

Homework LEC 04

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1) Classify the following Algebraic structures as group, ring or field. Show whether they are, or not, groups, rings or fields

a) (\mathbb{R}^*, \cdot) $a \cdot b \hat{=} a/b$. What is the neutral?

b) $(M_2^T, \text{matrix product}, I_2)$, M_2^T : 2×2 triang^{upper}-matrices

c) $(\bar{M}_2(\mathbb{R}), +, \text{matrix prod}, O_{2 \times 2}, I_2)$, \bar{M}_2 : 2×2 nonsing. matrices

d) $(\bar{M}_2^D(\mathbb{R}), +, \text{matrix prod}, D_{2 \times 2}, I_2)$, \bar{M}_2^D : diagonal 2×2 nonsing. matrices

e) $(\mathbb{Q}, +, \cdot, 0, 1)$

2) Create a non-trivial example and a counter example for: group, ring, field (6 cases total)

3) Solve problems Meyer 4.3.9, 4.3.11, 4.4.6

4) Determine $R(A)$ and $N(A)$ for A and A^T where

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 3 & 6 & 1 & 9 & 6 \\ 2 & 4 & 1 & 7 & 5 \end{bmatrix}$$