PSI5794 - Matrix Analysis - 2021

Homework 7 - Least Squares

1. A certain process $f\colon \mathbb{R} \to \mathbb{R}$ takes the form

$$f(x) = ax^2 + bx + c,$$

for some $a = 0.1$, $b = 1.0$ and $c = 1.5$. We take noisy measurements of this process and construct the following table:

x	0.1	0.5	1.0	2.0	2.5	3.0
$f(x)$	1.6912	1.9562	2.7460	3.9765	4.4972	5.3141

We want to use a mean-square framework to model this process.

(a) We know that the constant a is small, therefore we can approximate this s an affine function. Formulate the mean-square problem of finding the best polinomial $g(x) = px + q$ that approximate the process and find its solution. Compute the error.

(b) Now we want to model the system as a full quadratic function. Again, formulate the mean-square problem of finding the best polynomial $g(x) = rx^2 + sx + t$ that approximates the process and find its solution. Compute the error.

(c) Plot the graphs of the two solutions in a single figure. Also, plot a scatter graph of the set of measured points. Compare the results. (You may use software such as MATLAB or Octave to do this.)

(d) We make another mesurement and get $f(3.5) = 6.2250$. Find a way to compute the new solution from the previous one. Do so for both the affine case and the quadratic case. (Hint: the deterministic RLS algorithm.)

(e) Find the best (in a least-squares sense) degree 5 polynomial that approximates $f(x)$ using only the points in the table. Compute the error and plot the graph of the solution. Is it a good idea to use this solution to model $f(x)$?