Eletromagnetismo

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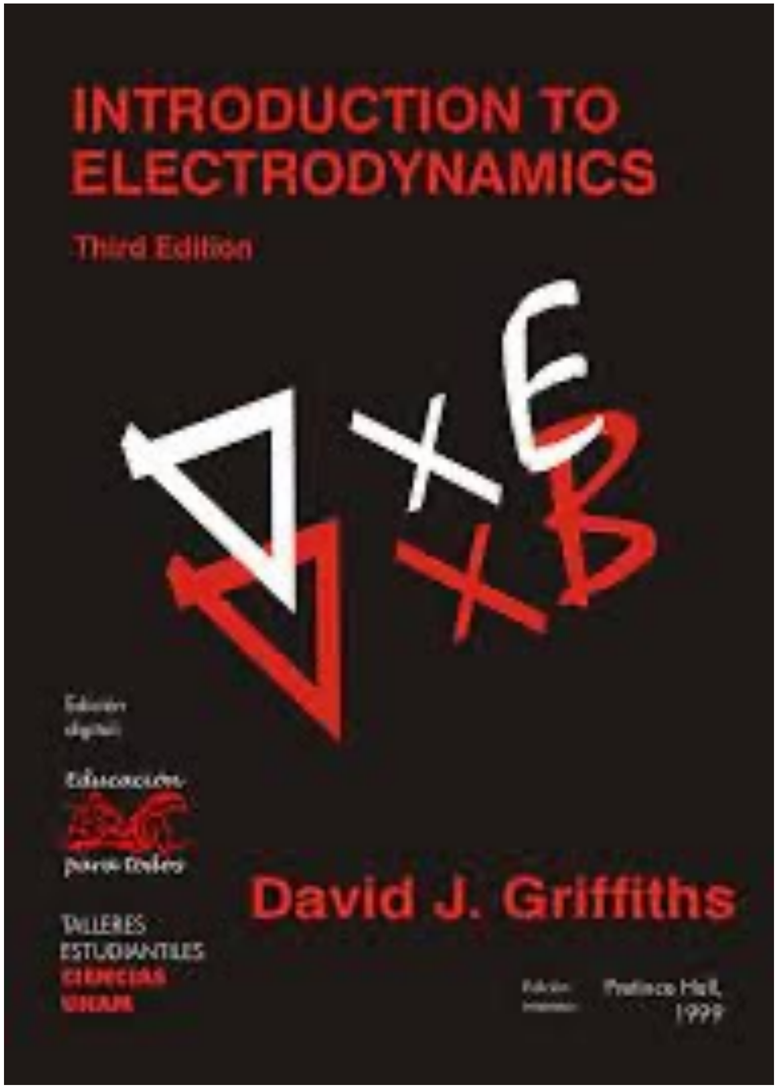
Plano do Curso

16/08	13/09	11/10	08/11
19/08	16/09	14/10	11/11
23/08	20/09 P1	18/10	15/11
26/08	23/09	21/10 P2	18/11
30/08	27/09	25/10	22/11
02/09	30/09	28/10	25/11
06/09	04/10	01/11	29/11 P3
09/09	07/10	04/11	02/12 ex
			06/12 Sub

Bibliografia



Capítulo 2 : eletrostática

Capítulo 5 : magnetostática

Capítulo 7 : eletrodinâmica

Capítulo 8 : leis de conservação

Capítulo 9 : ondas eletromagnéticas

Capítulo 10 : campos e potenciais

Capítulo 11 : radiação

Bibliografia

Física 3

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Instituto de Fisica da Universidade de São Paulo

São Paulo, 5 de fevereiro de 2020

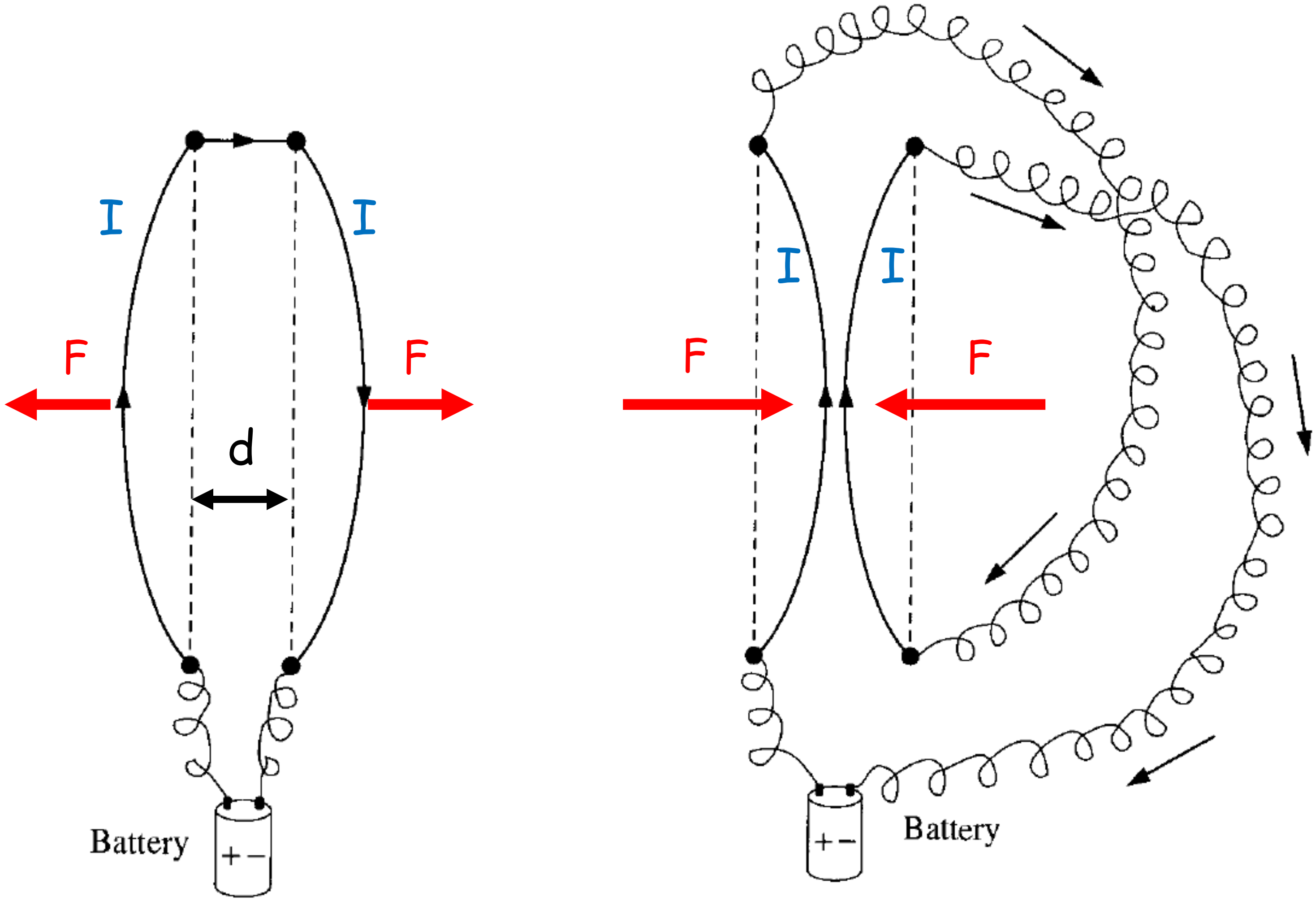
Aula 9

Magnetostática

Griffiths

Capítulo 5

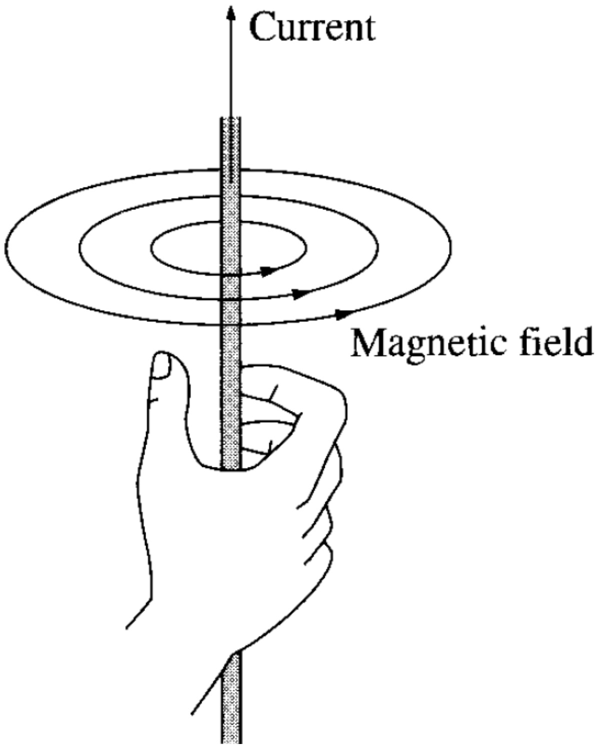
Força entre dois fios infinitos percorridos por correntes I

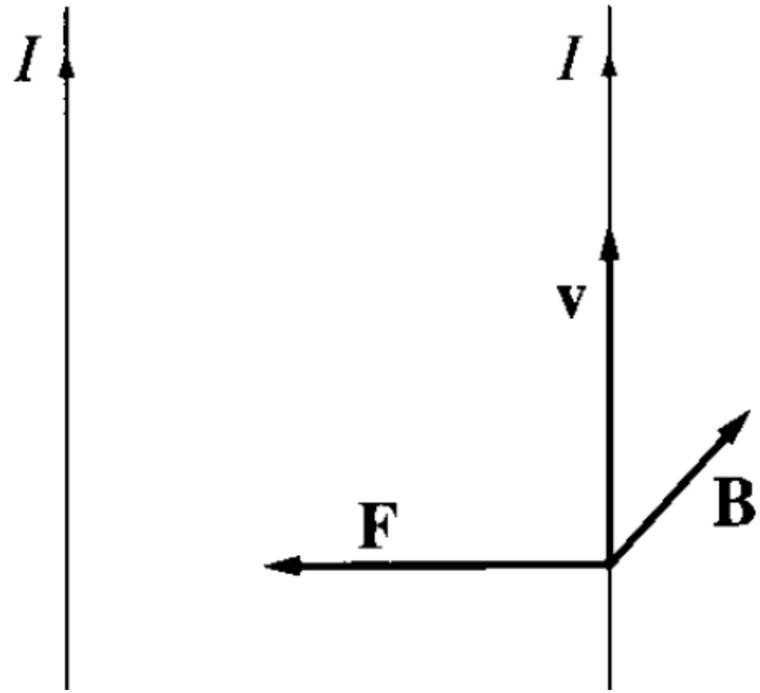


Empiricamente: $$F \propto \frac{I\,I}{d}$$

A corrente no fio 1 gera campo magnético

O campo magnético age sobre as cargas em movimento no fio 2





$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\imath}}}{\imath^2}.$$

Biot-Savart

$$\vec{F} = I \int d\vec{l} \times \vec{B}$$

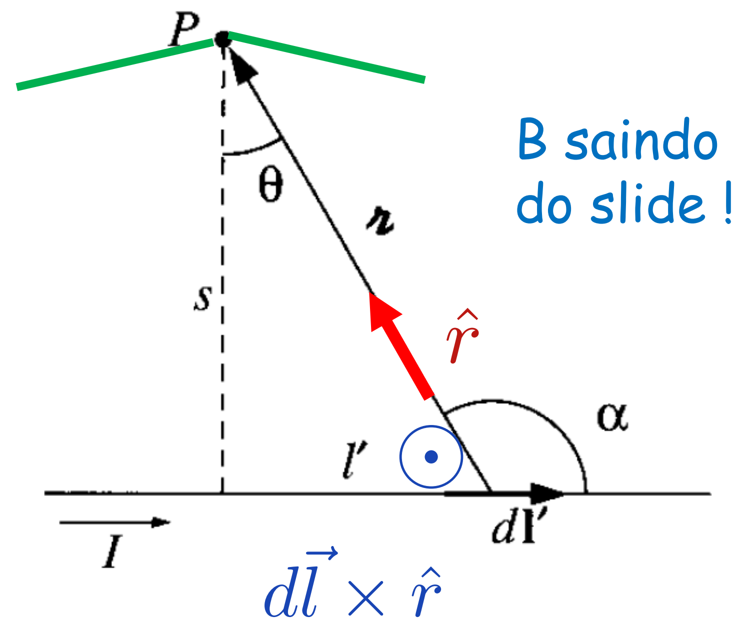
Campo magnético a uma distância s de um fio infinito com corrente I

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$|d\vec{l} \times \hat{r}| = dl' \, sen\alpha \quad = dl' \, cos\theta$$

$$dl' = \frac{s}{cos^2\theta} d\theta \qquad \frac{1}{r^2} = \frac{cos^2\theta}{s^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{s}{cos^2\theta} d\theta \cos\theta \frac{cos^2\theta}{s^2}$$



$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} d\theta \cos\theta \frac{1}{s}$$

$$\theta_1 = -\frac{\pi}{2} \qquad \theta_2 = +\frac{\pi}{2}$$

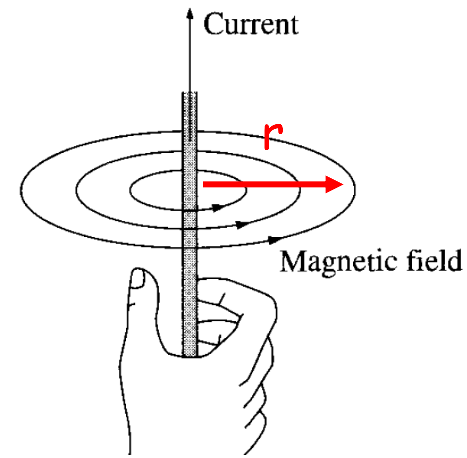
$$B = \frac{\mu_0 I}{4\pi s} (sen\,\theta_2 - sen\,\theta_1)$$

$$B = \frac{\mu_0 I}{2\pi s}$$

Lei de Ampère

A corrente no fio gera campo magnético :

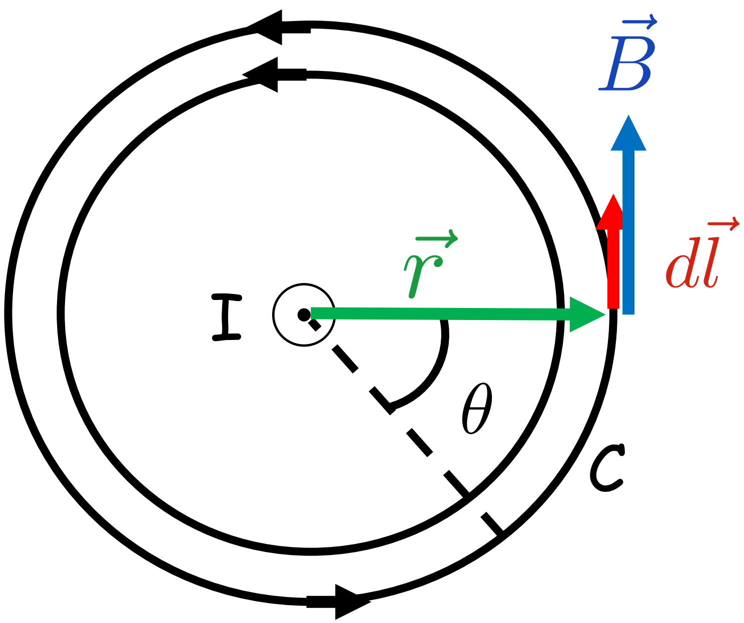
$$B = \frac{\mu_0 \, I}{2 \, \pi \, r}$$



Vamos calcular:

$$\oint_C \vec{B} \, . \, d\vec{l}$$

$$\vec{B} \, . \, d\vec{l} = B \, dl = B \, r \, d\theta$$



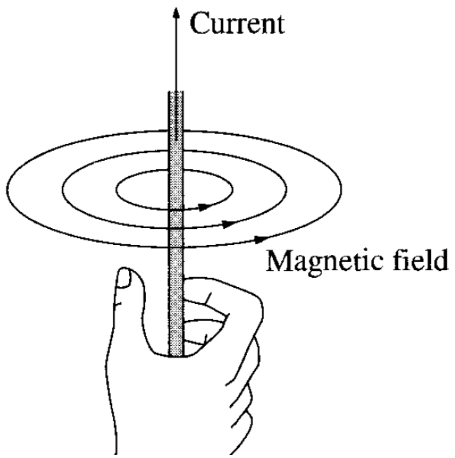
$$\oint_C \vec{B} \, . \, d\vec{l} = B \, r \int_0^{2\pi} d\theta = B \, r \, 2 \, \pi$$

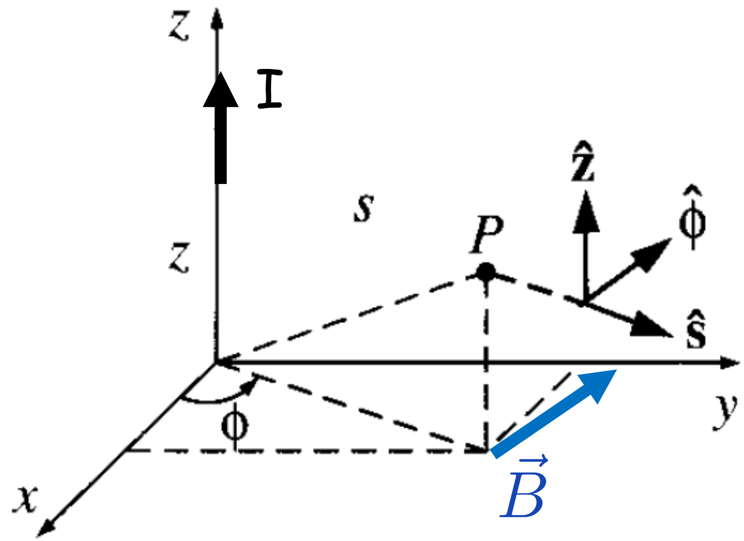
$$\oint_C \vec{B} \, . \, d\vec{l} = \frac{\mu_0 \, I}{2 \, \pi \, r} \, 2 \, \pi \, r$$

$$\oint_C \vec{B} \, . \, d\vec{l} = \mu_0 \, I$$

corrente enlaçada em C

Agora em coordenadas cilíndricas





$$\vec{B} = \frac{\mu_0 \, I}{2\,\pi\,s}\,\hat{\phi}$$

$$d\vec{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

$$\oint_C \vec{B}\,.\,d\vec{l} = \oint_0^{2\pi} \frac{\mu_0 \, I}{2\,\pi\,\cancel{s}}\,\cancel{s}\,d\phi$$

$$\oint_C \vec{B}\,.\,d\vec{l} = \mu_0 \, I$$

Lei de Ampère

Filho de família rica

Andre-Marie Ampère
(1775 - 1836)

Aos 18 anos perdeu o pai, decapitado na guilhotina

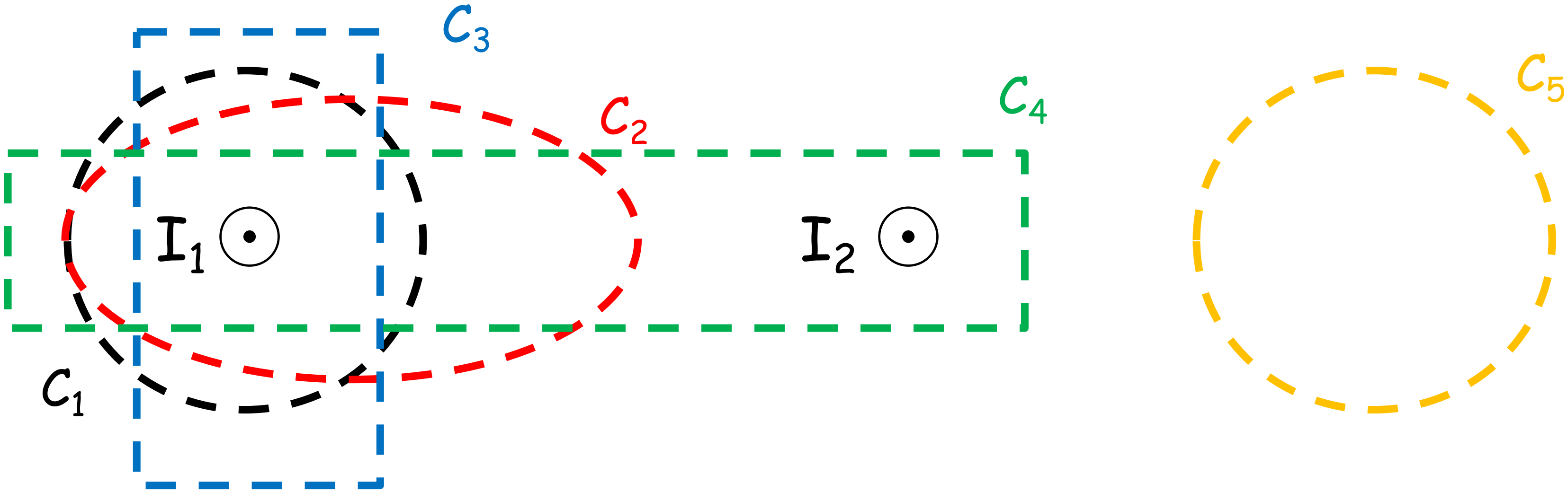
Um ano sem fazer nada...

Artigo famoso sobre a teoria dos jogos de azar

Professor do ensino médio por alguns anos

Depois na Ecole Polytechnique

1823: primeiros trabalho sobre eletricidade e magnetismo



$$\oint_{C_1} \vec{B}\,.\,d\vec{l} = \mu_0\, I_1$$

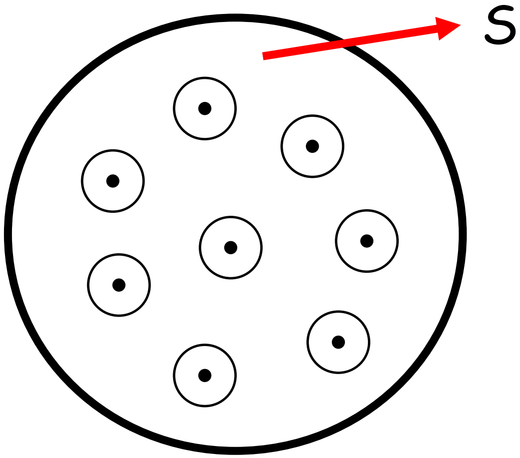
$$\oint_{C_5} \vec{B}\,.\,d\vec{l} = 0$$

$$\oint_{C_2} \vec{B}\,.\,d\vec{l} = \mu_0\, I_1$$

$$\oint_{C_3} \vec{B}\,.\,d\vec{l} = \mu_0\, I_1$$

$$\oint_{C_4} \vec{B}\,.\,d\vec{l} = \mu_0\, I_1\, +\, \mu_0\, I_2$$

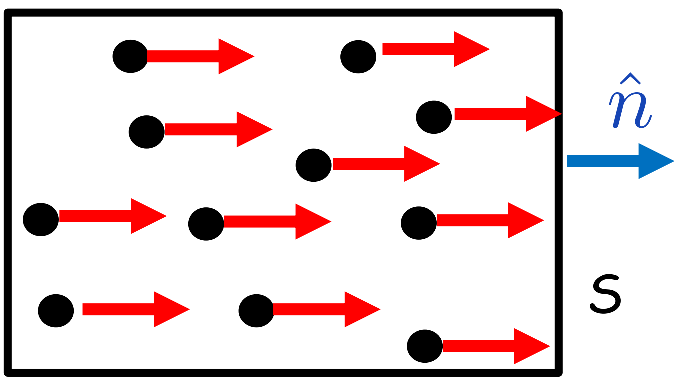
Fio com seção transversal grande = tubo de correntes



Densidade de corrente : $J = \dfrac{\text{corrente}}{\text{área}}$

$$J = \frac{dI}{da}$$

$$\vec{J} = \frac{d\vec{I}}{da}$$



$$d\vec{I} = \vec{J}\, da$$

$$d\vec{I}\,.\,\hat{n} = \vec{J}\,.\,\hat{n}\, da$$

$$dI = \vec{J}\,.\,d\vec{a}$$

$$I = \int_S \vec{J}\,.\,d\vec{a}$$

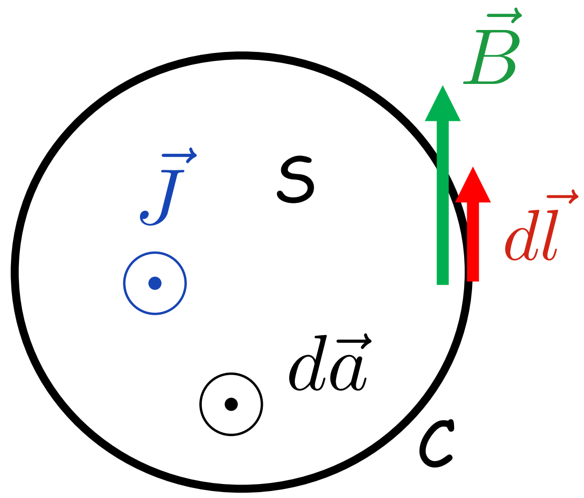
Teorema de Stokes

$$\oint_C \vec{B} \,.\, d\vec{l} = \mu_0 \, I$$

Lei de Ampère

$$\oint_C \vec{B} \,.\, d\vec{l} = \int_S (\vec{\nabla} \times \vec{B}) \,.\, d\vec{a}$$

$$I = \int_S \vec{J} \,.\, d\vec{a}$$

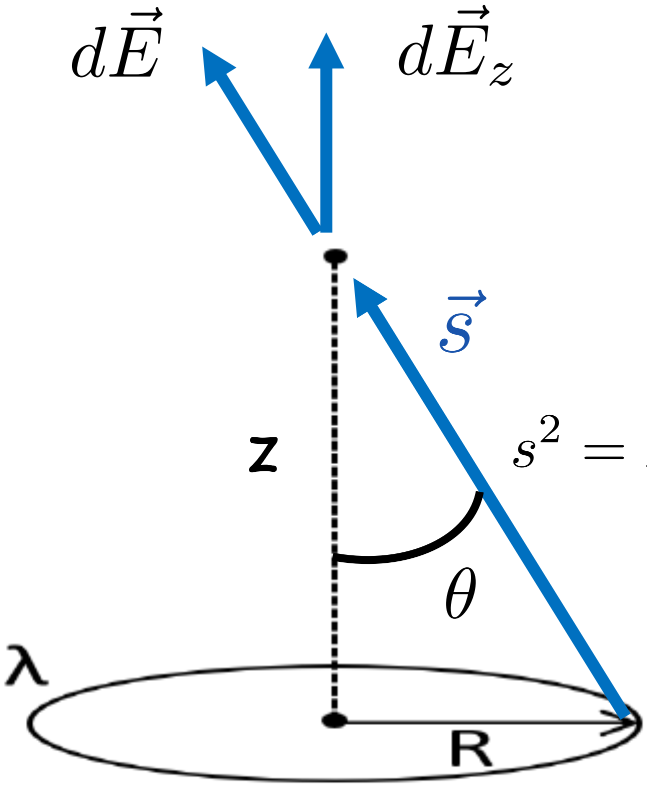


$$\int_S (\vec{\nabla} \times \vec{B}) \,.\, d\vec{a} = \mu_0 \int_S \vec{J} \,.\, d\vec{a}$$

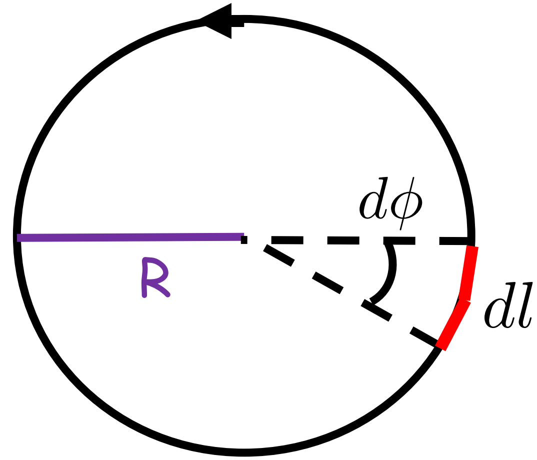
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \, \vec{J}}$$

Lei de Ampère na forma diferencial

Correção da 1ª Prova



$$s^2 = R^2 + z^2$$



$$dq = \lambda\, dl$$

$$dl = R\, d\phi$$

$$dq = \lambda\, R\, d\phi$$

$$dE_z = \frac{1}{4\,\pi\,\epsilon_0}\frac{\lambda\, R\, d\phi}{R^2 + z^2}\frac{z}{\sqrt{R^2 + z^2}}$$

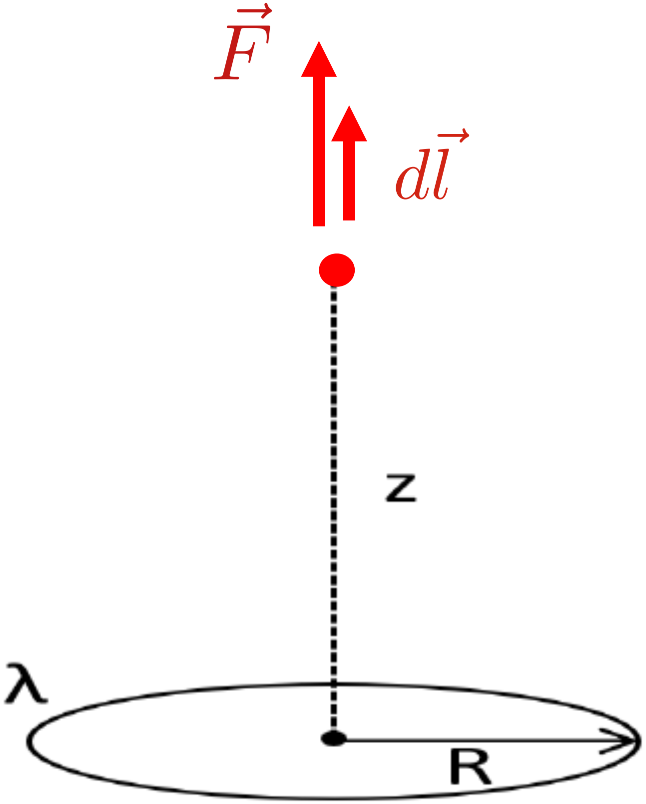
$$dE = \frac{1}{4\,\pi\,\epsilon_0}\frac{dq}{s^2}$$

$$E_z = \frac{\lambda\, R\, z}{4\,\pi\,\epsilon_0}\frac{1}{[R^2 + z^2]^{3/2}}\int_0^{2\pi} d\phi$$

$$dE_z = dE\, cos\theta$$

$$cos\,\theta = \frac{z}{\sqrt{R^2 + z^2}}$$

$$\vec{E}_z = \frac{\lambda\, R\, z}{2\,\epsilon_0}\frac{1}{[R^2 + z^2]^{3/2}}\hat{z}$$



$$\vec{F} = q\,\vec{E}_z \qquad \vec{E}_z = \frac{\lambda\,R\,z}{2\,\epsilon_0}\,\frac{1}{[R^2+z^2]^{3/2}}\,\hat{z}$$

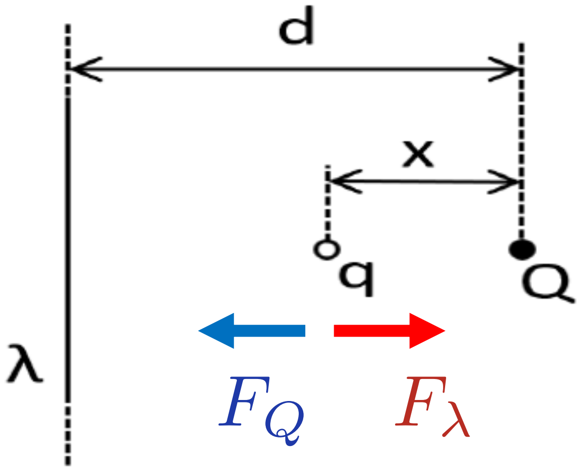
$$\vec{F} = \frac{q\,\lambda\,R\,z}{2\,\epsilon_0}\,\frac{1}{[R^2+z^2]^{3/2}}\,\hat{z}$$

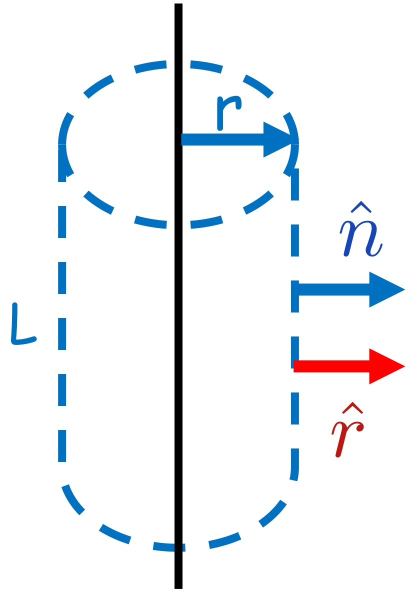
$$d\vec{l} = dz\,\hat{z}$$

$$\vec{F}\,.\,d\vec{l} = \frac{q\,\lambda\,R}{2\,\epsilon_0}\,\frac{z\,dz}{[R^2+z^2]^{3/2}}$$

$$W = \int_z^{\infty} \vec{F}\,.\,d\vec{l} \qquad = \frac{q\,\lambda\,R}{2\,\epsilon_0}\int_z^{\infty}\frac{z\,dz}{[\cancel{R^2}+z^2]^{3/2}} \qquad = \frac{q\,\lambda\,R}{2\,\epsilon_0}\int_z^{\infty}\frac{dz}{z^2}$$

$$\boxed{W = \frac{q\,\lambda\,R}{2\,\epsilon_0\,z}}$$





$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0}$$

$$\vec{E} = E\,\hat{r}$$

$$\vec{E}\,.\,\hat{n} = E$$

$$2\,\pi\,r\,L\,E = \frac{\lambda\,L}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\,\pi\,\epsilon_0\,r}}$$

$$F_Q = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{q\,Q}{x^2}$$

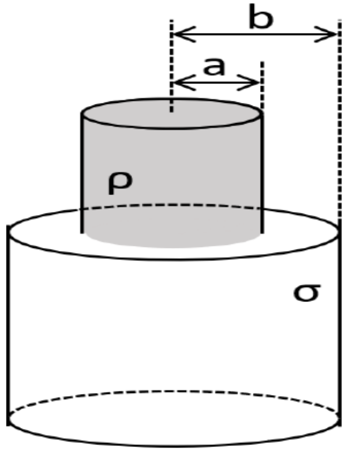
$$F_\lambda = q\,E = \frac{q\lambda}{2\,\pi\,\epsilon_0\,(d-x)}$$

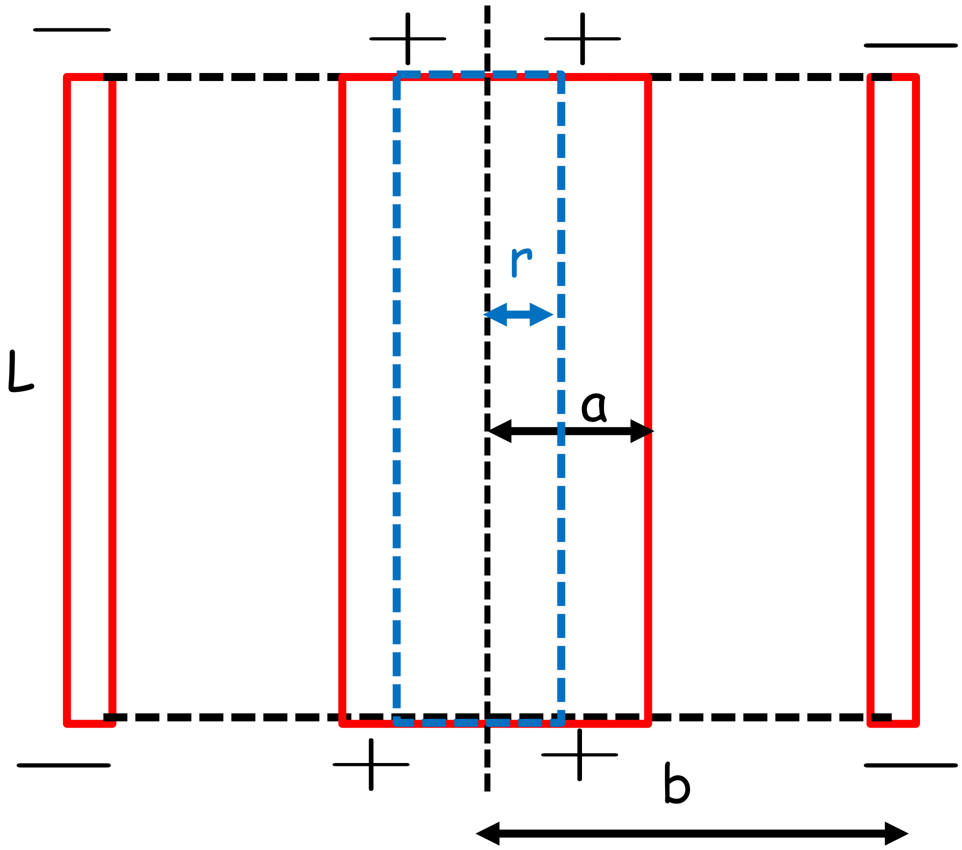
$$\frac{1}{4\,\pi\,\epsilon_0}\,\frac{q\,Q}{x^2} = \frac{q\lambda}{2\,\pi\,\epsilon_0\,(d-x)}$$

$$\frac{1}{2}\,\frac{2}{x^2} = \frac{1}{2-x}$$

$$x^2 + x - 2 = 0$$

$$\boxed{x = 1 \text{ m}}$$





$$q_+ = \pi\, a^2\, L\, \rho$$

$$q_- = 2\, \pi\, b\, L\, \sigma$$

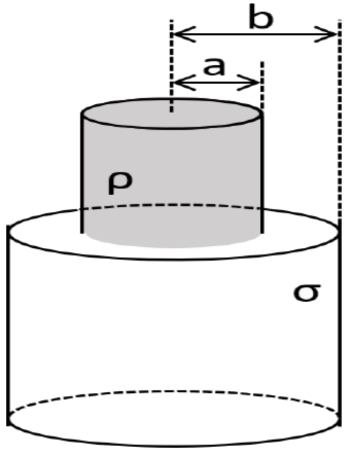
$$q_+ + q_- = 0$$

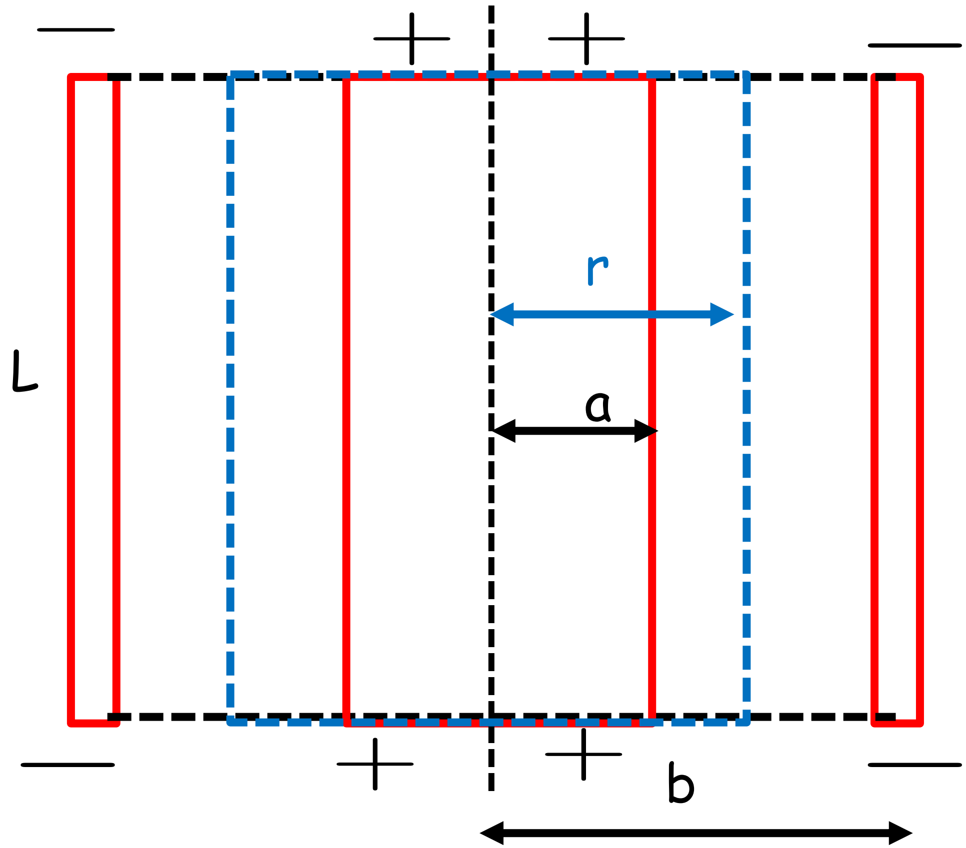
$$\sigma = -\frac{a^2\, \rho}{2\, b}$$

$$r < a$$

$$2\, \cancel{\pi}\, \cancel{r}\, \cancel{L}\, E = \frac{\cancel{\pi}\, r^{\cancel{2}}\, \cancel{L}\, \rho}{\epsilon_0}$$

$$E = \frac{\rho\, r}{2\, \epsilon_0}$$





$$q_+ = \pi\, a^2\, L\, \rho$$

$$q_- = 2\, \pi\, b\, L\, \sigma$$

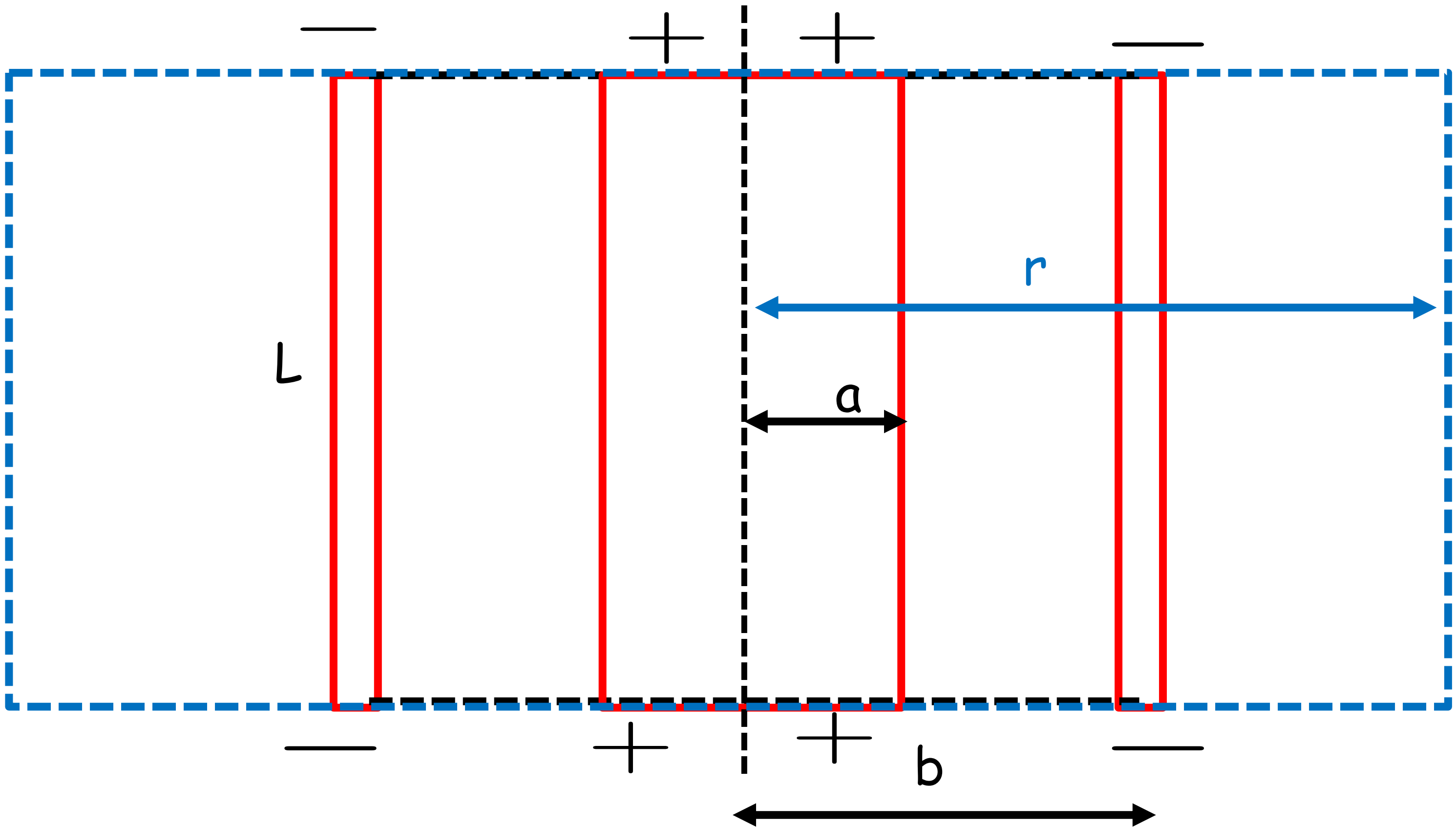
$$q_+ + q_- = 0$$

$$\sigma = -\frac{a^2\, \rho}{2\, b}$$

$$a < r < b$$

$$2\, \cancel{\pi}\, r\, \cancel{L}\, E = \frac{\cancel{\pi}\, a^2\, \cancel{L}\, \rho}{\epsilon_0}$$

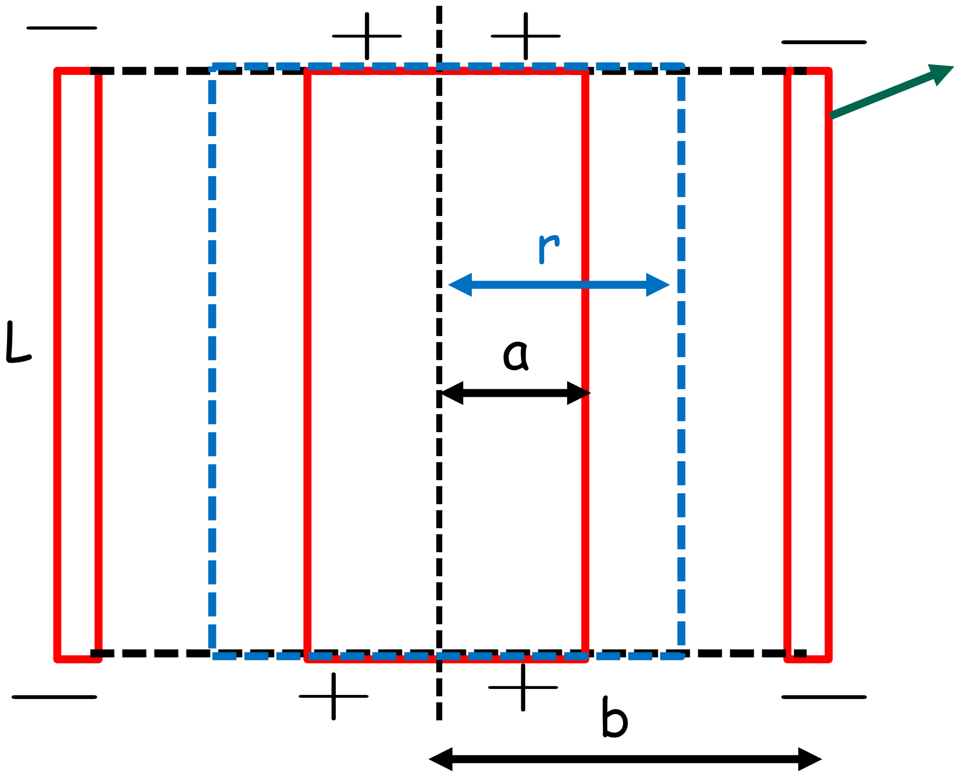
$$E = \frac{\rho\, a^2}{2\, \epsilon_0\, r}$$



$r > b$

$$2\,\pi\,r\,L\,E = \frac{\text{carga interna}}{\epsilon_0} = 0$$

$$E = 0$$



$$V(b) = 0$$

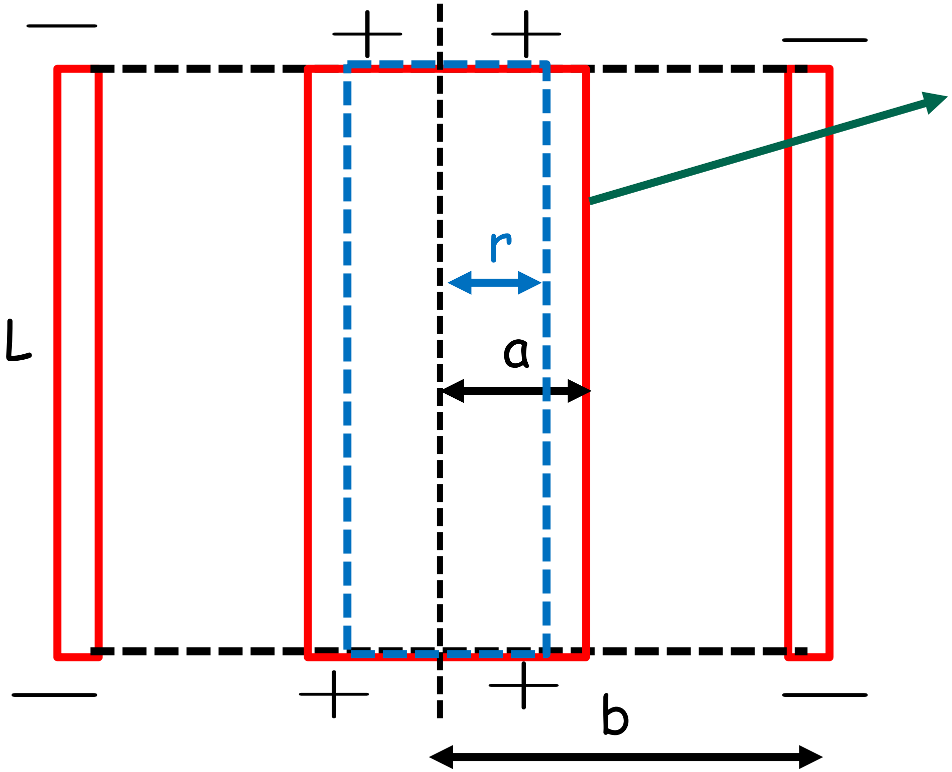
$$V(r) - V(b) = -\int_b^r \vec{E}\,.\,d\vec{l}$$

$$V(r) = -\int_b^r E\,.\,dr$$

$$r > b \qquad E = 0 \qquad \boxed{V(r) = 0}$$

$$a < r < b \qquad V(r) = -\int_b^r \frac{\rho\, a^2}{2\,\epsilon_0\, r} dr \;=\; -\frac{\rho\, a^2}{2\,\epsilon_0} ln\,\frac{r}{b} \;\boxed{= \frac{\rho\, a^2}{2\,\epsilon_0} ln\,\frac{b}{r}}$$

$$V(a) = \frac{\rho\, a^2}{2\,\epsilon_0} ln\,\frac{b}{a}$$



$$V(a) = \frac{\rho\, a^2}{2\,\epsilon_0} ln\, \frac{b}{a}$$

$$V(r) - V(a) = -\int_a^r \vec{E}\,.\,d\vec{l}$$

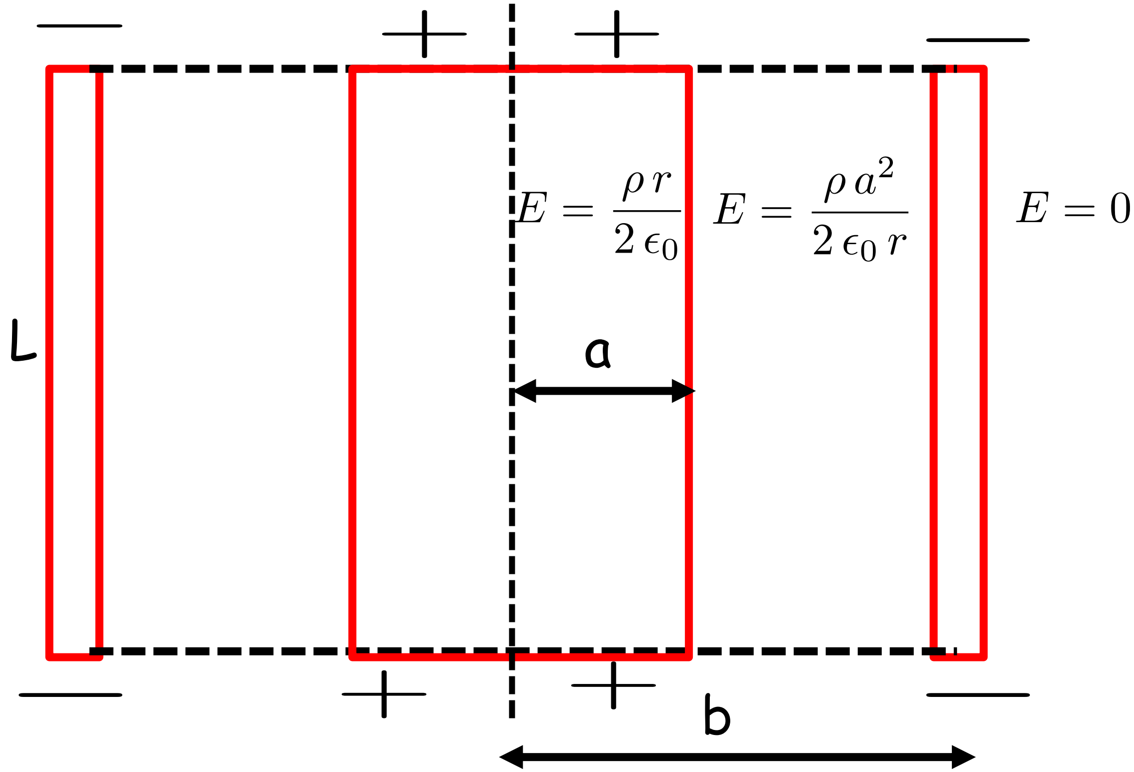
$$V(r) - V(a) = -\int_a^r E\,.\,dr$$

$r < a$

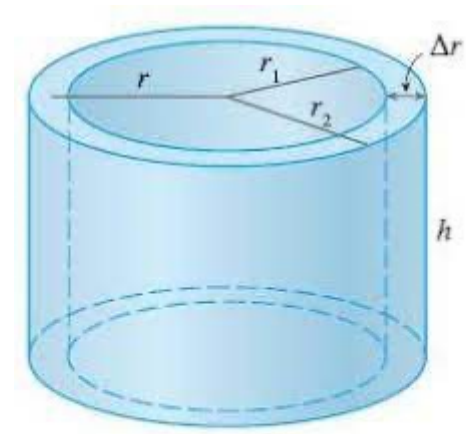
$$E = \frac{\rho\, r}{2\,\epsilon_0}$$

$$V(r) - \frac{\rho\, a^2}{2\,\epsilon_0} ln\, \frac{b}{a} = -\int_a^r \frac{\rho\, r}{2\,\epsilon_0} dr$$

$$V(r) = \frac{\rho\, a^2}{2\,\epsilon_0} ln\, \frac{b}{a} + \frac{\rho\, a^2}{4\,\epsilon_0} - \frac{\rho\, r^2}{4\,\epsilon_0}$$



$$W = \frac{\epsilon_0}{2} \int E^2 d^3 r$$



$$d^3 r = 2\,\pi\, r\, L\, dr$$

$$W = \frac{\epsilon_0}{2} \int_0^a \left(\frac{\rho\, r}{2\,\epsilon_0} \right)^2 2\,\pi\, L\, r\, dr + \int_a^b \left(\frac{\rho\, a^2}{2\,\epsilon_0\, r} \right)^2 2\,\pi\, L\, r\, dr$$

$$W = \frac{\pi\, L}{16} \frac{\rho^2\, a^4}{\epsilon_0} + \frac{\pi\, L}{4} \frac{\rho^2\, a^4}{\epsilon_0}\, ln\,(\frac{b}{a})$$

Fim