PSI 5794 – Principles of Matrix Analysis

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1. Contact information

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2. Syllabus

1. **Linear Vector Systems**
   Gauss Elimination and Gauss-Jordan. Scaling and Pivoting. Finite precision operations. Echelon forms and rectangular systems. Consistency. Homogeneous and non-homogeneous systems.
2. **Matrix Algebra**
   Elementary operations; matrix product. Block matrices and partitioning. Transposition, symmetries and structured matrices. Trace operator. Matrix inversion. Matrix inversion lemma. Elementary matrices. Equivalence. LU factorization.
3. **Vector Spaces**
   Groups, rings, fields, finite fields. Vector spaces. Subspaces. Operations with subspaces. Linear independence, basis and dimension. Range and null spaces.
4. **Linear Transformations**
   Matrix representation of a linear transformation (LT). Change of basis and vector coordinates. Similarity transformations. Equivalence transformations. Structure of LTs. The four fundamental spaces. Inverse linear transformation. Pseudo-inverse.
5. **Least-Squares**
   The least-squares problem. Orthogonality principle and the geometric argument. The algebraic argument. Weighted least squares. Regularized least-squares. Recursive least-squares.
6. **Eigenvalues and Eigenvectors; Canonical forms; Matrix Polynomials**
   Fundamentals. Eigenvalues and similarity. Multiplicity of eigenvalues: algebraic and geometric. Jordan form. Matrix polynomials. Matrix functions, series and sequences.
7. **Norms: vectors and matrices**
   Definition. Vector norms via inner products. Vector p-norms. Convergence of sequences. Matrix norms. Induced matrix norms. Condition number.
8. **Special matrices**
   Normal matrices. Unitary and orthogonal matrices. Hermitian and symmetric matrices. Positive definite matrices.
9. **QR factorization, Householder and Givens transforms**
   Orthonormal bases from generic bases. QR theorem. Least-squares and QR. QR implementation.
10. **Singular Value Decomposition (SVD)**
    Conceptual construction. The fundamental theorem. Outer product expansion. Pseudo-inverse revisited. Bases for the fundamental spaces. Linear systems solution. Image compression.
11. **Kronecker (tensor) product**
    The vec operator. Matrix linear systems. Sylvester and Lyapunov equations.

3. Grading

G = (a1***HW** + a2***E +** a3***P**)/(a1+a2+a3)

HW = homeworks (**No late** homework policy !)
E = exams (in class)
P = papers (in groups)

4. References

1. A.J. Laub, *Matrix Analysis for Scientists and Engineers*, SIAM: Society for Industrial and Applied Mathematics, 2004;

2. C.D. Meyer. *Matrix Analysis and Applied Linear Algebra*. SIAM, 2000;

3. R.A. Horn e C.R. Johnson. *Matrix Analysis*. Cambridge University Press, 2nd edition, 2013;

4. R.A. Horn e C.R. Johnson. *Topics in Matrix Analysis*. Cambridge University Press, 1991;

5. G.H. Golub e C.F. Van Loan. *Matrix Computations*. Johns Hopkins University Press, 3rd edition, 1996.

6. C. Pinter. *A Book of Abstract Algebra*. 2nd edition. Dover Publications;

7. N. Jacobson. *Basic Algebra I* . 2nd edition. Dover Publications, 2009;

7. N. Loehr. *Advanced Linear Algebra*. CRC Press, 2014;

8. B. N. Cooperstein. *Advanced Linear Algebra*. 2nd edition. CRC Press, 2015.