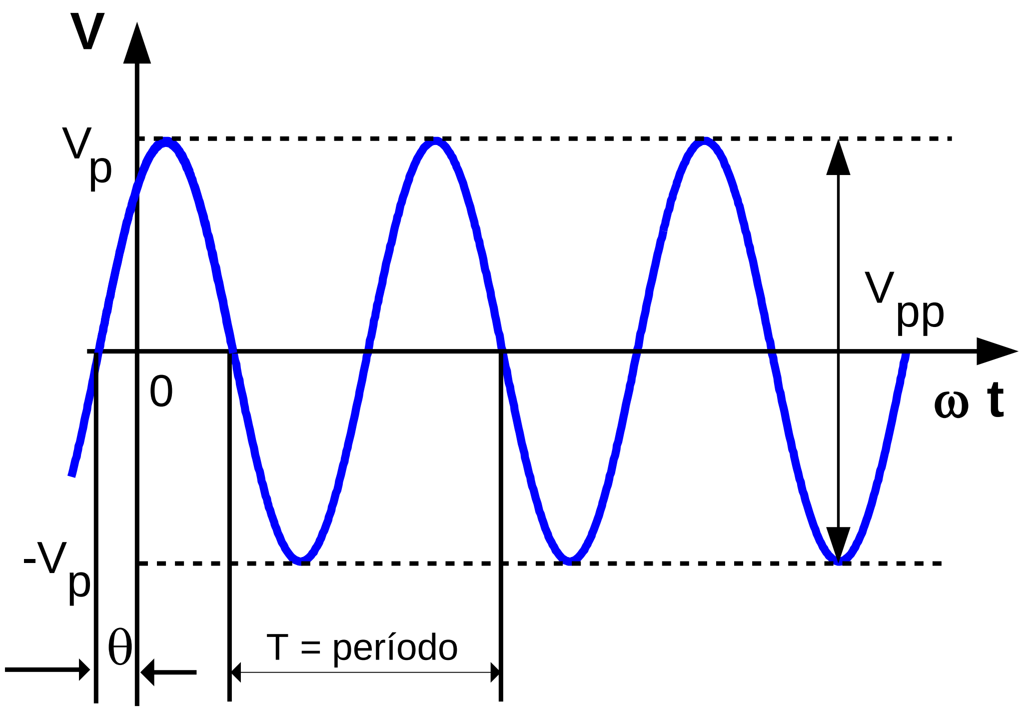
Circuitos CA
Resposta à Função Senoidal

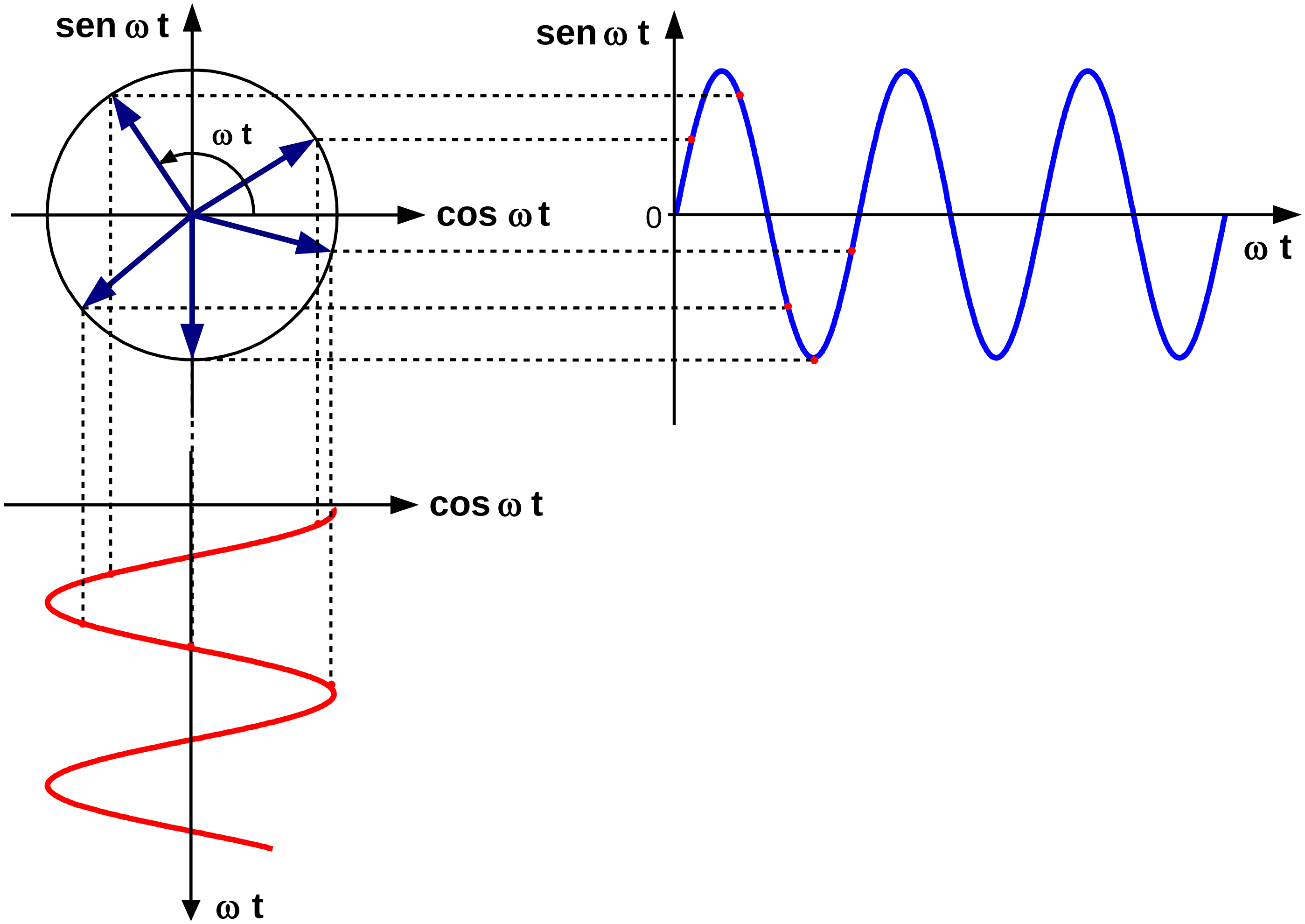
- Sinal senoidal
- Representação do sinal senoidal em variável complexa: fasores
- Impedância e Admitância
- Função de Transferência
- Resposta em frequência

Sinal Senoidal

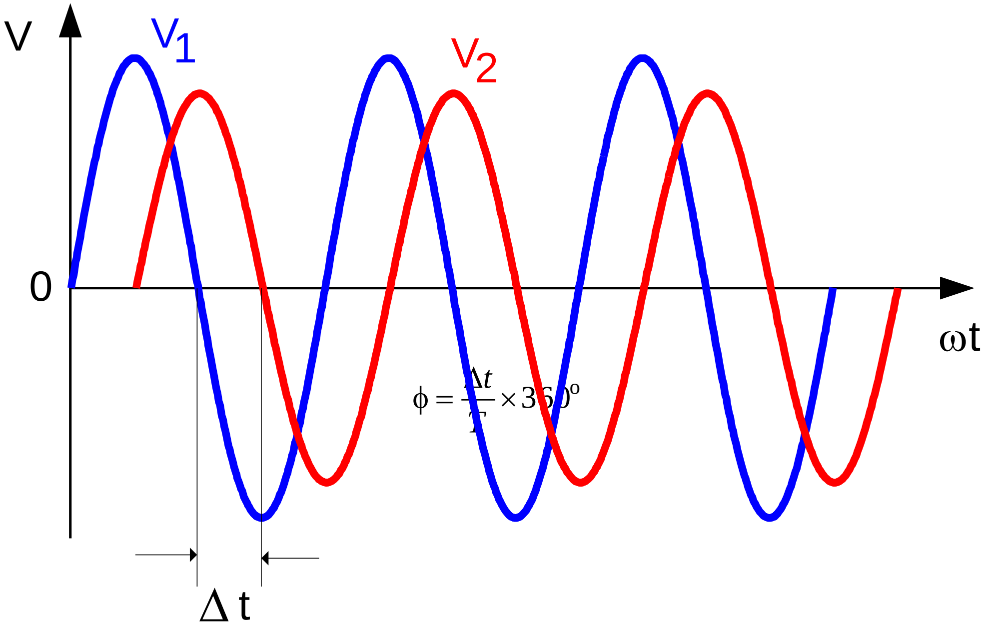


Forma geral da função senoidal: $v(t) = V_p\, sen(\omega t + \theta)$

Sinal Senoidal



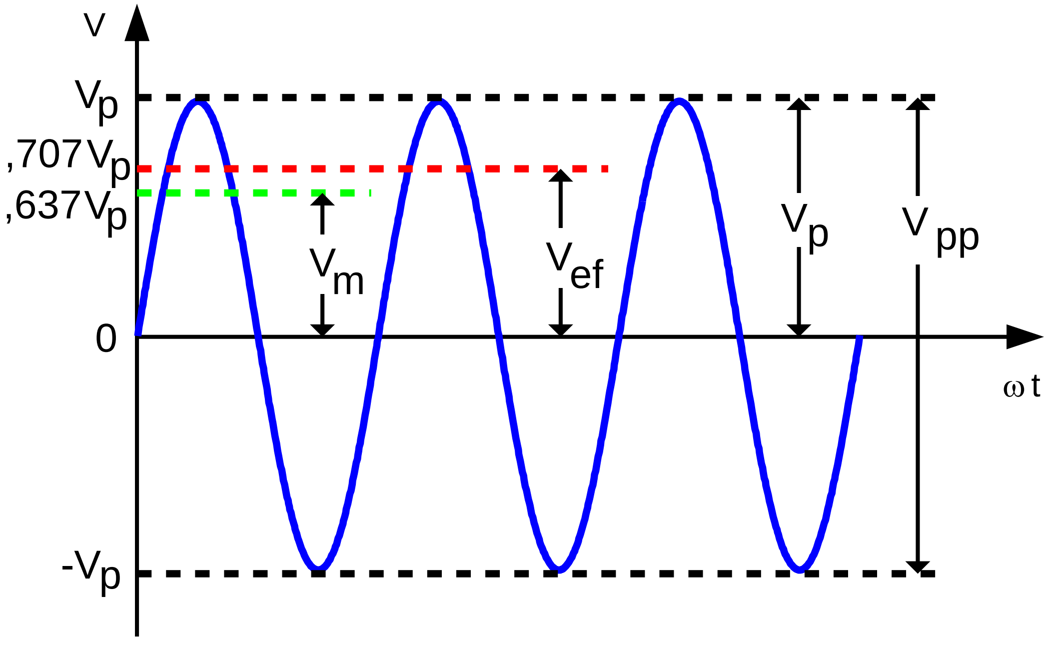
Diferença de Ângulo de Fase



Período: $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Diferença de fase:

Valor Médio e Valor Eficaz



Valor médio: $$V_m = \frac{2}{T}\int_0^{\frac{T}{2}} V_S dt = \frac{1}{\pi}\int_0^{\pi} V_p \,\mathrm{sen}\,\omega t = \frac{2V_p}{\pi} \cong 0{,}637V_p$$

Valor eficaz: $$V_{ef} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \,\mathrm{sen}^2\,\omega t dt} = \frac{V_p}{\sqrt{2}} \cong 0{,}707V_p$$

Ffunção senoidal

Seja a função senoidal: $v(t) = V_p \, sen(\omega t + \theta)$

Substituindo a relação trigonométrica (5):

$$v(t) = V_p \left(sen\omega t \cdot \cos\theta + sen\theta \cdot \cos\omega t \right) \quad = A \, sen\omega t + B \cos \omega t$$

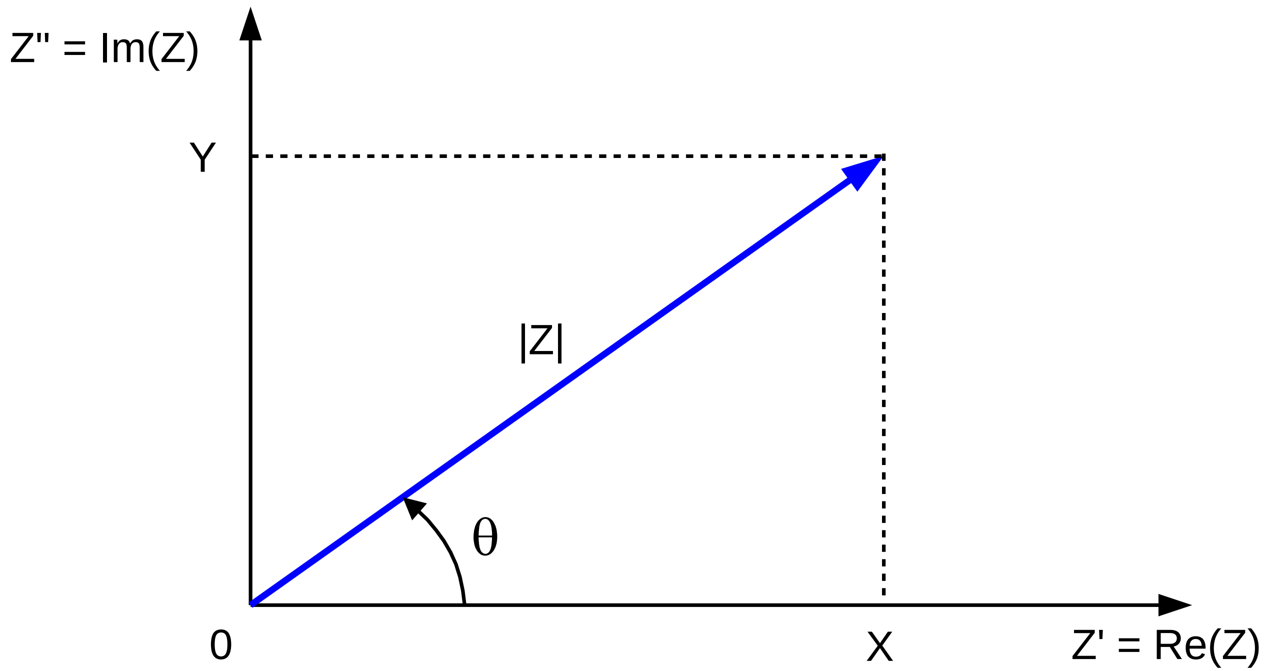
na qual:

$$v(t) = \sqrt{A^2 + B^2} \, sen\left[\omega t + arctg\left(\frac{B}{A} \right) \right]$$

$V_p = \sqrt{A^2 + B^2}$ e $\theta = arctg\left(\frac{B}{A} \right)$

Variável Complexa

Variável complexa Z em coordenadas retangulares:



Módulo:

$$|Z| = \sqrt{X^2 + Y^2}$$

Ângulo de fase:

$$\theta = arctg\frac{Y}{X}$$

Variável Complexa

Variável complexa: $Z = X + jY$

Componentes da variável complexa Z:

Componente real Z' = Re(Z): $X = |Z| \cdot cos\,\theta$

Componente imaginária Z'' = Im(Z): $Y = |Z| \cdot sen\,\theta$

Variável Complexa

Variável complexa Z em coordenadas polares:

$$Z = |Z| \cdot (\cos\theta + j \cdot sen\theta)$$

$$e^{j\theta} = \cos\theta + j \cdot sen\theta$$

Identidade de Euler:

Forma exponencial (polar) da variável complexa Z:

$$Z = |Z| \cdot e^{j\theta}$$

Variável Complexa

Variável complexa: $Z = X + jY$

Variável complexa conjugada: $Z^* = X - jY$

$$Z \cdot Z^* = (X + jY) \cdot (X - jY)$$

Módulo: $|Z| = \sqrt{Z \cdot Z^*} = \sqrt{X^2 + Y^2}$

Variável Complexa

Variável complexa Z em coordenadas polares:

$$Z = |Z| \cdot (cos\theta + j \cdot sen\theta)$$

Variável complexa conjugada Z* em coordenadas polares:

$$Z^* = |Z| \cdot (cos\theta - j \cdot sen\theta)$$

Forma exponencial (polar) da variável complexa Z conjugada:

$$Z^* = |Z| \cdot e^{-j\theta}$$

Variável Complexa

Fatoração de variáveis complexas

Seja a variável complexa: $$Z = \frac{1}{X + jY}$$

Fatoramos dividindo e multiplicando Z pelo complexo conjugado X - jY:

$$Z = \frac{1}{X + jY} \cdot \frac{X - jY}{X - jY} = \frac{X - jY}{X^2 + Y^2}$$

De modo que:

$$Z = \frac{X}{X^2 + Y^2} - \frac{jY}{X^2 + Y^2}$$

No qual:

$$Z' = Re(Z) = \frac{X}{X^2 + Y^2}$$

e

$$Z'' = Im(Z) = -\frac{Y}{X^2 + Y^2}$$

Fasores

Quantidade física que é expressa como uma variável complexa:

$$Z = Z' + j \cdot Z''$$ Coordenadas retangulares

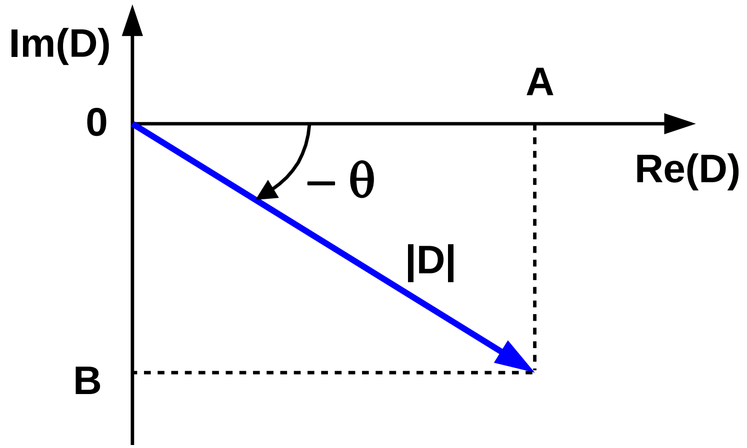
$$Z = |Z| \cdot (cos\,\theta + j \cdot sen\,\theta)$$ Coordenadas polares

Notação polar do fasor Z:

$$Z = |Z| \cdot e^{j\theta} = |Z| \angle \theta$$

Fasores

Equação na forma fasorial: $A - jB = |D| \angle \theta$



Módulo: $|D| = \sqrt{A^2 + B^2}$

Ângulo de fase: $\theta = -arctg(B / A)$

Fasores

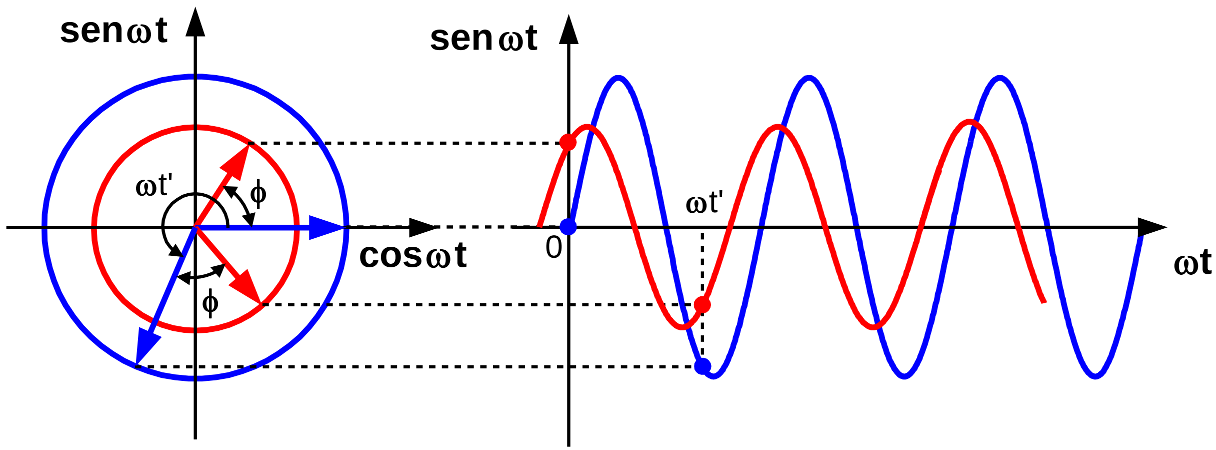
Função senoidal na forma complexa:

$$v(t) = V_p\, sen(\omega t + \theta) = V_p\,(A sen\omega t + B\cos\omega t)$$

$$V_p = \sqrt{A^2 + B^2}$$

$$\theta = arctg\left(\frac{B}{A}\right)$$

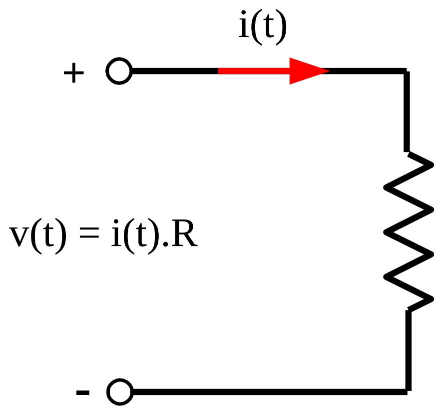
Fasores



Relação de Fasores para Elementos de Circuito

- Circuito Resistivo
- Circuito Capacitivo
- Circuito Indutivo

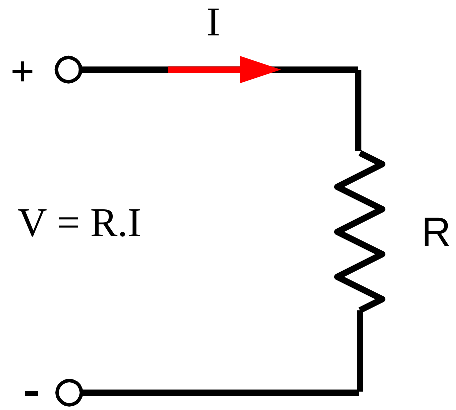
Fasores para o Circuito Resistivo



$$v(t) = R \cdot i(t)$$

$$V_p\, e^{j(\omega t + \theta_v)} = R \cdot I_p\, e^{j(\omega t + \theta_i)}$$

$$V_p\, e^{j\theta_v} = R \cdot I_p\, e^{j\theta_i}$$



Forma fasorial:

$$V = R \cdot I$$

$$V = V_p \angle \theta_v$$

$$I = I_p \angle \theta_i$$

Fasores para o Circuito Resistivo

Forma de onda temporal

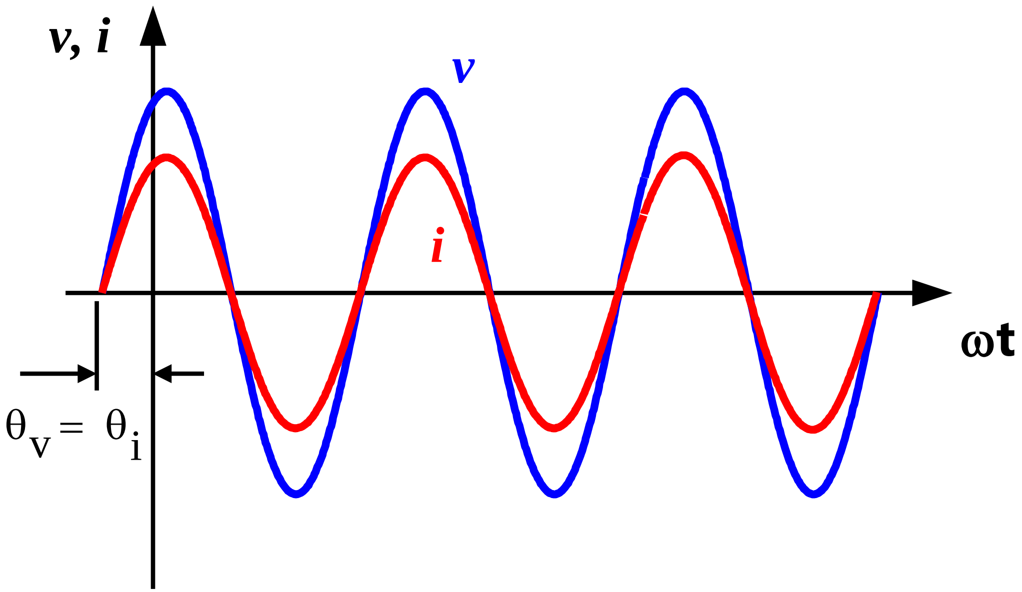
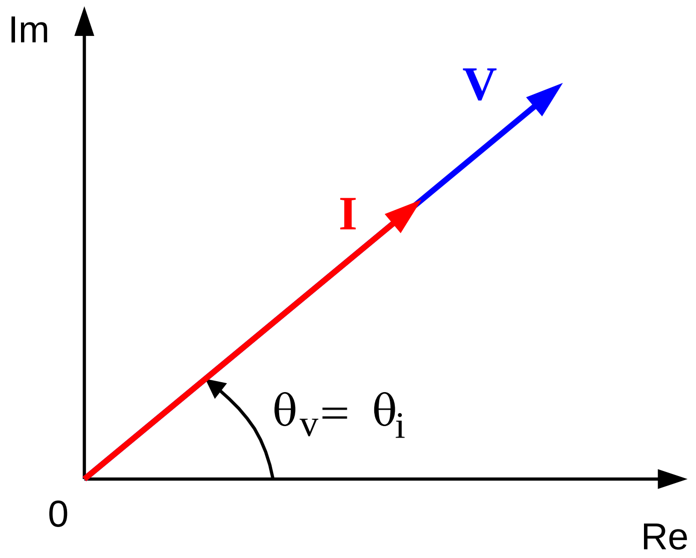
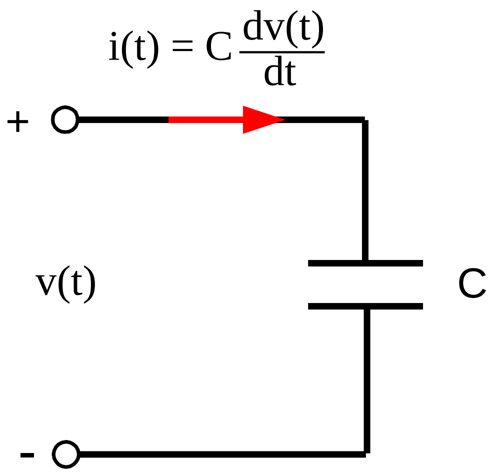
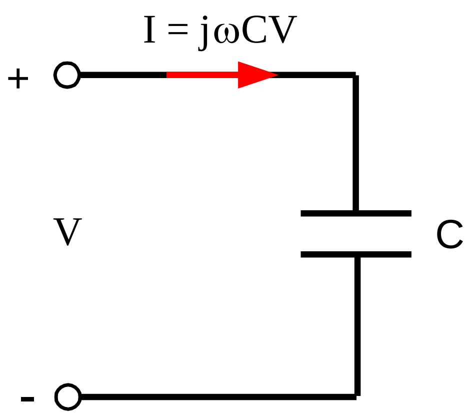


Diagrama fasorial



Fasores para o Circuito Capacitivo





$$i(t) = C \cdot \frac{dv(t)}{dt}$$

$$I_p\, e^{j(\omega t + \theta_i)} = C \cdot \frac{d}{dt} V_p\, e^{j(\omega t + \theta_v)}$$

$$I_p\, e^{j\theta_i} = j\omega C V_p\, e^{j\theta_v}$$

Forma fasorial:

$$I = j\omega C V$$

$$V = V_p \angle \theta_v$$

$$j = 1\, e^{j \cdot 90^o}$$

$$I = I_p \angle \theta_v + 90^o$$

Fasores para o Circuito Capacitivo

Forma de onda temporal

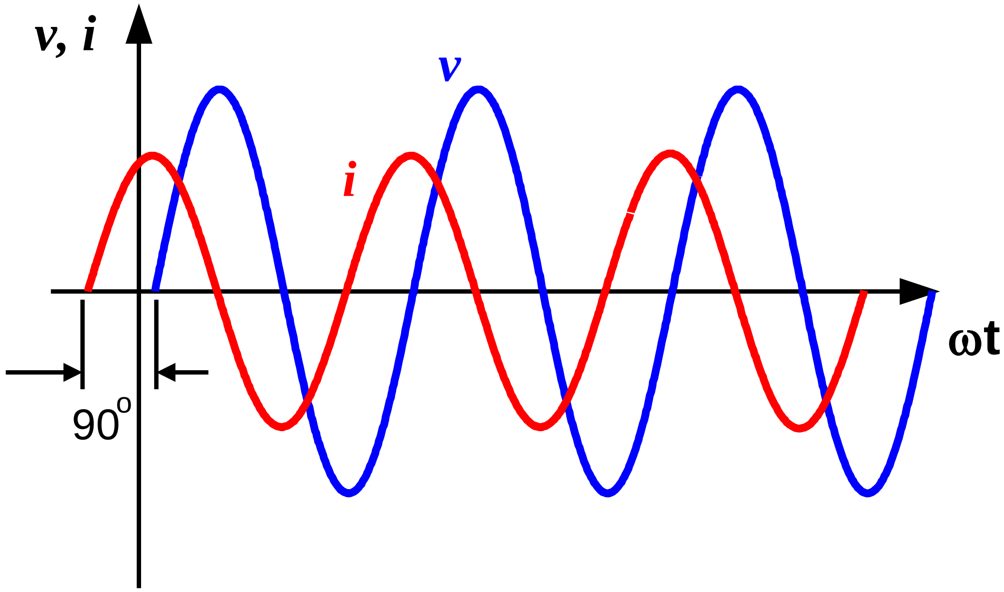
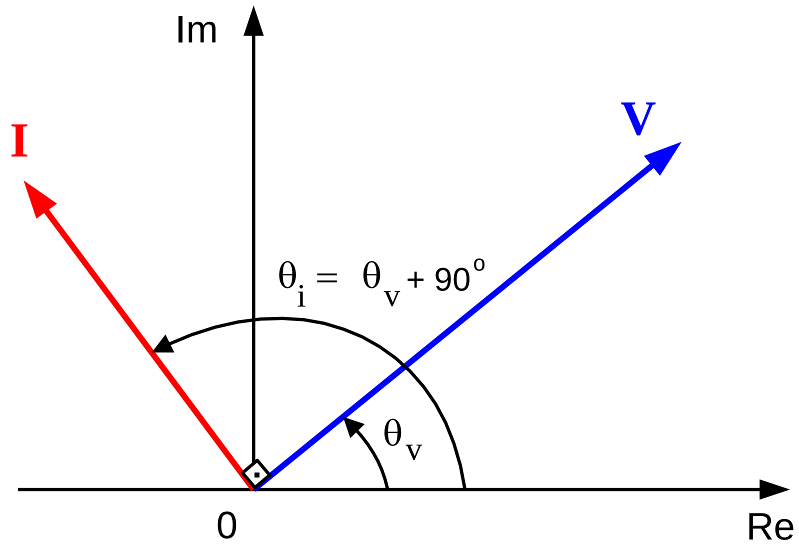
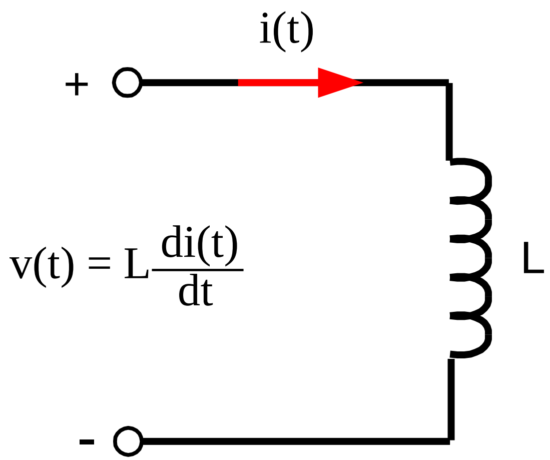


Diagrama fasorial



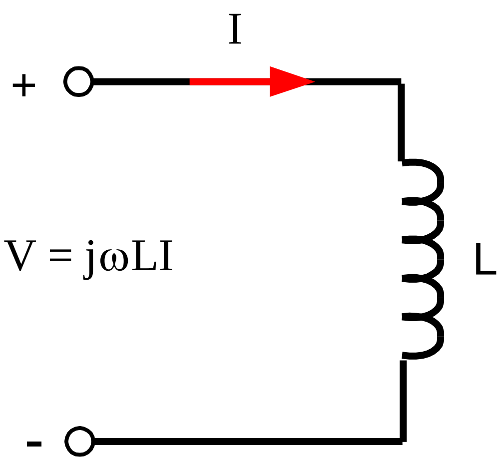
Fasores para o Circuito Indutivo



$$v(t) = L \cdot \frac{di(t)}{dt}$$

$$V_p\, e^{j(\omega t + \theta_v)} = L \cdot \frac{d}{dt} I_p\, e^{j(\omega t + \theta_i)}$$

$$V_p\, e^{j\theta_v} = j\omega L\, I_p\, e^{j\theta_i}$$



Forma fasorial:

$$V = j\omega LI$$

$$V = V_p \angle \theta_v$$

$$-j = 1\, e^{-j \cdot 90^{o}}$$

$$I = I_p \angle \theta_v - 90^{o}$$

Fasores para o Circuito Indutivo

Forma de onda temporal

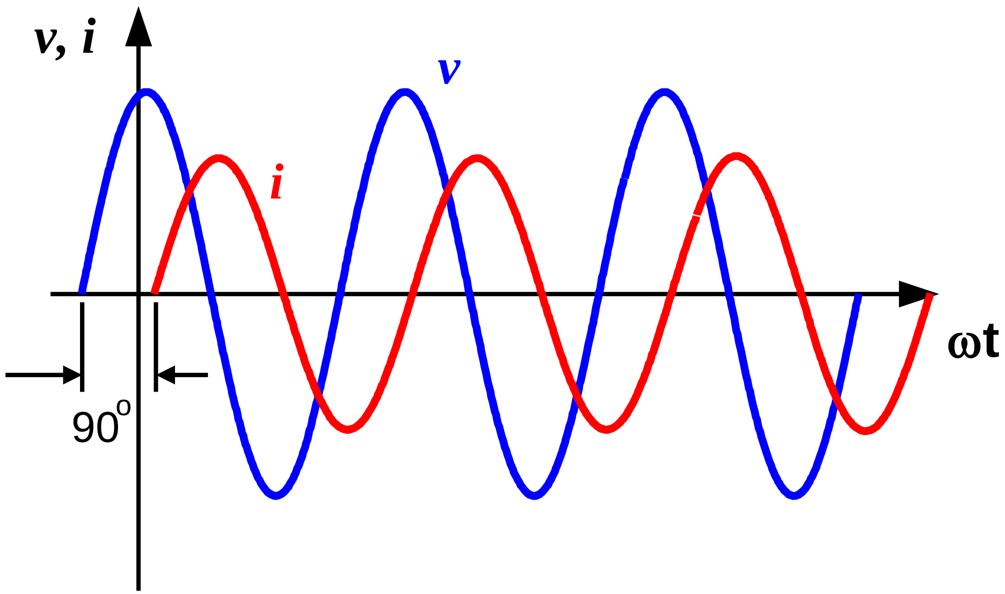
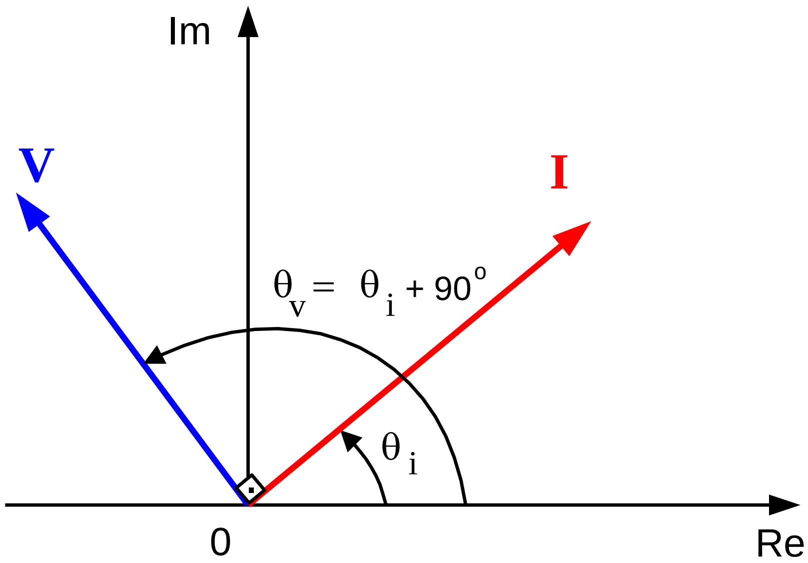


Diagrama fasorial



Impedância

Definição de impedância: $Z = \frac{V}{I}$ (Ohm = Ω)

Impedância como fasor: $$Z = \frac{V_p \angle \theta_v}{I_p \angle \theta_i} = \frac{V_p}{I_p} \cdot \angle \theta_v - \theta_i = |Z| \angle \theta_Z$$

$$Z(\omega) = R(\omega) + j \cdot X(\omega)$$

Componente real ou ativa: $R(\omega)$ (Resistência)

Componente imaginária ou reativa: $X(\omega)$ (Reatância)

Impedância

$$Z = |Z| \angle \theta_Z = R + j\,X$$

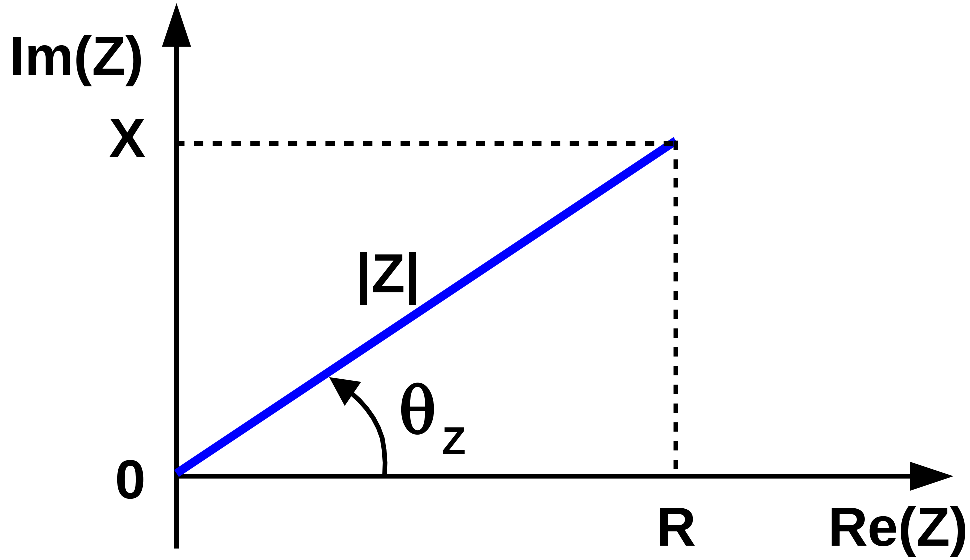
na qual:

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_Z = arctg\,\frac{X}{R}$$

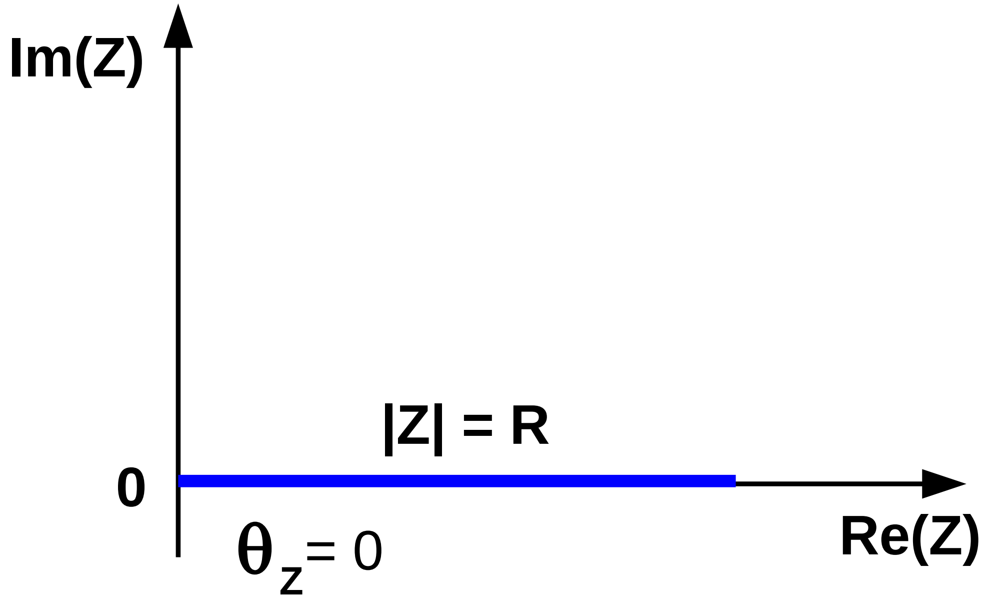
$$R = |Z| \cdot cos\,\theta_Z$$

$$X = |Z| \cdot sen\theta_Z$$



Impedância de Elementos Passivos

Resistor, R $Z_R = R\angle 0^{o}$



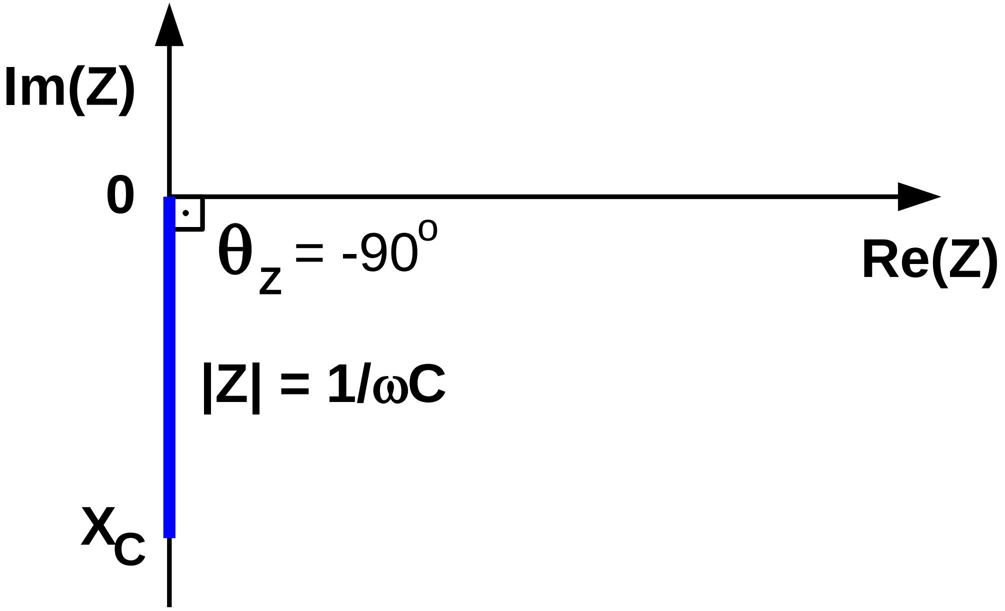
Impedância de Elementos Passivos

Capacitor, C

$$Z_C = \frac{1}{j\omega C} = -j\,X_C = \frac{1}{\omega C}\angle -90^o$$

Reatância capacitiva:

$$X_C = \frac{1}{\omega C}$$



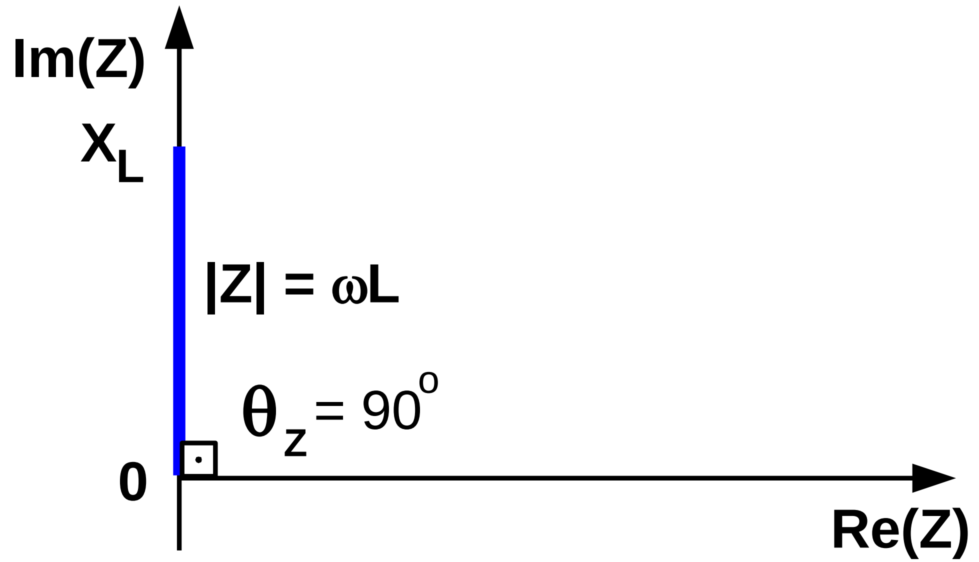
Impedância de Elementos Passivos

Indutor, L

$$Z_L = j\omega L = j\,X_L = \omega L \angle 90^o$$

Reatância indutiva:

$$X_L = \omega L$$



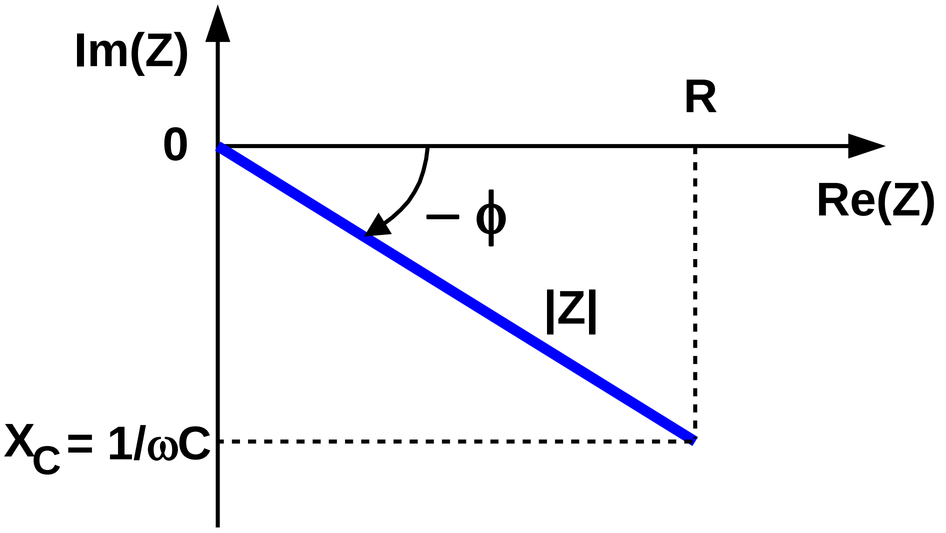
Impedância de Circuito RC em Série

Impedância na forma retangular: $$Z = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

Impedância na forma fasorial: $Z = |Z| \angle \phi$

$$|Z| = \sqrt{R^2 + 1/(\omega C)^2}$$

$$\phi = -arctg\left(\frac{1}{\omega RC}\right)$$



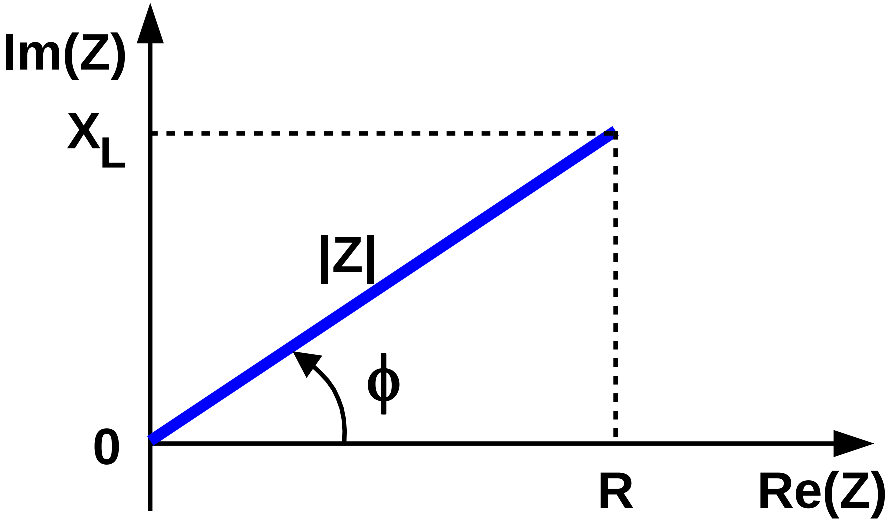
Impedância de Circuito RL em Série

Impedância na forma retangular: $Z = R + j\omega L$

Impedância na forma fasorial: $Z = |Z| \angle \phi$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\phi = arctg\left(\frac{\omega L}{R}\right)$$



Admitância

Definição de admitância: $Y = \frac{I}{V} = \frac{1}{Z}$ (siemens = S)

Admitância como fasor: $Y = \frac{I_p \angle \theta_i}{V_p \angle \theta_v} = \frac{I_p}{V_p} \cdot \angle \theta_i - \theta_v = |Y| \angle \theta_Y$

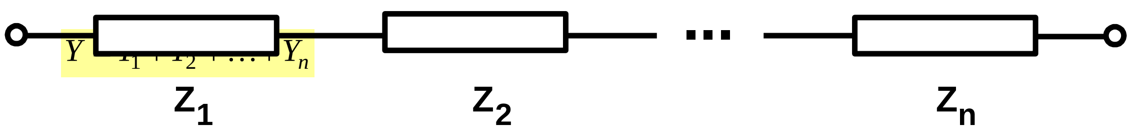
$$Y(\omega) = G(\omega) + j \cdot B(\omega)$$

Condutância: $G(\omega) = \frac{1}{R}$

Susceptância: $B(\omega) = \frac{1}{X}$

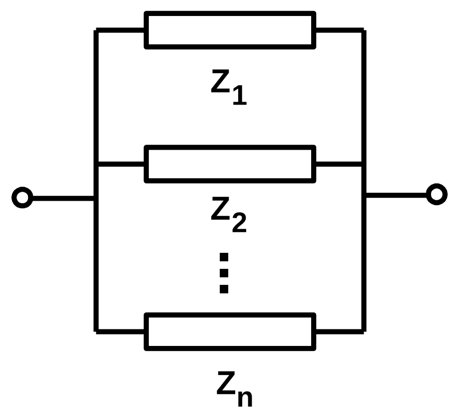
Impedância Equivalente

Associação de Impedâncias em Série: $Z = Z_1 + Z_2 + \ldots + Z_n$



Associação de Impedâncias em Paralelo:

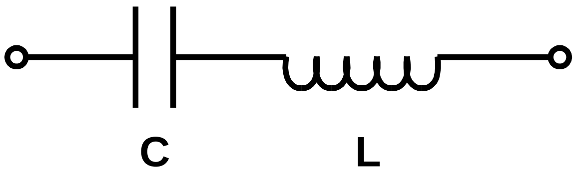
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}$$



Impedância Equivalente

$$\phi = arctg\left(\frac{Z''}{Z'}\right) = \begin{cases} -90^o, & se \quad 1/\omega C > \omega L \\ 90^o, & se \quad 1/\omega C < \omega L \end{cases}$$

Exemplo: Associação de Impedâncias em Série



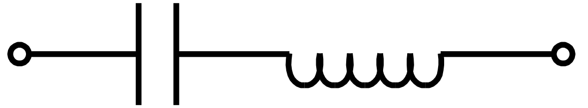
Impedância na forma retangular:

$$Z = Z_C + Z_L = -\frac{j}{\omega C} + j\omega L = j \cdot \left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2}$$

Impedância Equivalente

Exemplo: Associação de Impedâncias em Série (cont.)



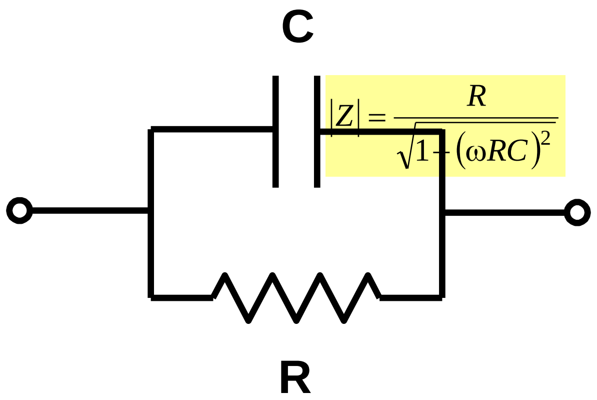
Impedância na forma polar e fasorial:

$$Z = Z_C + Z_L = \left(\omega L - \frac{1}{\omega C} \right) \cdot e^{\pm j \cdot 90^{\circ}} = \left(\omega L - \frac{1}{\omega C} \right) \angle \pm 90^{\circ}$$

Impedância Equivalente

Exemplo: Associação de Impedâncias em Paralelo

$$|Z| = \frac{R}{\sqrt{1+(\omega RC)^2}}$$



$$Y = Y_R + Y_C = \frac{1}{R} + j\omega C = \frac{1 + j\omega RC}{R}$$

$$Z = \frac{1}{Y} = \frac{R}{1 + j\omega RC} \cdot \frac{1 - j\omega RC}{1 - j\omega RC} = \frac{R - j\omega CR^2}{1 + (\omega RC)^2}$$

$$Z = \frac{R}{1 + (\omega RC)^2} - \frac{j\omega CR^2}{1 + (\omega RC)^2}$$

$$\phi = -arctg(\omega RC)$$

Impedância Equivalente

Exemplo: Associação de Impedâncias em Paralelo (cont.)

$$Y = Y_R + Y_C = \frac{1 + j\omega RC}{R} = \frac{\sqrt{1 + (\omega RC)^2} \cdot \angle arctg(\omega RC)}{R\angle 0^o}$$

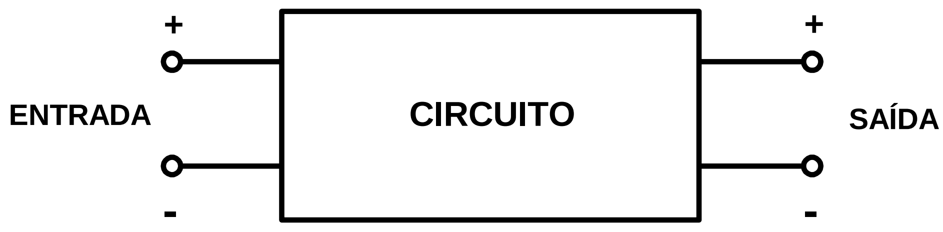
$$Z = \frac{1}{Y} = \frac{R\angle 0^o}{\sqrt{1 + (\omega RC)^2}\ \angle arctg(\omega RC)} = \frac{R}{\sqrt{1 + (\omega RC)^2}} \angle - arctg(\omega RC)$$

$$Z = |Z| \cdot \angle \phi$$

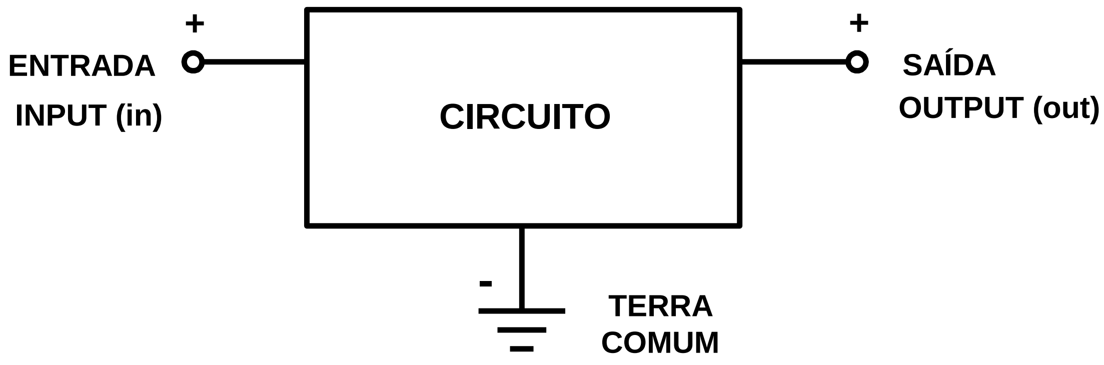
$$\phi = - arctg(\omega RC)$$

Função de Transferência

Representação simplificada de um circuito:

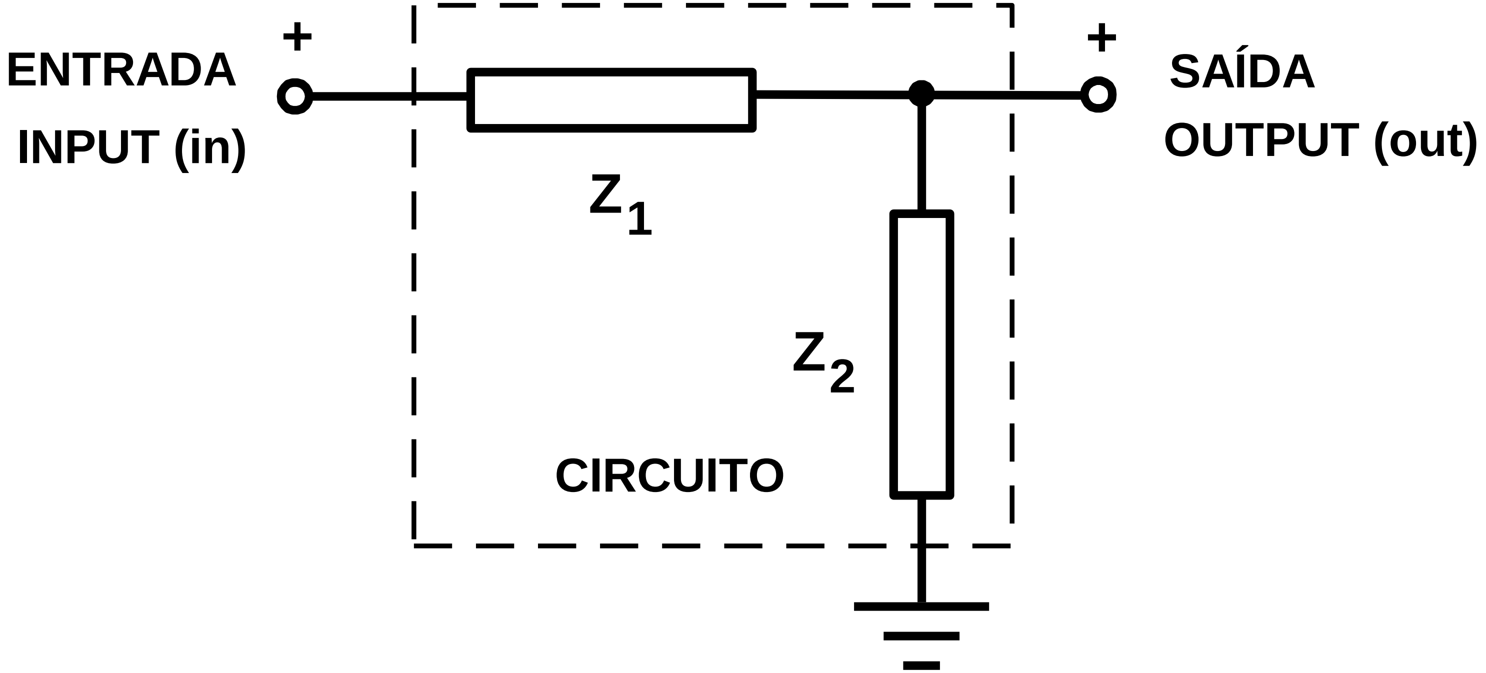


Representação simplificada de um circuito com terra comum:

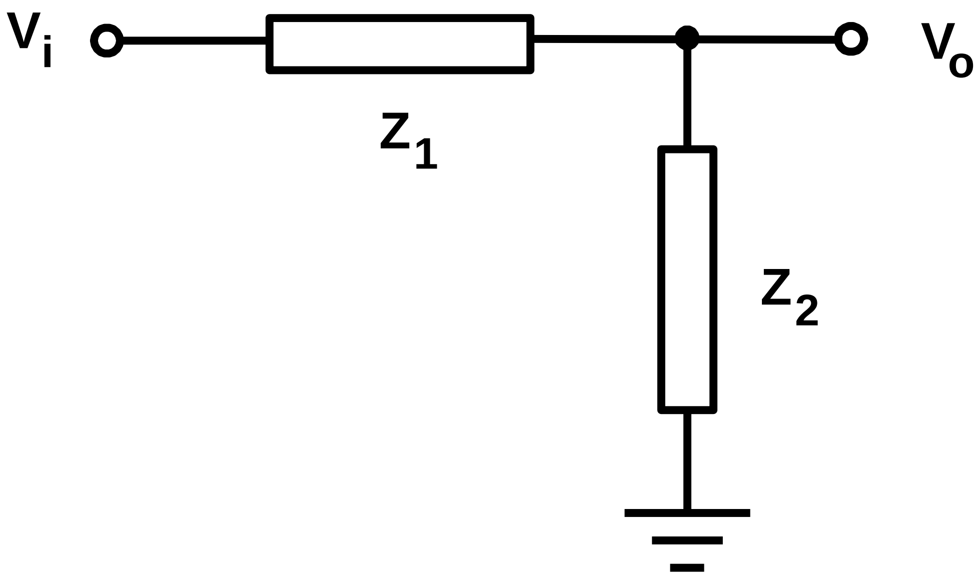


Função de Transferência

Representação simplificada de um circuito:



Função de Transferência

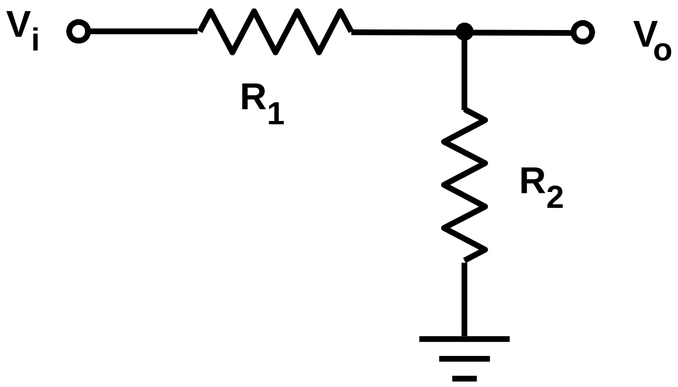


Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

Função de Transferência

Circuito Divisor de Tensão Resistivo

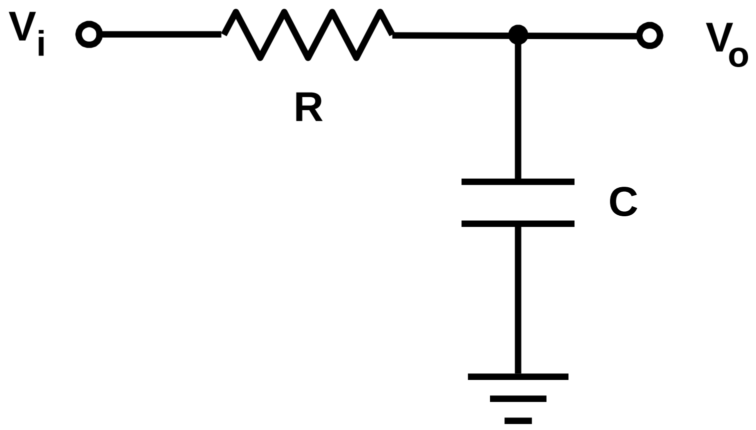


Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

Função de Transferência

Circuito RC em Série



Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{Z_C}{R + Z_C} = \frac{-j/\omega C}{R - j/\omega C}$$

$$A_V = \frac{-j}{\omega RC - j} \cdot \frac{\omega RC + j}{\omega RC + j} = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

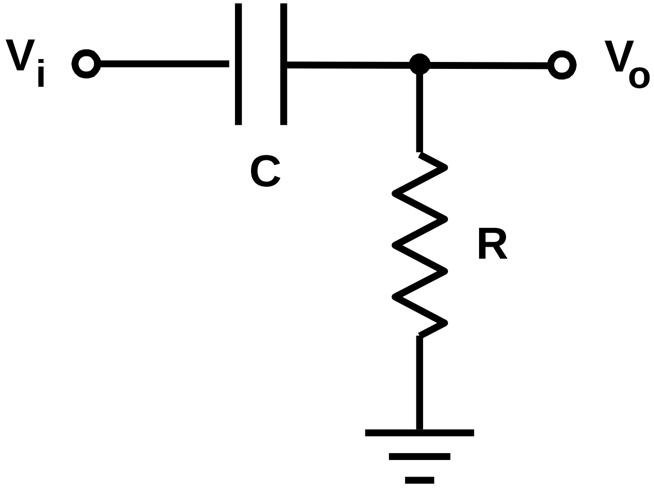
Fasor $A_V = |A_V| \angle \phi$

$$|A_V| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi = -arctg(\omega RC)$$

Função de Transferência

Circuito RC em Série



Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{R}{R + Z_C} = \frac{R}{R - j/\omega C}$$

$$A_V = \frac{\omega RC}{\omega RC - j} \cdot \frac{\omega RC + j}{\omega RC + j} = \frac{(\omega RC)^2 + j\omega RC}{1 + (\omega RC)^2}$$

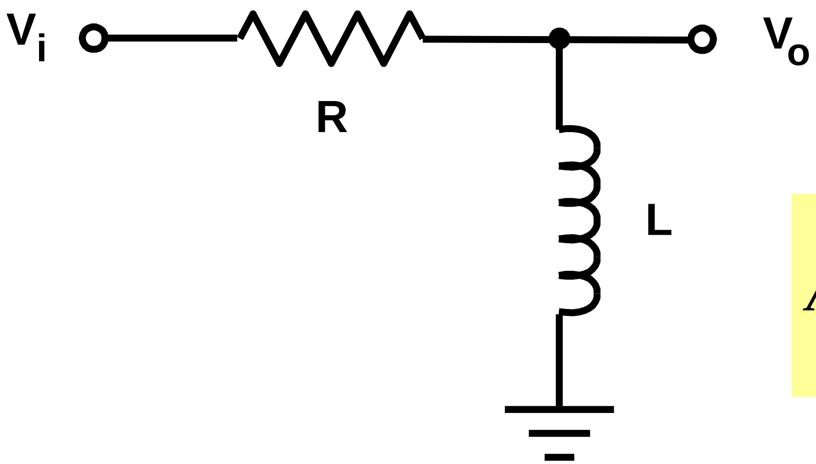
Fasor $A_V = |A_V| \angle \phi$

$$|A_V| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi = arctg\left(\frac{1}{\omega RC}\right)$$

Função de Transferência

Circuito RL em Série



Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{Z_L}{R + Z_L} = \frac{j\omega L}{R + j\omega L}$$

$$A_V = \frac{j\omega L}{R + j\omega L} \cdot \frac{R - j\omega L}{R - j\omega L} = \frac{(\omega L)^2 + j\omega RL}{R^2 + (\omega L)^2}$$

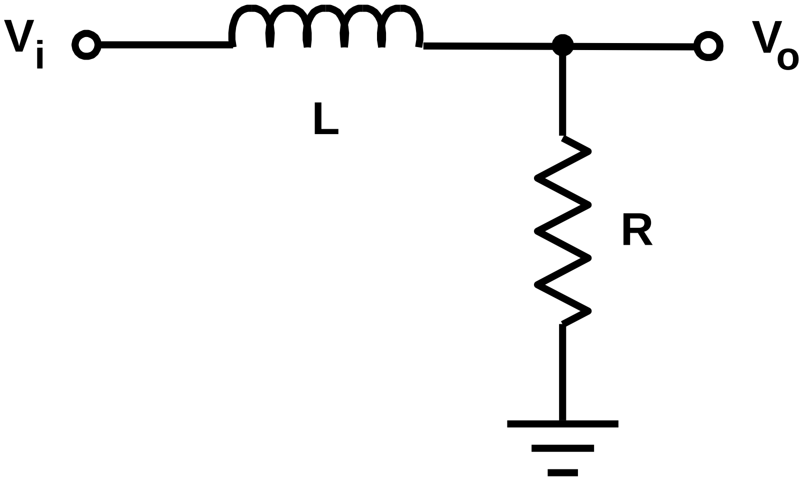
Fasor $A_V = |A_V| \angle \phi$

$$|A_V| = \frac{\omega (L/R)}{\sqrt{1 + (\omega L/R)^2}}$$

$$\phi = arctg\left(\frac{1}{\omega L/R}\right)$$

Função de Transferência

Circuito RL em Série



Razão de transferência, A_V:

$$A_V = \frac{V_o}{V_i} = \frac{R}{R+Z_L} = \frac{R}{R+j\omega L}$$

$$A_V = \frac{R}{R+j\omega L} \cdot \frac{R-j\omega L}{R-j\omega L} = \frac{R^2 - j\omega RL}{R^2 + (\omega L)^2}$$

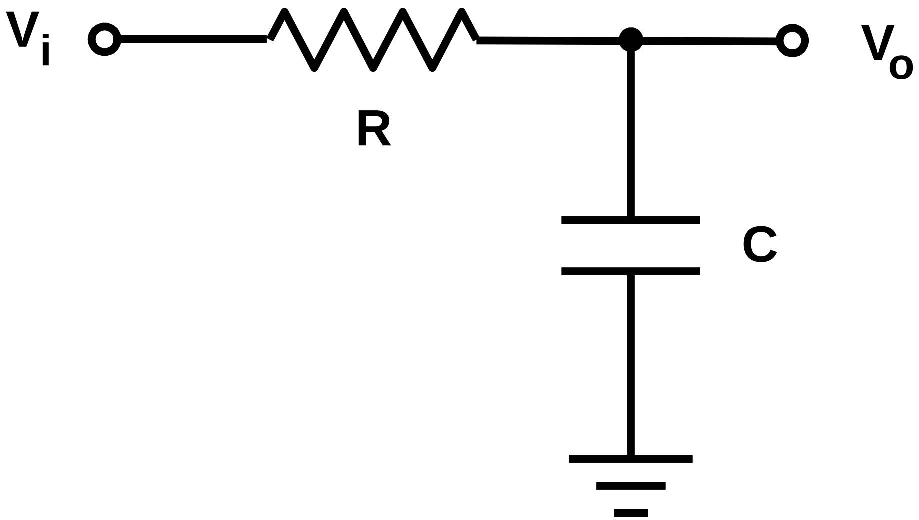
Fasor $A_V = |A_V| \angle \phi$

$$|A_V| = \frac{1}{\sqrt{1+(\omega L/R)^2}}$$

$$\phi = -arctg(\omega L/R)$$

Resposta em Frequência

Circuito RC Filtro
Passa Baixa



Constante de tempo: $\tau = RC$

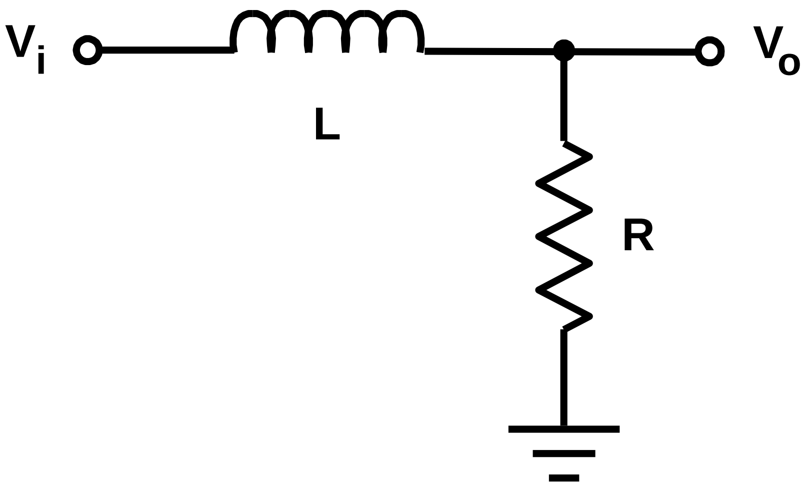
$$|A_V| = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

$$\phi = -arctg(\omega\tau)$$

$|A_V| = f(\omega)$ $\phi = f(\omega)$

Resposta em Frequência

Circuito RL Filtro
Passa Baixa



Constante de tempo: $\tau = \dfrac{L}{R}$

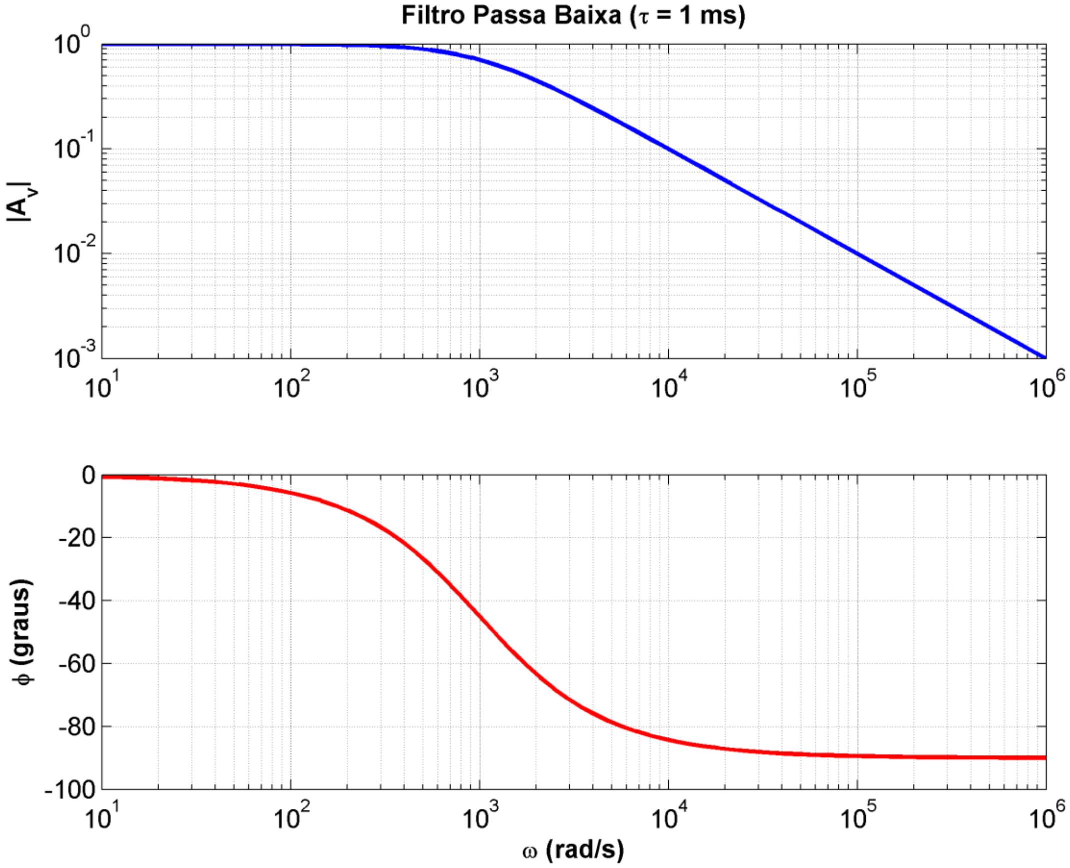
$$|A_V| = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

$$\phi = -arctg(\omega\tau)$$

$|A_V| = f(\omega)$ $\phi = f(\omega)$

Resposta em Frequência

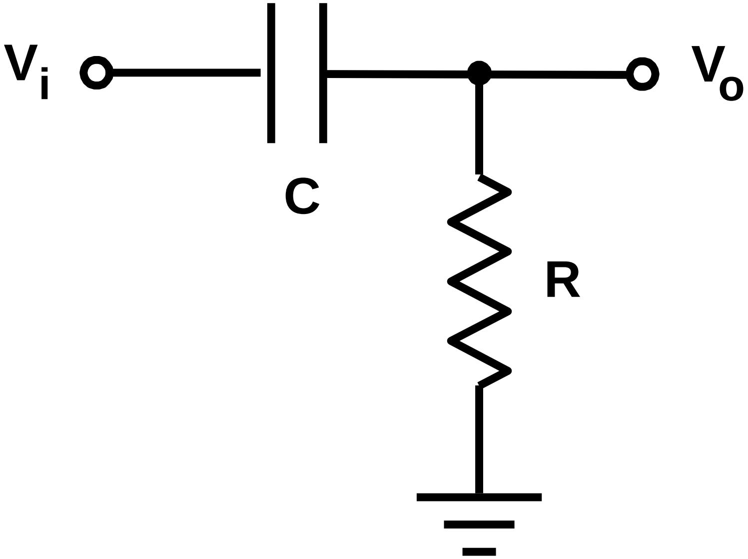
Diagrama de Bode: diagrama de resposta em frequência



Resposta em Frequência

Circuito RC

Filtro Passa Alta



Constante de tempo: $\tau = RC$

$$|A_V| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

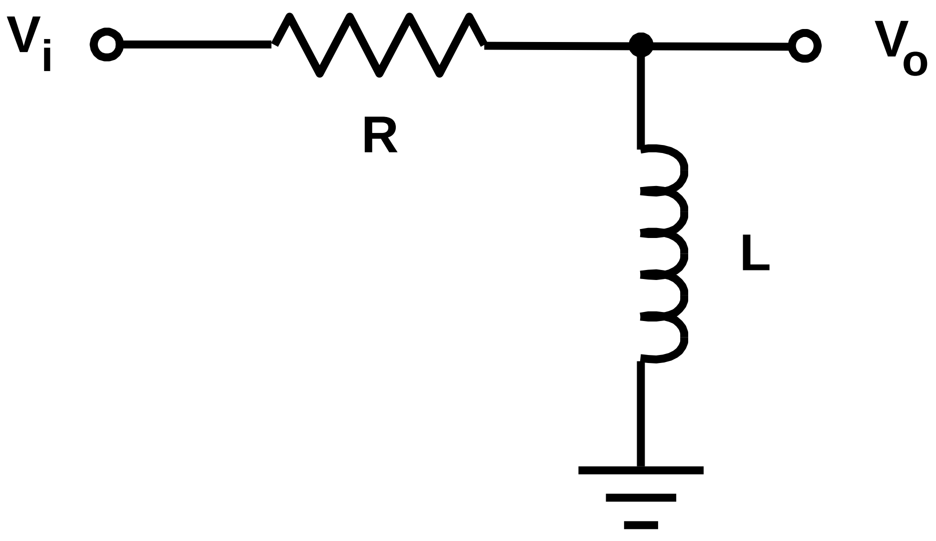
$$\phi = arctg(1/\omega\tau)$$

$|A_V| = f(\omega)$ $\phi = f(\omega)$

Resposta em Frequência

Circuito RL

Filtro Passa Alta



Constante de tempo: $\tau = \dfrac{L}{R}$

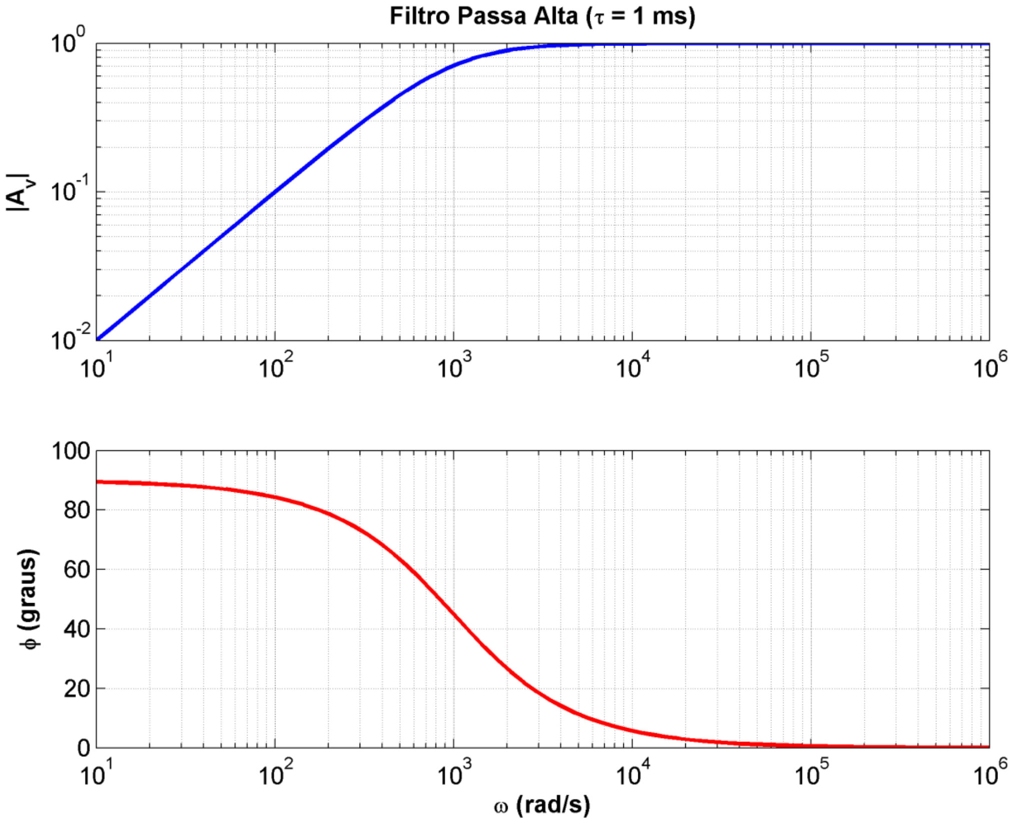
$$|A_V| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\phi = arctg(1/\omega\tau)$$

$|A_V| = f(\omega)$ $\phi = f(\omega)$

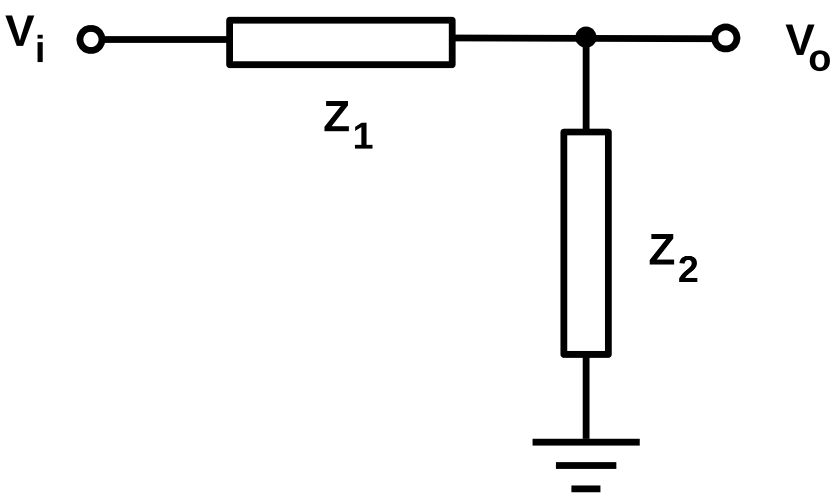
Resposta em Frequência

Diagrama de Bode: diagrama de resposta em frequência



Resposta em Frequência

Definição de frequência de corte, ω_c



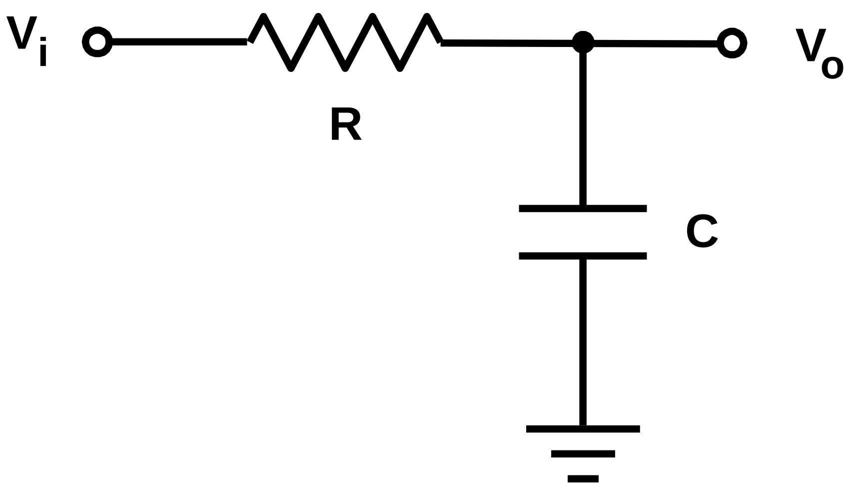
Frequência na qual: $X_1 = X_2$ ou $|A_V| = 1/\sqrt{2}$ ou $\phi = 45^o$

Resposta em Frequência

Frequência de corte do circuito RC em série

$$\left|A_V\right| = \frac{1}{\sqrt{1+(\omega_c\,\tau)^2}} = \frac{1}{\sqrt{2}}$$

$$R = X_C = \frac{1}{\omega_c \cdot C}$$

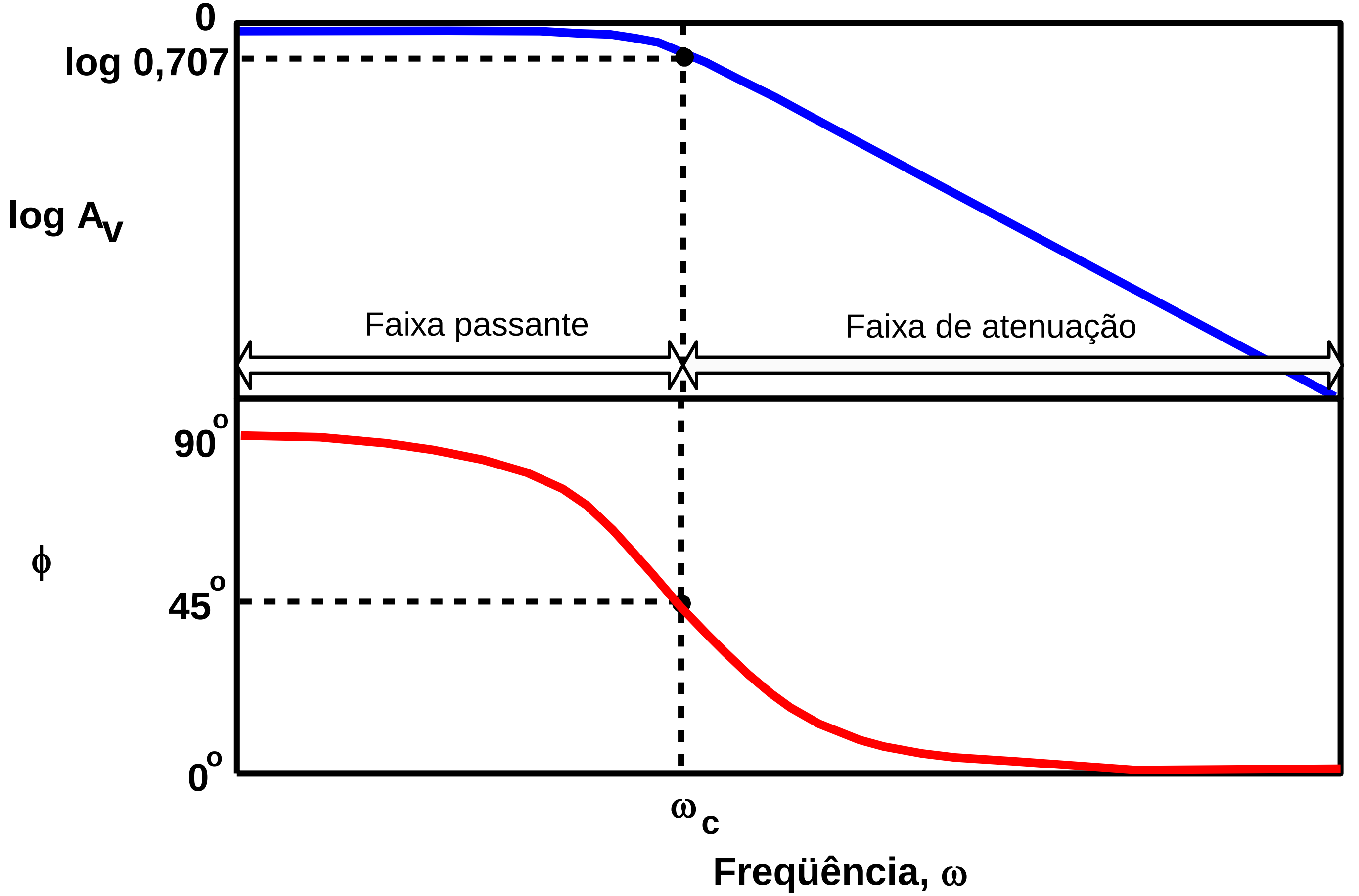


$$\omega_c = \frac{1}{RC} = \frac{1}{\tau}$$

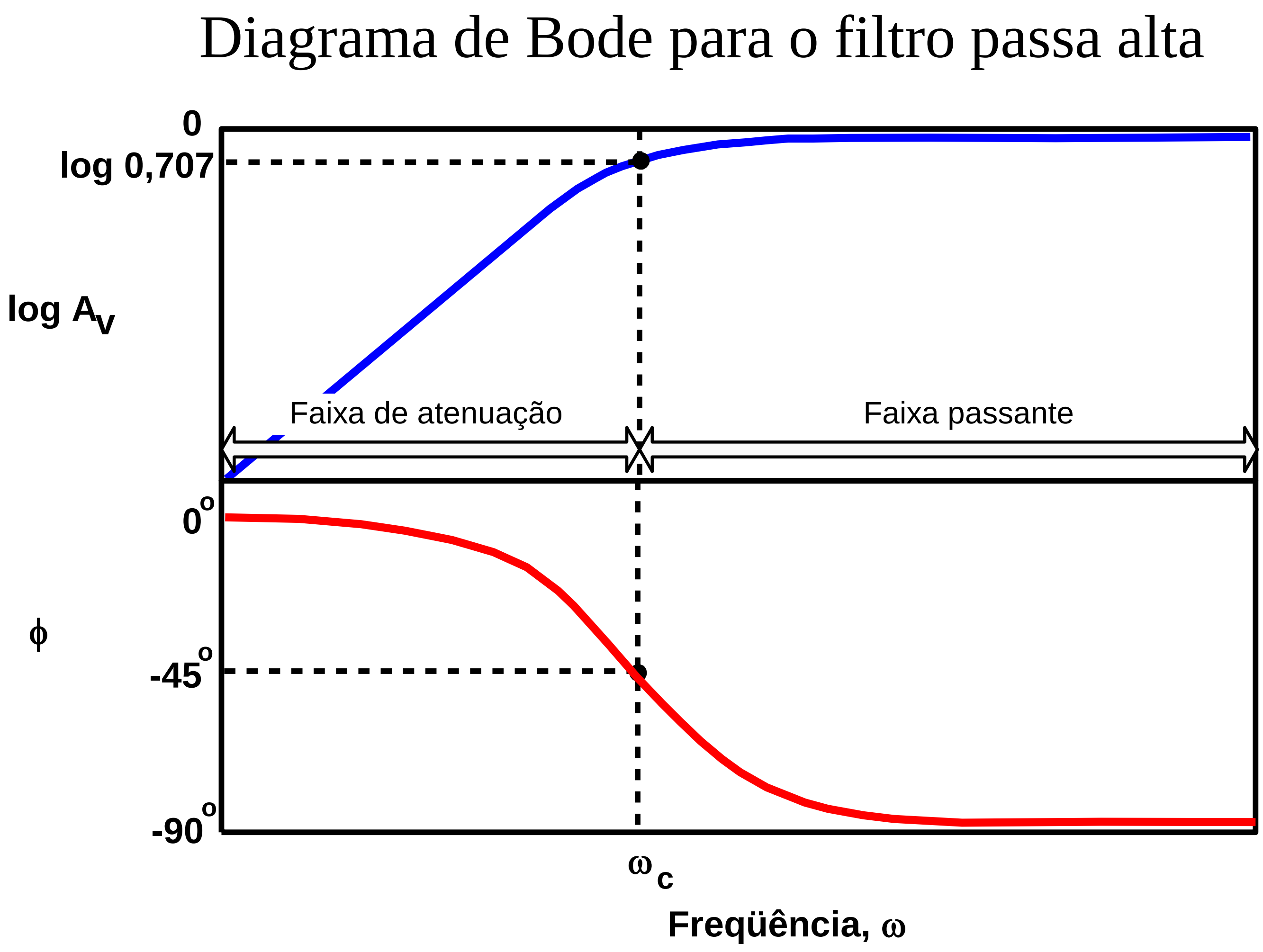
$$\phi = -arctg\left(\omega_c \tau\right) = -45^{o}$$

Resposta em Frequência

Diagrama de Bode para o filtro passa baixa

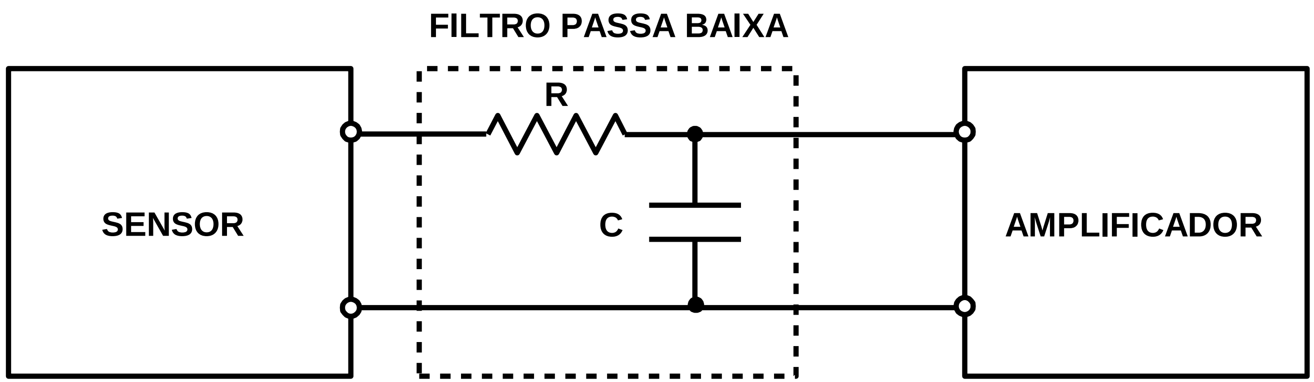


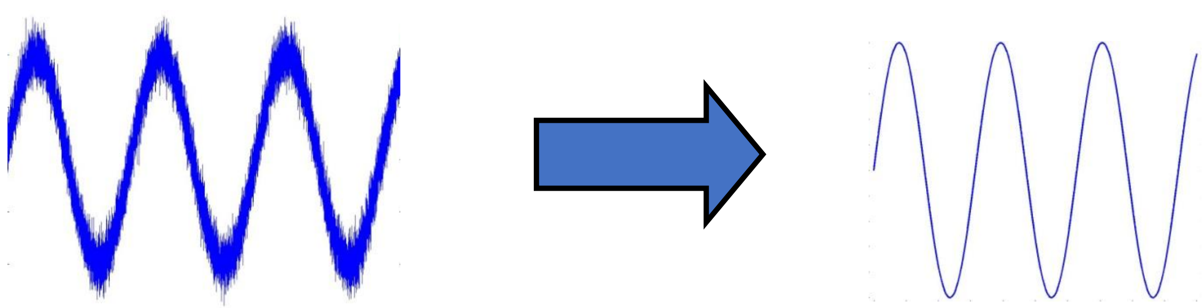
Resposta em Frequência



Aplicação

Aplicação do circuito RC filtro passa baixa





Sinal com ruído

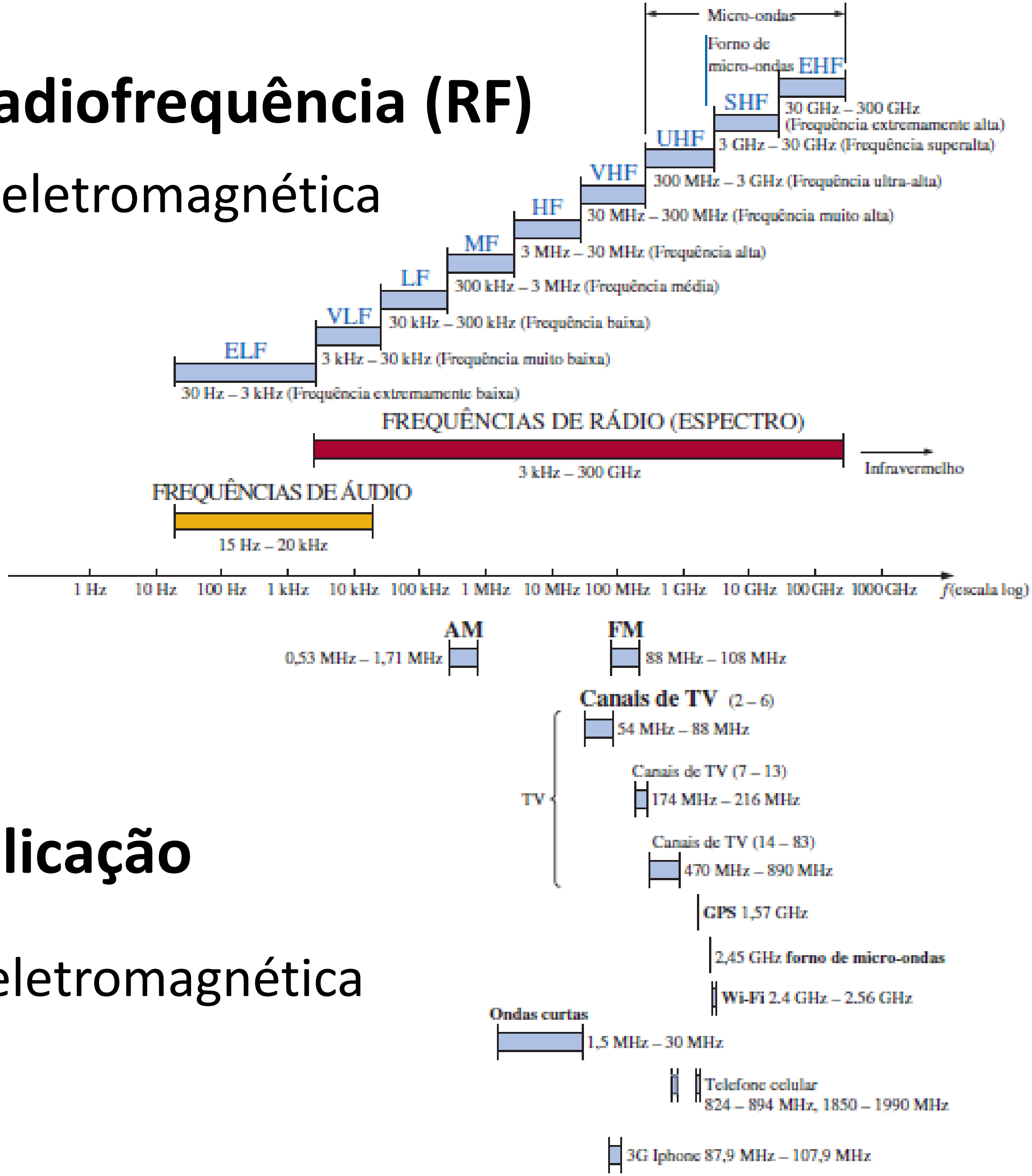
Sinal de baixa frequência filtrado

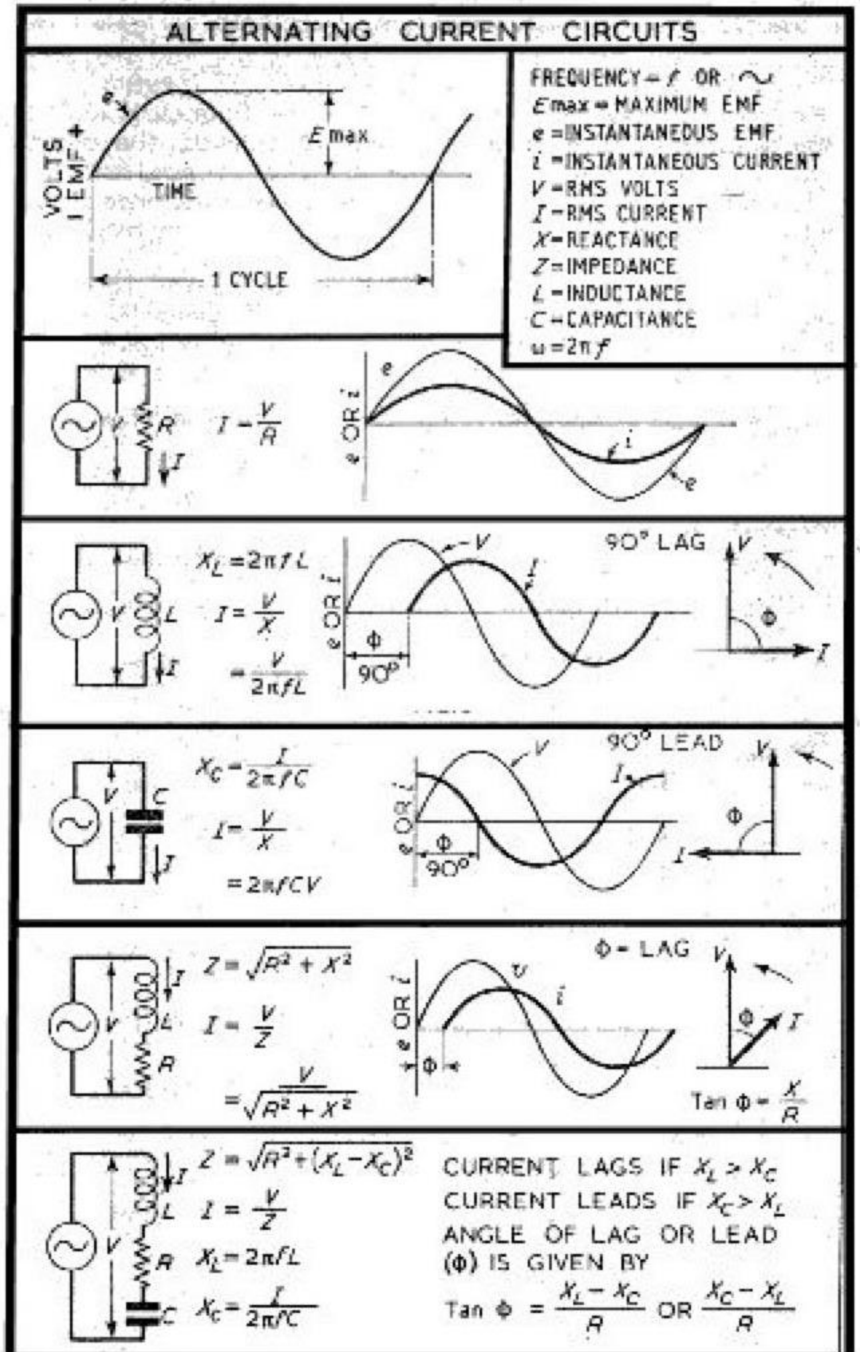
Faixas de áudio e radiofrequência (RF)

Espectro de radiação eletromagnética

Faixas de aplicação

Espectro de radiação eletromagnética





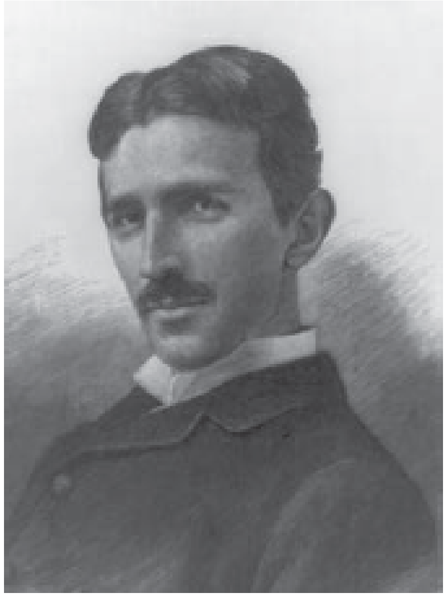
Quadro resumo

Circuitos em corrente alternada

Histórico que vale a pena conhecer

Nikola Tesla (1856–1943) was a Croatian-American engineer whose inventions—among them the induction motor and the first polyphase ac power system—greatly influenced the settlement of the ac versus dc debate in favor of ac. He was also responsible for the adoption of 60 Hz as the standard for ac power systems in the United States.

Born in Austria-Hungary (now Croatia), to a clergyman, Tesla had an incredible memory and a keen affinity for mathematics. He moved to the United States in 1884 and first worked for Thomas Edison. At that time, the country was in the "battle of the currents" with George Westinghouse (1846–1914) promoting ac and Thomas Edison rigidly leading the dc forces. Tesla left Edison and joined Westinghouse be - cause of his interest in ac. Through Westinghouse, Tesla gained the reputation and acceptance of his polyphase ac generation, transmission, and distribution system. He held 700 patents in his lifetime. His other inventions include high-voltage apparatus (the tesla coil) and a wireless transmission system. The unit of magnetic flux density, the tesla, was named in honor of him.



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ALEXANDER, C.K.; SADIKU, M.N.O. Fundamentals of electric circuit. 6th. Ed. New York: McGraw-Hill, 2013.

Histórico que vale a pena conhecer

George Westinghouse. Photo

Nikola Tesla (1856–1943) and **George Westinghouse** (1846–1914) helped establish alternating current as the primary mode of electricity transmission and distribution.

Today it is obvious that ac generation is well established as the form of electric power that makes widespread distribution of electric power efficient and economical. However, at the end of the 19th century, which was the better—ac or dc—was hotly debated and had extremely outspoken supporters on both sides. The dc side was led by Thomas Edison, who had earned a lot of respect for his many contributions. Power generation using ac really began to build after the successful contributions of Tesla. The real commercial success in ac came from George Westinghouse and the outstanding team, including Tesla, he assembled. In addition, two other big names were C. F. Scott and B. G. Lamme.

The most significant contribution to the early success of ac was the patenting of the polyphase ac motor by Tesla in 1888. The induction motor and polyphase generation and distribution systems doomed the use of dc as the prime energy source.

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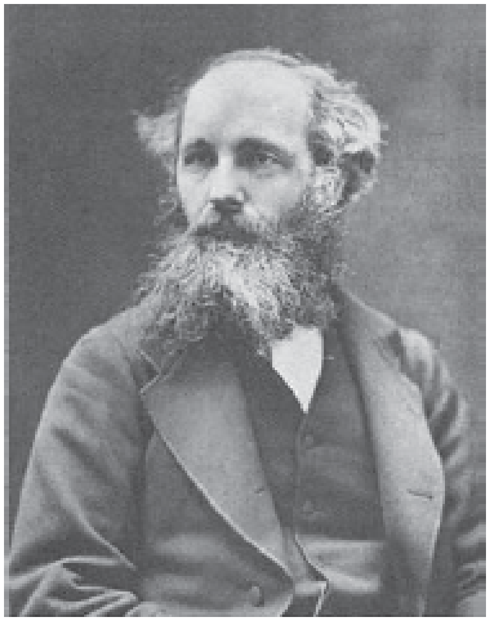
Histórico que vale a pena conhecer



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James Clerk Maxwell (1831–1879), a graduate in mathematics from Cambridge University, in 1865 wrote a most remarkable paper in which he mathematically unified the laws of Faraday and Ampere. This relationship between the electric field and magnetic field served as the basis for what was later called electromagnetic fields and waves, a major field of study in electrical engineering. The Institute of Electrical and Electronics Engineers (IEEE) uses a graphical representation of this principle in its logo, in which a straight arrow represents current and a curved arrow represents the electromagnetic field. This relationship is commonly known as *the right-hand rule*. Maxwell was a very active theoretician and scientist. He is best known for the "Maxwell equations." The maxwell, a unit of magnetic flux, was named after him.

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Histórico que vale a pena conhecer

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Heinrich Rudorf Hertz (1857–1894), a German experimental physicist, demonstrated that electromagnetic waves obey the same fundamental laws as light. His work confirmed James Clerk Maxwell's celebrated 1864 theory and prediction that such waves existed.

Hertz w as born into a prosperous f amily in Hamb urg, German y. He attended the Uni versity of Berlin and did his doctorate under the prominent physicist Hermann von Helmholtz. He became a professor at Karlsruhe, where he be gan his quest for electromagnetic w aves. Hertz successfully generated and detected electromagnetic w aves; he w as the first to show that light is electromagnetic ener gy. In 1887, Hertz noted for the first time the photoelectric effect of electrons in a molecular structure. Although Hertz only lived to the age of 37, his discovery of electromagnetic waves paved the way for the practical use of such waves in radio, television, and other communication systems. The unit of frequency, the hertz, bears his name.

ALEXANDER, C.K.; SADIKU, M.N.O. Fundamentals of electric circuit. 6th. Ed. New York: McGraw-Hill, 2013.

Histórico que vale a pena conhecer

Charles Proteus Steinmetz (1865–1923), a German-Austrian mathematician and engineer, introduced the phasor method (covered in this chapter) in ac circuit analysis. He is also noted for his work on the theory of hysteresis.

Steinmetz was born in Breslau, Germany, and lost his mother at the age of one. As a youth, he was forced to leave Germany because of his political activities just as he was about to complete his doctoral dissertation in mathematics at the University of Breslau. He migrated to Switzerland and later to the United States, where he was employed by General Electric in 1893. That same year, he published a paper in which complex numbers were used to analyze ac circuits for the first time. This led to one of his many textbooks, *Theory and Calculation of ac Phenomena*, published by McGraw-Hill in 1897. In 1901, he became the president of the American Institute of Electrical Engineers, which later became the IEEE.

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