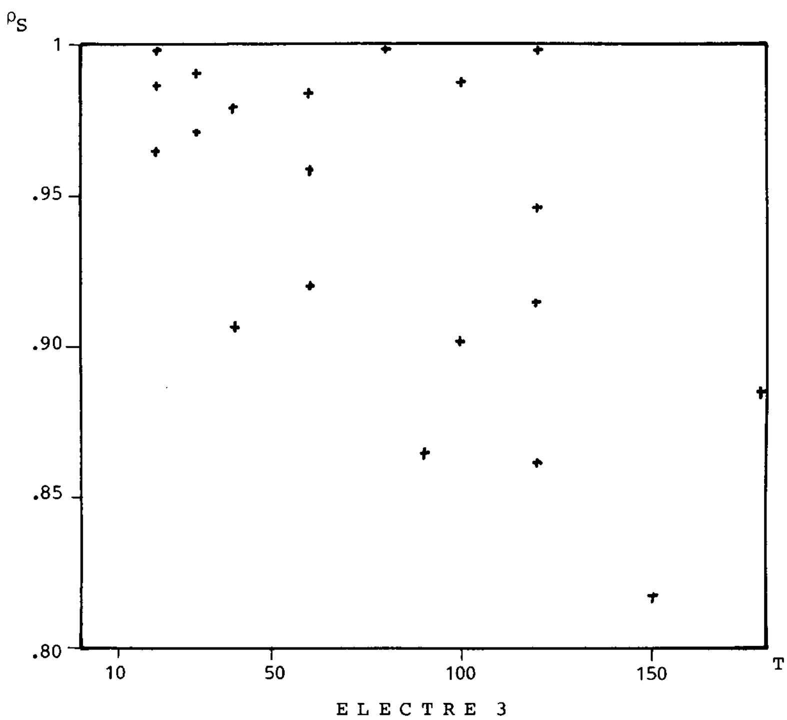


Figure 16

maker can fix them more easily and more precisely. This was one of the principal purposes of PROMETHEE.

Further we have numerically studied the stability of the PROMETHEE rankings and compare the results to those given by ELECTRE III [6].

The methods have been tested on a set of 21 various problems. For each problem, small deviations on the initial thresholds were randomly considered. The corresponding complete preorders

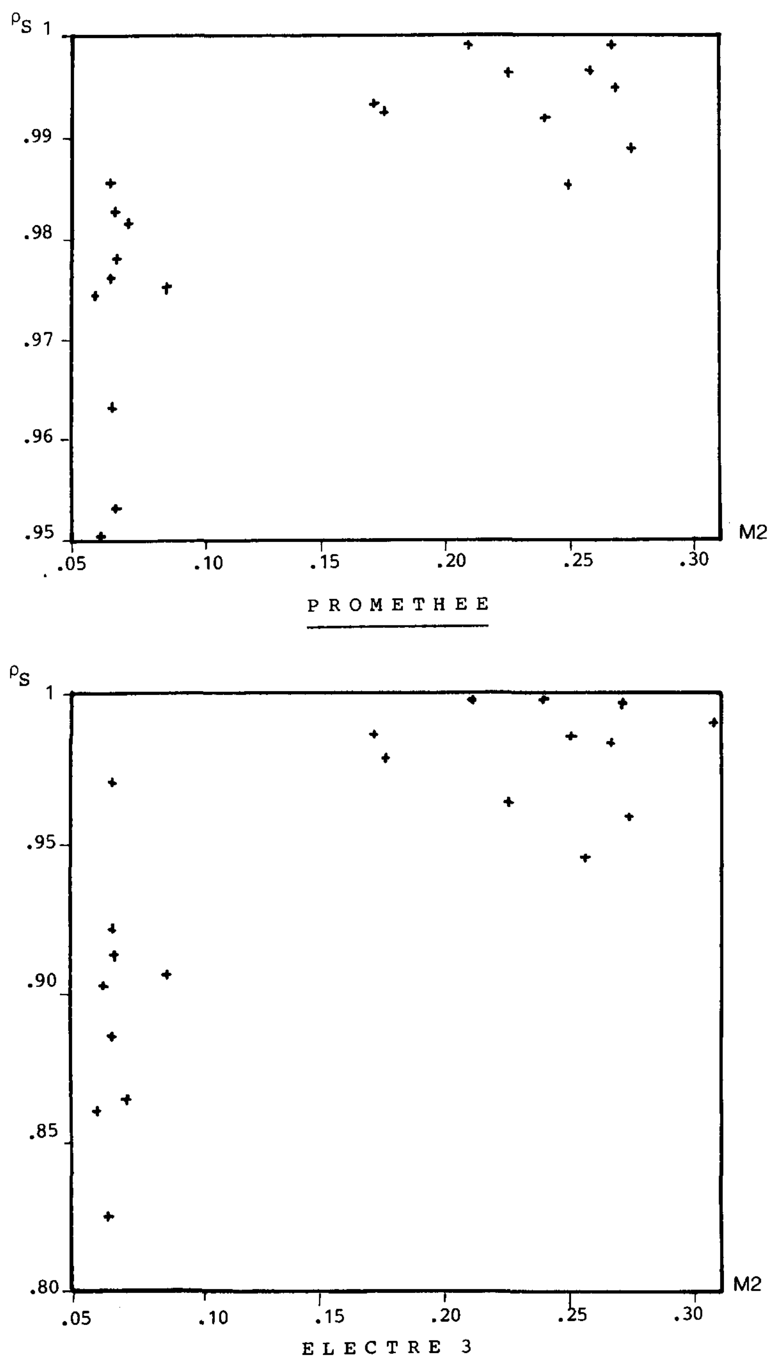
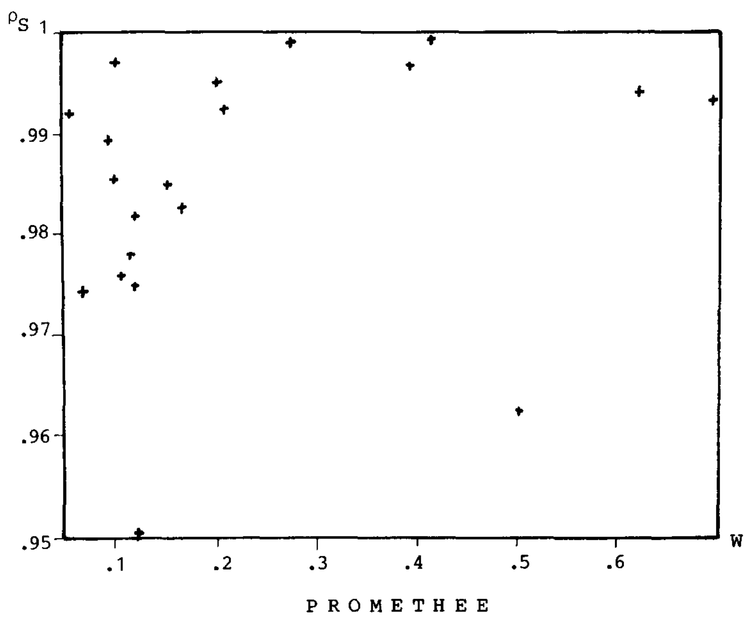


Figure 17

were compared to the initial ones by means of the Spearman's rank correlation coefficient ρ_S. The partial preorders were compared by means of an index p being the proportion of arcs which do not appear in the graphs of both preorders.

The stability was first considered for generalized criteria with linear preference and indifference area. After 300 random variations of the thresholds in a range of 20% of their initial values, we obtained the average results of Table 6, where s



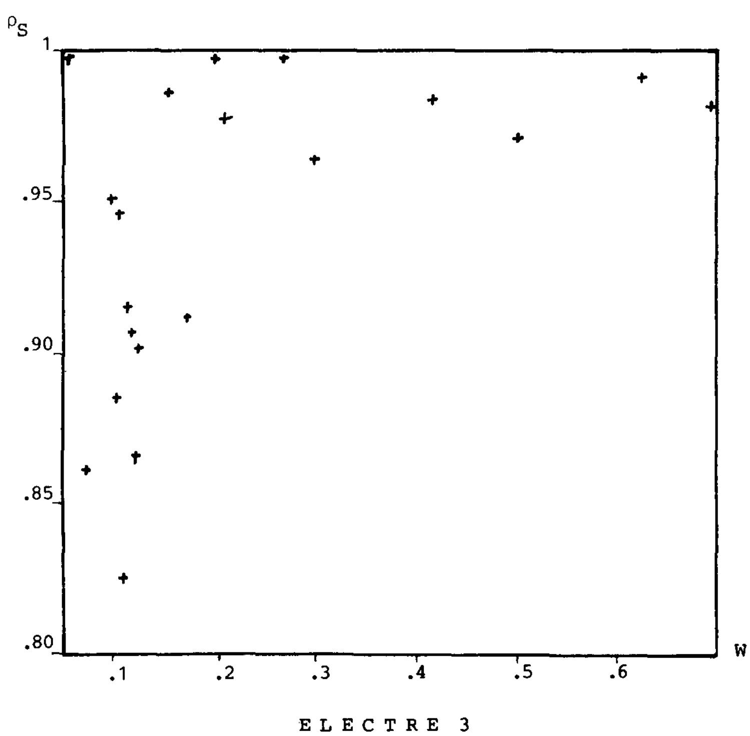


Figure 18

is a discrimination threshold needed by the ELECTRE III method.

It appears clearly that PROMETHEE is more stable than ELECTRE III. This can be explained by the 'discrete' character of the notion of λ-qualification, in ELECTRE III.

Another reason for weaker stability is due to discontinuities in the preference functions or their derivatives.

A second analysis was done by considering level criteria with discontinuous preference functions. The average results after having applied 100 random deviations of the parameters (Table 7) are meaningful.

At the other hand the results given by Gaussian criteria, with very 'smooth' preference functions, are still better.

We have also analysed stability as a function of the characteristics of the multicriteria problem. Three characteristics have been considered:
- the *size* T of the problem (i.e. the number of actions times the number of criteria);
- the *difficulty* of the problem (i.e. the disagreement between the criteria), measured by the Kendall's coefficient of concordance $W (W = 1$ if all the criteria are in perfect agreement);
- the *proximity* coefficient M2, which measures the average distance between a difference of two evaluations and the thresholds.

The results are presented in Figures 16, 17 and 18. It appears that the size of the problem does not influence the sensitivity of the rankings. But the dificulty and especially the proximity of the problem are two determinant factors of instability.

We can conclude that the contribution of PROMETHEE is relevant in matter of stability and that such 'smooth' preference functions as the Gaussian one would have to be more often used in the future.

6. Further developments and open problems

Two other methods in the PROMETHEE family have been developed: In PROMETHEE III, intervals are considered instead of the flows in order to emphasize the role of indifference in the rankings. The method is giving either preorders or, more generally, interval orders on a finite set of actions. The PROMETHEE IV method solves a choice problematic for an infinite set of actions; it uses the same outranking relation but the flows are defined on a compact subset of $\mathbb{R}^n$.

Other improvements could be considered:
- new preference functions, more adapted to practical problems or more meaningful to the decision maker;
- other types of multicriteria preference indices;
- variable thresholds and their implications;
- other methods, such as cluster analysis, for the exploitation of the PROMETHEE flows.

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