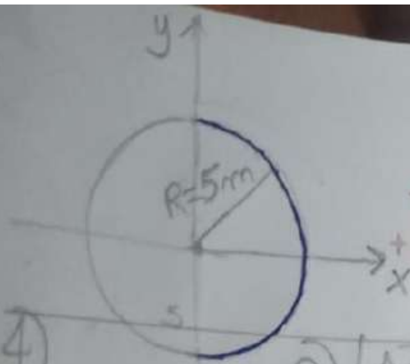


4) a) $|\Delta\vec{r}| = |\vec{r}_f - \vec{r}_0|$ $\vec{r}_0 = 0\hat{i} + 5\hat{j} = 5\hat{j}$

$\vec{r}_f = 0\hat{i} - 5\hat{j} \Rightarrow \vec{r}_f = -5\hat{j}$

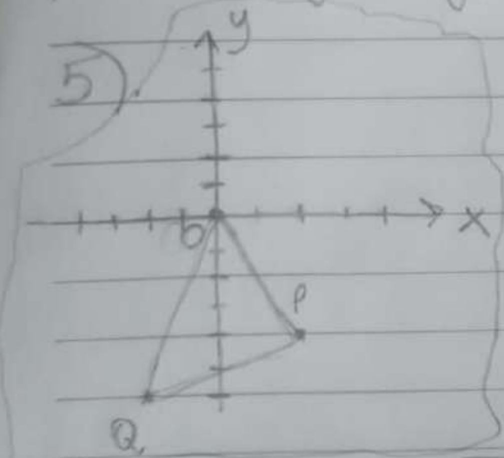
$|\Delta\vec{r}| = |\vec{r}_f - \vec{r}_0| = |-5\hat{j} - 5\hat{j}| \Rightarrow |\Delta\vec{r}| = |-10\hat{j}| = 10\,m$

b) distância percorrida $= \frac{2\pi R}{2} = \pi R = 5\pi m$

c) $|\Delta\vec{r}| = |\vec{r}_f - \vec{r}_0|$ $\vec{r}_f = 5\hat{j}$; $\vec{r}_0 = 5\hat{j}$

$|\Delta\vec{r}| = |5\hat{j} - 5\hat{j}| = 0$

5)

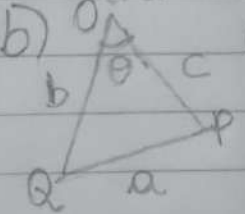


a) $d_{PQ} = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

$d_{PQ} = \sqrt{(2-(-2))^2 + (-4-(-6))^2}$

$d_{PQ} = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$ Km

b)



$a^2 = b^2 + c^2 - 2bc\cos\theta$

Lei dos Cossenos

$b = d_{OQ} = \sqrt{(x_Q - x_O)^2 + (y_Q - y_O)^2}$

$b = d_{OQ} = \sqrt{(-2-0)^2 + (-6-0)^2}$

$b = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ km

$c = d_{OP} = \sqrt{(2-0)^2 + (-4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ Km

$20 = 40 + 20 - 2 \cdot 2\sqrt{10} \cdot 2\sqrt{5} \cdot \cos\theta$

$8\sqrt{50}\cos\theta = 40 \Rightarrow 5\sqrt{2}\cos\theta = 5 \Rightarrow \cos\theta = 1/\sqrt{2} = \sqrt{2}/2$

$\theta = 45°$

Dedução:

$a = \frac{dv}{dt} \Rightarrow dv = a\,dt$

$\int dv = \int a\,dt$

$v = at + \text{constante}$

$\text{constante} = v_0$

$v = v_0 + a \cdot t$

$v = \frac{ds}{dt}$

$ds = v\,dt$

$\int ds = \int v\,dt$

$\int ds = \int (v_0 + at)\,dt$

$s = v_0 t + \frac{at^2}{2} + \text{constante}$

$s = s_0 + v_0 t + \frac{at^2}{2}$

Eq. de Torricelli:

$\begin{cases} v = v_0 + at \\ s = s_0 + v_0 t + \frac{at^2}{2} \end{cases}$

$t = \frac{v - v_0}{a}$

$s = s_0 + v_0 \frac{(v - v_0)}{a} + \frac{a}{2}\left(\frac{v - v_0}{a}\right)^2$

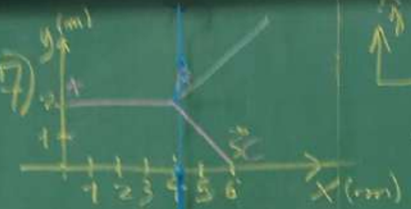
$\Delta s = \frac{v_0 v - v_0^2}{a} + \frac{(v - v_0)^2}{2a}$

$2a\Delta s = 2v_0 v - 2v_0^2 + v^2 - 2v_0 v + v_0^2$

$v^2 = v_0^2 + 2a\Delta s$

$t)\hat{j}$

7)



$V = 2\,m/s$

a) $V = V_x + V_y \quad \sqrt{2^2+0^2} = |v|$

$\vec{V} = 2\hat{i} + 0\hat{j} = (2,0)$

b) $V = \frac{dx}{dt} \rightarrow \int dx = \int v\,dt \Rightarrow \vec{X} = \vec{X_0} + \vec{V}t$

$\hat{j}$, $\hat{i}$

$\vec{x} = x_0\hat{i} + 2t\hat{i}$

$\boxed{\vec{x} = 2t\hat{i}}$

$\vec{y} = y_0\hat{j} + vt\hat{j}$

$\boxed{\vec{y} = 2\hat{j}}$

$\vec{S} = 2t\hat{i} + 2\hat{j} = (2t, 2)$

c) $V = \frac{\Delta S}{\Delta t} =$

d) $|V| = \sqrt{\ }$

$V = 2 \cdot \sqrt{\ }$

$V = \sqrt{2}\,\hat{i}$

$\ldots 2t\,\hat{i}$

$+ v t\,\hat{j}$

$+2\hat{j} = (2t, 2)$

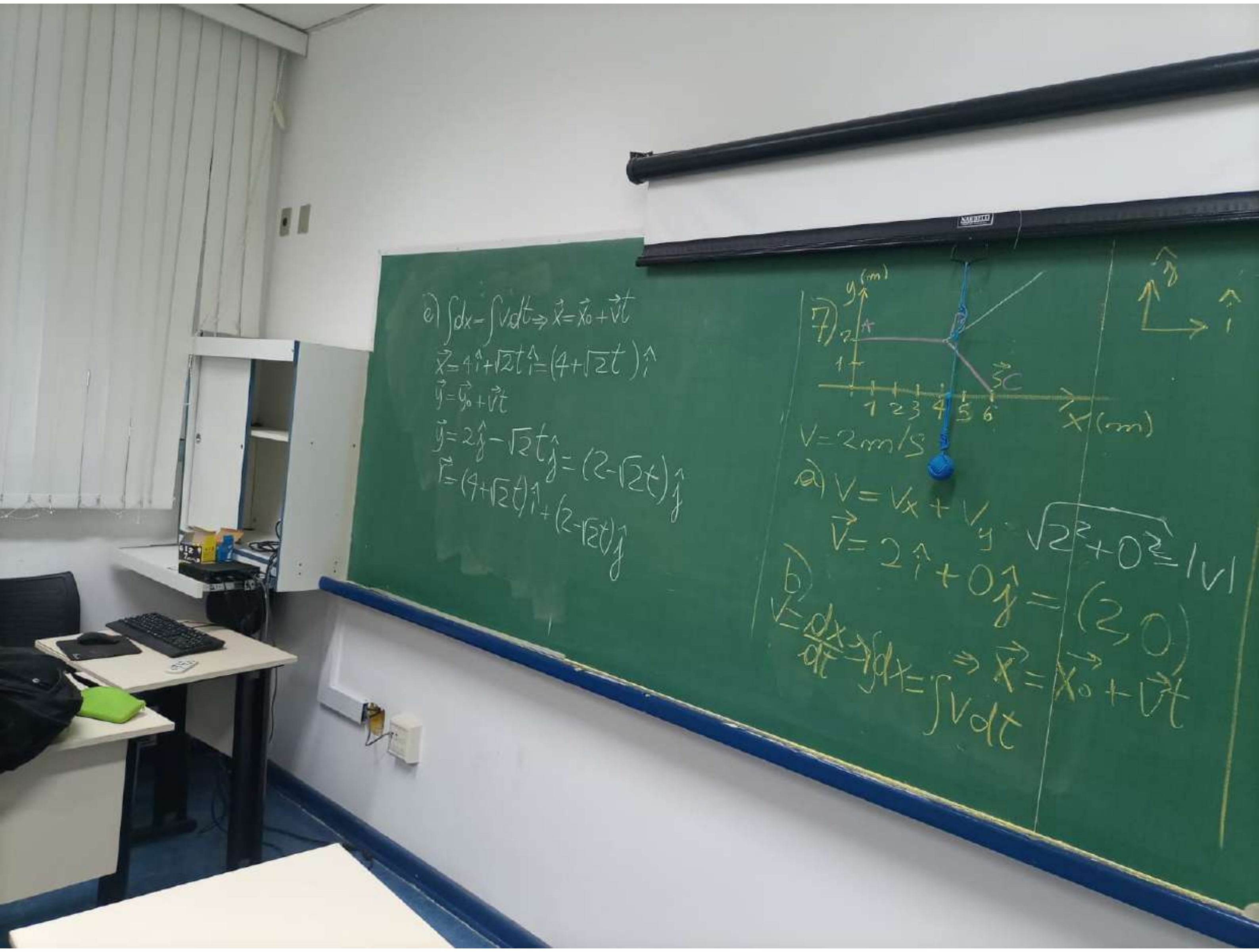
c) $v = \dfrac{\Delta S}{\Delta t} \Rightarrow \Delta t = \dfrac{\Delta S}{v} = \dfrac{4}{2} = 2s$

d) $|v| = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$

$v = 2\dfrac{\sqrt{2}}{2}\,\hat{i} + 2\dfrac{\sqrt{2}}{2}\,\hat{j}$

$v = \sqrt{2}\,\hat{i} - \sqrt{2}\,\hat{j}$

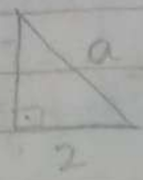
$\sqrt{\sqrt{2}^2 + \sqrt{2}^2} = \sqrt{2+2}$
$= 2$



e) ∫dx = ∫v dt ⇒ x = x₀ + vt
x = 4î + √2 t î = (4 + √2 t) î
y = y₀ + vt
y = 2ĵ − √2 t ĵ = (2 − √2 t) ĵ
r = (4 + √2 t) î + (2 − √2 t) ĵ
7)
y (m)
x (m)
1 2 3 4 5 6
V = 2 m/s
a) V = Vx + Vy
V = 2î + 0ĵ = (2, 0)
√(2² + 0²) = |v|
b) V = dx/dt ⇒ ∫dx = ∫v dt ⇒ x = x₀ + vt

Passive voice. I was sent a letter. / A letter was sent to me

7) f)



$a^2 = 4 + 4$

$a = \sqrt{8} = 2\sqrt{2}$

dist. total $= 4 + 2\sqrt{2}$ m

$v = 2$ m/s

$t = \dfrac{4 + 2\sqrt{2}}{2} = 2 + \sqrt{2}$ s

g) $|\Delta \vec{r}| = |(\sqrt{2} + 4 + \sqrt{2}t)\hat{i} + (-\sqrt{2} + 2 - \sqrt{2}t)\hat{j}|$

$|\Delta \vec{r}| = \sqrt{2 + 8\sqrt{2} + 16 + 2(\sqrt{2}+4)\sqrt{2}t + 2t^2 + 2 + 2(-\sqrt{2}\cdot 2) + 4 + 2(-\sqrt{2}+2)(-\sqrt{2}t) + 2t^2}$

$|\Delta \vec{r}| = \sqrt{24 + 8\sqrt{2} + 4t + 8\sqrt{2}t + 2t^2 - 4\sqrt{2} + 4 + 4t - 4\sqrt{2}t + 2t^2}$

$|\Delta \vec{r}| = \sqrt{28 + 4\sqrt{2} + 8t + 4\sqrt{2}t + 4t^2}$

h) $v = \dfrac{d}{t} \Rightarrow d = v \cdot t = 2 \cdot 3 = 6$ m

15) a) $\vec{r}(t) = 18{,}0t\,\hat{i} + (4{,}00t - 4{,}90t^2)\,\hat{j}$

$\vec{v}(t) = 18{,}0\,\hat{i} + (4{,}00 - 9{,}80t)\,\hat{j}$

$\vec{a}(t) = -9{,}80\,\hat{j}$

b) $\vec{r}(3) = 18{,}0 \cdot 3\,\hat{i} + (4{,}00 \cdot 3 - 4{,}90 \cdot 9)\,\hat{j}$

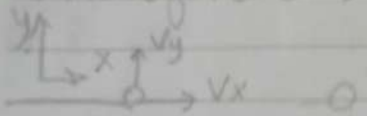
$\vec{r}(3) = 54{,}0\,\hat{i} - 32{,}1\,\hat{j}$

$\vec{v}(3) = 18{,}0\,\hat{i} + (4{,}00 - 9{,}80 \cdot 3)\,\hat{j}$

$\vec{v}(3) = 18{,}0\,\hat{i} - 25{,}40\,\hat{j}$

$\vec{a}(3) = -9{,}80\,\hat{j}$

c) Horizontal $\Rightarrow$ movimento em x



Em x: $x = 18t$

Em y: $y = 4t - 4{,}9t^2$

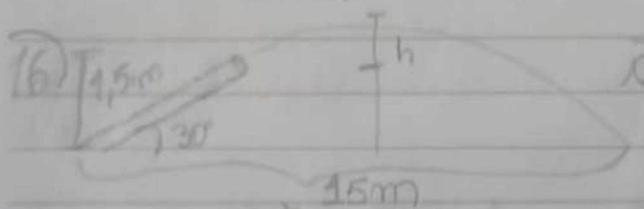
$v = v_0 + at$

$0 = 4 - 9{,}8t \Rightarrow t_s = 4/9{,}8\ s$

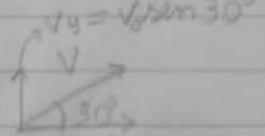
$t_s = t_d \quad t_{tot} = 8/9{,}8\ s$

$x = 18 \cdot 8/9{,}8 \cong 14{,}7\ m$

16)



a)



Em x:

$S = S_0 + v \cdot t$

$X = X_0 + V_x t$

$x = x_0 + |\vec{v}_0| \cos\theta\, t \qquad x_0 = 0$

$t = (x - x_0)/|\vec{v}_0| \cos\theta$

Em y: $y = y_0 + |\vec{v}| \operatorname{sen}\theta\, t - gt^2/2$

$y = y_0 + |\vec{v}_0| \operatorname{sen}\theta \cdot \dfrac{(x - x_0)}{|\vec{v}_0| \cos\theta} - \dfrac{g\,(x - x_0)^2}{2\,|\vec{v}_0|^2 \cos^2\theta}$

Alcance máximo: $y = 0$:

$\dfrac{g(x - x_0)^2}{2|v_0|^2 \cos^2\theta} - x \operatorname{tg}\theta = 0$

Resolvendo por bhaskara:

$x - x_0 = \dfrac{2 \operatorname{tg}\theta\, |\vec{v}_0|^2 \cos^2\theta}{g}$

$\operatorname{sen}2\theta = 2 \operatorname{sen}\theta \cos\theta$

$x = x_0 + \dfrac{|\vec{v}_0|^2}{g} \operatorname{sen}2\theta \qquad x_0 = 0$

$1{,}5 = \dfrac{v_0^2}{10} \cdot \operatorname{sen}60^\circ = v_0^2 \cdot \dfrac{\sqrt{3}}{20}$

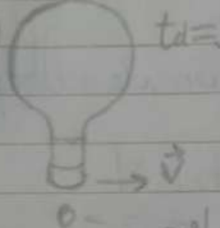
$v_0^2 = 30/\sqrt{3} = 30\sqrt{3}/3 = 10\sqrt{3}$

$V_0 = \sqrt{10\sqrt{3}}\ m/s$

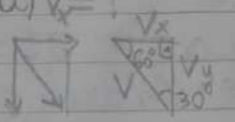
b) $y - y_0 = \dfrac{V_0^2 sen^2\alpha}{2g} \Rightarrow y = 1{,}5 + \dfrac{1\cancel{0}\sqrt{3}\,(\sqrt{3}/2)^2}{2\cancel{0}} = 1{,}5 + \dfrac{3\sqrt{3}}{8}\ m$

$V_y^2 = V_{0y}^2 - 2g\,\Delta y \Rightarrow 0 = V_0^2 sen^2\theta - 2g(y - y_0)$

$y - y_0 = \dfrac{V_0^2 sen^2\theta}{2g}$

17) 

$t_d = 3s$ a) $V_x = ?$



$V_x = V_0 \cos 60°$

$X = X_0 + V_x \cdot t \Rightarrow X = V_0 \cos 60° \cdot t$

$X = \dfrac{|V_0|^2 sen 2\theta}{g}$

$t = 3s$, $X = X \Rightarrow \cancel{V_0} \cos 60° t = \dfrac{|V_0|^{\cancel{2}} \cdot sen 120°}{g}$

$g = 10 m/s^2$

$\dfrac{1}{2} \cdot 3 = \dfrac{V_0}{10} \cdot \dfrac{\sqrt{3}}{2} \Rightarrow V_0 = 30\sqrt{3}\ m/s$

$V_x = V_0 \cos 60° \Rightarrow V_x = 30\sqrt{3} \cdot \dfrac{1}{2} = 15\sqrt{3}\ m/s$

Velocidade do balão: $V_x = 15\sqrt{3}\ m/s$

b) $S = S_0 + V_0 t + \dfrac{a \cdot t^2}{2} \Rightarrow h = 0 + \dfrac{10}{2} \cdot 3^2 \Rightarrow h = 45\ m$

tilibra

c) $x = V_0 \cos 60° \cdot t = 15\sqrt{3} \cdot 3 = 45\sqrt{3}\ m$

d) $V = V_0 + a t \Rightarrow V = 0 + 10 \cdot 3 = 30\ m/s$

/ /

18) $V_{0x} = 20\,m/s$ $\quad V_{0y} = -15\,m/s$ $\quad a_x = 4\,m/s^2$

a) $V = V_0 + at$

$\vec{V}(t) = (20 + 4t)\hat{i} - 15\hat{j}$

b) $\vec{V}(5) = (20 + 4\cdot 5)\hat{i} - 15\hat{j}$

$\vec{V}(5) = 40\hat{i} - 15\hat{j}$ $\quad |\vec{V}| = \sqrt{40^2 + 15^2} \cong 42{,}7\,m/s$

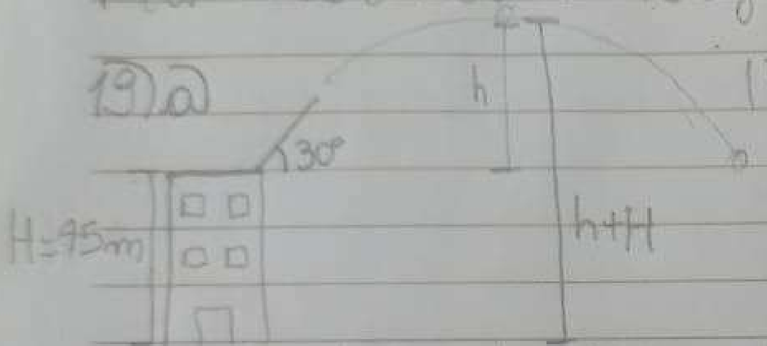
c) $\vec{r}(t) = \vec{r}_0 + \vec{V}_0 t + \vec{a}/2\, t^2$ e $\vec{r}(t) = \vec{r}_0 + \vec{V}t$

Parte da origem $r_{0x} = 0$; $r_{0y} = 0$

$\vec{r}(t) = (20t + 4/2\, t^2)\hat{i} - 15t\hat{j}$

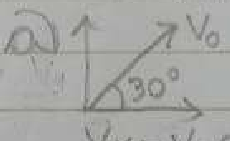
$\vec{r}(t) = (20t + 2t^2)\hat{i} - 15t\hat{j}$

19) a)



$|\vec{V}_0| = 20\,m/s$

a)



$V_y = V_0 \,sen 30°$

$V_x = V_0 \cos 30°$

Movimento vertical:

$V^2 = V_0^2 + 2a\Delta S$

$0 = (20\,sen 30°)^2 - 2\cdot 10\cdot h$

$0 = 400\cdot\frac{1}{4} - 20h$

$20h = 100 \Rightarrow h = 5m$

Altura Total da descida:

$45 + 5 = 50m$

$S = S_0 + V_0 t + \frac{a}{2}t^2$

$\Delta S = V_0 t + (a/2)t^2$

Na descida o movimento começa depois que a pedra para no alto, logo $V_0 = 0$

$50 = 0 + (10/2)\,t_d^2$

$50 = 5t_d^2 \Rightarrow t_d^2 = 10 \Rightarrow t_d = \sqrt{10}\,s$

Movimento vertical subida:

$V = V_0 + at$

$0 = 20\,sen 30° - 10t_s$

$10t_s = 20\cdot\frac{1}{2}$

$t_s = 1\,s$

$t_{TOTAL} = 1 + \sqrt{10}\,s$

b) $V = V_0 + at_d$

$V = 0 + 10\cdot\sqrt{10} = 10\sqrt{10}\,m/s$

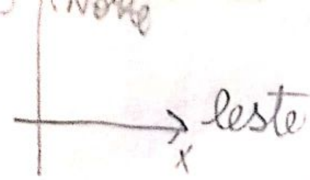
c) $X = X_0 + V\cdot t_{TOT}$

$X = V_0 \cos 30°\cdot t_{TOT}$

$X = 20\cdot\frac{\sqrt{3}}{2}\cdot(1 + \sqrt{10})$

$X = 10\sqrt{3}(1 + \sqrt{10})\,m$

tilibra



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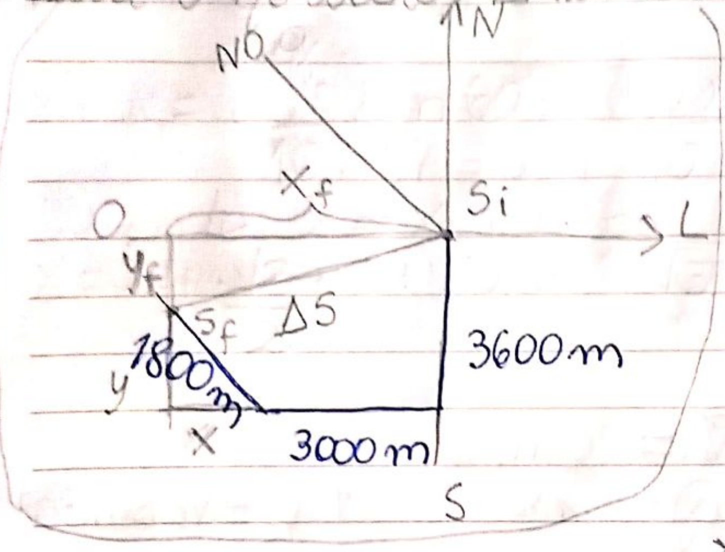
8) a) Para o Sul: $\Delta S_S = V \cdot \Delta t = 20 \cdot (3 \cdot 60) = 3600\,m$

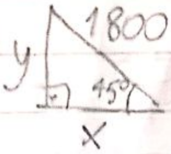
Para o Oeste: $\Delta S_O = 25 \cdot 2 \cdot 60 = 3000\,m$

Para o Noroeste: $\Delta S_{NO} = 30 \cdot 1 \cdot 60 = 1800\,m$



1º) $\vec{\Delta S} = \vec{S_f} - \vec{S_i}$

$\vec{S_i} = (0,0)$



$\text{sen}\,45° = \frac{y}{1800}$

$y = 1800 \cdot \sqrt{2}/2$

$y = 900\sqrt{2}\,m$

$\cos 45° = \frac{x}{1800}$

$x = 900\sqrt{2}\,m$

2º) $x_f = x + 3000 = 900\sqrt{2} + 3000\,m = 300(3\sqrt{2}+10)\,m$

$y_f = 3600 - y = 3600 - 900\sqrt{2} \Rightarrow y_f = 300(12 - 3\sqrt{2})\,m$

$\vec{S_f} = (x_f, y_f) \Rightarrow \vec{S_f} = (300(3\sqrt{2}+10), 300(12-3\sqrt{2}))$

$\vec{\Delta S} = S_f - S_i = (300(3\sqrt{2}+10), 300(12-3\sqrt{2})) - (0,0)$

$\vec{\Delta S} = (300(3\sqrt{2}+10), 300(12-3\sqrt{2}))\,m$

b) $|\vec{\Delta S}| = \sqrt{90000(9 \cdot 2 + 2 \cdot 3 \cdot 10\sqrt{2} + 100) + 90000(144 - 2 \cdot 12 \cdot 3\sqrt{2} + 9 \cdot 2)}$

$|\vec{\Delta S}| = 300\sqrt{118 + 60\sqrt{2} + 162 - 72\sqrt{2}}$

$|\vec{\Delta S}| = 300\sqrt{280 - 12\sqrt{2}} = 600\sqrt{70 - 3\sqrt{2}}\,m$

$v = \frac{|\vec{\Delta S}|}{\Delta t} = \frac{\cancel{600}\sqrt{70-3\sqrt{2}}}{6 \cdot \cancel{60}} \Rightarrow v = \frac{5}{3}\sqrt{70 - 3\sqrt{2}}\,m/s$

c) $\vec{V} = \frac{\vec{\Delta S}}{\Delta t} = \frac{(\cancel{300}(3\sqrt{2}+10), \cancel{300}(12-3\sqrt{2}))}{6 \cdot \cancel{60}}$

$\vec{V} = \left(\frac{5}{6}(3\sqrt{2}+10), \frac{5}{6}(12 - 3\sqrt{2})\right)\,m/s$

9) a) $\vec{V} = \frac{\Delta \vec{r}}{\Delta t}$ $\quad \vec{r}(t) = (1t+1)\hat{i} + (0{,}125t^2 + 1)\hat{j}$

$\vec{r}(4) = (4+1)\hat{i} + (0{,}125 \cdot 16 + 1)\hat{j} \Rightarrow r(4) = 5\hat{i} + 3\hat{j}$

$\vec{r}(2) = (2+1)\hat{i} + (0{,}125 \cdot 4 + 1)\hat{j} \Rightarrow r(2) = 3\hat{i} + 1{,}5\hat{j}$

$\Delta\vec{r} = \vec{r}(4) - \vec{r}(2) = 5\hat{i} + 3\hat{j} - 3\hat{i} - 1{,}5\hat{j}$

$\Delta\vec{r} = 2\hat{i} + 1{,}5\hat{j}$

/ /

$\Delta t = 4 - 2 = 2s$

$\vec{v} = \frac{2\hat{i} + 1{,}5\hat{j}}{2} \Rightarrow \vec{v} = 1\hat{i} + 0{,}75\hat{j}$ m/s

b) $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{v}(t) = \frac{d}{dt}\left[(1t+1)\hat{i} + (0{,}125t^2 + 1)\hat{j}\right]$

$\vec{v}(t) = 1\hat{i} + 0{,}25t\hat{j}$

$\vec{v}(2) = 1\hat{i} + 0{,}25 \cdot 2\hat{j} \Rightarrow \vec{v}(2) = 1\hat{i} + 0{,}5\hat{j}$ m/s

10) a) $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$\vec{r}(1) = (2 \cdot 1 - 3 \cdot 1)\hat{i} + (1 - 2 + 1)\hat{j} \Rightarrow \vec{r}(1) = -1\hat{i}$

b) $\vec{v}_x(t) = \frac{dx}{dt} \Rightarrow \vec{v}_x(t) = (6t^2 - 6t)\hat{i}$

$\vec{v}_y(t) = \frac{dy}{dt} \Rightarrow \vec{v}_y(t) = (2t - 2)\hat{j}$

c) $v(0) = v_x(0) + v_y(0) \Rightarrow \vec{v}(0) = -2\hat{j}$

$\vec{v}(1) = 0$

d) $\begin{cases} 0 = (6t^2 - 6t) \\ 0 = 2t - 2 \end{cases} \Rightarrow \begin{cases} 6t^2 = 6t \\ 2t = 2 \end{cases} \Rightarrow \begin{cases} t^2 = t \\ t = 1 \end{cases}$ $t = 1s$

e) $\vec{a}(t) = \frac{dv}{dt}$

$\vec{a}_x(t) = \frac{dv_x}{dt} \Rightarrow \vec{a}_x(t) = (12t - 6)\hat{i}$

$\vec{a}_y(t) = \frac{dv_y}{dt} \Rightarrow \vec{a}_y(t) = 2\hat{j}$

f) Paralela ao eixo y quando $a_x = 0$

$12t - 6 = 0 \Rightarrow t = 2s$

11) a) $\vec{r}(t) = \int \vec{v}\,dt \Rightarrow x(t) = \int 2\,dt$

$x(t) = (2t + \vec{r}_0)\hat{i} \Rightarrow x(t) = 2t\hat{i}$ $x_0 = 0$

$y(t) = \int (4t^3 + 4t)\,dt \Rightarrow y(t) = 4 \cdot \frac{t^4}{4} + \frac{4t^2}{2} + y_0$ $y_0 = 2$

$y(t) = (t^4 + 2t^2 + 2)\hat{j}$

$\vec{r}(t) = (2t)\hat{i} + (t^4 + 2t^2 + 2)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{a}(t) = (12t^2 + 4)\hat{j}$

b)

13) a) $\vec{r}(t) = -5{,}0\,\sin(\omega t)\,\hat{\imath} + (4{,}0 - 5{,}0\cos(\omega t))\,\hat{\jmath}$

$\vec{v}(t) = -5{,}0\,\omega\cos(\omega t)\,\hat{\imath} + 5{,}0\,\omega\sin(\omega t)\,\hat{\jmath}$

$\vec{a}(t) = 5{,}0\,\omega^2\sin(\omega t)\,\hat{\imath} + 5{,}0\,\omega^2\cos(\omega t)\,\hat{\jmath}$

b) $\vec{r}(0) = -5{,}0\,\sin(0)\,\hat{\imath} + (4{,}0 - 5{,}0\cos(0))\,\hat{\jmath}$

$\vec{r}(0) = -1{,}0\,\hat{\jmath}$

$\vec{v}(0) = -5{,}0\,\omega\cos 0\,\hat{\imath} + 5{,}0\,\omega\sin 0\,\hat{\jmath}$

$\vec{v}(0) = -5{,}0\,\omega\,\hat{\imath}$

$\vec{a}(0) = 5{,}0\,\omega^2\,\hat{\jmath}$

14) a) $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{(9{,}00\hat{i} + 7{,}00\hat{j}) - (3{,}00\hat{i} - 2{,}00\hat{j})}{3}$

$\vec{a} = \dfrac{6{,}00\hat{i} + 9{,}00\hat{j}}{3} \Rightarrow \vec{a} = 2{,}00\hat{i} + 3{,}00\hat{j}$

b) $\vec{v}(t) = \int \vec{a}\,dt \Rightarrow \vec{v}(t) = \int (2{,}00\hat{i} + 3{,}00\hat{j})\,dt$

$\vec{v}(t) = 2{,}00t\hat{i} + 3{,}00t\hat{j} + \vec{v}_i$; $\vec{v}_i = (3{,}00\hat{i} - 2{,}00\hat{j})$

$\vec{v}(t) = (3{,}00 + 2{,}00t)\hat{i} + (-2{,}00 + 3{,}00t)\hat{j}$

$$\vec{r}(t) = \int \vec{v}\, dt = \int [(3{,}00 + 2{,}00t)\hat{i} + (-2{,}00 + 3{,}00t)\hat{j}]\, dt$$

$$\vec{r}(t) = \left(3{,}00t + \frac{2{,}00t^2}{2}\right)\hat{i} + \left(-2{,}00t + \frac{3{,}00t^2}{2}\right)\hat{j} + \vec{r}_i$$

$\vec{r}_i = \vec{0}$ pois está na origem em $t=0$

$$\vec{r}(t) = (3{,}00t + 1{,}00t^2)\hat{i} + (-2{,}00t + 1{,}50t^2)\hat{j}$$