Specialty Experiment: PIV-plus-Feedforward Control

Magnetic Levitation (MagLev)

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All of Quanser's systems have an inherent open architecture design. It should be noted that the following experimental setup, accompanying files, and configuration are merely one of the many possible uses of this product.

Instructor Manual

Table of Contents

1. Laboratory Objectives

This laboratory requires the student to design, through pole placement, two control loops in order to levitate in air and track a steel ball to a desired trajectory. First, a "low-level" Proportional-plus-Integral (PI) scheme is used to control the MAGLEV electromagnet current from the power amplifier voltage. This closed-loop system is nested inside a Proportional-plus-Integral-plus-Velocity (PIV) position control scheme with a feedforward action, as fully described in Reference [5]. The Quanser One-Degree-Of-Freedom (DOF) Magnetic Levitation experiment is an example of control of a physically unstable system.

2. Purpose of the Instructor Manual

The present manual is an instructor's manual which provides additional information to the person running the *Magnetic Levitation (MAGLEV) – Specialty Experiment: PIV-plus-Feedforward Control* student laboratory**. It comes as a complement of Reference [5]**. Therefore, the instructor is assumed fully aware of the content of Reference [5] prior to starting reading this manual. The laboratory instructor is also expected to be completely familiar with at least References [1], [2], and [3].

⚠ **This document is not intended for the students as it provides solutions to the pre-lab assignments and contains typical experimental results from the laboratory procedure.**

3. Contents

This document contains the following:
- ❏ A list and description of all the computer files accompanying the experiment.
- ❏ How to customize the laboratory design files to your particular configuration.
- ❏ The solutions to the pre-lab assignments questions.
- ❏ Some typical experimental results expected from the laboratory procedure.

4. References

[1] *MAGLEV User Manual.*
[2] *Data Acquisition Card User Manual.*
[3] *Universal Power Module User Manual*
[4] *WinCon User Manual*.
[5] *Magnetic Levitation (MAGLEV) – Specialty Experiment: PIV-plus-Feedforward Control – Student Handout.*

5. Laboratory Configuration

5.1. Wiring and Hardware Setup

As also mentioned in Reference [5], please refer to Reference [1] for the setup information required to carry out the present magnetic levitation control laboratory. Reference [1] also provides the specifications and a description of the main components composing your system, as well as the wiring conventions used for the MAGLEV plant.

5.2. Experiment Design Files

Table 1, below, lists and describes the various computer files coming with the experiment.

File Name	Description
MAGLEV_PIV_Student_526.pdf	The student handout for the *Magnetic Levitation (MAGLEV) – Specialty Experiment: PIV-plus-Feedforward Control* laboratory. It contains pre-lab assignment questions, as well as in-lab experimental procedure and questions.
MAGLEV_PIV_Instructor_527.pdf	The instructor manual for the *Magnetic Levitation (MAGLEV) – Specialty Experiment: PIV-plus-Feedforward Control* laboratory. It contains information to setup and run the lab, the solutions to the pre-lab assignment questions, and some typical experimental results expected from the experimental procedure.
MAGLEV Equations.mws	Maple worksheet used to analytically derive the Equations of Motion as well as the Laplace and State-Space models corresponding to the experiment's system and both controllers design equations. The Waterloo Maple's Maple 8, or later release, is required to open, modify, and execute this file.

File Name	*Description*
MAGLEV Equations.html	HTML presentation of the *MAGLEV Equations.mws* file. It allows to view the content of the Maple worksheet without having Maple 8 installed. Of course in this case, no modification to the equations can be done.
quanser_tools.mws	Executing this worksheet generates the *quanser* repository containing the *Quanser_Tools* package. The two package files are named: *quanser.ind* and *quanser.lib*. The *Quanser_Tools* module defines generic procedures used in Lagrangian mechanics. It also contains data processing routines to save the obtained state-space matrices into a Matlab-readable file.
quanser_tools.rtf	Rich Text Format presentation of the *quanser_tools.mws* file. It allows to view the content of the Maple worksheet without having Maple 8 installed. Of course in this case, no modification to the described Maple procedures can be done.
setup_lab_maglev_piv.m	Matlab file. It contains the configuration flags describing the user's system. **It is the only design file to execute.** It calls the other design files: *setup_maglev_configuration.m*, and *d_maglev_pi_piv.m*.
setup_maglev_configuration.m	Matlab file. It returns the MAGLEV model parameters accordingly to the user-configuration set in *setup_lab_maglev_piv.m*.
d_maglev_pi_piv.m	Matlab controller design file. It automatically calculates the system's PI, PIV, and feedforward gains, satisfying the user-defined pole placement. It can also carry out some system analysis and simulate and plot various time responses.

File Name	Description
q_cal_maglev_ZZ.mdl	Simulink controller file. It implements a closed-loop system to control the current in the MAGLEV electromagnet, in a way that can be used for the position sensor calibration procedure. The diagram interfaces to the *'ZZ'* data acquisition board. The name substitutions stood for by *'ZZ'* are listed and described in Table 2, below. Real-time code is generated from this controller file and associated with the WinCon project *q_cal_maglev_ZZ.wcp*.
q_cal_maglev_ZZ.wcp	WinCon calibration project containing the position sensor calibration control panel, and two digital meters, and associated with a real-time current controller.
q_pi_maglev_ZZ.mdl	Simulink controller file. It implements the real-time PI closed-loop to control the actual MAGLEV electromagnet current. The hardware is connected to the *'ZZ'* data acquisition board. The name substitutions stood for by *'ZZ'* are listed and described in Table 2, below. The real-time code can be generated from this controller file and run with WinCon.
q_piv_maglev_ZZ.mdl	Simulink file. It implements the real-time PIV-plus-Feedforward closed-loop controlling the MAGLEV ball position. The diagram interfaces to the *'ZZ'* data acquisition board. The name substitutions stood for by *'ZZ'* are listed and described in Table 2, below. Real-time code can be generated from this controller file and run with WinCon.

Table 1 Laboratory Design Files

The possible substitution names stood for by *'ZZ'* in the controller design files described in Table 1 are listed below:

'ZZ' Substitution Name	Description
mq3	The corresponding controller file interfaces to the Quanser's MultiQ-3 board.

'ZZ' Substitution Name	*Description*
mqp	The corresponding controller file interfaces to the Quanser's MultiQ-PCI board.
q8	The corresponding controller file interfaces to the Quanser's Q8 board.
nie	The corresponding controller file interfaces to the National Instruments' E-Series boards.

Table 2 *'ZZ'* Substitution Names

All the calculations and equations derived in Reference [5] (i.e. the student handout for this lab) and the present manual (i.e. the instructor manual) are entirely carried out in the Maple worksheet called *MAGLEV Equations.mws* and provided with this laboratory. Using Maple 8, the worksheet equations can be edited and re-calculated by executing the worksheet.

5.3. Matlab File Configuration: *setup_lab_maglev_piv.m*

Prior to the starting the laboratory with the students, the Matlab file *setup_lab_maglev_piv.m* should be manually edited (i.e. configured) to reflect your actual experimental equipment and setup that you have, and the controller design that you wish to implement.

The Matlab script *setup_lab_maglev_piv.m* initializes all the Matlab workspace variables and system parameters required by the Simulink diagrams and for further analysis. Open *setup_lab_maglev_piv.m* in order to edit it.

5.3.1. File Configuration to Reflect your Hardware Setup

In the *setup_lab_maglev_piv.m* file, there is a section that must be set according to your lab equipment and its hardware configuration. This section is reproduced below:

```
% ##### USER-DEFINED MAGLEV CONFIGURATION #####
% Cable Gain used: set to 5
K_CABLE = 5;
% Universal Power Module (UPM) Type: set to 'UPM_2405'
UPM_TYPE = 'UPM_2405';
% Digital-to-Analog Maximum Voltage (V); for MultiQ cards set to 10
VMAX_DAC = 10;
% ##### END OF USER-DEFINED MAGLEV CONFIGURATION #####
```

The settings seen above are the **default configuration parameters** of this experiment. It is up to you, the instructor, to ensure that these settings match the configuration of your actual experimental setup, as you present it to your students. If you are unsure about your configuration, please refer to References [1], [2], and [3].

5.3.2. File Configuration to Reflect your Controller Implementation

Further in the Matlab script *setup_lab_maglev_piv.m*, there is a section where the controller design is defined by the user, i.e. the instructor. This section is reproduced below:

```
% ##### USER-DEFINED CONTROLLER DESIGN #####
% Type of Controller: set it to 'AUTO', 'MANUAL'
CONTROLLER_TYPE = 'AUTO';    % controller design: automatic mode
%CONTROLLER_TYPE = 'MANUAL';    % controller design: manual mode
% Selected quiescent point of operation: ( xb0, Ic0 )
% Operating Air Gap (m)
xb0 = 6e-3;
% Rising (and Falling) Setpoint Slew Rate (m/s)
XB_REF_RATE = 0.005;
% Specifications of the second-order low-pass filter
% for the ball (differentiated) velocity signal
wcf = 2 * pi * 100;    % filter cutting frequency
zetaf = 0.9;        % filter damping ratio
% Specifications of the first-order low-pass filter
% for the ball position feedback signal
tau_xb = 1 / ( 2 * pi * 80 );    % filter time constant
% first-order low-pass filter specifications
% used before plotting the command voltage (Vc) and the coil current (Ic)
tau_c = 1 / ( 2 * pi * 10 );    % filter time constant
% Integral anti-windup maximum for the limiter integrator
% in the inner current loop (V)
MAX_IC_WDUP = 24;
% in the ball position loop (A)
MAX_XB_WDUP = 1;
% ##### END OF USER-DEFINED CONTROLLER DESIGN #####
```

In this section, setting the *CONTROLLER_TYPE* flag to *'AUTO'* triggers the call to the controller design file *d_maglev_pi_piv.m* for both control loops by specifying their respective closed-loop pole locations, which are also set automatically in this mode. The *'AUTO'* mode is the turn-key mode to run the magnetic levitation experiment. The user can also call the *d_maglev_pi_piv.m* script by specifying his/her own set of closed-loop poles. The controller design file *d_maglev_pi_piv.m* calculates and returns the feedforward and PI and PIV feedback controllers gains (i.e. K_{ff_b}, K_{p_c}, K_{i_c}, K_{p_b}, K_{i_b}, and K_{v_b}).

When set to *'MANUAL'*, the *CONTROLLER_TYPE* flag only initializes the MAGLEV

system model parameters. The user (e.g. student) is then free to implement the current and/or position controller(s) of her or his choice. It lets the students carry out the calculations themselves and set the feedback controllers gains (i.e. K_{p_c}, K_{i_c}, K_{ff_b}, K_{p_b}, K_{i_b}, and K_{v_b}) in the Matlab workspace.

It is suggested that the instructor first runs the *setup_lab_maglev_piv.m* script in the *'AUTO'* mode in order to get an idea of realistic values for the both controllers set of closed-loop pole locations, as well as of the system's diverse controller gain values. The instructor may then have the students run this script file in the *'MANUAL'* mode, since the manual tuning of the PI and Feedforward-plus-PIV controllers is part of the lab procedure, as detailed in Reference [5].

Alternatively, the controller design file *d_maglev_pi_piv.m* also has flags to have it design a PV position controller as opposed to PIV. The pole placement of both current and position controllers can either be done by specifying the pole locations themselves (e.g. if on the real-axis) or by specifying their peak/settling time and percent overshoot (for a complex and conjugate pair of poles). **These design possibilities are left up to the instructor for possible extensions of the scope of the presented laboratory.** Finally, also carries out some system's simulations and plotting.

6. Solutions to the Pre-Lab Assignments

⚠ **The students are required to include the solutions to all the pre-lab assignments in their lab report.**

6.1. Solution To Assignment #1: Electrical System Modelling

The solutions to the questions of Assignment #1 are as follows:

1. Using Kirchhoff's voltage law, we obtain the following first-order differential equation:

$$V_c(t) = (R_c + R_s) I_c(t) + L_c \left(\frac{d}{dt} I_c(t) \right) \qquad [1]$$

2. Applying the Laplace transform to Equation [1] and rearranging yields the desired open-loop transfer function for the MAGLEV electrical system, such that:

$$G_c(s) = \frac{Kc_{dc}}{\tau_c s + 1} \qquad [2]$$

with:

$$Kc_{dc} = \frac{1}{R_c + R_s} \qquad \text{and} \qquad \tau_c = \frac{L_c}{R_c + R_s} \qquad [3]$$

Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, $G_c(s)$ is of type zero.

6.2. Solution To Assignment #2 - Electro-Mechanical System Modelling: Equation Of Motion (EOM)

The solutions to the questions of Assignment #2 are as follows:

1. The force due to gravity applied on the ball is expressed by:

$$F_g = M_b g \qquad [4]$$

Using Equation [3] of Reference [5], the total external force experienced by the ball

using the electromagnet is given by:

$$F_c + F_g = -\frac{1}{2} \frac{K_m I_c^2}{x_b^2} + M_b g \qquad [5]$$

Applying then Newton's second law of motion to the ball, the following non-linear Equation Of Motion (EOM) comes:

$$\frac{\partial^2}{\partial t^2} x_b = -\frac{1}{2} \frac{K_m I_c^2}{M_b x_b^2} + g \qquad [6]$$

2. At equilibrium, all time derivative terms equate zero and Equation [6] becomes:

$$-\frac{1}{2} \frac{K_m I_{c0}^2}{x_{b0}^2} + M_b g = 0 \qquad [7]$$

As a remark, it can be seen from Equation [7] that at the equilibrium point the electromagnet force equals the ball weight.

The coil current at equilibrium, I_{c0}, then comes from Equation [7] as being a function of x_{b0} and K_m, as expressed below:

$$I_{c0} = \sqrt{2} \sqrt{\frac{M_b g}{K_m}} x_{b0} \qquad [8]$$

Using the system's specifications given in Reference [1] and the design requirements given in Reference [5], the evaluation of Equation [8] results to be:

$$I_{c0} = 0.86 \, [A] \qquad [9]$$

Finally, the electromagnet force constant, K_m, also comes from Equation [7] as being a function of the nominal pair (x_{b0}, I_{c0}), as shown below:

$$K_m = \frac{2 M_b g x_{b0}^2}{I_{c0}^2} \qquad [10]$$

6.3. Solution To Assignment #3 - Electro-Mechanical System Modelling: EOM Linearization and Transfer Function

The solutions to the questions of Assignment #3 are as follows:

1. Applying the Taylor's series approximation about (x_{b0}, I_{c0}) as given by Equation [6] of Reference[5], Equation [6] becomes linearized as represented below:

$$\frac{\partial^2}{\partial t^2} x_{b1} = -\frac{1}{2} \frac{K_m I_{c0}^2}{M_b x_{b0}^2} + g + \frac{K_m I_{c0}^2 x_{b1}}{M_b x_{b0}^3} - \frac{K_m I_{c0} I_{c1}}{M_b x_{b0}^2} \tag{11}$$

Substituting K_m with its expression given in Equation [10], simplifying, and re-arranging, the system linearized EOM becomes:

$$\frac{\partial^2}{\partial t^2} x_{b1} = \frac{2 g x_{b1}}{x_{b0}} - \frac{2 g I_{c1}}{I_{c0}} \tag{12}$$

2. Applying the Laplace transform to Equation [12] and rearranging yields the desired open-loop current-to-position transfer function for the MAGLEV electro-mechanical system, such that:

$$G_{b1}(s) = -\frac{Kb_{dc} \, \omega_b^2}{s^2 - \omega_b^2} \tag{13}$$

with:

$$Kb_{dc} = \frac{x_{b0}}{I_{c0}} \qquad \text{and} \qquad \omega_b = \sqrt{2} \sqrt{\frac{g}{x_{b0}}} \tag{14}$$

Equation [13] shows a second-order system of type zero. The two open-loop poles are located on the real axis at $s = \pm\omega_b$. Having one pole in the Right-Half Plane (RHP), the open-loop system is unstable and feedback control is required.

6.4. Assignment #4 – Coil Current Controller Design: Pole Placement

The solutions to the questions of Assignment #4 are as follows:

1. The coil current closed-loop transfer function is expressed by:

$$T_c(s) = \frac{K_{p\_c}\, s + K_{i\_c}}{L_c\, s^2 + (R_c + R_s + K_{p\_c})\, s + K_{i\_c}}$$

[15]

2. Using Equation [15], it comes that the normalized characteristic polynomial of the electrical system is such as:

$$s^2 + \frac{(R_c + R_s + K_{p\_c})\, s}{L_c} + \frac{K_{i\_c}}{L_c} = 0$$

[16]

3. The system's desired characteristic equation is:

$$s^2 + (-p_{c2} - p_{c1})\, s + p_{c1}\, p_{c2} = 0$$

[17]

4. Solving the system of two equations and two unknowns resulting from identifying the coefficients of Equation [16] with those of Equation [17], the PI controller gains can be expressed by:

$$K_{p\_c} = -(p_{c1} + p_{c2})\, L_c - R_c - R_s$$

[18]

and:

$$K_{i\_c} = p_{c1}\, p_{c2}\, L_c$$

[19]

6.5. Solution To Assignment #5 – Ball Position Controller Design: Pole Placement

The solutions to the questions of Assignment #5 are as follows:

1. By definition, at the static equilibrium point (x_{b0}, I_{c0}):

$$(x_b = x_{b\_des}) = x_{b0} \qquad \text{and} \qquad (I_c = I_{c\_des}) = I_{c0}$$

[20]

It comes from Equation [18] that:

$$I_{c1} = 0$$

[21]

Using Equations [9] and [10] of Reference [5], the current feedforward gain results as it follows:

$$K_{ff} = \frac{I_{c0}}{x_{b0}} \qquad \text{or} \qquad K_{ff} = \frac{1}{Kb_{dc}} \qquad [22]$$

2. The ball position closed-loop transfer function is expressed by:

$$T_b(s) = -\frac{2\,g\,((K_{ff\_b} + K_{p\_b})\,s + K_{i\_b})}{I_{c0}\left(s^3 - \frac{2\,g\,K_{v\_b}\,s^2}{I_{c0}} + \left(-\frac{2\,g}{x_{b0}} - \frac{2\,g\,K_{p\_b}}{I_{c0}}\right)s - \frac{2\,g\,K_{i\_b}}{I_{c0}}\right)} \qquad [23]$$

3. It comes from Equation [23] that the normalized characteristic polynomial of the electro-mechanical system is such as:

$$s^3 - \frac{2\,g\,K_{v\_b}\,s^2}{I_{c0}} + \left(-\frac{2\,g}{x_{b0}} - \frac{2\,g\,K_{p\_b}}{I_{c0}}\right)s - \frac{2\,g\,K_{i\_b}}{I_{c0}} = 0 \qquad [24]$$

4. The system's desired characteristic equation is:

$$s^3 + (-p_{b1} - p_{b3} - p_{b2})\,s^2 + (p_{b1}\,p_{b3} + p_{b1}\,p_{b2} + p_{b2}\,p_{b3})\,s - p_{b1}\,p_{b2}\,p_{b3} = 0 \qquad [25]$$

5. Solving the system of three equations and three unknowns resulting from identifying the coefficients of Equation [24] with those of Equation [25], the PIV controller gains are determined to be such as:

$$K_{p\_b} = -\frac{1}{2}\frac{\left(p_{b1}\,p_{b3} + p_{b1}\,p_{b2} + p_{b2}\,p_{b3} + \frac{2\,g}{x_{b0}}\right)I_{c0}}{g} \qquad [26]$$

and:

$$K_{v\_b} = \frac{1}{2}\frac{(p_{b1} + p_{b2} + p_{b3})\,I_{c0}}{g} \qquad [27]$$

and:

$$K_{i\_b} = \frac{1}{2}\frac{p_{b1}\,p_{b2}\,p_{b3}\,I_{c0}}{g} \qquad [28]$$

7. Typical In-Lab Results

7.1. Real-Time Implementation: Tuning Of The PI Current Control Loop

Before the students run the Matlab script called *setup_lab_maglev_piv.m*, the instructor should edit it and ensure that the *CONTROLLER_TYPE* flag is properly set to *'MANUAL'*.

Students are expected to have determined Equations [18] and [19] from their pre-lab assignments. When evaluating this set of Equations with the desired pair of closed-loop pole locations, as defined by Equation [16] in Reference [5], the PI current controller gains result to be such as shown below:

$$ K_{p\_c} = 66.8 \left[\frac{V}{A} \right] \qquad \text{and} \qquad K_{i\_c} = 23.3 \left[\frac{V}{s\,A} \right] \qquad [29] $$

For safety reasons, the values of the PI feedback controller gains, as determined and calculated by the students, should be checked by the instructor (i.e. yourself) before they start the real-time controller.

It can be observed that the actual current response presented in Reference [5] is clearly over-damped and has a rise time of less than 0.35 seconds with no steady-state error. Therefore the controller gains of Equation [29] meet the current loop design requirements.

For the purpose of determining the current closed-loop pole locations, it is a good insight to evaluate the open-loop pole of the coil voltage-to-current transfer function, as determined in Equation [2]. Substituting in Equation [3] with the electromagnet specification values given in Reference [1] and rearranging, $G_c(s)$ open-loop pole results to be equal to:

$$ -\frac{1}{\tau_c} = -26.7 \left[\frac{rad}{s} \right] \qquad [30] $$

As a remark, the Matlab plot shown in Figure 1 has been generated by calling the script named *d_maglev_pi_piv.m* with the flag *PLOT_RESPONSE* set to *'YES'*. The Matlab Control System Toolbox function *'step'* has been used for the simulation. Figure 1 represents the simulated current PI-closed-loop response.
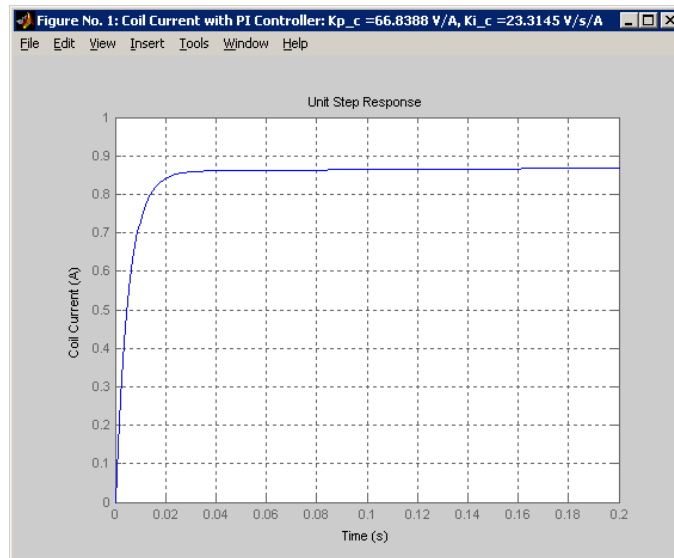
Figure 1 PI Current Loop: Simulated Step Response

7.2. Ball Position Sensor Calibration

Before starting the actual experiment, the instructor should ensure that the position sensor has been properly calibrated. Such a calibration procedure for the photodarlington is fully described in Reference [1].

However, as an alternative and an extension to the presented laboratory, the instructor can have the students run the calibration procedure as well, as described in Reference [1].

7.3. Real-Time Implementation: Tuning Of The Feedforward-Plus-PIV Position Control Loop

Students are expected to have determined Equation [22] from pre-lab Assignment #5. Evaluating Equation [22] at the desired operating point leads to the following feedforward gain:

$$K_{ff\_b} = 142.9 \left[\frac{A}{m} \right] \tag{31}$$

Students are expected to have determined Equations [26], [27], and [28] from their pre-lab assignments. When evaluating this set of Equations with the desired set of closed-loop pole

locations, as defined by Equation [17] in Reference [5], the PIV position controller gains result to be such as shown below:

$$K_{p\_b} = -252.6 \left[\frac{A}{m} \right] \quad \text{and} \quad K_{v\_b} = -4.3 \left[\frac{A\,s}{m} \right] \quad \text{and} \quad K_{i\_b} = -248.1 \left[\frac{A}{s\,m} \right] \quad [32]$$

As a remark, the controller gains negative sign is a characteristic of a positive feedback loop in the MAGLEV model.

⚠ For safety reasons, the values of the feedforward gain as well as of the PIV feedback controller gains, as determined and calculated by the students, should be checked by the instructor (i.e. yourself) before they start the real-time controller.

It can be observed from the actual position response presented in Reference [5] that it has a maximum settling time of 0.9 seconds, a Percent Overshoot less than 15 %, and no static (or average) steady-state tracking error. Some of the observed position oscillations are due to the ball swaying right and left rather than vertically. In such a case, students may touch the ball lightly to dampen out these vibrations, as they occur in the system's uncontrolled degrees of freedom (e.g. horizontal plane). As a remark, it is reminded that when the ball position value increases and its trace goes up on the WinCon Scope, the steel ball actually moves down and away from the electromagnet core, and vice-versa.

From the same experimental run as the one presented in Reference [5], the corresponding command voltage sent to the power amplifier are displayed in Figure 2.
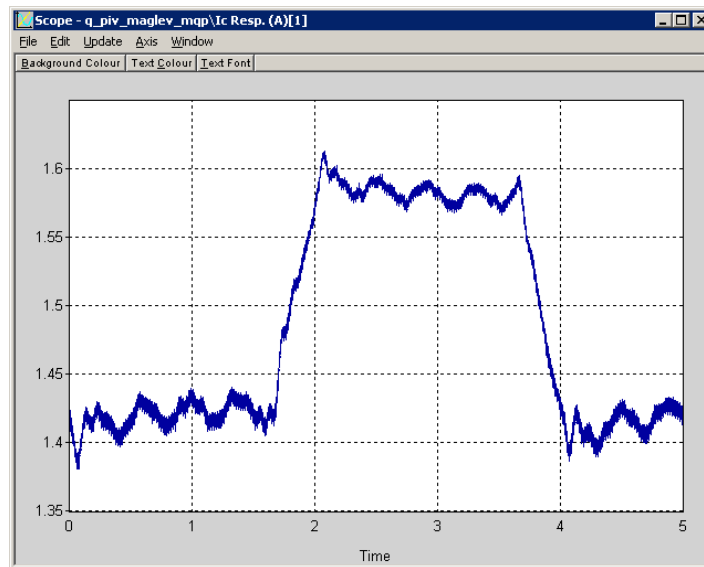


Figure 2 Actual Coil Voltage Command

Now considering the actual coil current presented in Reference [5] and command voltage response shown in Figure 2, above, it can be observed that they never go into saturation, preserving the controller's and closed-loop system's linear behaviour. Specifically to avoid saturation of the power amplifier (e.g. UPM-2405), the command voltage should always be between ±4.4 Volts. As a side note, it is reminded that both current and command voltage are filtered just before being plotted in order to offer a better visualization of their actual behaviours. One can also observe, as expected, that less coil current is required to regulate the ball position as it gets closer to the electromagnet.

Therefore the controller gains evaluated in Equations [31] and [32] meet the position loop design requirements.

For the purpose of determining the position closed-loop pole locations, it is a good insight to evaluate the two open-loop poles of the current-to-position transfer function, as determined in Equation [13]. Substituting in Equation [14] with the gravitational constant given in Reference [1] and the operating position, x_{b0}, given in the controller design specifications and rearranging, $G_b(s)$ open-loop poles result to be equal to:

$$57.2 \left[\frac{rad}{s} \right] \qquad \text{and} \qquad -57.2 \left[\frac{rad}{s} \right] \qquad\qquad [33]$$

As a remark, the Matlab plot shown in Figure 3, below, has been generated by calling the script named *d_maglev_pi_piv.m* with the flag *PLOT_RESPONSE* set to *'YES'*. The Matlab Control System Toolbox function *'lsim'* has been used for the simulation. Figure 3 represents the simulated position closed-loop response obtained from the Feedforward-plus-PIV scheme when assuming instantaneous coil current response.
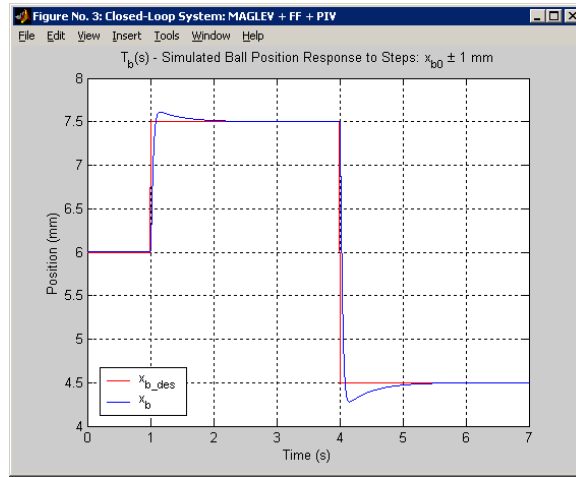
Figure 3 T_b(s) - PIV Position Loop: Step Responses

7.4. Resulting System's Actual Closed-Loop Poles

Assuming feedforward action and taking into account the closed-loop current dynamics, $G_m(s)$ can be determined from Equations [13] and [15] as shown below:

$$G_m(s) = -2\ g\ x_{b0}\ (K_{p\_c}\ s - K_{i\_c})\ \big/\ (L_c\ I_{c0}\ x_{b0}\ s^4 + (R_c + R_s + K_{p\_c})\ I_{c0}\ x_{b0}\ s^3$$
$$+ (K_{i\_c}\ I_{c0}\ x_{b0} - 2\ L_c\ I_{c0}\ g)\ s^2 - 2\ (R_c + R_s + K_{p\_c})\ I_{c0}\ g\ s - 2\ K_{i\_c}\ I_{c0}\ g) \quad\quad [34]$$

Using Matlab to evaluate $G_m(s)$ results in the following expression:

$$G_m(s) = \frac{-139\ s - 48.5}{0.0375\ s^4 + 7.08\ s^3 - 120.5\ s^2 - 23140\ s - 6931} \quad\quad [35]$$

A system's block diagram reduction using Matlab's Control System Toolbox functions is presented in the *d_maglev_pi_piv.m* script. It leads to the following relationship:

$$T_m(s) = \frac{863\ s^2 + 2253\ s + 681}{0.002122\ s^5 + 0.4\ s^4 + 26.9\ s^3 + 689.7\ s^2 + 2253\ s + 680.9} \qu\quad [36]$$

The actual closed-loop zeros result to be:
   -2.3 , -0.3 $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ [37]

The actual closed-loop poles result to be:
   -74.3 , $-55.3 + 26.6\ i$, $-55.3 - 26.6\ i$, -3.4, -0.3 $\quad\quad\quad\quad\quad\quad$ [38]

As a remark, $T_m(s)$ is analytically derived in the Maple worksheet titled *MAGLEV Equations.mws*. For reference, it is expressed below:

$$T_m(s) = (-2 \, K_{p\_c} \, g \, x_{b0} \, (K_{ff\_b} + K_{p\_b}) \, s^2 + (-2 \, K_{i\_c} \, g \, x_{b0} \, (K_{ff\_b} + K_{p\_b}) - 2 \, g \, x_{b0} \, K_{p\_c} \, K_{i\_b}) \, s$$

$$- 2 \, g \, x_{b0} \, K_{i\_c} \, K_{i\_b}) \, \Big/ \, (s^5 \, I_{c0} \, L_c \, x_{b0} + (I_{c0} \, x_{b0} \, R_c + I_{c0} \, x_{b0} \, R_s + I_{c0} \, x_{b0} \, K_{p\_c}) \, s^4$$

$$+ (-2 \, L_c \, I_{c0} \, g - 2 \, g \, x_{b0} \, K_{p\_c} \, K_{v\_b} + K_{i\_c} \, I_{c0} \, x_{b0}) \, s^3$$

[39]

$$+ (-2 \, I_{c0} \, g \, R_c - 2 \, I_{c0} \, g \, R_s - 2 \, I_{c0} \, g \, K_{p\_c} - 2 \, g \, x_{b0} \, K_{p\_c} \, K_{p\_b} - 2 \, g \, x_{b0} \, K_{i\_c} \, K_{v\_b}) \, s^2$$

$$+ (-2 \, g \, x_{b0} \, K_{p\_c} \, K_{i\_b} - 2 \, g \, x_{b0} \, K_{i\_c} \, K_{p\_b} - 2 \, K_{i\_c} \, I_{c0} \, g) \, s - 2 \, g \, x_{b0} \, K_{i\_c} \, K_{i\_b})$$

In addition, the Matlab plot shown in Figure 4, below, has been generated by calling the script named *d_maglev_pi_piv.m* with the flag *PLOT_RESPONSE* set to *'YES'*. The Matlab Control System Toolbox function *'lsim'* has been used for the simulation. Figure 4 represents the simulated position response obtained from the Feedforward-plus-PIV scheme when taking into account the current control system dynamics. As observed in Figure 4, this results in a slight deterioration of the position response performance.
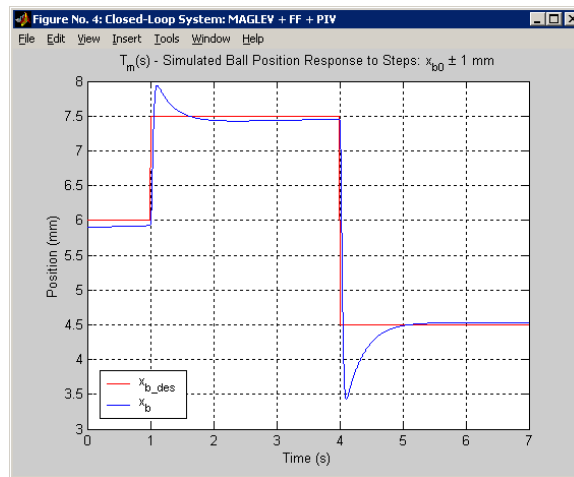

Figure 4 $T_m(s)$ - PIV Position Loop: Step Responses

8. Lab Report

Instructions regarding the students' lab report together with its corresponding marking scheme are left up to the instructor's discretion.