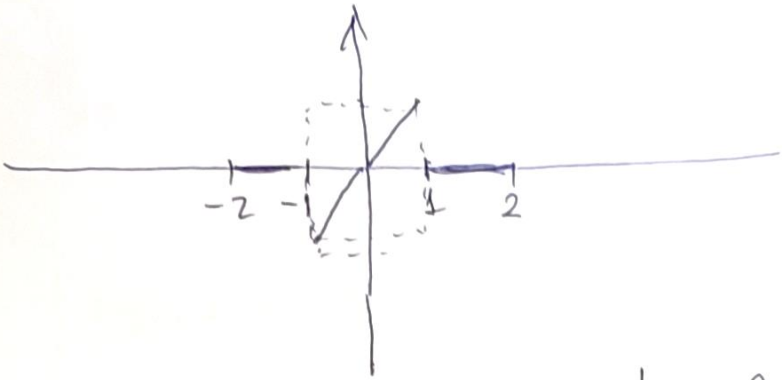
Série de Fourier

Exercício B — Slide 1.1

$$f(x) = \begin{cases} 0 & p/ \ -2 \leq x \leq -1 \\ x & p/ \ -1 < x < 1 \\ 0 & 1 \leq x \leq 2 \end{cases} \qquad f(x+4) = f(x)$$



Função é ímpar $\quad L = 2 \quad P = 4$

$$b_n = \frac{1}{2}\int_{-1}^{1} x \cdot \operatorname{sen}\frac{n\pi x}{2}\,dx = \int_0^1 x \operatorname{sen}\frac{n\pi x}{2}\,dx$$

$$x = u \qquad du = dx$$

$$dv = \operatorname{sen}\frac{n\pi x}{2}\,dx \rightarrow v = -\frac{2}{n\pi}\cos\frac{n\pi x}{2}$$

$$b_n = -\frac{2}{n\pi} \cdot x \cdot \cos\frac{n\pi x}{2}\Big|_0^1 + \frac{2}{n\pi}\int_0^1 \cos\frac{n\pi x}{2}\,dx$$

$$b_n = -\frac{2}{n\pi}\left\{\left(\cos\frac{n\pi}{2} + 0\right) - \frac{2}{n\pi}\operatorname{sen}\frac{n\pi x}{2}\Big|_0^1\right\}$$

$$\boxed{b_n = -\frac{2}{n\pi}\left\{\left(\cos\frac{n\pi}{2}\right) - \frac{2}{n\pi}\cdot\left(\operatorname{sen}\frac{n\pi}{2} + \cancel{0}\right)\right\}} \quad (1)$$

Logo: $\boxed{b_n = -\frac{2}{n\pi}\left\{\cos\left(\frac{n\pi}{2}\right) - \frac{2}{n\pi}\cdot sen\left(\frac{n\pi}{2}\right)\right\}}$

Análise:

1) n ímpar $\Rightarrow n = (2m-1)$, $m = 1, 2, 3, \dots$

$$\begin{cases} \cos\frac{(2m-1)\pi}{2} = 0 \quad p/\ \forall m \\ sen\,\frac{(2m-1)\pi}{2} = (-1)^{m+1} \end{cases}$$

$$b_{2m-1} = -\frac{2}{(2m-1)\pi}\cdot -\frac{2}{(2m-1)\pi}\cdot (-1)^{m+1}$$

$$\boxed{b_{2m-1} = \frac{4}{\pi^2}\cdot\frac{(-1)^{m+1}}{(2m-1)^2}}$$

2) n par $\Rightarrow n = 2m$, $m = 1, 2, 3, \dots$

$$\begin{cases} \cos\frac{2m\pi}{2} = \cos m\pi = (-1)^m \\ sen\,\frac{2m\pi}{2} = sen\, m\pi = 0 \quad \forall m \end{cases}$$

$$\boxed{b_{2m} = \overset{(-1)^1}{\ominus}\frac{\cancel{2}}{\cancel{2}m\pi}\times\left\{(-1)^m\right\} = \frac{(-1)^{m+1}}{m\pi}}$$

A série de Fourier seria: $f(x) = \sum_{n=1}^{\infty} b_n \cdot sen\,\frac{n\pi x}{2}$

ou $f(x) = \sum_{n\,par} b_n \cdot sen\,\frac{n\pi x}{2} + \sum_{n\,ímpar} b_n \cdot sen\,\frac{n\pi x}{2}$

$\Downarrow \qquad\qquad \Downarrow$

$$f(x) = \sum_{m=1}^{\infty} b_{2m}\, sen\,\frac{\cancel{2}m\pi x}{\cancel{2}} + \sum_{m=1}^{\infty} b_{(2m-1)}\cdot sen\frac{(2m-1)\pi x}{2}$$

Substituindo-se b_{2m} e b_{2m+1}

$$f(x) = \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m\pi} \cdot \operatorname{sen} n\pi x + \sum_{m=1}^{\infty} \frac{4}{\pi^2} \frac{(-1)^{m+1}}{(2m-1)^2} \cdot \operatorname{sen} \frac{(2m-1)\pi x}{2}$$

$$\boxed{f(x) = -\sum_{m=1}^{\infty} \left[\frac{(-1)^m}{m\pi} \operatorname{sen} m\pi x + \frac{4}{\pi^2 (2m-1)^2} (-1)^m \operatorname{sen} \frac{(2m-1)\pi x}{2} \right]}$$