# Lecture 6
# Thermodynamics

Part II

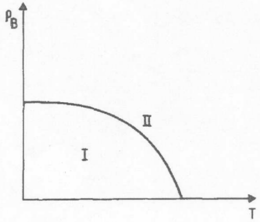
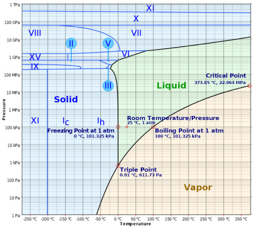



Fig. 1. Schematic phase diagram of hadronic matter. $\rho_B$ is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

First QCD phase diargam: N. Cabibbo and G. Parisi, Phys. Lett. B59 (1975) 67

# Phase diagram

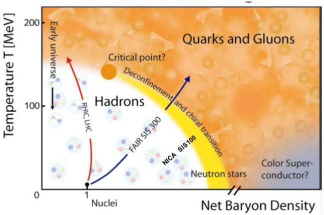



- Water can exist in various phases: solid, liquid, vapor, depending on the value of $p$ and $T$.
- Continuous line: phase boundary where water can exist in two phases simultaneously $\Rightarrow$ First order phase transition
  Phase transition (Ehrenfest classification):

  First order: first derivative of free energy has discontinuity, for example there is an entropy jump and latent heat. Second-order: first derivative continuous, second derivative not.
- Critical point: end of a phase boundary. After that, water will pass from liquid to vapor in a continuous way without doing a phase transition (no discontinuity in free energy or change of symmetry) At pressures and/or temperatures beyond the critical point, no physical distinction between liquid and gas.

  Density of the gas and thermal motion of the liquid so great that gas and liquid are the same.

# What do we know for QCD?



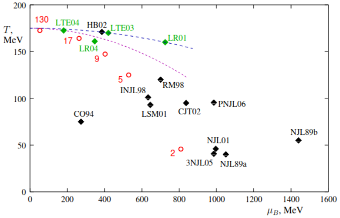




**Figure 4:** Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. Green points are lattice predictions: LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.

1. For $T$ or $\mu_b$ large: use thermodynamics for non-interacting particles
2. For $T$ small and $\mu_b$ large: use phenomenological models $\rightarrow$ first order phase transition line Stephanov "QCD phase diagram: an overview" in proc. LAT06
3. For $T$ large and $\mu_b$ small: use lattice QCD $\rightarrow$ crossover

$\Rightarrow$ There must be a critical point at the end of phase boundary

Note: Only 1 and 3 are rigorous calculations.

## Exercise: Toy model to obtain $T_{deconf}$

The MIT bag model was constructed to incorporate confinement and asymptotic freedom in a simple phenomenological model. Hadrons are pictured as drops of massless quarks asymptotically free maintained together by a (vacuum) pressure.

The idea can be extrapolated to a large number of quarks: $\epsilon_{qg} = \frac{\pi^2}{30}(g_g + \frac{7}{8}g_q)T^4 + B$ and $p_{qg} = \frac{\pi^2}{90}(g_g + \frac{7}{8}g_q)T^4 - B$. Assuming that this QGP becomes a gas of massless pions at low temperature, what are the deconfinement temperature and latent heat for $\mu = 0$? Use $B = 0.5\text{GeV}/fm^3$

MIT bag model: Chodos et al. Phys. Rev. D10 (1974) 2599; DeGrand et al. hys. Rev. D12 (1975) 2060

Caveats: 1) The pion mass cannot be ignored around 150-200 MeV. 2) $B$ should depend on $T$ and $\mu_b$ and go to 0 for high values of these quantities.

For a first order phase transition:
$p_{qg}(T_{deconf}, \mu_{deconf}) = p_\pi(T_{deconf}, \mu_{deconf})$.

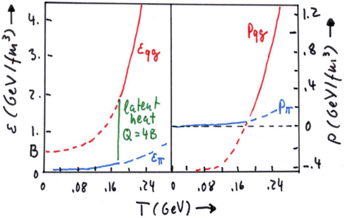
Here $\mu_{deconf} = 0$, so we can get $T_{deconf}$ from
$p_{qg}(T_{deconf}, 0) = p_\pi(T_{deconf}, 0)$.

We use $\epsilon_\pi = g_\pi \times \frac{\pi^2}{30} T^4$ and $p_\pi = \epsilon_\pi/3$.
We get:

$T_{deconf} = \left(\frac{90B}{(g_{qg} - g_\pi)\pi^2}\right)^{1/4}$, so $T_{deconf} \sim 180\,\text{MeV}$

The latent heat is $T(s_{qg} - s_\pi)$.
Using $\epsilon = Ts + \mu\, n - p$ (with $\mu = o$ and equal pressures), we get
$T(s_{qg} - s_\pi) = \epsilon_{qg} - \epsilon_\pi = \frac{\pi^2}{30}(g_g + \frac{7}{8}g_q)T^4_{deconf} + B - g_\pi \times \frac{\pi^2}{30} T^4_{deconf} =$
$4B = 2\,\text{GeV}\,fm^{-3}$

Note: $p$ and $T$ are equal in both phases but $s$ and $\epsilon$ have a jump.

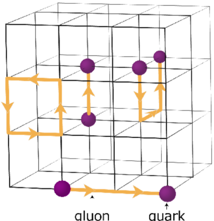


# Lattice QCD

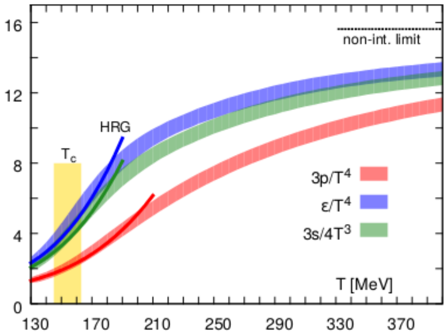
An alternative to compute the QGP equation fo state is to start from the grand canonical function $Z = \sum_{states} < a|e^{-\frac{\hat{H}-\mu\hat{N}}{T}}|a >= Tre^{-\frac{\hat{H}-\mu\hat{N}}{T}}$.

For massless bosons $Z_B = \frac{g_B V}{90\pi^2} T^4$ and for massless fermions $Z_{F\bar{F}} = g_f V(\frac{7\pi^2}{360} T^4 + \frac{\mu_q}{12} T^2 + \frac{mu_q^4}{24\pi^2})$.

Thermodynamics quantities are obtained with: $n = (T/V)\frac{\partial \ln Z}{\partial \mu}$, $\epsilon = (T^2/V)\frac{\partial \ln Z}{\partial T} + \mu n$, $p = T \ln Z/V$, $s = \frac{\partial T \ln Z}{\partial T}$.
For lattice QCD at $\mu = 0$, one can write
$Z = \int dAd\Psi d\bar{\Psi} exp\left(-\int_V d^3x \int_0^{1/T} d\tau \mathcal{L}(A, \psi, \bar{\Psi})\right)$.
The $\tau$ integral is on imaginary time between 0 and $1/T$.
Spacetime is then discretized with quarks at lattice sites and gluons at lattice links.



## Equation of state at $\mu = 0$




- $T_{deconf} = (154 \pm 9)\, MeV$, $\epsilon_c = (0.18 - 0.5)\ GeV/fm^3$, $\epsilon_{mat.nucl.} \sim 0.15\ GeV/fm^3$
- Cross-over: rapid increase in degrees of freedom
- Tends to hadron gas at low $T$
- Tends to ideal quark gluon gas at very high $T$: non-pertubative effects must be present near $T_{deconf}$

### What about $\mu \neq 0$?

Sign problem:

- Z contains a complex determinant
- No positive weight for Monte Carlo simulations

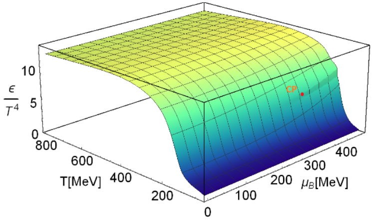
$\longrightarrow$ Use Taylor expansion to extrapolate into region of finite chemical potential

# Modern equations of state

Parotto et al. Phys. Rev. C 101, 034901 (2020); arXiv:2103.08146

$$p(T\mu_b) = T^4 \sum_n c_{2n}(T)\left(\frac{\mu_b}{T}\right)^{2n} \text{ where } c_n(T) = \frac{1}{n!}\frac{\partial^n p/T^4}{\partial(\mu_b/T)^n}\Big|_{\mu_b=0}$$

The critical point can be introduced phenomenologically by analogy to the 3D Ising model (QCD is in the same universality class




M.Stephanov "QCD phase diagram and the critical point" Prog.Theor.Phys.Suppl.15 (2004)139:

*This is a consequence of the fact that at $m_q \neq 0$ no symmetry remains which would require the order parameter to have more than just one component. The field theory which describes the static critical behavior, the one-component $\phi^4$ theory in 3 dimensions, has the critical exponents of the Ising model.*

## Challenge


Write the equation that gives the phase boundary between a nucleon gas (in the Boltzmann approximation) and QGP (treated with the MIT bag model).

Homework

For the toy model, what should be the range of values of $B$ to get a deconfinement temperature in agreement with lattice QCD?

Other references on this topic


- R. Vogt, Ultrarelativistic Heavy-ion Collisions, Elsevier, 2007
- C.Y. Wong, Introduction to High-Energy Heavy-Ion Collisions, World Scientific, 1994