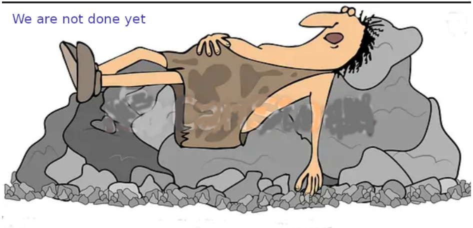
Lecture 3
Kinematic variables

Part II



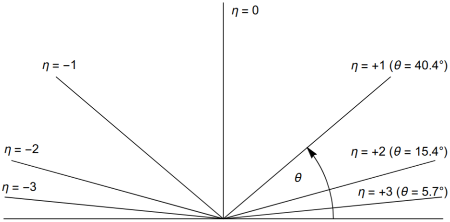
From CartoonStock

Pseudorapidity

Introduce θ, angle of momentum wrt collision axis: $p_z = |\vec{p}| \cos\theta$

$$\boxed{\eta \equiv \frac{1}{2} \ln \frac{|\vec{p}| + p_z}{|\vec{p}| - p_z}}$$

- So $\eta = \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta} = \frac{1}{2} \ln \frac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)} \boxed{= -\ln[\tan\frac{\theta}{2}]}$
- For $m \to 0$, $y = \eta$



$\theta \longrightarrow 0, \eta \longrightarrow \infty$

η is used when one does not know the particle mass

Exercise: write $|\vec{p}|$ and p_z in term of pseudorapidity and p_T

Summing and subtracting

$e^{\eta} = \sqrt{\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}}$, $e^{-\eta} = \sqrt{\frac{|\vec{p}|-p_z}{|\vec{p}|+p_z}}$

$\Rightarrow \boxed{|\vec{p}| = p_T \cosh\eta, p_z = p_T \sinh\eta}$

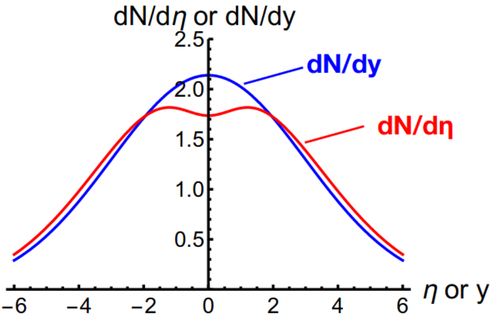
Relationship between rapidity and pseudorapidity

$m \to 0, y = \eta$, what happens for finite mass?

$$y = \frac{1}{2}\ln\frac{\sqrt{p_T^2\cosh^2\eta + m^2} + p_T\sinh\eta}{\sqrt{p_T^2\cosh^2\eta + m^2} - p_T\sinh\eta} \text{ and } \eta = \frac{1}{2}\ln\frac{\sqrt{m_T^2\cosh^2 y - m^2} + m_T\sinh y}{\sqrt{m_T^2\cosh^2 y - m^2} - m_T\sinh y}$$

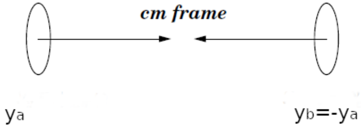
$$\Rightarrow \frac{dN}{d\eta} = \sqrt{1 - \frac{m^2}{m_T^2\cosh^2 y}}\frac{dN}{dy} = \frac{|\vec{p}|}{E}\frac{dN}{dy}$$

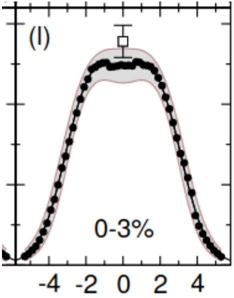
At midrapidity $y = 0 = \eta$ and $\frac{dN}{d\eta}_{|\eta=0} = \frac{p_T}{m_T}\frac{dN}{dy}_{|y=0} \leq \frac{dN}{dy}_{|y=0}$



Examples of rapidity and pseudorapidity distributions

Au+Au $dN/d\eta$

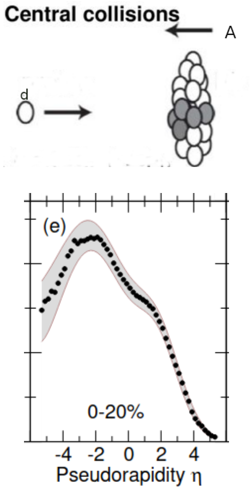




Pseudorapidity distribution for central Au+Au collisions at 200 GeV: PHOBOS Phys. Rev. C 83, 024913 (2011)

$$y_{beam} \sim \pm acosh 100 = 5.3 \text{ and } < N >= \int d\eta \frac{dN}{d\eta} \sim 5000$$

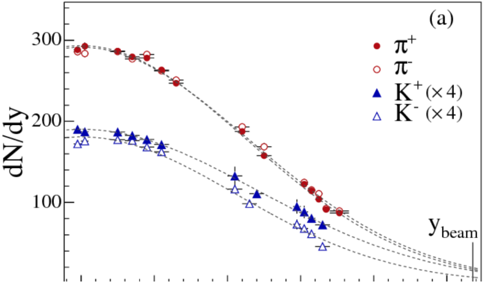
d+Au $dN/d\eta$



Pseudorapidity distribution for central d+Au collisions at 200 GeV: PHOBOS Phys. Rev. C 83, 024913 (2011)

Note the asymmetry

Au+Au dN/dy



Rapidity distribution for central Au+Au collisions at 200 GeV: BRAHMS Phys. Rev. Lett. 94 (2005) 16

Note no “hole” at $y = 0$ compared to $dN/d\eta$

Transverse momentum and transverse mass disitrbutions

$\frac{d^3N}{dp^3}$ is the number of particles with momentum in $[p_x, p_x + dp_x]$,$[p_y, p_y + dp_y]$, $[p_z, p_z + dp_z]$.

It is not Lorentz-invariant (= independent of the inertial frame), so instead we use the Lorentz-invariant

$$\boxed{E\frac{d^3N}{dp^3}}$$

It is more convenient to use the following forms:

$$E\frac{d^3N}{dp^3} = E\frac{d^3N}{p_T dp_T dp_z d\phi} = E\frac{d^3N}{p_T dp_T dy d\phi}\frac{dy}{dp_z} = E\frac{d^3N}{p_T dp_T dy d\phi}\frac{1}{m_T \cosh y} \overset{m_T \cosh y = E}{=}$$

$$= \frac{d^3N}{d\phi p_T dp_T dy}$$

$$= \frac{d^3N}{d\phi p_T dm_T dy}\frac{dm_T}{dp_T} = \frac{d^3N}{d\phi p_T dm_T dy}\frac{p_T}{m_T}$$

$$= \frac{d^3N}{d\phi m_T dm_T dy}$$

Exercise:

Let us check that $E\frac{d^3N}{d^3p}$ is Lorentz-invariant.

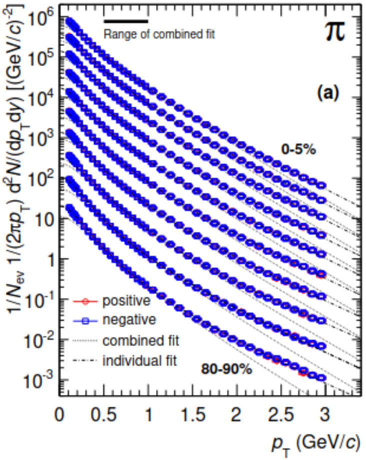
$$E'\frac{d^3N}{dp'_x dp'_y dp'_z} = E'\frac{\partial(p_x,p_y,p_z)}{\partial(p'_x,p'_y,p'_z)}\frac{d^3N}{dp_x dp_y dp_z} \equiv E'\begin{vmatrix} \frac{\partial p_x}{\partial p'_x} & \frac{\partial p_x}{\partial p'_y} & \frac{\partial p_x}{\partial p'_z} \\ \frac{\partial p_y}{\partial p'_x} & \frac{\partial p_y}{\partial p'_y} & \frac{\partial p_y}{\partial p'_z} \\ \frac{\partial p_z}{\partial p'_x} & \frac{\partial p_z}{\partial p'_y} & \frac{\partial p_z}{\partial p'_z} \end{vmatrix}\frac{d^3N}{dp_x dp_y dp_z}$$

We use: $p'_x = p_x, p'_y = p_y, p'_z = \gamma(p_z - vE), E' = \gamma(E - vp_z)$

$$\frac{\partial(p_x,p_y,p_z)}{\partial(p'_x,p'_y,p'_z)} = \begin{vmatrix} \frac{\partial p_x}{\partial p'_x} & 0 & 0 \\ 0 & \frac{\partial p_y}{\partial p'_y} & 0 \\ 0 & 0 & \frac{\partial p_z}{\partial p'_z} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma(1 + p'_z/E') = E/E' \end{vmatrix} = \frac{E}{E'}$$

So $E'\frac{d^3N}{dp'_x dp'_y dp'_z} = E\frac{d^3N}{dp_x dp_y dp_z}$

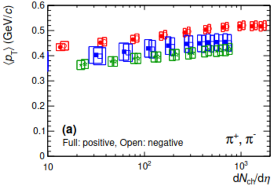
Example of momentum distributions



ALICE (mid-rapidity) Pb+Pb at 2.76 TeV: Phys. Rev. C 88 (2013) 044910

Challenge

In the paper mentioned in previous slide, Phys. Rev. C 88 (2013) 044910, the ALICE Collaboration fits their data for $1/p_t \, dN/dp_T$ with functions of the type $e^{-p_T/T}$. For $p_T < 2\,GeV$ and the plot shown in previous slide, $T \sim .2 - .3\,GeV$ (in the 0-5% centrality bin). Compute $< p_T >$ and compare with the ALICE plot below (red points).



Homework

1) At what angle (in degrees) with respect to the beam axis is emitted a particle that has $\eta = 2$?

2) Show that for a beam of energy $E >> m$, the rapidity is $y \sim \ln 2E/m$. What is the beam rapidity for $E = 100\,\mathrm{GeV}$? For $E = 3.5\,\mathrm{TeV}$?

Other references on this topic

- W. Florkowski, Phenomenology of Ultra-Relativistic Heavy-Ion Collisions, World Scientific, 2010
- R. Vogt, Ultrarelativistic Heavy-ion Collisions, Elsevier, 2007
- C.Y. Wong, Introduction to High-Energy Heavy-Ion Collisions, World Scientific, 1994
- `https://www.physi.uni-heidelberg.de/~reygers/lectures/2019/qgp/qgp2019_02_kinematics.pdf`