Calcular a TF da função $f(x)$

$$f(x) = \begin{cases} 1 - \dfrac{|x|}{a} & \text{se } |x| < a \\ 0 & \text{se } |x| > a \end{cases}$$

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2} \cdot \int_{-\infty}^{\infty} f(x) \cdot e^{+iwx}\, dx$$

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-a}^{a} f'(x)\, e^{iwx}\, dx$$

$$f'(x) = \begin{cases} 1 + \dfrac{x}{a} & -a < x < 0 \\ 1 - \dfrac{x}{a} & 0 < x < a \end{cases}$$

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2} \cdot \left\{ \int_{-a}^{0} \left(1 + \frac{x}{a}\right) \cdot e^{iwx}\, dx + \int_{0}^{a} \left(1 - \frac{x}{a}\right) e^{iwx}\, dx \right\}$$

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2} \left\{ \underbrace{\int_{-a}^{0} e^{iwx}\, dx + \int_{0}^{a} e^{iwx}\, dx} + \frac{1}{a} \int_{-a}^{0} x e^{iwx}\, dx \right.$$

$$\left. - \frac{1}{a} \int_{0}^{a} x\, e^{iwx}\, dx \right\}$$

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2} \left\{ \int_{-a}^{a} e^{iwx}\, dx + \frac{1}{a} \left[\int_{-a}^{0} x e^{iwx}\, dx - \int_{0}^{a} x e^{iwx}\, dx \right] \right\}$$

$(\ast)$

$$\boxed{\int_{-a}^{a} e^{iwx}\,dx} = \frac{e^{iwx}}{iw}\Bigg|_{-a}^{a} = \frac{2\times 1}{w}\left[\frac{e^{iwa}-e^{-iwa}}{2i}\right] = \underline{\frac{2}{w}\,\text{sen}\,wa} \qquad (1)$$

Resolvendo: $\int x\,e^{iwx}\,dx \quad \begin{cases} x = u \rightarrow du = dx \\ e^{iwx}\,dx = dv \rightarrow v = \dfrac{e^{iwx}}{iw} \end{cases}$

$$\int x\,e^{iwx}\,dx = \frac{x\cdot e^{iwx}}{iw} - \frac{1}{iw}\int e^{iwx}\,dx = \underbrace{\frac{x\,e^{iwx}}{iw} + \frac{1}{w^2}\,e^{iwx}}$$

$$\int_{-a}^{a} x\cdot e^{iwx}\,dx = \cancel{\frac{0\cdot e^{iw0}}{iw}}^{\,0} + \frac{1}{w^2}\cdot e^{iw0} - \left[\frac{-a\cdot e^{-iwa}}{iw} + \frac{1}{w^2}\cdot e^{-iwa}\right]$$

$$\boxed{\int_{-a}^{0} x\cdot e^{iwx}\,dx = \frac{1}{w^2} + \frac{a}{iw}\,e^{-iwa} - \frac{1}{w^2}\,e^{-iwa}} \qquad (2)$$

$$\int_{0}^{a} e^{iwx}\,dx = \frac{a\cdot e^{iwa}}{iw} - \cancel{\frac{0\cdot e^{iw0}}{iw}} + \frac{1}{w^2}\left[e^{iwa} - 1\right]$$

$$\boxed{\int_{0}^{a} e^{iwx}\,dx = \frac{a\cdot e^{iwa}}{iw} + \frac{1}{w^2}\,e^{iwa} - \frac{1}{w^2}} \qquad (3)$$

Substituindo (1), (2) e (3) em (*):

$$F(w) = \left(\frac{1}{2\pi}\right)^{1/2}\cdot\left\{\frac{2}{w}\,\text{sen}\,wa + \frac{1}{a}\left[\cancel{\frac{1}{w^2}} + \cancel{\frac{a}{iw}\,e^{-iwa}} - \frac{1}{w^2}\,e^{-iwa} - \cancel{\frac{a}{iw}\,e^{iwa}} - \frac{1}{w^2}\,e^{iwa} + \cancel{\frac{1}{w^2}}\right]\right.$$

$$F(w)=\left(\frac{1}{2\pi}\right)^{1/2}\left\{\frac{2}{w}\,\mathrm{sen}\,wa+\frac{2}{aw^2}-\frac{2\times 1}{iw}\left(\frac{e^{iwa}-e^{-iwa}}{i2}\right)-\frac{1\times 2}{aw^2}\left(\frac{e^{iwa}+e^{-iwa}}{2}\right)\right\}$$

$$F(w)=\left(\frac{1}{2\pi}\right)^{1/2}\left\{\cancel{\frac{2}{w}\,\mathrm{sen}\,wa}+\frac{2}{aw^2}-\cancel{\frac{2}{w}\,\mathrm{sen}\,wa}-\frac{2}{aw^2}\cos wa\right\}$$

$$F(w)=\left(\frac{1}{2\pi}\right)^{1/2}\left\{\frac{2}{aw^2}-\frac{2}{aw^2}\cos wa\right\}$$

$$\cos wa=\cos\left(\frac{wa}{2}+\frac{wa}{2}\right)=1-2\,\mathrm{sen}^2\frac{wa}{2}$$

$$F(w)=\left(\frac{1}{2\pi}\right)^{1/2}\left\{\frac{2}{aw^2}\left[1-1+2\,\mathrm{sen}^2\frac{wa}{2}\right]\right\}$$

$$F(w)=\left(\frac{1}{2\pi}\right)^{1/2}\cdot\frac{2}{aw^2}\cdot 2\,\mathrm{sen}^2\frac{wa}{2}$$

$$F(w)=\frac{2}{aw^2}\left(\frac{\cancel{4}\,2}{2\pi}\right)^{1/2}\cdot\mathrm{sen}^2\frac{wa}{2}$$

$$\boxed{F(w)=2\sqrt{\frac{2}{\pi}}\;\frac{\mathrm{sen}^2\frac{wa}{2}}{aw^2}}$$