## MATLAB

## An Introduction

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## Why Matlab?

Friendly environment
Simple programming language
Lots of tools
Can be applied in several areas of knowledge

## First steps



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## Interface

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## First steps

Programming language

$$
\begin{aligned}
& \gg a=1+1 \\
& \gg b=[1,1,2,3,5,8,13,21] \\
& \gg x=a+b \\
& x= \\
& 3,3,4,5,7,10,15,23
\end{aligned}
$$

## First steps

## Matlab's tools

- Vectors and matrices
- Plotting and graphics
- Symbolic calculus
- Differential equations
- Transforms
- Model fitting
- Simulink
- A lot of other tools...


## Hands on!

## Try calculating those math operations:

$$
\begin{gathered}
5\left(\frac{3}{4}\right)+\frac{9}{5}=5.55 \\
4^{3}\left(\frac{3}{4}-\frac{9}{2 * 3}\right)=-48
\end{gathered}
$$

## Hands on!

Find the volume of a beer can (consider the can as a cylinder): The volume of a beer can be calculated by:

$$
\begin{aligned}
& V=\pi r^{2} h \\
& r=3 \mathrm{~cm} \\
& h=12.5 \mathrm{~cm}
\end{aligned}
$$

$$
\begin{aligned}
& \gg r=3 ; \\
& \gg h=12.5 ; \\
& \gg V=p i^{*} 3^{\wedge} 2 * 12.5 \\
& V= \\
& \quad 353.4292
\end{aligned}
$$

## Other operators

## Natural logarithm <br> $\gg \log (\mathrm{a})$; <br> Base ten log <br> $\gg \log 10(\mathrm{a}) ;$ <br> Exponential: <br> $\gg \exp (\mathrm{a})$;

Trigonometric functions
$\gg \cos (\mathrm{pi})$;
$\gg \sin (\mathrm{pi})$;
$\gg \tan (\mathrm{pi})$;

$$
\begin{aligned}
& \gg \operatorname{acos}(\mathrm{pi}) ; \\
& \gg \operatorname{asin}(\mathrm{pi}) ; \\
& \gg \operatorname{atan}(\mathrm{pi}) ;
\end{aligned}
$$

Complex numbers
$\gg y=5 i ;$
$\gg \mathrm{z}=1+3^{*} \mathrm{i}$;
$\gg \mathrm{w}=3 \mathrm{j}$;

## Script file

Using script files, it's possible to save the work for later use or for recording data

It's very useful when there is a long sequence of operations

Let's create a script file:

- File -> New -> Script
- Or click on the New file icon on the toolbar at the top of the screen


## Script file

Type in the script file:

$$
\begin{aligned}
& \text { \% Example 1: Using script file } \\
& x=[1,2,3,4] \\
& y=\exp (x)
\end{aligned}
$$

Save the file as example1.m

## Script file

## At the command window, type: <br> >> example1

## Vector and Matrices

When you work with data, you need to handle them sometimes.
Vectors are one-dimensional arrays.

| $\gg \mathrm{a}=[1,2,3]$ | >> $\mathrm{a}=[1 ; 2 ; 3]$ | >> $a^{\prime}$ |
| :---: | :---: | :---: |
| $\mathrm{a}=$ | $\mathrm{a}=$ | = |
| 123 | 1 | 123 |
|  | 2 |  |
|  | 3 |  |

## Vector and Matrices

Matlab allows you to append vectors together to create new ones.
Let $\mathbf{u}$ and $\mathbf{v}$ be two column vectors $m$ and $n$ respectively.

What happens if I type:

$$
\gg w=[\mathbf{u} ; \mathbf{v}]
$$

## Vector and Matrices

$$
\begin{aligned}
& \text { >> w = }[\mathbf{u} ; \mathbf{v}] ; \\
& \text { >> size }(\mathrm{w}) \\
& \text { ans }= \\
& \quad \mathrm{m}+\mathrm{n}
\end{aligned}
$$

The same works for row vectors as well

## Vector and Matrices

It is possible to create uniformly spaced vector using colons:

$$
\begin{aligned}
& \gg t=[0: 10] \\
& t= \\
& 0
\end{aligned} 1
$$

## Vector and Matrices

You can also change the step size of the vector using the syntax:

$$
\begin{aligned}
& \text { >> t = [0:2:10] } \\
& \mathrm{t}= \\
& \begin{array}{llllll}
0 & 2 & 4 & 6 & 8 & 10
\end{array}
\end{aligned}
$$

## Hands on!

Using a script file, try to create a time vector $t$ from 0 to 10 using 1 as step size. Then, create a vector $\mathrm{y}=1-\exp (-\mathrm{t})$.

After that, create an vector $\mathbf{t 2}$ from 0 to 10 using 0.1 as step size and a vector y2 = 1-exp(-t2).

## Hands on!

First, the vector t :

$$
\gg t=[0: 10] ;
$$

Then, the vector y :
>> y = 1-exp(-t);

The vector $\mathbf{t 2}$ and $\mathbf{y} 2$ :
>> t2 = [0:0.1:10]; y = 1-exp(-t2);

## Hands on! (Plus)

Using the command plot, try to plot txy and t 2 xy 2 in the same figure.

## Tips:

The syntax for plot is plot(a,b).
$\mathbf{a}$ and $\mathbf{b}$ must be the same length.
You can plot more than one couple using the syntax ( $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ ).

## Extracting information of the vectors

There are several commands to get information from vectors.
Some examples are:
>> max(f)
>> f = [1 4 -6 3 7 9-2 6 3-7...
49 19];
ans =
>> length(f)
ans =
$\gg \min (f)$
13 19
ans =

## Extracting information of the vectors

First of all, we need the dot product of the vector $\mathbf{V}$.
Let's define $\mathbf{v}=\left[\begin{array}{ll}4 & 6 \\ 9\end{array}\right]$.
The array product of $\mathbf{v}$ is given by:

$$
\begin{aligned}
& \gg v . * v \\
& \text { ans }= \\
& 16 \quad 36 \quad 81
\end{aligned}
$$

## Extracting information of the vectors

Then, we need to sum the dot product of the vector $\mathbf{v}$ :

$$
\begin{aligned}
& \gg a=\operatorname{sum}(v . * v) \\
& a=
\end{aligned}
$$

$$
133
$$

The magnitude of $v$ is the square root of $a$.

$$
\begin{aligned}
& \gg \text { mag }=\text { sqrt }(a) \\
& \text { mag }= \\
& 11.5325
\end{aligned}
$$

## Operation with matrices

A matrix is a two-dimensional array of numbers. To create a matrix in Matlab, we enter each row as a sequence of comma (or space), and then use semicolons to mark the end of each row.

For example:

$$
\begin{array}{lll}
\gg A=\left[\begin{array}{ll}
1,4 ; 5 & 2
\end{array}\right] & \gg 2^{*} A \\
A= \\
1 & 4 & \text { ans }= \\
5 & 2 & 2
\end{array}
$$

## Operation with matrices

If two matrices have the same size, we can add or subtract them:

$$
\begin{aligned}
& \text { >> } B=[13 ;-1-4] ; \\
& \gg A+B \\
& \text { ans }= \\
& \begin{array}{rr}
2 & 7 \\
4 & -2
\end{array}
\end{aligned}
$$

## Operation with matrices

We can also compute the transpose of a matrix. The transpose operation switch the rows and columns in a matrix.

$$
\begin{array}{cc}
\gg A^{\prime} \\
\text { ans }= & \\
1 & 5 \\
4 & 2
\end{array}
$$

## Operation with matrices

If the matrix contains complex elements, the transpose will compute the conjugates:

$$
\begin{array}{ll}
\gg C=[1+i, 4-i ; 5+2 i, 3-3 i] & \gg C^{\prime} \\
C= & \text { ans }= \\
1+1 i \quad 4-1 & 1-1 i \\
5+2 i \quad 3-3 i & 4+i
\end{array}
$$

## Operation with matrices

If you want to compute the transpose of a matrix with complex elements without computing the conjugate, you use (.'):

$$
\begin{aligned}
& \gg \text { C.' } \\
& \text { ans }= \\
& 1+1 i \quad 5+2 i \\
& 4-i \quad 3-3 i
\end{aligned}
$$

## Operation with matrices

The array multiplication works with matrix as well. It is important to recognize that this is not matrix multiplication.

$$
\begin{aligned}
& \gg A=[1,4 ; 5 \quad 2] ; B=[13 ;-1-4] ; \\
& \gg A . * B \\
& \text { ans }= \\
& \begin{array}{rl}
1 & 12 \\
-5 & -8
\end{array}
\end{aligned}
$$

## Matrix multiplication

Let's consider two matrices:

$$
\gg C=[2,1 ; 1,2] ; D=[3,4 ; 5,6] ;
$$

The multiplication between them will be:

$$
\begin{array}{ll}
\gg \text { C*D } & \\
\text { ans }= \\
11 & 14 \\
13 & 16
\end{array}
$$

## Special matrix types

The identify matrix is a square matrix that has ones along the diagonal and zeros elsewhere. To create a n-order identify matrix, type:

$$
\begin{aligned}
& \text { >> eye(n); } \\
& \text { >> eye(2) } \\
& \text { ans }= \\
& 1
\end{aligned}
$$

## Special matrix types

To create a matrix of zeros, type:
>> zeros(n) \% n-order matrix of zeros
>> zeros(m,n) \% mxn matrix of zeros

To create a matrix of ones, type ones(n) or ones(m,n).

## Referencing matrix elements

Individuals elements and columns in a matrix can be referenced using Matlab. Consider the matrix:

$$
\left.\begin{array}{l}
\gg A=[123 ; 456 ; 789] \\
A= \\
1 \\
1 \\
4 \\
5
\end{array}\right] \begin{array}{lll} 
& 2 & 6 \\
7 & 8 & 9
\end{array} l
$$

## Referencing matrix elements

We can pick out the element at row position $m$ and column position $n$ by typing $A(m, n)$.

For example:

$$
\begin{aligned}
& \gg A(2,3) \\
& \text { ans }=
\end{aligned}
$$

6

## Referencing matrix elements

To reference all elements in the ith column, we write $A(:, i)$.

$$
\begin{gathered}
\gg A(:, 2) \\
\text { ans }= \\
2 \\
5 \\
8
\end{gathered}
$$

## Referencing matrix elements

To pick out the elements in the ith through jth column, we type $A(:, i: j)$.

$$
\begin{aligned}
& \gg A(:, 2: 3) \\
& \text { ans }= \\
& 2
\end{aligned}
$$

## Referencing matrix elements

We can pick out pieces or sub matrices as well.

```
>> A(2:3,1:2)
ans =
    4
    7
```


## Referencing matrix elements

We can change the value of matrix elements using these references as well.

$$
\begin{aligned}
& \gg A(1,1)=-8 \\
& \text { ans = } \\
& -8 \quad 2 \quad 3 \\
& 456 \\
& 789
\end{aligned}
$$

## Three-Dimensional Plots



## Hands on!

example, if the coordinates $x, y$, and $z$ are given as a function of the parameter t by:

$$
\begin{gathered}
x=v t * \sin \left(2^{*} t\right) \\
y=v t^{*} \cos \left(2^{*} t\right) \\
z=0.5^{*} t \\
\text { For } 0 \leq t \leq 6^{*} \pi
\end{gathered}
$$

Using the command plot3, try to plot $\mathrm{tx}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ in the same figure.

## Hands on!

$$
\begin{aligned}
& \mathrm{t}=0: 0.1: 6^{*} \mathrm{pi} ; \\
& \mathrm{x}=\operatorname{sqrt}(\mathrm{t}) . .^{*} \sin (2 * \mathrm{t}) ; \\
& \mathrm{y}=\operatorname{sqrt}(\mathrm{t}) .{ }^{*} \cos \left(2{ }^{*} \mathrm{t}\right) ; \\
& \mathrm{z}=0.5^{*} \mathrm{t} ; \\
& \text { plot3(x,y,z,'k','linewidth',1) } \\
& \text { grid on } \\
& \text { xlabel(' } \mathrm{x} \text { '); ylabel('y'); zlabel('z') }
\end{aligned}
$$

## Determinants and Linear Systems

To calculate the determinant of a matrix A in Matlab, simply write $\operatorname{det}(A)$.
For example:

$$
\begin{aligned}
& \gg A=[13 ; 45] ; \operatorname{det}(A) \\
& \text { ans }= \\
& -7
\end{aligned}
$$

## Determinants and Linear Systems

Consider the following set of equations:

$$
\begin{aligned}
& 5 x+2 y-9 z=44 \\
& -9 x-2 y+2 z=11 \\
& 6 x+7 y+3 z=44
\end{aligned}
$$

To find a solution to a system of equations like this, we can use two steps.

## Determinants and Linear Systems

First, we find the determinant of the coefficient matrix A:

$$
A=\left(\begin{array}{ccc}
5 & 2 & -9 \\
-9 & -3 & 2 \\
6 & 7 & 3
\end{array}\right)
$$

>> A = [5 2-9; -9-3 2; 673 3]; $\operatorname{det}(\mathrm{A})$
ans =
368
When the determinant is nonzero, a solution exists. This solution is the column vector:

$$
X=\left(\begin{array}{l}
x \\
y \\
z
\end{array}\right)
$$

## Determinants and Linear Systems

Matlab allows us to generate the solution readily using left division. First we need create a column of the numbers on righthand side of the system. We find:

$$
\begin{aligned}
& \gg b=[44 ; 11 ; 5] ; \\
& \gg A \backslash b \\
& \text { ans = } \\
& {[-5.12507 .6902-6.0272]^{\prime}}
\end{aligned}
$$

## Determinants and Linear Systems

Another way to solve linear system problems is check the rank of the system. Let's consider the linear system of equations with $m$ equations and n unknowns:

$$
\mathbf{A x}=\mathbf{b}
$$

The augmented matrix is formed by concatenating the vector $\mathbf{b}$ onto the matrix A .
[A b]

## Determinants and Linear Systems

The system has a solution if and if only $\operatorname{rank}(A)=\operatorname{rank}([A b])$. If the rank is equal to $n$, then the system has a unique solution.

If $\operatorname{rank}(A)=\operatorname{rank}([A b])$ but the rank $<n$, there are infinite number of solutions. If we denote the rank by $r$, then $r$ of the unknown variables can be expressed as linear combination of $n-r$ the other variables.

## Determinants and Linear Systems

To compute the rank of a matrix, you can use the Matlab command $\operatorname{rank}(\mathrm{A})$, for example.
>> rank(A);

## Hands on!

Let's consider the linear system

$$
\begin{gathered}
x-2 y+z=12 \\
3 x+4 y+5 z=20 \\
-2 x+y+7 z=11
\end{gathered}
$$

Find the solution using the Matlab command rank and the left division.

## Inverse and pseudoinverse of a matrix

Matlab has commands to compute the inverse and pseudoinverseof a matrix. The syntax is:

$$
\begin{aligned}
& \gg A=[1,2 ; 3,4] ; \\
& \gg \operatorname{inv}(A) ; \% \text { For inverse of the matrix } A \\
& \gg \operatorname{pinv}(A) ; \% \text { For the pseudoinverse of } \\
& \text { the matrix } A
\end{aligned}
$$

## Decomposition of a matrix

Matlab can computes the LU decomposition of a matrix using the command lu.

$$
\begin{aligned}
& \gg A=[123 ; 321 ; 7511] ; b=[4 ; 2 ;-1] ; \\
& \gg[L, U]=\operatorname{lu}(A) ;
\end{aligned}
$$

$$
\mathrm{L}=
$$

0.14291 .00000
0.4286 -0.1111
1.0000
1.000000

U =
7.00005 .000011 .0000
01.28571 .4286

0 0-3.555659

## Decomposition of a matrix

Matlab can computes the LU decomposition of a matrix using the command lu.

$$
\begin{aligned}
& \gg A=[123 ; 321 ; 7511] ; b=[4 ; 2 ;-1] ; \\
& \gg[L, U]=\operatorname{lu}(A) ;
\end{aligned}
$$

| $\mathrm{L}=$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1429 | 1.0000 | 0 | $\mathrm{U}=$ |  |  |
| 0.4286 | -0.1111 | 1.0000 | 7.0000 | 5.0000 | 11.0000 |
| 1.0000 | 0 | 0 | 0 | 1.2857 | 1.4286 |
|  |  | 0 | 0 | -3.5556 |  |

## Decomposition of a matrix

To solve the linear system, you need to solve the equation:

$$
\begin{aligned}
& x=U \backslash(L \backslash \mathrm{~b}) \\
& \\
& \gg \mathrm{x}=U \backslash(\mathrm{~L} \backslash \mathrm{~b}) \\
& \mathrm{x}= \\
& -1.8125 \\
& 4.1250 \\
& -0.8125
\end{aligned}
$$

## Checkpoint

Let's put our hands on practical programming things. Go to EESC Moodle's website and download the file Checkpoint 1.pdf

## References

[1] Matlab Product Help.
[2] Matlab Demystified. A Self-Teaching Guide, David McMahon, McGraw Hill.
[3] Matlab: An Introduction with Applications, Amos Gilat, Fourth Edition, JOHN WILEY \& SONS.

