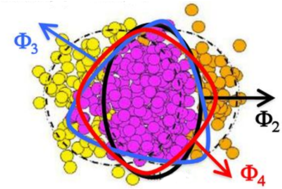
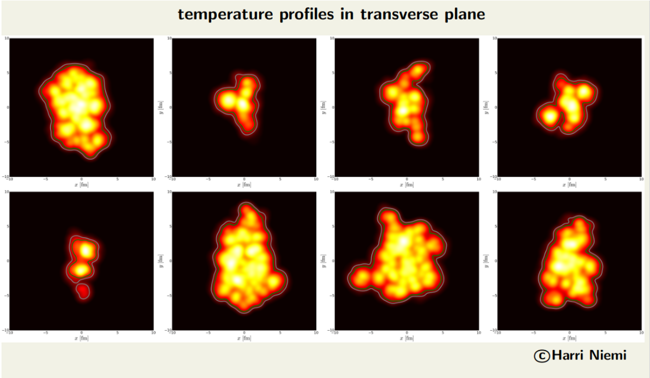
Lecture 15
Anisotropic flow



What is anisotropic flow?



In lecture 13, we studied the radial expansion of the fluid, assuming it what the same in all azimuthal directions
In fact, there is a small azimuthal angular dependance, this is the anisotropic flow.

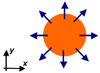
So we write:

$$E\frac{d^3N}{dp^3} = \frac{d^3N}{dyd^2p_\perp} = \frac{d^2N}{2\pi dyp_\perp dp_\perp}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_\perp, y)\cos(n(\phi_p - \psi_n(p_\perp, y)))\right]$$

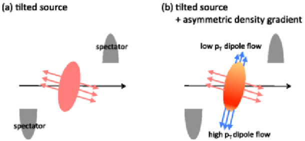
and

$$\frac{d^2N}{d\phi_p dy} = \frac{dN}{2\pi dy}\left[1 + \sum_{n=1}^{\infty} 2v_n(y)\cos(n(\phi_p - \psi_n(y)))\right]$$

- The term [1+ before the sum corresponds to radial flow (independant of ϕ_p) that we studied:

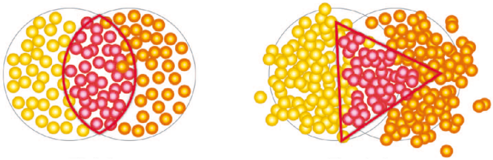


- The term n=1 in the sum, corresponds to directed flow:



Left v_1 : comes from different number of nucleon collisions and breaking in the overlap region. Right v_1 : comes from the fluctuations (hot spots) in a transverse slice.

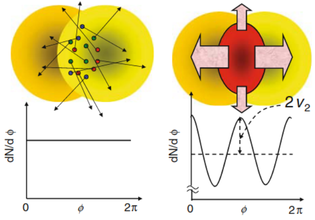
In the following, we concentrate on n>1, where fluctuations have an important part



- The term n=2 is elliptic flow: it comes mostly from the existing shape of overlap region but also on its hot spots.
- n>1 odd terms would not exist without the fluctuations.

A signature of collectivity

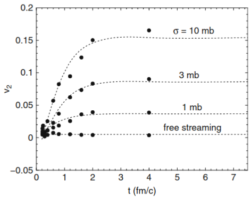
The presence of elliptic flow and its large value are signal of collectivity



(a) Large mean free path (b) Small mean free path

T.Hirano,N. van der Kolk, A. Bilandzic arXiv:0808.2684

If the mean free path was larger than the system size, particles would be emitted isotropically. On the other side, if matter is thermalized, pressure gradients are larger horitally (smaller distance) than vertically (larger distance) and flow is enhanced horizontally

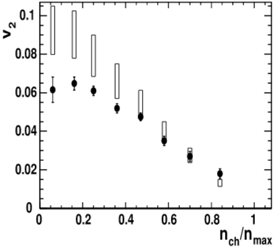


Gluons undergoing elastic collisions in an almond shape region: from B. Zhang, M. Gyulassy, C.M. Ko: Phys. Lett. B 455 (1999) 45

In the simple case above, we see that:

- v_2 is not generated in the free-streaming case, so elliptic flow is generated indeed through secondary collisions
- elliptic flow is generated in the early stage of the collision and saturates after the first 2 to 3 fm/c
- the saturated value of v_2 is sensitive to the cross section among the particles so, in the kinetic theory of gases, to η
  $\sigma \propto 1/l_{mfp} \propto 1/\eta$ (see lecture 11)
- for large cross sections, the system is expected to reach the ideal hydrodynamic result ($l_{mfp} \to 0$)

The observation of elliptic flow saturating the hydro limit (STAR collboration, 2000), was a major step to establish the validity of the hydrodynamic description. It also became soon clear, that a quark gluon phase was necessary for a proper description.



Black dots are STAR data and white boxes are various ideal hydro predictions

Mass ordering

In lecture 13, we saw that for the freeze out of a cylindrical perfect fluid with constant transverse expansion and boost invariance

$$\frac{d^3N}{m_\perp dm_\perp d\phi_p dy} = \frac{gR^2/2}{(2\pi)^3}\tau_{f.out}\sum_{n=1}^{\infty}(\pm)^{n+1}e^{\frac{n\mu}{T_{f.out}}}\int_{f.out} d\eta_s d\phi \cosh(y-\eta_s)$$

$$m_\perp\, exp(-\frac{nm_\perp \cosh\rho\cosh(y-\eta_s)}{T_{f.out}}+\frac{np_\perp \sinh\rho\cos(\phi_p-\phi)}{T_{f.out}})$$

So ignoring angular dependance we had derived

$$\frac{d^2N}{m_\perp dm_\perp dy} = \frac{gR^2}{2\pi}\tau_{f.out}m_\perp\sum_{n=1}^{\infty}(\pm)^{n+1}e^{\frac{n\mu}{T_{f.out}}}K_1(nm_\perp\cosh\rho/T_{f.out})I_0(np_\perp\sinh\rho/T_{f.out})$$

$$\sim \frac{gR^2}{2\pi}\tau_{f.out}m_\perp e^{\frac{\mu}{T_{f.out}}}K_1(m_\perp\cosh\rho/T_{f.out})I_0(p_\perp\sinh\rho/T_{f.out})$$

To study elliptic flow, angular dependance must be included. Instead of a constant ρ, we use $\rho=\rho_0+\rho_a\cos(2\phi)$ so

$$\frac{d^3N}{m_\perp dm_\perp d\phi_p dy} = \frac{gR^2/2}{(2\pi)^3}\tau_{f.out}\sum_{n=1}^{\infty}(\pm)^{n+1}e^{\frac{n\mu}{T_{f.out}}}\int_{f.out} d\phi$$

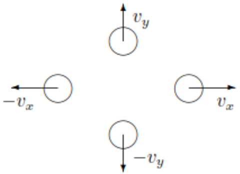
$$m_\perp\, K_1(\frac{nm_\perp\cosh\rho}{T_{f.out}})exp(\frac{np_\perp\sinh\rho\cos(\phi_p-\phi)}{T_{f.out}})$$

Therefore: $\frac{d^3N}{m_\perp dm_\perp d\phi_p dy} \sim$
$$\frac{gR^2/2}{(2\pi)^3} \tau_{f.out} e^{\frac{\mu}{T_{f.out}}} \int_{f.out} d\phi m_\perp \, K_1(\frac{m_\perp \cosh\rho}{T_{f.out}}) exp(\frac{p_\perp \sinh\rho \cos(\phi_p - \phi)}{T_{f.out}})$$
To compute v_2, we note the general property (using the expression in blue p.3):

$$v_m(p_\perp, y) = \frac{\int d\phi_p \cos(m(\phi_p - \psi_m)) \frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}{\int d\phi_p \frac{d^3N}{m_\perp dm_\perp d\phi_p dy}}$$

So in our particular wave case:

$$v_2(p_\perp, y) = \frac{\int_0^{2\pi} d\phi cos(2(\phi - \psi_2)) K_1(\frac{m_\perp \cosh\rho}{T_{f.out}}) I_2(\frac{p_\perp \sinh\rho}{T_{f.out}})}{\int_0^{2\pi} d\phi K_1(\frac{m_\perp \cosh\rho}{T_{f.out}}) I_0(\frac{p_\perp \sinh\rho}{T_{f.out}})}$$



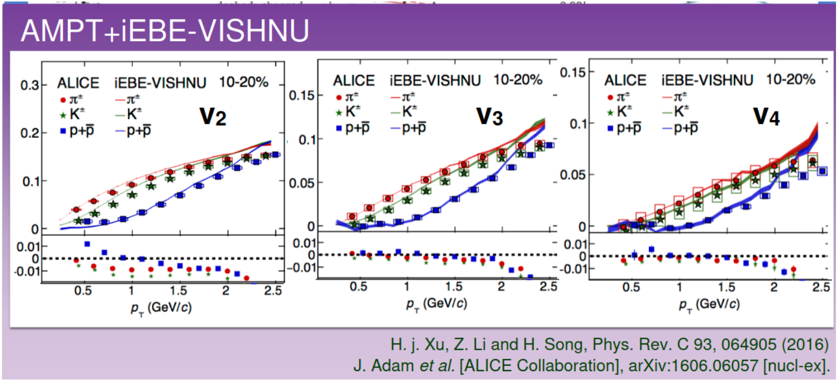
To go farther, we approximate the elliptic fireball by 4 sources. We get:

$$v_2(p_\perp, y) = \frac{I_2(\frac{\gamma_x v_x p_\perp}{T_{f.out}}) - e^{\frac{E}{T_{f.out}}(\gamma_x - \gamma_y)} I_2(\frac{\gamma_y v_y p_\perp}{T_{f.out}})}{I_0(\frac{\gamma_x v_x p_\perp}{T_{f.out}}) + e^{\frac{E}{T_{f.out}}(\gamma_x - \gamma_y)} I_0(\frac{\gamma_y v_y p_\perp}{T_{f.out}})}$$

The particle mass enters only in the term $e^{\frac{E}{T_{f.out}}(\gamma_x - \gamma_y)}$, so if all other variables are held fixed, v_2 decreases with increasing mass.

This model is presented in P.Huovinen et al. Phys.Lett.B503 (2001) 58, arXiv:hep-ph/0101136

This prediction has indeed been confirmed. Mass ordering seem to hold also for $n > 2$. This is another fact that established the hydrodynamical description.



Challenge

Using the 4 sources model, show that

a) at midrapidity $v_2(p\perp) \sim \mathtt{tanh}\left(\frac{1}{2}\left(\frac{(\kappa p_\perp - \lambda m_\perp}{T_{f.out}} + \mu\right)\right)$ where $\kappa = \gamma_x v_x - \gamma_y v_y$, $\lambda = \gamma_x - \gamma_y$ and $\mu = \ln\sqrt{\gamma_x v_x/(\gamma_y v_y)}$,

b) for intermediate $p_\perp$, v_2 is approximately linear in $p_\perp$ (use $T_{f.out}$ = 140 MeV, v_x = 0.6, and v_y = 0.5).

Homework

At midrapidity, an angular distribution is given by:

$$\frac{d^2N}{d\phi_p dy}_{|y=0} = \frac{dN}{2\pi dy}\left[1 + 2\times 0.1\cos(2\phi_p) - 2\times 0.05\cos(3(\phi_p - \pi/6))\right]$$

What are the values of v_2 and v_3?

Other references on this topic

- T.Hirano,N. van der Kolk, A. Bilandzic arXiv:0808.2684
- P.F.Kolb and U. Heinz nucl-th/0305084.pdf