

FORMULÁRIO

Balanço diferencial de energia térmica $\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla}T - \alpha \nabla^2 T - \frac{\dot{q}_V}{\rho \cdot Cp} = 0$	Lei de Fourier: $\vec{q}'' = -k \vec{\nabla}T$ Lei de resf. de Newton: $q_{conv} = h \cdot A_s (T_s - T_\infty)$ Difusividade térmica: $\alpha = \frac{k}{\rho \cdot Cp}$
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Operadores diferenciais no espaço tridimensional

S C	Gradiente	Divergente	Laplaciano (divergente do gradiente)
1	$\vec{\nabla}s = \left(\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z} \right)$	$\vec{\nabla} \cdot \vec{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$	$\nabla^2 s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$
2	$\vec{\nabla}s = \left(\frac{\partial s}{\partial r}, \frac{1}{r} \frac{\partial s}{\partial \phi}, \frac{\partial s}{\partial z} \right)$	$\vec{\nabla} \cdot \vec{u} = \frac{1}{r} \frac{\partial}{\partial r} (r \cdot u_r) + \frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z}$	$\nabla^2 s = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 s}{\partial \phi^2} + \frac{\partial^2 s}{\partial z^2}$
3	$\vec{\nabla}s = \left(\frac{\partial s}{\partial r}, \frac{1}{r} \frac{\partial s}{\partial \theta}, \frac{1}{r \cdot \text{sen} \theta} \frac{\partial s}{\partial \phi} \right)$	$\vec{\nabla} \cdot \vec{u} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot u_r) + \frac{1}{r \cdot \text{sen} \theta} \frac{\partial}{\partial \theta} (u_\theta \cdot \text{sen} \theta) + \frac{1}{r \cdot \text{sen} \theta} \frac{\partial u_\phi}{\partial \phi}$	$\nabla^2 s = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial s}{\partial r} \right) + \frac{1}{r^2 \cdot \text{sen} \theta} \frac{\partial}{\partial \theta} \left(\text{sen} \theta \cdot \frac{\partial s}{\partial \theta} \right) + \frac{1}{r^2 \cdot \text{sen}^2 \theta} \frac{\partial^2 s}{\partial \phi^2}$

Tabela: Perfis de temperatura e taxas de calor para aletas com área de seção transversal uniforme.

Condição de contorno na ponta	Perfil de temperatura $\frac{\theta(x)}{\theta_b} = \frac{T(x) - T_\infty}{T_b - T_\infty}$	Taxa de calor para a aleta $q_a = -k A_x \frac{dT}{dx} \Big _{x=0}$
Convecção	$\frac{\cosh(m(L-x)) + \left(\frac{h}{mk}\right) \cdot \text{senh}(m(L-x))}{\cosh(mL) + \left(\frac{h}{mk}\right) \cdot \text{senh}(mL)}$	$M \frac{\text{senh}(mL) + \left(\frac{h}{mk}\right) \cdot \cosh(mL)}{\cosh(mL) + \left(\frac{h}{mk}\right) \cdot \text{senh}(mL)}$
Temperatura conhecida	$\frac{(\theta_L/\theta_b) \cdot \text{senh}(mx) + \text{senh}(m(L-x))}{\text{senh}(mL)}$	$M \frac{\text{senh}(mL) - (\theta_L/\theta_b)}{\text{senh}(mL)}$
Adiabática	$\frac{\cosh(m(L-x))}{\cosh(mL)}$	$M \tanh(mL)$
Aleta infinita $L(\mathbf{m}) \gtrsim \frac{2,65}{m}$	e^{-mx}	M
Substituições:	$m = \sqrt{\frac{hp}{kA_x}}$	$M = k \cdot A_x \cdot m \cdot \theta_b$ $\theta_b = T_b - T_\infty$ $\theta_L = T_L - T_\infty$

- Equações na tabela acima originadas a partir de $\frac{d^2 T}{dx^2} = \frac{hp}{kA_x} (T - T_\infty)$ ou $\frac{d^2 \theta}{dx^2} = m^2 \theta$

Funções hiperbólicas:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{senh}(x) = \frac{e^x - e^{-x}}{2} \quad \tanh(x) = \frac{\text{senh}(x)}{\cosh(x)}$$