PREFACE

This manual contains solutions to Exercise Problems presented at the end of each chapter in Thermodynamic Loop Applications in Materials Systems (Volume One). Although answers are given in this book, the specific methods used to derive those answers are presented in this manual. Data not provided in a problem statement can be found in the book appendices.

Due to the difficulty in reading graphs or variations in the computational path used to obtain an answer, the reader may obtain slightly different numerical values (not regarded as incorrect) than those given. Likewise, certain assumptions made by the reader may yield somewhat more significant differences than those obtained using assumptions made in this manual. Again, such differences are not regarded as errors as long as the assumptions used can be justified. Throughout the book, an attempt is made to avoid this difficulty by making plausible engineering assumptions wherever possible. As will be noted, data limitations are often the reason that thermodynamics cannot be applied in a given situation.

The authors have used great care to eliminate computational errors. However, if any are found, we would be grateful to readers for information about such errors including misprints in both the book and the solutions manual.

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GLENN B. STRACHER

Chapter 1

[1.1] Calculate the coefficient of thermal expansion, α, at 273 K for an ideal gas.
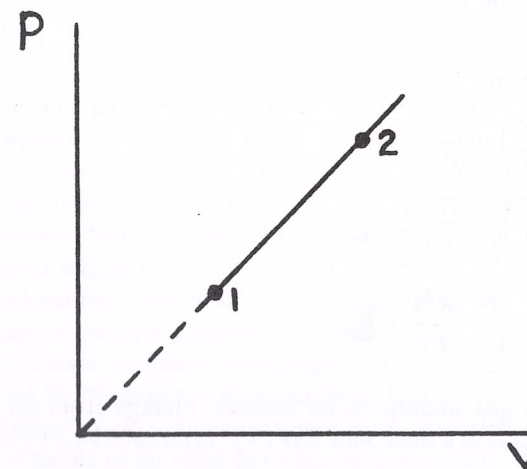
Solution

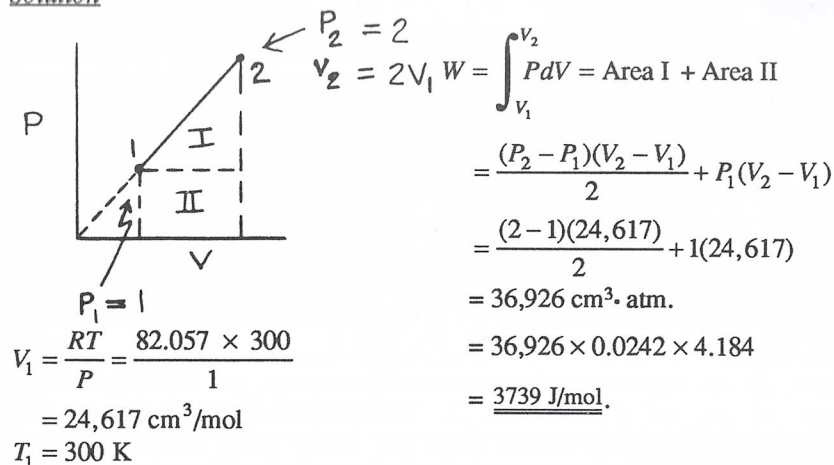$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \qquad\qquad V = \frac{RT}{P}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$\alpha = \frac{P}{RT}\cdot\frac{R}{P} = \frac{1}{T} = 273^{-1} = \underline{3.66\times 10^{-3}\,\text{K}^{-1}}.$$

[1.2] Consider the reversible expansion process represented by the straight line 1-2 in the figure below. The working substance is 1 mole of an ideal gas. $P_1 = 1$ atm, $P_2 = 2$ atm, $V_2 = 2V_1$ and $T_1 = 300$ K. Find W for the process.

Solution

$$W = \int_{V_1}^{V_2} P\,dV = \text{Area I} + \text{Area II}$$

$$= \frac{(P_2 - P_1)(V_2 - V_1)}{2} + P_1(V_2 - V_1)$$

$$= \frac{(2-1)(24,617)}{2} + 1(24,617)$$

$$= 36,926 \text{ cm}^3 \cdot \text{atm.}$$

$$= 36,926 \times 0.0242 \times 4.184$$

$$= \underline{3739 \text{ J/mol}}.$$

$$V_1 = \frac{RT}{P} = \frac{82.057 \times 300}{1}$$

$$= 24,617 \text{ cm}^3/\text{mol}$$

$$T_1 = 300 \text{ K}$$

[1.3] The generalized expression

$$C_p - C_v = T\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

can be used to calculate $C_p - C_v$ for any substance. Show that $C_p - C_v = R$ for an ideal gas.

Solution

$PV = RT$ where $n = 1$

$$V = \frac{RT}{P} \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$P = \frac{RT}{V} \Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

$$\therefore C_p - C_v = \frac{TR}{P} \cdot \frac{R}{V} = \frac{R^2 T}{PV} = \underline{\underline{R}}.$$

[1.4] An ideal gas undergoes an isobaric change from an initial State 1:(P_1, V_1', T_1) to a final State 2:(P_1, V_2', T_2). Prove $V_2' - V_1' = (nR/P) \times (T_2 - T_1)$.

Solution

$$PV_1' = nRT_1 \qquad\qquad \text{and} \qquad\qquad PV_2' = nRT_2$$

$$V_1' = \frac{nRT_1}{P} \qquad\qquad\qquad\qquad\qquad\qquad V_2' = \frac{nRT_2}{P}$$

$$V_2' - V_1' = \frac{nRT_2}{P} - \frac{nRT_1}{P} = \underline{\underline{\frac{nR}{P}(T_2 - T_1)}}.$$

[1.5] Show that the total work performed during the reversible isothermal expansion or compression of an ideal gas from a volume V_1' to V_2' is given by the equivalent expressions $W' = nRT \ln(V_2'/V_1')$ and $W' = nRT \ln(P_1/P_2)$.

Solution

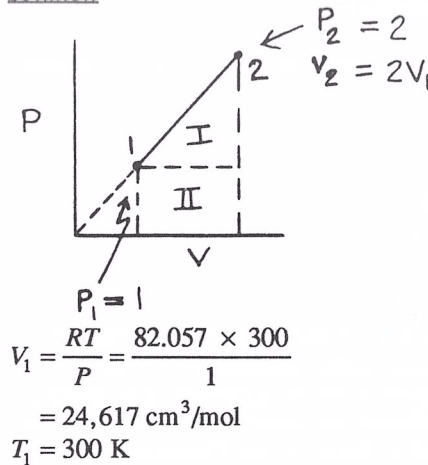$$W' = \int_{V_1'}^{V_2'} P\,dV' = \underline{\underline{nRT \ln(V_2'/V_1')}}$$

or $V_2'/V_1' = \dfrac{nRT/P_2}{nRT/P_1} = P_1/P_2$

$$W' = \underline{\underline{nRT \ln(P_1/P_2)}}.$$

[1.6] The reading on a pressure gauge relative to vacuum has to be added to atmospheric pressure in order to determine absolute pressure, i.e., *absolute pressure = gauge pressure + atmospheric pressure*.
  (a) Calculate the work done *by* 10 moles of an ideal gas expanding reversibly from 58.784 to 14.696 psig (lb_f/in^2 gauge) in a piston-cylinder arrangement at a constant temperature of 125°F. Perform the calculation *without* using numerical gas volumes at absolute pressure. Express the answer in joules.
  (b) Calculate the volume occupied by 10 moles of an ideal gas at 14.696 and 58.784 psig at a temperature of 125°F.
  (c) Using the gas volumes obtained in (b), calculate the work done *by* 10 moles of an ideal gas expanding reversibly at a constant temperature of 125°F. Express the answer in joules. How does the answer compare to the one in (a)? Why are they the same or different?

Solution

$$P_2 = 2$$
$$V_2 = 2V_1$$

$$W = \int_{V_1}^{V_2} P\,dV = \text{Area I} + \text{Area II}$$

$$= \frac{(P_2 - P_1)(V_2 - V_1)}{2} + P_1(V_2 - V_1)$$

$$= \frac{(2-1)(24{,}617)}{2} + 1(24{,}617)$$

$$= 36{,}926 \ \text{cm}^3 \cdot \text{atm.}$$

$$= 36{,}926 \times 0.0242 \times 4.184$$

$$= \underline{3739 \ \text{J/mol}}.$$

$$P_1 = 1$$

$$V_1 = \frac{RT}{P} = \frac{82.057 \times 300}{1}$$

$$= 24{,}617 \ \text{cm}^3/\text{mol}$$

$$T_1 = 300 \ \text{K}$$

[1.3] The generalized expression

$$C_p - C_v = T\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

can be used to calculate $C_p - C_v$ for any substance. Show that $C_p - C_v = R$ for an ideal gas.

Solution

$$PV = RT \quad \text{where } n = 1$$

$$V = \frac{RT}{P} \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$$

$$P = \frac{RT}{V} \Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

$$\therefore C_p - C_v = \frac{TR}{P} \cdot \frac{R}{V} = \frac{R^2 T}{PV} = \underline{\underline{R}}.$$

[1.4] An ideal gas undergoes an isobaric change from an initial State 1:(P_1, V_1', T_1) to a final State 2:(P_1, V_2', T_2). Prove $V_2' - V_1' = (nR/P) \times (T_2 - T_1)$.

Solution

$$PV_1' = nRT_1 \qquad \text{and} \qquad PV_2' = nRT_2$$

$$V_1' = \frac{nRT_1}{P} \qquad\qquad\qquad\qquad V_2' = \frac{nRT_2}{P}$$

$$V_2' - V_1' = \frac{nRT_2}{P} - \frac{nRT_1}{P} = \frac{nR}{P}(T_2 - T_1).$$

[1.5] Show that the total work performed during the reversible isothermal expansion or compression of an ideal gas from a volume V_1' to V_2' is given by the equivalent expressions $W' = nRT \ln(V_2'/V_1')$ and $W' = nRT \ln(P_1/P_2)$.

Solution

$$W' = \int_{V_1'}^{V_2'} P\,dV' = \underline{\underline{nRT \ln(V_2'/V_1')}}$$

or $V_2'/V_1' = \dfrac{nRT/P_2}{nRT/P_1} = P_1/P_2$

$$W' = \underline{\underline{nRT \ln(P_1/P_2)}}.$$

[1.6] The reading on a pressure gauge relative to vacuum has to be added to atmospheric pressure in order to determine absolute pressure, i.e., *absolute pressure = gauge pressure + atmospheric pressure*.
 (a) Calculate the work done *by* 10 moles of an ideal gas expanding reversibly from 58.784 to 14.696 psig (lb$_f$/in^2 gauge) in a piston-cylinder arrangement at a constant temperature of 125°F. Perform the calculation *without* using numerical gas volumes at absolute pressure. Express the answer in joules.
 (b) Calculate the volume occupied by 10 moles of an ideal gas at 14.696 and 58.784 psig at a temperature of 125°F.
 (c) Using the gas volumes obtained in (b), calculate the work done *by* 10 moles of an ideal gas expanding reversibly at a constant temperature of 125°F. Express the answer in joules. How does the answer compare to the one in (a)? Why are they the same or different?

Solution

(a) Absolute gas pressures can be calculated from

$$P_{abs} = P_{gauge} + P_{atm}$$

$$P_2 = 14.696 \text{ psig} + 14.696 \text{ psi} \Rightarrow P_2 = 29.392 \frac{lb_f}{in^2}$$

$$P_1 = 58.784 \text{ psig} + 14.696 \text{ psi} \Rightarrow P_1 = 73.48 \frac{lb_f}{in^2}$$

The work done by the gas is given by

$$W' = \int PdV'.$$

Since the calculation is to be done without using numerical gas volumes at absolute pressures, an expression for dV' must be derived.

$$PV' = nRT$$
$$V' = nRTP^{-1}$$
$$dV' = -nRTP^{-2}dP$$

Substituting for dV' in the work integral,

$$W' = \int_{P_1}^{P_2} P(-nRTP^{-2})dP = -nRT \int_{P_1}^{P_2} \frac{dP}{P} = -nRT \ln\left(\frac{P_2}{P_1}\right)$$

Converting 125°F to K,
°C = 5/9(°F − 32) and K = °C + 273.16° ⇒ K = 5/9(125°F − 32) + 273.16°
⇒ K = 324.83°

Hence,

$$W' = -10 \text{ mol}\left(\frac{8.3144 \text{ J}}{\text{mol} \cdot \text{K}}\right)(324.83 \text{ K})\ln\left(\frac{29.392}{73.48}\right)$$

$$W' \approx 24,750 \text{ J}$$

(b) From the solution to (a),
$P_{gauge} = 14.696$ psig, $P_{abs} = 29.392$ lb$_f$/in² and
$P_{gauge} = 58.784$ psig, $P_{abs} = 73.48$ lb$_f$/in²
From the ideal gas law, $PV' = nRT \Rightarrow V' = nRT/P$.

Thus,

$$V'_{14.696} = \frac{(10 \text{ mol})\left(8.3144\frac{N \cdot m}{mol \cdot K}\right)(324.83 \text{ K})}{29.392 \frac{lb_f}{in^2}\left(\frac{4.448 \text{ N}}{1 \text{ lb}_f}\right)\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)^2\left(\frac{10^2 \text{ cm}}{m}\right)^2}$$

$$V'_{14.696} = \underline{0.133 \text{ m}^3}$$

$$V'_{58.784} = \frac{(10 \text{ mol})\left(8.3144\frac{N \cdot m}{mol \cdot K}\right)(324.83 \text{ K})}{73.48 \frac{lb_f}{in^2}\left(\frac{4.448 \text{ N}}{1 \text{ lb}_f}\right)\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)^2\left(\frac{10^2 \text{ cm}}{m}\right)^2}$$

$$V'_{58.784} = \underline{0.053 \text{ m}^3}$$

(c) The work done by the gas is given by

$$W' = \int PdV'.$$

Since the calculation is to be done using numerical gas volumes at absolute pressure, an expression for P must be derived.

$$PV' = nRT \Rightarrow P = nRT/V'.$$

Substituting for P in the work integral,

$$W' = \int_{V_i}^{V_2} \frac{nRT}{V'} dV' = nRT \int_{V_i}^{V_2} \frac{dV'}{V'} = nRT \ln\left(\frac{V'_2}{V'_1}\right)$$

$W' \approx \underline{24,750 \text{ J}}$. This is the same answer as in part (a) because the path between identical initial and final states is the same.

[1.7] Calculate the work done when the hydrostatic pressure on a cube of Cu measuring 2 cm on an edge is increased reversibly and isothermally at 0°C from 1 to 100 atm. Assume a negligible change in V over the pressure interval.

Solution

$$W' = \int PdV' = -\beta V' \int_1^{100} PdP$$

$$= -7.6 \times 10^{-6} \times 8 \text{ cm}^3 \times \left[\frac{P^2}{2}\right]_1^{100} \text{atm}^2$$

$$= -7.6 \times 10^{-6} \times 8 \times \frac{10{,}000 - 1}{2}$$

$$= -0.304 \text{ cm}^3 \cdot \text{atm}$$

$$= \frac{-0.304 \text{ cm}^3 \cdot \text{atm} \cdot \text{cal}}{41.32 \text{ cm}^3 \cdot \text{atm}} \times 4.184 \text{ J/cal}$$

$$= -\underline{0.031 \text{ J}}$$

or

$$W = \frac{0.031 \text{ J}}{8 \text{ cm}^3} \times \frac{\text{cm}^3}{8.96 \text{ gm}} \times \frac{63.54 \text{ gm}}{\text{mol}}$$

$$= -\underline{0.027 \text{ J/mol}}.$$

[1.8] Calculate the work done on the surroundings when one mole of liquid potassium expands to vapor reversibly and isothermally at 1000 K and 0.753 bar in a piston – cylinder arrangement.

Solution

$$W = P \int_{V_1}^{V_g} dV$$

$$= 0.753 \text{ bar} (2592 - 1.493)\frac{L}{kg} \times \frac{8.3144 \text{kJ}}{83.144 L \cdot \text{bar}} \times \frac{39.10 \text{kg}}{\text{kg mol}}$$

$$= \underline{7627 \text{ kJ/kg mol}}$$

[1.9] Calculate the work done (internal compression) when a 350 lb mass of low carbon steel transforms from γ (austenite) to α (ferrite) during an air quench. Given that $V'_\gamma = 0.0486$ nm^3 and $2V'_\alpha = 0.0493$ nm^3, the basis for these volumes is 4 atoms of iron (one unit cell of γ or two unit cells of α [Van Vlack, 1985, p. 76]).

Solution

$$W' = P \int_{V'_\alpha}^{V'_\gamma} dV' \quad \text{where } P \approx 1 \text{ bar}, \ V'_\gamma = 0.0486 \text{ nm}^3; \ V'_\alpha = 0.0493 \ \text{nm}^3$$

$$W' = 1 \text{ bar}(0.0493 - 0.0486)\frac{\text{nm}^3}{4 \text{ atoms}} \times \frac{\text{cm}^3}{10^{21} \text{ nm}^3} \times \frac{8.3144}{83.057 \text{ cm}^3 \cdot \text{bar}}$$

$$\times \frac{6.024 \times 10^{23} \text{ atoms}}{\text{gm} \cdot \text{mol}} \times \frac{350 \text{ lb}_m \times 453.6 \text{ gm/lb}_m}{55.85 \text{ gm/(gm} \cdot \text{mol})} = \underline{30 \text{ J}}.$$

[1.10] Calculate the isobaric temperature change necessary to produce a molar volume change of 0.02 cm^3/mol in a pyrope garnet, Mg$_3$Al$_2$Si$_3$O$_{12}$, crystal. Assume $\alpha V_{1 \text{ bar}, 298K}$ is constant over the temperature interval.

Solution

$$\alpha = \frac{1}{V_{1,298}}\left(\frac{\partial V}{\partial T}\right)_P$$

$$dV = \alpha V_{1,298} \, dT$$

$$\int dT = \frac{1}{\alpha V_{1,298}} \int dV$$

$$\Delta T = \frac{1}{\alpha V_{1,298}} \Delta V$$

$$\Delta T = \frac{0.02 \text{ cm}^3/\text{mol}}{2.98 \times 10^{-3} \text{ cm}^3/(\text{mol} \cdot \text{K})} = \underline{6.71 \text{ K}}$$

where $V_{1 \text{ bar}, 298 \text{ K}} = 113.18$, $\alpha V_{1\text{bar}, 298K} = 2.98 \times 10^{-3}$.

[1.11] Calculate the molar volume of the pyroxenoid wollastonite, CaSiO$_3$, at a pressure of 200 bar and temperature of 298 K. Assume $\beta V_{1 \text{ bar}, 298K}$ is constant over the pressure interval. The initial pressure is 1 bar.

Solution

$$\beta = -\frac{1}{V_{1,298}}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\int_{V_i}^{V_f} dV = -\beta V_{1,298}\int_{P_i}^{P_f} dP \Rightarrow \Delta V = -\beta V_{1,298}\Delta P$$

$$V_f = V_i + \Delta V$$

$$V_f = 39.93 \text{ cm}^3/\text{mol} - 3.6\times10^{-5}\frac{\text{cm}^3}{\text{mol}\cdot\text{bar}}(200-1)\text{bar}$$

$$= 39.93 - 0.01 = \underline{39.92 \text{ cm}^3/\text{mol}}$$

where $V_{1,298} = 39.93$, $\beta V_{1,298} = 3.6\times10^{-5}$.

[1.12] Express $(\partial P/\partial T)_V$ in terms of the volume thermal expansion (α) and isothermal compressibility (β) coefficients of a mineral. Assume $\Delta\alpha$ and $\Delta\beta$ are negligible for small changes in P and T.

Solution
$V = f(P,T)$

$$dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT$$

$$0 = -\beta V dP + \alpha V dT$$

At constant V,

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{\beta}.$$

[1.13] Calculate the pressure on a crystal of spinel, $MgAl_2O_4$, heated isochorically from 273 to 308 K. The pressure at 273 K is 1 bar. Assume negligible changes in α and β over the P-T interval. The volume remains constant because of external constraint.

Solution

$$V = f(P,T) \Rightarrow dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT$$

$$\alpha = \frac{1}{V_{1,298}}\left(\frac{\partial V}{\partial T}\right)_P, \quad \beta = -\frac{1}{V_{1,298}}\left(\frac{\partial V}{\partial P}\right)_T$$

$$dV = -\beta V_{1,298}\int_{P_1}^{P_2} dP + \alpha V_{1,298}\int_{T_1}^{T_2} dT = 0$$

Rearranging and substituting where $\alpha V_{1,298} = 1.03\times10^{-3}$ cm^3/(mol·K), $\beta V_{1,298} = 1.9\times10^{-5}$ cm^3/(mol·bar)

$$P_2 = P_1 + \frac{\alpha}{\beta}(T_2 - T_1) = P_1 + \frac{\alpha V_{1,298}}{\beta V_{1,298}}(T_2 - T_1)$$

$$= 1 \text{ bar} + \frac{1.03\times10^{-3} \text{ cm}^3/(\text{mol}\cdot\text{K})}{1.9\times10^{-5} \text{ cm}^3/(\text{mol}\cdot\text{bar})}(308-298)\text{K}$$

$$= 1 + 542.12 = \underline{543\text{bar}}.$$

[1.14] A beam in a truss bridge is subject to uniaxial tension and compression (σ) in such a manner that the beam undergoes infinitesimal changes from one *thermodynamic equilibrium* state (T_1,σ_1) to another (T_2,σ_2). The total differential of the dependent variable ε involves two parameters known from experiments to be nearly constant for small temperature changes. These parameters are the coefficient of linear thermal expansion α and Young's Modulus E. Mathematical definitions and additional experimental properties are as follows:

$$\alpha = \left(\frac{\partial\varepsilon}{\partial T}\right)_\sigma, \quad \alpha \text{ is nearly independent of } \sigma$$

$$E = \left(\frac{\partial\sigma}{\partial\varepsilon}\right)_T, \quad \text{for } \sigma < \text{the elastic proportional limit}$$

(a) Write an equation of state for the beam as a functional relationship.
(b) Express the exact differential of ε in terms of α and E.
(c) Show that if ε is constant, $(\partial\sigma/\partial T)_\varepsilon = -\alpha E$.

Solution

(a) Since ε is the dependent variable and T and σ are independent variables, $\underline{\varepsilon = f(T,\sigma)}$.

(b) $d\varepsilon = \left(\frac{\partial\varepsilon}{\partial T}\right)_\sigma dT + \left(\frac{\partial\varepsilon}{\partial\sigma}\right)_T d\sigma$

and substituting from the problem statement,

$$d\varepsilon = \alpha dT + \left(\frac{1}{E}\right)d\sigma$$

(c) $d\varepsilon = 0$

$$0 = \alpha dT + \left(\frac{1}{E}\right)d\sigma$$

or

$$\left(\frac{\partial \sigma}{\partial T}\right)_\varepsilon = -\alpha E.$$

[1.15] Beginning with the expression $P = f(V,T)$
  (a) Express the total differential of P in terms of partial derivatives and name the partial differentials.
  (b) Use the condition of exactness to demonstrate that dP is an exact differential.

Solution

(a) Applying the chain rule to $P = f(V,T)$

$$dP = \underbrace{\left(\frac{\partial P}{\partial V}\right)_T dV}_{1} + \underbrace{\left(\frac{\partial P}{\partial T}\right)_V dT}_{2}$$

Expressions 1 & 2 are the "partial differentials."

(b) If dP is an exact differential, the condition for exactness must hold true. Applying the condition to

$$dP = \left(\frac{\partial P}{\partial V}\right)_T dV + \left(\frac{\partial P}{\partial T}\right)_V dT$$

$$\frac{\partial}{\partial T}\left[\left(\frac{\partial P}{\partial V}\right)_T\right]_V \overset{?}{=} \frac{\partial}{\partial V}\left[\left(\frac{\partial P}{\partial T}\right)_V\right]_T \Rightarrow \frac{\partial^2 P}{\partial T \partial V} = \frac{\partial^2 P}{\partial V \partial T}$$

Thus dP is exact.

[1.16] Uniaxial tensile first cycle stress-strain loops for the magnesium alloy Dowmetal A-T4 are illustrated below. Small partial cycles beginning at the lower end of each loop are on a constant slope of modulus $E = 6.5 \times 10^6$ ksi since the first part of each loop side is a line parallel to the original modulus line. For each such partial cycle, assume total elastic return upon the removal of stress. Compute the total work performed in ft · lb$_f$ when a 10.0 in. long Dowmetal A-T4 cylinder of radius 2.50 in. undergoes the partial cycles for loops 1 and 2 shown.
  (a) Loop 1: $\sigma_1 = 1$ ksi, $\sigma_2 = 5$ ksi.
  (b) Loop 2: $\varepsilon_1 = 2000$ μin/in, $\varepsilon_2 = 2500$ μin/in.
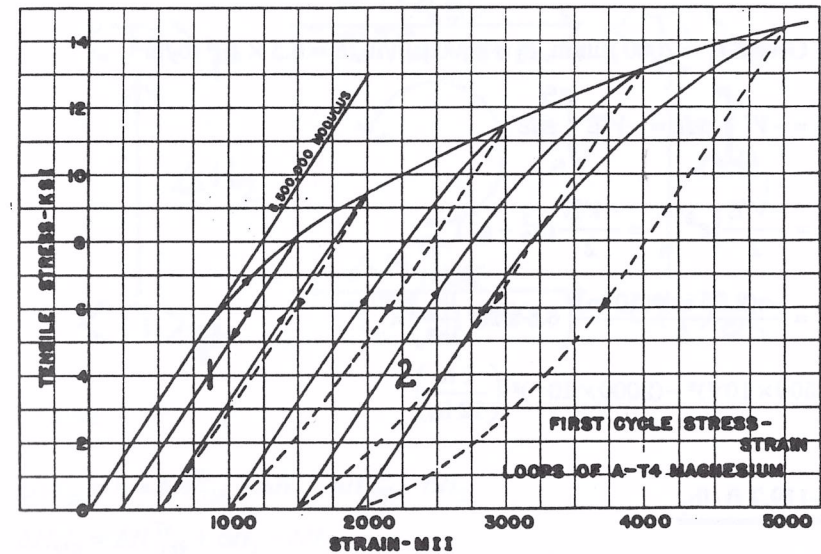
First cycle uniaxial tensile stress-strain curves for Dowmetal A-T4 magnesium alloy. Loops 1 and 2 are labeled. MII = microinches/inch. (From M.H. Polzin, 1951, Fig. 9 with modifications. Reprinted by permission of the Society for Experimental Mechanics, Inc., Bethel, CT.)

Solution

(a) Given $\sigma_1 = 1$ ksi, $\sigma_2 = 5$ ksi, $E = 6.5 \times 10^6$ lb$_f$/in^2

$$W_1 = -\int \sigma d\varepsilon \quad \text{(work per unit volume) thus}$$

$$W_1' = -V'\int \sigma d\varepsilon \quad \text{(assume } V' \text{ is approximately constant)}$$

$$E = \frac{\sigma}{\varepsilon} \Rightarrow \varepsilon = \frac{\sigma}{E} \Rightarrow d\varepsilon = \frac{1}{E}d\sigma \text{ and thus}$$

$$W_1' = \frac{-V'}{E}\int_{\sigma_1}^{\sigma_2} \sigma d\sigma = \frac{-V'}{2E}\left[\sigma^2\right]_{\sigma_1}^{\sigma_2} = \frac{-V'}{2E}(\sigma_2^2 - \sigma_1^2)$$

$$W_1' = \frac{-\pi(2.50 \text{ in})^2(10 \text{ in})}{2(6.5 \times 10^6 \text{ lb}_f/\text{in}^2)}\left\{\left(5 \times 10^3 \frac{\text{lb}_f}{\text{in}^2}\right)^2 - \left(10^3 \frac{\text{lb}_f}{\text{in}^2}\right)^2\right\}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$W_1' = -30.21 \text{ ft} \cdot \text{lb}_f$$

(b) Given: $\varepsilon_1 = 2000 \text{ }\mu\text{in/in}$, $\varepsilon_2 = 2500 \text{ }\mu\text{in/in}$, $E = 6.5 \times 10^6 \text{ lb}_f/\text{in}^2$

$$W_2' = -V'\int \sigma d\varepsilon = -V'E\int_{\varepsilon_1}^{\varepsilon_2} \varepsilon d\varepsilon$$

$$W_2' = \frac{-V'E}{2}\left[\varepsilon^2\right]_{\varepsilon_1}^{\varepsilon_2} = \frac{-V'E}{2}(\varepsilon_2^2 - \varepsilon_1^2)$$

$$W_2' = \frac{-\pi(2.50 \text{ in})^2(10 \text{ in})}{2}\left(6.5 \times 10^6 \frac{\text{lb}_f}{\text{in}^2}\right)\times$$

$$\{(2500 \times 10^{-6})^2 - (2000 \times 10^{-6})^2\}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= -119.7 \text{ ft} \cdot \text{lb}_f$$

The work is negative, hence work is done on the cylinder.