Série de Fourier

Retificador de meia onda.

$$f(t) = \begin{cases} \operatorname{sen}\omega t & 0 \leq \omega t \leq \pi \rightarrow 0 \leq t \leq \frac{\pi}{\omega} \\ 0 & \pi \leq \omega t \leq 2\pi \rightarrow \frac{\pi}{\omega} \leq t \leq \frac{2\pi}{\omega} \end{cases}$$

$$T = 2\pi/\omega$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \operatorname{sen}(n\omega t) \right)$$

$$a_n = \frac{2}{T} \cdot \int_0^T f(t) \cos n\omega t \, dt \; ; \; b_n = \frac{2}{T} \int_0^T f(t) \operatorname{sen}(n\omega t) \, dt$$

$$a_n = \frac{2}{\frac{2\pi}{\omega}} \cdot \int_0^{\pi/\omega} \operatorname{sen}\omega t \cdot \cos n\omega t \, dt$$

$$a_n = \frac{\omega}{\pi} \int_0^{\pi/\omega} \operatorname{sen}\omega t \cdot \cos n\omega t \, dt$$

$$n = 0 \longrightarrow \cos 0 = 1$$

$$a_0 = \frac{\omega}{\pi} \int_0^{\pi/\omega} \operatorname{sen}\omega t \, dt = \frac{\cancel{\omega}}{\pi} \cdot - \left\{ \frac{\cos \omega t}{\cancel{\omega}} \Big|_0^{\pi/\omega} \right\}$$

$$a_0 = -\frac{1}{\pi} \cdot \left\{ \overset{-1}{\cos \pi} - \overset{1}{\cos 0} \right\} \Rightarrow a_0 = \frac{2}{\pi}$$

(1)

Para o cálculo de a_n p/ $n \geq 1$:

$\operatorname{sen}\omega t \cdot \cos n\omega t \Rightarrow$

$\operatorname{sen}(n\omega t + \omega t) = \operatorname{sen} n\omega t \cdot \cos\omega t + \operatorname{sen}\omega t \cos n\omega t$

$\operatorname{sen}(n\omega t - \omega t) = \operatorname{sen} n\omega t \cdot \cos\omega t - \operatorname{sen}\omega t \cos n\omega t$

$$\operatorname{sen}\omega t \cdot \cos n\omega t = \frac{\operatorname{sen}[(n+1)\omega t] - \operatorname{sen}[(n-1)\omega t]}{2}$$

$$a_n = \frac{\omega}{2\pi} \cdot \left\{ \int_0^{\pi/\omega} [\operatorname{sen}(n+1)\omega t - \operatorname{sen}(n-1)\omega t]\, dt \right\}$$

Ao integrar estes termos aparecerá um termo $(n-1)$ no denominador do segundo termo que é problemático para $n=1$.
então calculamos p/ $n=1$

$$a_1 = \frac{\omega}{2\pi} \cdot \left\{ \int_0^{\pi/\omega} \operatorname{sen} 2\omega t\, dt - \int_0^{\pi/\omega} \operatorname{sen} 0\, dt \right\}$$

$$a_1 = \frac{\cancel{\omega}}{2\pi} \cdot -\frac{\cos 2\omega t}{2\cancel{\omega}} \Big|_0^{\pi/\omega} = -\frac{1}{4\pi}\left[\underbrace{\cos 2\pi}_{1} - \underbrace{\cos 0}_{1}\right] = 0$$

$n \geq 2$

$$a_n = \frac{\cancel{\omega}}{2\pi} \cdot -\left\{ \left[\frac{\cos(n+1)\omega t}{(n+1)\cancel{\omega}} - \frac{\cos(n-1)\omega t}{(n-1)\cancel{\omega}}\right]\Bigg|_0^{\pi/\omega} \right\}$$

$$a_n = -\frac{1}{2\pi}\left\{ \frac{\cos(n+1)\pi}{(n+1)} - \frac{\cos 0}{(n+1)} - \frac{\cos(n-1)\pi}{n-1} - \frac{(-\cos 0)}{(n-1)} \right\}$$

$$\cos(n+1)\pi = \cos(n-1)\pi = (-1)^{n-1} = (-1)^{n+1}$$

$$a_n = -\frac{1}{2\pi}\left\{ \frac{(-1)^{n-1}}{(n+1)} - \frac{1}{(n+1)} - \frac{(-1)^{n-1}}{(n-1)} + \frac{1}{(n-1)} \right\} \quad (2)$$

$$a_n = -\frac{1}{2\pi}\left\{ \frac{(-1)^{n-1} - 1}{(n+1)} - \frac{(-1)^{n-1} - 1}{(n-1)} \right\}$$

$$a_n = -\frac{1}{2\pi}\left\{ \frac{(n-1)\left((-1)^{n-1} - 1\right) - (n+1)\left((-1)^{n-1} - 1\right)}{(n+1)(n-1)} \right\}$$

$$a_n = -\frac{1}{2\pi}\left\{\frac{\left[(-1)^{n-1}-1\right]\left(\cancel{n}-1-\cancel{n}-1\right)}{n^2-1}\right\}$$

$$a_n = -\frac{1}{\cancel{2}\pi}\left\{-\frac{\cancel{2}\left[(-1)^{n-1}-1\right]}{n^2-1}\right\}$$

$$a_n = \frac{1}{\pi}\left\{\frac{(-1)^{n-1}-1}{n^2-1}\right\} = \begin{cases} 0 & n \text{ ímpar} \\ -2 & n \text{ par}\end{cases}$$

$$n = 2m$$

$$\boxed{a_{2m} = \frac{-2}{\pi(4m^2-1)}}$$

$n \geq 2$ ou seja $m \geq 1$

$$b_n = \frac{\omega}{\pi}\cdot\int_0^{\pi/\omega} \operatorname{sen}\omega t \cdot \operatorname{sen} n\omega t \, dt$$

(3)

$$n=1 \Rightarrow b_1 = \frac{\omega}{\pi}\cdot\int_0^{\frac{\pi}{\omega}} \operatorname{sen}^2\omega t \, dt$$

$$\cos 2\omega t = \cos^2\omega t - \operatorname{sen}^2\omega t = 1 - 2\operatorname{sen}^2\omega t$$

$$\operatorname{sen}^2\omega t = \frac{1-\cos 2\omega t}{2}$$

$$b_1 = \frac{\omega}{\pi 2}\left\{\int_0^{\pi/\omega} dt - \int_0^{\pi/\omega}\cos 2\omega t \, dt\right\}$$

$$b_1 = \frac{\omega}{2\pi}\left\{\left(\frac{\pi}{\omega}-0\right) - \cancel{\left.\frac{\operatorname{sen} 2\omega t}{2\omega}\right|_0^{\pi/\omega}}^{\,0}\right\} = \frac{1}{2}$$

$n \geq 2$

$$bn = \frac{2}{\frac{2\pi}{\omega}} \int_0^{\frac{2\pi}{\omega}} f(t) \cdot sen(n\omega t)\, dt = \frac{\omega}{\pi} \int_0^{\frac{\pi}{\omega}} sen\omega t \cdot sen n\omega t \, dt$$

$$\underbrace{sen\omega t}_{b} \cdot \underbrace{sen n\omega t}_{a} = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$= \frac{\cos[(n-1)\omega t] - \cos[(n+1)\omega t]}{2}$$

$$bn = \frac{\omega}{\pi} \left\{ \int_0^{\frac{\pi}{\omega}} \frac{\cos([n-1]\omega t)}{2} dt - \int_0^{\frac{\pi}{\omega}} \frac{\cos[n-1]\omega t}{2} dt \right\}$$

$$bn = \frac{\omega}{2\pi} \left\{ \frac{sen(n-1)\omega t}{(n-1)\omega} \Big|_0^{\pi/\omega} - \frac{sen(n+1)\omega t}{(n+1)\omega} \Big|_0^{\frac{\pi}{\omega}} \right\}$$ (4)

$bn = 0$ $\quad sen(n-1)\pi$ e $sen(n+1)\pi = 0$

$a_0 = \frac{2}{\pi}$ $\quad a_1 = 0$ $\quad a_n = a_{2m} = -\frac{2}{\pi \cdot (4m^2 - 1)}$

$m \geq 1$

$b_1 = \frac{1}{2}$ $\quad b_n = 0 \quad n \geq 2$

Qual a série?

$$f(t) = \frac{a_0}{2} + a_1\left[\cos\omega t\right] + b_1\left[\operatorname{sen}\omega t\right] + \sum_{n=2}^{\infty}\left(a_n \cos n\omega t + \underbrace{b_n}_{0} \operatorname{sen} n\omega t\right)$$

$n \to 2m$

$n = 2 \to m = 1$

$$f(t) = \frac{1}{\pi} + 0.\cos\omega t + \frac{1}{2}\operatorname{sen}\omega t - \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{1}{(4m^2-1)} \cdot \cos 2m\omega t$$

$$\boxed{f(t) = \frac{1}{\pi} + \frac{1}{2}\operatorname{sen}\omega t - \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\cos 2m\omega t}{(4m^2-1)}}$$