4302212 – Física IV

Transformação do Campo Eletromagnético

Transformação de Lorentz Inversa

– Anteriormente, definimos a TL para 4-vetores (contravariantes) a^μ:

$$\begin{pmatrix} a'^0 \\ a'^1 \\ a'^2 \\ a'^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a^0 \\ a^1 \\ a^2 \\ a^3 \end{pmatrix}$$

$$a'^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu \, a^\nu \qquad \text{ou} \qquad \mathrm{a}' = \hat{\Lambda}\mathrm{a} \implies \mathrm{a} = \hat{\Lambda}^{-1}\mathrm{a}'$$

– A seguir, obteremos a TL inversa em forma matricial. Há métodos mais elegantes, mas sabendo que a inversão segue essencialmente de $V \to -V$, apenas verificaremos o resultado.

– É imediato verificar que (a rigor, a inversão combina transposição e inversão do sinal):

$$\begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(\Lambda^{-1})^{\mu}_{\ \rho}\,\Lambda^{\rho}_{\ \nu} \;=\; \delta^{\mu}_{\nu}$$

$$\hat{\Lambda}^{-1}\hat{\Lambda} = \hat{1}$$

– Portanto: $$\mathbf{a}' = \hat{\Lambda}\mathbf{a} \implies \mathbf{a} = \hat{\Lambda}^{-1}\mathbf{a}'$$

$$\begin{pmatrix} a^0 \\ a^1 \\ a^2 \\ a^3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a'^0 \\ a'^1 \\ a'^2 \\ a'^3 \end{pmatrix}$$

Força de Lorentz: 4-vetores

– Vamos construir o 4-vetor associado à Força de Lorentz:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

$$\mathbf{F} \cdot \mathbf{u} = q\mathbf{E} \cdot \mathbf{u}$$

– Portanto:

$$K^0 = \gamma(u)\frac{q}{c}\mathbf{E} \cdot \mathbf{u}$$

$$K^1 = \gamma(u)q[E_x + (\mathbf{u} \times \mathbf{B})_x] = \gamma(u)q[E_x + (u_y B_z - u_z B_y)]$$

$$K^2 = \gamma(u)q[E_y + (\mathbf{u} \times \mathbf{B})_y] = \gamma(u)q[E_y + (u_z B_x - u_x B_z)]$$

$$K^3 = \gamma(u)q[E_z + (\mathbf{u} \times \mathbf{B})_z] = \gamma(u)q[E_z + (u_x B_y - u_y B_x)]$$

– O 4-vetor K^μ pode ser reescrito na forma abaixo (verifique!):

$$\begin{pmatrix} K^0 \\ K^1 \\ K^2 \\ K^3 \end{pmatrix} = \frac{q}{c} \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix} \gamma(u) \begin{pmatrix} c \\ u_x \\ u_y \\ u_z \end{pmatrix}$$

$$= \frac{q}{c} \underbrace{\begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}}_{\hat{\mathrm{F}}} \begin{pmatrix} \eta^0 \\ \eta^1 \\ \eta^2 \\ \eta^3 \end{pmatrix}$$

– Em notação compacta (perceba que $\hat{\mathrm{F}}$ descreve os campos):

$$\mathrm{K} = \frac{q}{c}\,\hat{\mathrm{F}}\,\eta$$

– Transformação de K^{μ}:

$$\mathrm{K} = \frac{q}{c}\,\hat{\mathrm{F}}\,\eta \Longrightarrow \hat{\Lambda}\mathrm{K} = \frac{q}{c}\hat{\Lambda}\hat{\mathrm{F}}\eta \Longrightarrow \hat{\Lambda}\mathrm{K} = \frac{q}{c}\hat{\Lambda}\hat{\mathrm{F}}\hat{\Lambda}^{-1}\hat{\Lambda}\eta$$

$$\boxed{\mathrm{K}' = \frac{q}{c}\hat{\mathrm{F}}'\eta'} \qquad (i)$$

$$\boxed{\hat{\mathrm{F}}' = \hat{\Lambda}\hat{\mathrm{F}}\hat{\Lambda}^{-1}} \qquad (ii)$$

– A expressão (i) é o 4-vetor que descreve a Força de Lorentz no referencial *S*'.

– A expressão (ii) estabelece a TL para a matriz (tensor) que descreve os campos.

– Vamos expressar os elementos de $\hat{F}'$ em função dos elementos de $\hat{F}$. Lembre-se que $\gamma = \gamma(V)$ na expressão de $\widehat{\Lambda}$.

$$\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & E_x & \gamma(E_y - VB_z) & \gamma(E_z + VB_y) \\ E_x & 0 & c\gamma(B_z - VE_y/c^2 & -c\gamma(B_y + VE_z/c^2) \\ \gamma(E_y - VB_z) & -c\gamma(B_z - VE_y/c^2) & 0 & cB_x \\ \gamma(E_z + VB_y) & c\gamma(B_y + VE_z/c^2) & -cB_x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & E'_x & E'_y & E'_z \\ E'_x & 0 & cB'_z & -cB'_y \\ E'_y & -cB'_z & 0 & cB'_x \\ E'_z & cB'_y & -cB'_x & 0 \end{pmatrix}$$

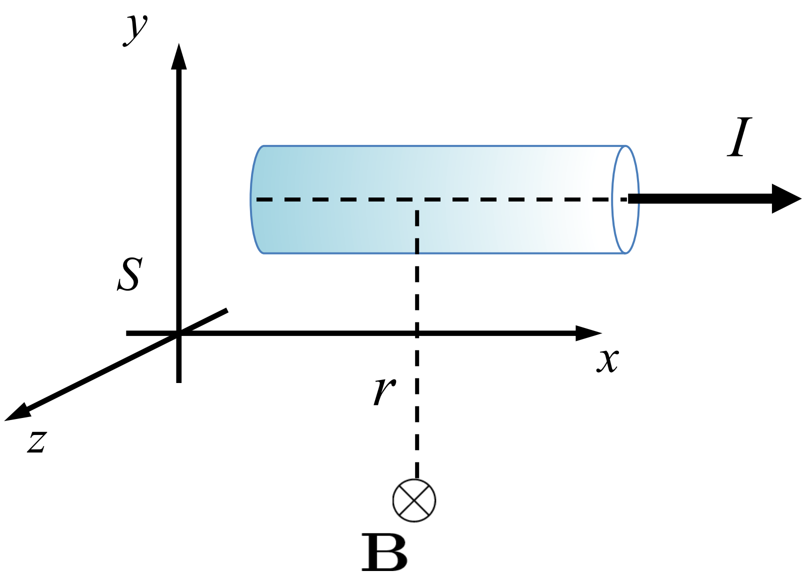
Transformação dos Campos

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \gamma(E_y - VB_z) \qquad B'_y = \gamma(B_y + \frac{V}{c^2}E_z)$$

$$E'_z = \gamma(E_z + VB_y) \qquad B'_z = \gamma(B_z - \frac{V}{c^2}E_y)$$

Cilindro Condutor (Corrente Estacionária)



$$\mathbf{E} = (0, 0, 0)$$

$$\mathbf{B} = (0, 0, B_z)$$

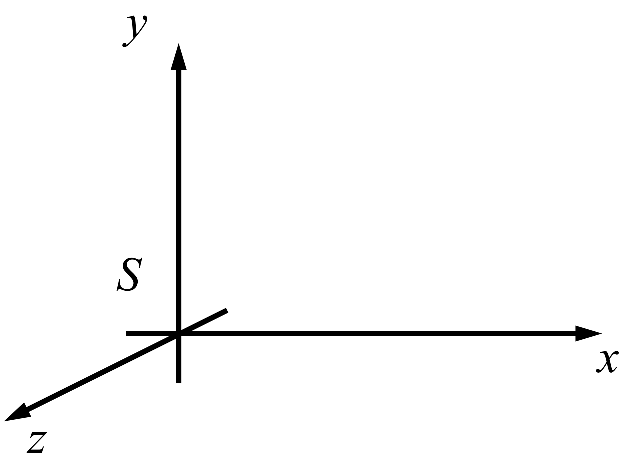
$$B_z = -\frac{\mu_0}{2\pi}\frac{I}{r}$$

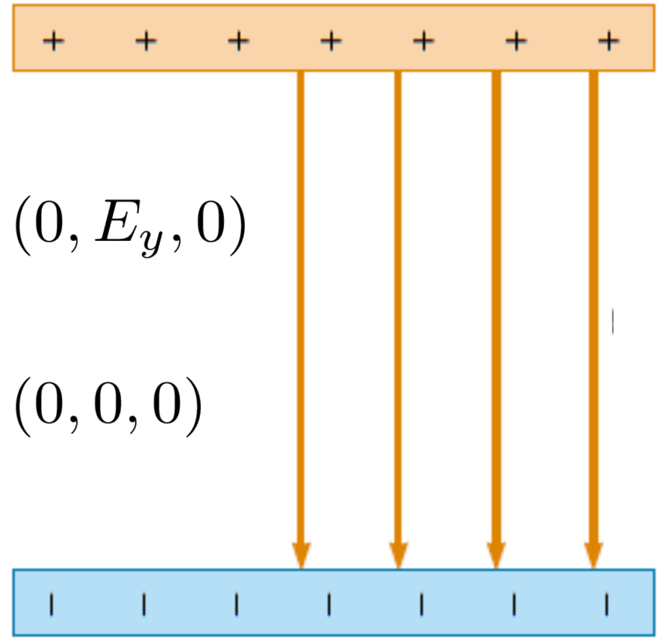
$$E'_x = 0 \qquad B'_x = 0$$

$$E'_y = -\gamma V B_z \qquad B'_y = 0$$

$$E'_z = 0 \qquad B'_z = \gamma B_z$$

Capacitor de Placas Paralelas





$$\mathbf{E} = (0, E_y, 0)$$

$$\mathbf{B} = (0, 0, 0)$$

$$E'_x = 0 \qquad B'_x = 0$$

$$E'_y = \gamma E_y \qquad B'_y = 0$$

$$E'_z = 0 \qquad B'_z = -\gamma \frac{V}{c^2} E_y$$