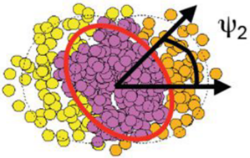
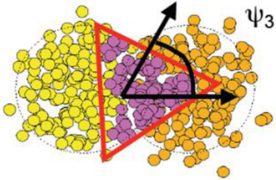
Lecture 16
Anisotropic flow (part II)





Last time, we saw that anisotropic flow has been important to establish the hydrodynamic part of the “ Heavy-Ion Standard Model”. In this lecture, we will see how it helps to extract information on initial conditions and viscosity.

Characterizing the initial conditions

We saw that the momentum disitribution can be written

$$E\frac{d^3N}{dp^3} = \frac{d^3N}{dyd^2p_\perp} =$$

$$\frac{d^2N}{2\pi dyp_\perp dp_\perp}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_\perp, y)\cos(n(\phi_p - \psi_n(p_\perp, y)))\right]$$

so that $v_m(p_\perp, y) = \frac{\int d\phi_p \cos(m(\phi_p - \psi_m))\frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}{\int d\phi_p \frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}$

and $\psi_m(p_\perp, y) = (1/m)\arctan\frac{\int d\phi_p \sin(m\phi_p)\frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}{\int d\phi_p \cos(m\phi_p)\frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}$

It is usual to introduce the compact notation:

$$v_m e^{im\psi_n} = \frac{\int d\phi_p e^{im\phi_p}\frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}{\int d\phi_p \frac{d^3N}{m_\perp dm_\perp d\phi_p dy}} \equiv < e^{im\phi_p} >_p$$

Similarly, we introduce to characterize the initial conditions

$$\epsilon_m = \frac{\int dxdyr^m \cos(m(\phi - \Phi_m))\epsilon(x,y,\tau_0)}{\int dxdyr^m \epsilon(x,y,\tau_0)}$$

and

$$\Phi_m = (1/m)\arctan\frac{\int dxdyr^m \sin(m\phi)\epsilon(x,y,\tau_0)}{\int dxdyr^m \cos(m\phi)\epsilon(x,y,\tau_0)} + (\pi/m)$$

or more compactly:

$$\epsilon_m e^{im\Phi_m} = -\frac{\int dxdyr^m e^{im\phi}\epsilon(x,y,\tau_0)}{\int dxdyr^m \epsilon(x,y,\tau_0)} \equiv -\frac{<r^m e^{im\phi}>_x}{<r^m>_x}$$

The origin of the coordinates is chosen so that $< x >_x = < y >_x = 0$

Exercise:

a) Show that $\epsilon_2 = \frac{\sqrt{<y^2-x^2>_x^2+4<xy>_x^2}}{<y^2+x^2>_x}$

b) If the axes Ox and Oy correspond to the main axes of the ellipse and the distribution of matter is symmetric with respect to the main axis, show that $\epsilon_2 = \frac{<y^2-x^2>_x}{<y^2+x^2>_x}$ (which parallels the definition of third eccentricity for an ellipse).

a)

$$\epsilon_2 = |\epsilon_2 e^{i2\Phi_2}| = |-\frac{< r^2 e^{i2\phi} >_x}{< r^2 >_x}|$$

$$= \frac{\sqrt{< r^2\cos(2\phi) >^2 + < r^2\sin(2\phi) >^2}}{< r^2 >}$$

$$= \frac{\sqrt{< r^2\cos^2\phi - r^2\sin^2\phi >^2 + < r^2 2\cos\phi\sin\phi) >^2}}{< r^2\cos^2\phi + r^2\sin^2\phi >}$$

$$= \frac{\sqrt{< y^2 - x^2 >^2 + 4 < xy >^2}}{< x^2 + y^2 >}$$

b) $< xy >= \int_{-b}^{b} dyy \int_{-a}^{a} dxx\epsilon(x,y) = 0$ since $\epsilon(x,y) = \epsilon(-x,y)$ so $\int_{-a}^{a} dxx\epsilon(x,y) = 0$

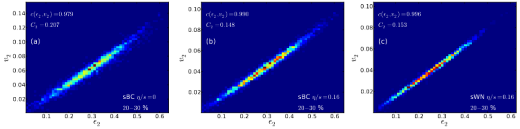


FIG. 1. ϵ_2 and v_2 of pions in the 20 − 30 % centrality class using different initializations and viscosities. a) sBC and $\eta/s = 0$, b) sBC and $\eta/s = 0.16$ and c) sWN and $\eta/s = 0.16$.

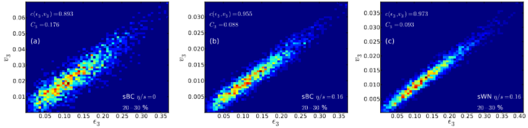


FIG. 2. ϵ_3 and v_3 of pions in the 20 − 30 % centrality class using different initializations and viscosities. a) sBC and $\eta/s = 0$, b) sBC and $\eta/s = 0.16$ and c) sWN and $\eta/s = 0.16$.

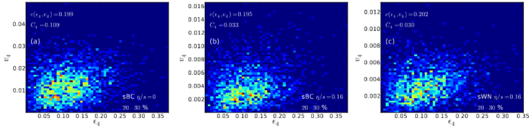


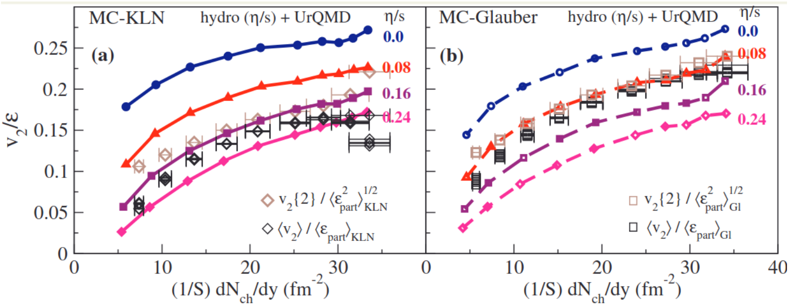
FIG. 3. ϵ_4 and v_4 of pions in the 20 − 30 % centrality class using different initializations and viscosities. a) sBC and $\eta/s = 0$, b) sBC and $\eta/s = 0.16$ and c) sWN and $\eta/s = 0.16$.

H. Niemi, G.Denicol, H.Holopainen, P.Huovinen Phys. Rev. C 87 (2013) 054901 arXiv:1212.1008

For the three different choices of initial conditions, in a centrality class, there is a strong connection: $v_2 \propto \epsilon_2$ (also $\psi_2 \propto \Phi_2$) and $v_3 \propto \epsilon_3$ (also $\psi_3 \propto \Phi_3$), it is more complicated for $n \geq 4$.

For each centrality class, an average $< v_2 >_c / < \epsilon_2 >_c$ (v_2/ϵ in the plots below) can be computed and compared to data (devided by a theoretical average ϵ_2).

For different choices of initial conditions, the best value of η/s is different



C.Shen et al. J.Phys.G38 (2011) 124045: v_2/ϵ as function of centrality

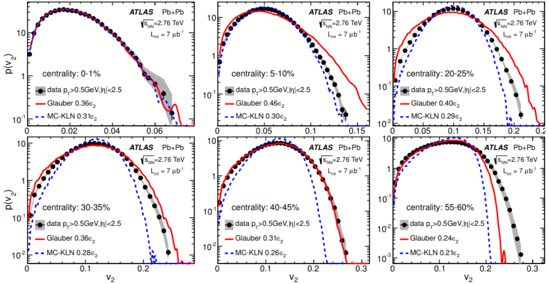
For initial conditions MC-KLN, the best η/s=0.2.

For initial conditions MC-Glauber, the best η/s=0.08.

How to disentangle initial conditions and viscosity effects?

Instead of looking at the average $< v_2 >_c$ in a centrality class, one can look at the probability to have a certain value of v_2 in data. Since $v_2 \propto \epsilon_2$, this can be compared to the probability to have a certain value of ϵ_2 (this avoid having to run hydro for each initial conditions model).

The ATLAS collaboration has shown that indeed it is possible to eliminate initial conditions models this way.

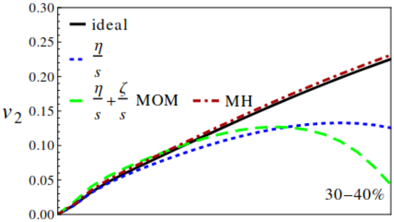


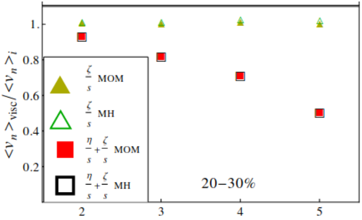
ATLAS JHEP 11 (2013) 183

MC-Glauber initialization is too wide and MC-KLN initialization too narrow. Both models are eliminated!

Sensitivity on viscosity

Like the momentum spectra, the anisotropic flow coefficients $v_n(p_\perp)$ (left) are sensitive to δf but the integrated values v_n (right) are much less sensitive





J. Noronha-Hostler, J. Noronha, FG Phys. Rev. C 90 (2014) 034907 arXiv:1406.3333

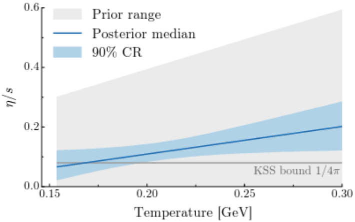
- v_n's are more useful to extract values for η/s
- v_n's are more supressed due to shear viscosity as n increases.

A systematic tool to get η/s (and more): Bayesian analysis

- Choose a set of parameters $\theta = \theta_1, \theta_2, ...$ of the hydro problem related to the initial conditions (entropy deposition, normalization, etc) and to the QGP properties (hadronization temperature, slope of linear increase of specific shear viscosity vs. temperature,...).
- Choose a set of observed data $y = y_1, y_2, ...$ such as dN_π/dy, v_2, in various centrality bins.
- Find the probability (posterior) that the parameters assume some values θ given that the values y are observed:
  $p(\theta|y) = N \times \mathcal{L}(y|\theta) \times p(\theta)$ (Bayes's theorem)
  where $\mathcal{L}(y|\theta)$ likelihood to observe y for a given θ,
  $p(\theta)$ probability of θ (prior) before data comparison,
  $N = 1/\int_\theta \mathcal{L}(y|\theta)p(\theta)$ (normalization).

$\mathcal{L}(y|\theta)$ is constructed using hydro runs (and interpolating between them) and $p(\theta)$ is somehow a guess (e.g. uniform value in a certain range).

Example of result



Benhard et al. Phys. Rev. C 94 (2016) 024907 arXiv:1605.03954

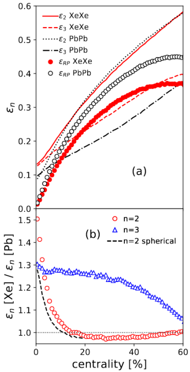
Challenge

Fluctuations in the energy density (hot spots) are expected to scale roughly as the inverse square root of the number of participants (R.S. Bhalerao et al. Phys.Rev.C84 (2011) 054901, arXiv:1107.5485). Use this to explain the differences in ϵ_2 for central collisions Xe+Xe compared to Pb+Pb and those in ϵ_3 for all centralities, for the figure in the homework.

Homework

At the LHC, runs were made for Pb+Pb and then Xe+Xe (at comparable $\sqrt{s}$). Between these, theoreticians made predictions on how anisotropic flow would be for Xe+Xe compared to Pb+Pb.

One group (G.Giacalone et al. Phys. Rev. C 97 (2018) 034904 arXiv:1711.08499) found that the excentricities would be as in the figure below. Make predictions on how v_2 and v_3 would be as function of centrality for Xe+Xe compared to Pb+Pb. Look for the data and check your predictions.



Other references on this topic

- T.Hirano,N. van der Kolk, A. Bilandzic arXiv:0808.2684
- P.F.Kolb and U. Heinz nucl-th/0305084.pdf