

- Mais algumas leis de ótica explicadas pelo eletrromagnetismo...
- Reflexão e refração
- Lei de Snell Descartes
- Lei de Brewster
- O que acontece em uma interface?

Condições de Contorno

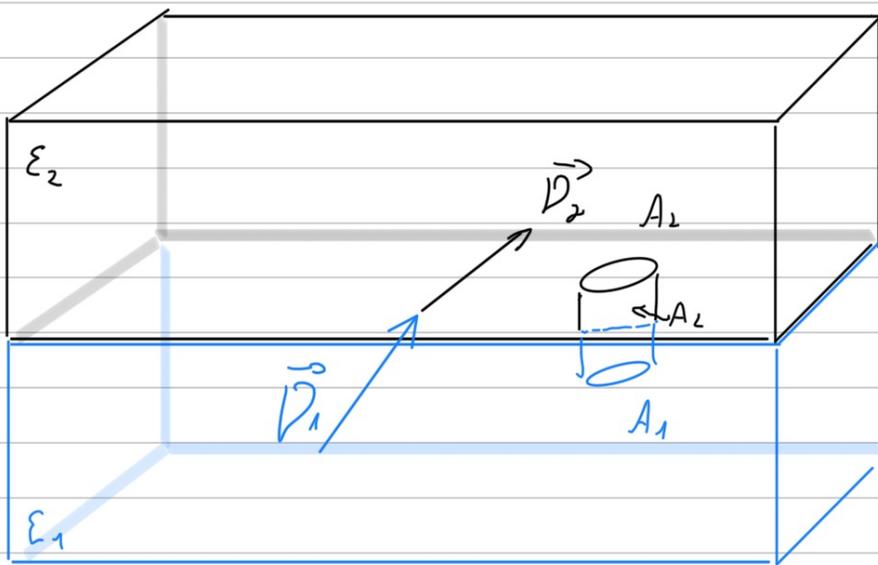
Campo do Vector Deslocamento $\vec{D} \Rightarrow$ Cargas livres

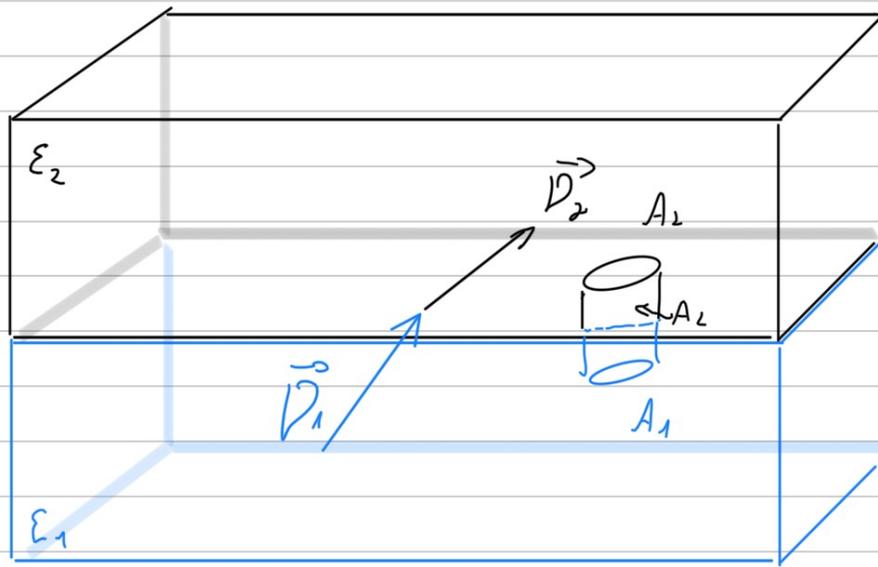
$$\nabla \cdot \vec{D} = \rho \Rightarrow \oint_S \vec{D} \cdot \hat{n} da = Q$$

Campo Elétrico $\vec{E} = -\nabla V \Rightarrow \nabla \times \vec{E} = 0$

$$\Rightarrow V(\vec{r}_B) - V(\vec{r}_A) = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{l}$$

O que ocorre entre dois dielétricos?





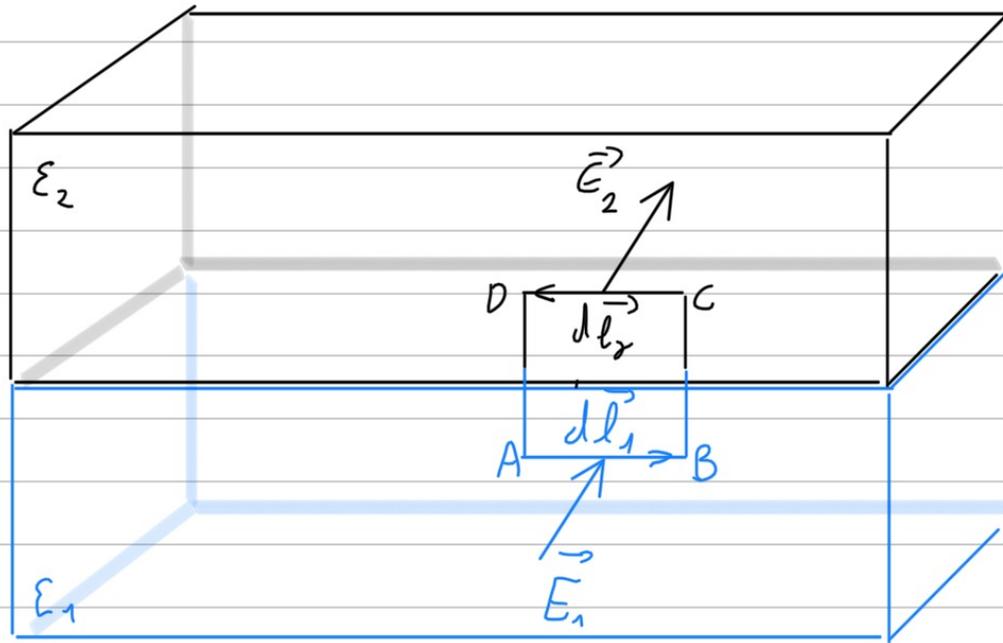
$$\oint \vec{D} \cdot \vec{n} \, da = \int_{A_1} \vec{D}_1 \cdot \vec{n}_1 \, da_1 + \int_{A_2} \vec{D}_2 \cdot \vec{n}_2 \, da_2 + \int_{A_L} \vec{D}_L \cdot \vec{n}_L \, da_L$$

$$\vec{n}_1 = -\vec{n}_2 ; \, da_1 = da_2 \quad \rightarrow 0$$

$$\oint \vec{D} \cdot \vec{n} \, da = \int_{A_2} (\vec{D}_2 - \vec{D}_1) \cdot \vec{n}_2 \, da_2$$

Carque nula (livre) $\Rightarrow \underline{(\vec{D}_2 - \vec{D}_1) \cdot \vec{n}_2 = 0}$

$$\Rightarrow (\epsilon_2 \vec{E}_2 - \epsilon_1 \vec{E}_1) \cdot \vec{n}_2 = 0$$



$$V_{BA} = V_B - V_A = - \int_A^B \vec{E}_1 \cdot d\vec{l}_1 \quad ; \quad V_{CB} = - \int_B^C \vec{E}_2 \cdot d\vec{l}$$

$$V_{DC} = V_D - V_C = - \int_C^D \vec{E}_2 \cdot d\vec{l}_2 \quad ; \quad V_{AD} = - \int_D^A \vec{E}_2 \cdot d\vec{l}$$

$$V_{BA} = V_B - V_A = - \int_A^B \vec{E}_1 \cdot d\vec{l}_1 \quad ; \quad V_{CB} = - \int_B^C \vec{E} \cdot d\vec{l}$$

$$V_{DC} = V_D - V_C = - \int_C^D \vec{E}_2 \cdot d\vec{l}_2 \quad ; \quad V_{AD} = - \int_D^A \vec{E} \cdot d\vec{l}$$

$$L \rightarrow D \Rightarrow V_{CB} = V_{AD} = 0$$

$$V_{BA} + V_{CB} + V_{DC} + V_{AD} = 0 \rightarrow \text{volta completa}$$

$$\Rightarrow V_{BA} - V_{DC} = 0 \quad d\vec{l}_1 = -d\vec{l}_2$$

$$\Rightarrow \int_A^B (\vec{E}_1 - \vec{E}_2) \cdot d\vec{l}_2 = 0$$

$$\left\{ \begin{array}{l} \text{Componente transversal: } \vec{E}_1 \cdot d\vec{l}_2 = \vec{E}_2 \cdot d\vec{l}_2 \\ \text{Componente perpendicular: } \vec{D}_1 \cdot \hat{n} = \vec{D}_2 \cdot \hat{n} \end{array} \right.$$

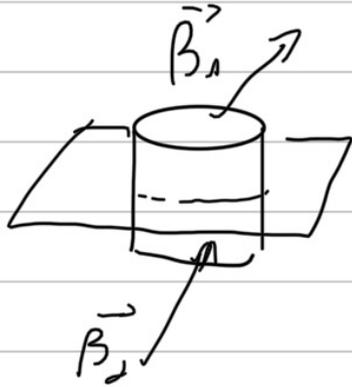
Outra consequência: $\oint \vec{D} \cdot d\vec{l}$ não é nulo na interface

$$\text{Met-1: } \vec{E}_1 \cdot d\vec{l}_2 = \vec{E}_2 \cdot d\vec{l}_2$$

$$\text{meio 1} \rightarrow \text{condutor} \rightarrow \vec{E}_1 = 0 \Rightarrow \vec{E}_2 \cdot d\vec{l}_2 = 0 \Rightarrow \vec{E}_2 = E \cdot \hat{n}_2$$

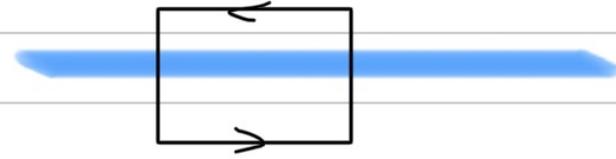
Interface : $\nabla \cdot \vec{B} = 0$

$$\nabla \times \vec{H} = \vec{J}$$



$$\vec{B}_1 \cdot \vec{n}_1 = \vec{B}_2 \cdot \vec{n}_2$$

$$B_{1n} = B_{2n}$$



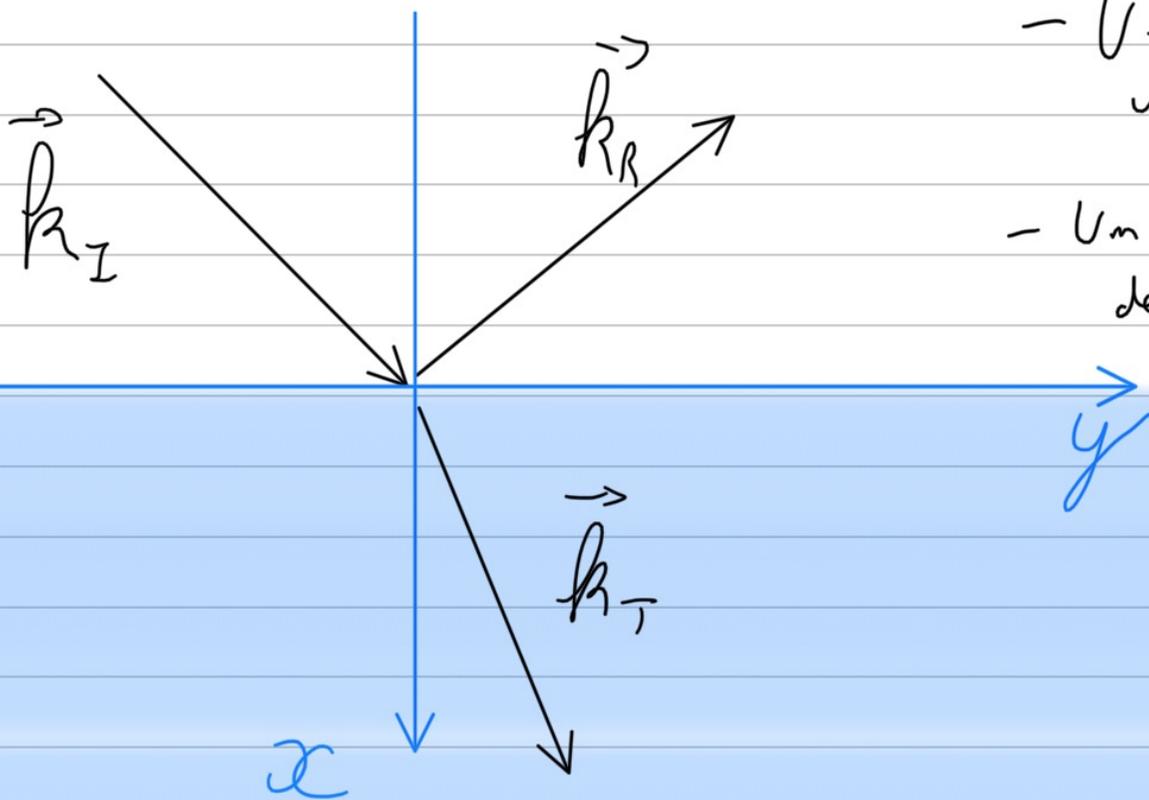
$$H_{1\parallel} = H_{2\parallel}$$

$$\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$$

$$B_{2\parallel} = \frac{\mu_2}{\mu_1} B_{1\parallel} ; B_{2n} = B_{1n}$$

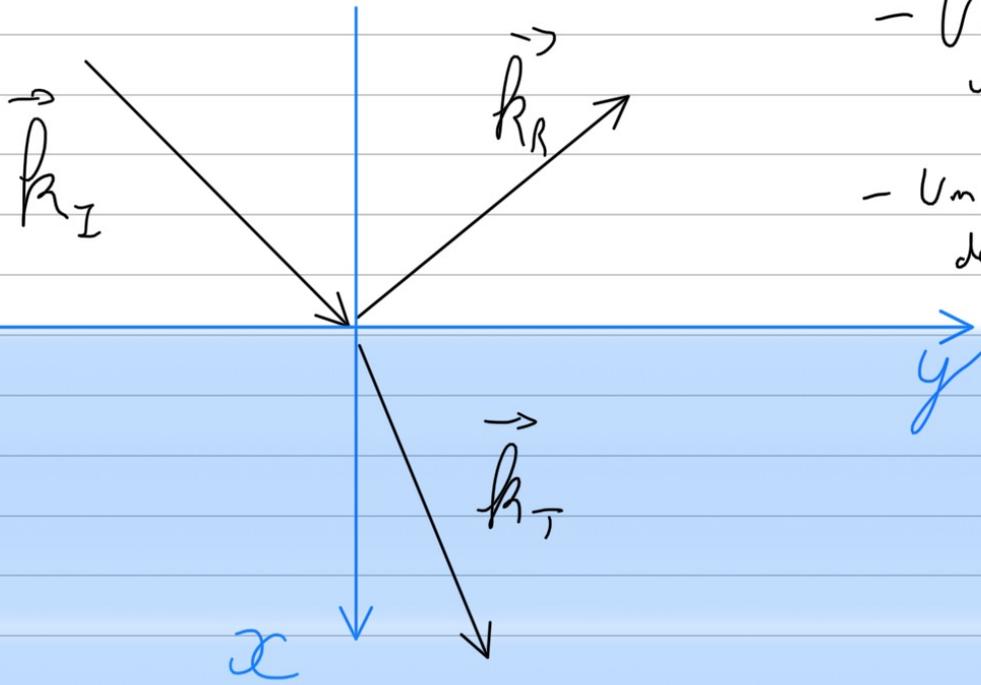
Interface: $E_{1||} = E_{2||} ; \epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$

$$\frac{B_{1||}}{\mu_1} = \frac{B_{2||}}{\mu_2} ; B_{1\perp} = B_{2\perp}$$



- Um vetor define um conjunto de planos

- Um vetor em um ponto define um plano



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$$\vec{E}_I(\vec{r}, t) = \text{Re} \left[\vec{E}_{I0} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} \right]; \quad \vec{B}_I = \frac{1}{v_I} \vec{k}_I \times \vec{E}_I$$

$$\vec{E}_R(\vec{r}, t) = \text{Re} \left[\vec{E}_{R0} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} \right]; \quad \vec{B}_R = \frac{1}{v_R} \vec{k}_R \times \vec{E}_R$$

$$\vec{E}_T(\vec{r}, t) = \text{Re} \left[\vec{E}_{T0} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)} \right]; \quad \vec{B}_T = \frac{1}{v_T} \vec{k}_T \times \vec{E}_T$$

1) Na fronteira, a resposta temporal deve ser a mesma de um lado e de outro: $\omega_1 = \omega_2 = \omega_3 = \omega$

2) Os módulos de \vec{k} se relacionam:

$$\omega = k_I v_I = k_R v_R = k_T v_T$$

3) A onda incidente e a refletida estão no mesmo meio

$$v_I = v_R = v_1 \quad v_T = v_2$$

4) Condições dos campos na fronteira: $\vec{r} = y \hat{y} + z \hat{z}$ ($x=0$)

$$E_{1||} = E_{2||} ; \epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$\forall t$

$$\frac{B_{1||}}{\mu_1} = \frac{B_{2||}}{\mu_2} ; B_{1\perp} = B_{2\perp}$$

$$\Rightarrow \alpha \vec{E}_1 + \beta \vec{E}_3 = \gamma \vec{E}_2$$

$$\vec{E}_I(\vec{r}, t) = \text{Re} \left[\vec{E}_{I0} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} \right]$$

$$\vec{k}_I \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r}$$

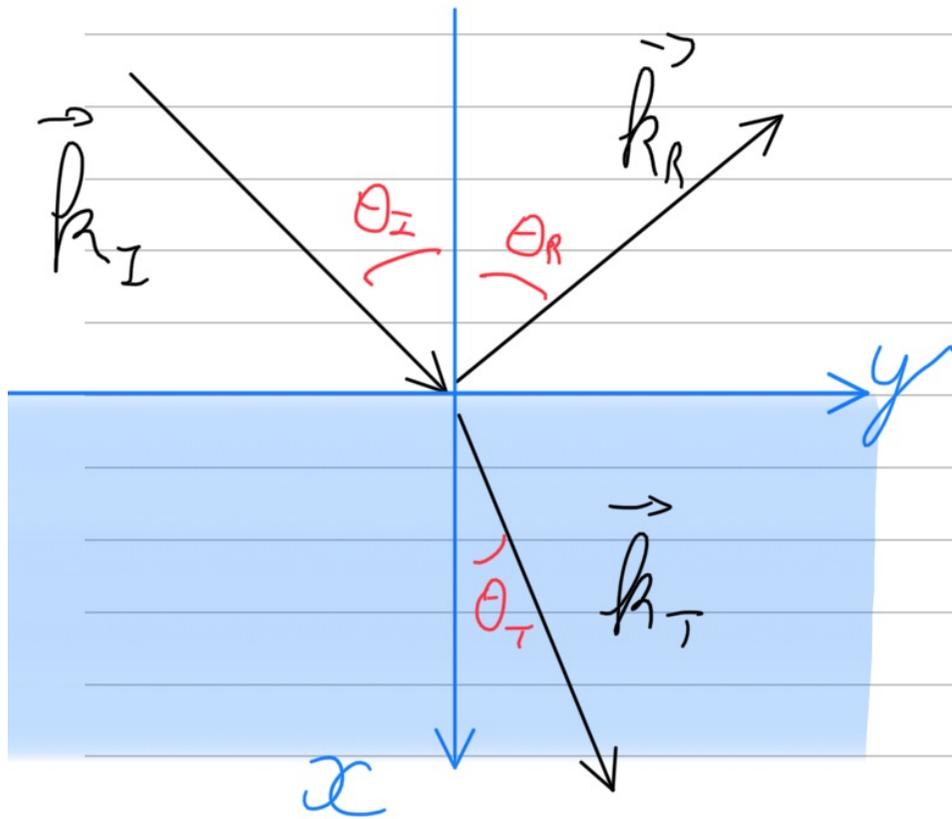
$$\vec{E}_R(\vec{r}, t) = \text{Re} \left[\vec{E}_{R0} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} \right]$$

$$\odot \quad x=0 \quad \nabla \quad \circ$$

$$\vec{E}_T(\vec{r}, t) = \text{Re} \left[\vec{E}_{T0} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)} \right]$$

$$\vec{k}_z \cdot \vec{r} = k_{Iz} \cdot x + k_{Iy} \cdot y + k_{Iz} \cdot z$$

$$\Rightarrow k_{Iy} = k_{Ry} = k_{Ty} ; k_{Iz} = k_{Rz} = k_{Tz}$$



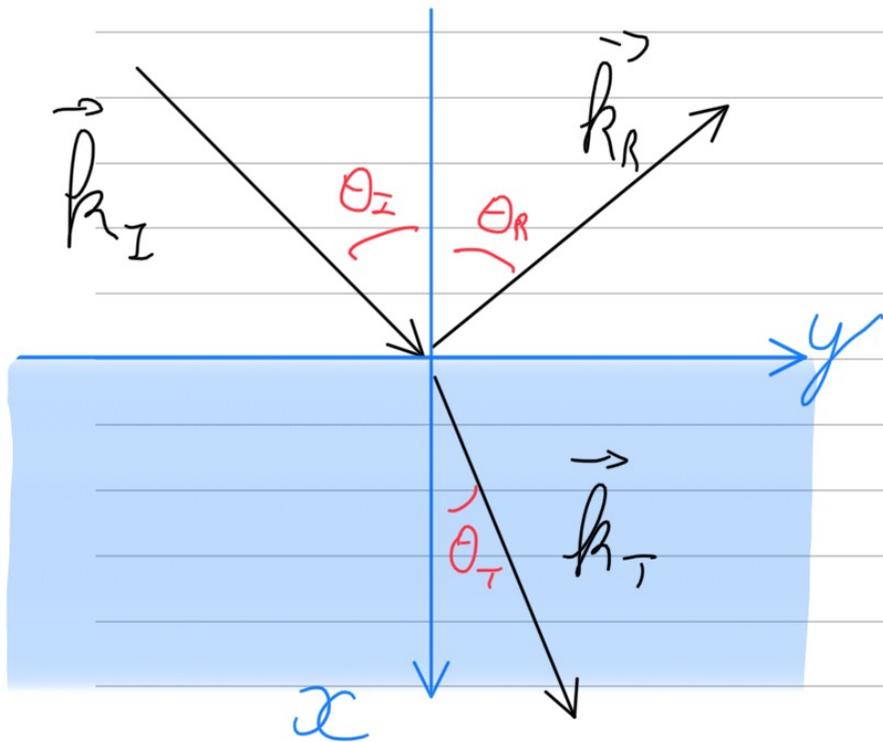
$$\Rightarrow k_I \sin \theta_I = k_R \sin \theta_R = k_T \sin \theta_T$$

$$k_I = \frac{\omega}{v_1} = k_T = k_R$$

$$\Rightarrow \sin \theta_I = \sin \theta_R$$

$$\Rightarrow \theta_I = \theta_R = \theta_1$$

Lei da Reflexão



$$\Rightarrow k_I \sin \theta_I = k_R \sin \theta_R = k_T \sin \theta_T$$

$$k_I = \frac{\omega}{v_1} = k_R = k_T$$

$$\Rightarrow \sin \theta_I = \sin \theta_R$$

$$\Rightarrow \underline{\theta_I = \theta_R = \theta_1}$$

Lei da Reflexão

$$k_1 \sin \theta_1 = k_2 \sin \theta_2$$

$$c \cdot \frac{\omega}{v_1} \sin \theta_1 = \frac{\omega}{v_2} \cdot \sin \theta_2 \quad c. \Rightarrow \underline{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$

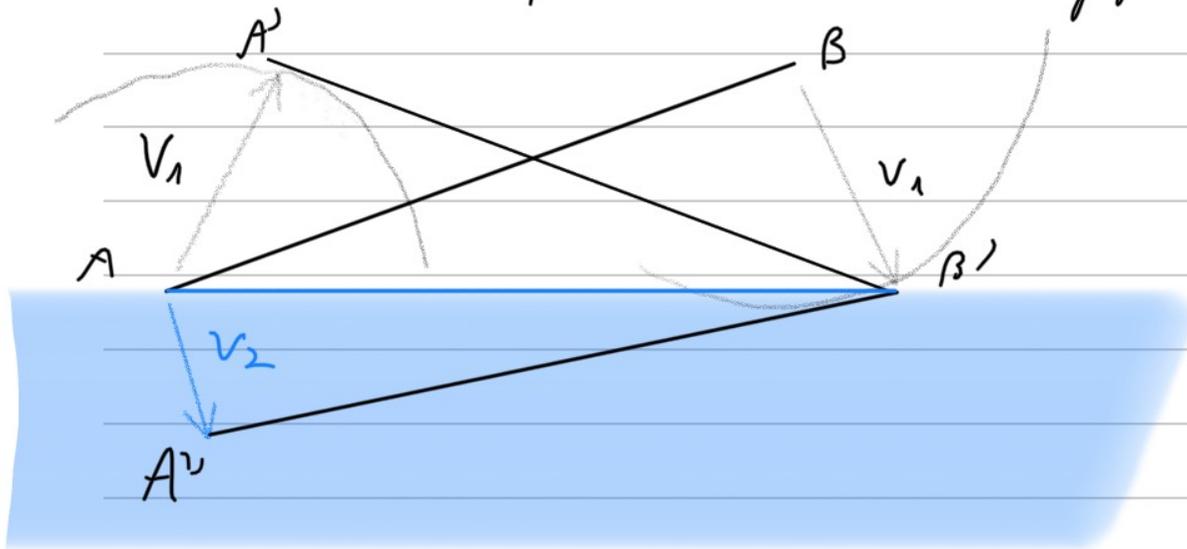
Eqs. Maxwell \rightarrow

Lei de Snell-Descartes

Princípio de Huygens

Eqs. Maxwell \rightarrow Lei de Snell-Descartes

Princípio de Huygens



$$AA' = v_1 t = BB'$$

$$AA' = AB' \sin \theta_I$$

$$BB' = AB' \sin \theta_R$$

$$\underline{\sin \theta_I = \sin \theta_R}$$

$$AA'' = v_2 t \Rightarrow AA'' = AB' \sin \theta_T$$

$$\Rightarrow \frac{v_2 t}{\sin \theta_T} = \frac{v_1 t}{\sin \theta_I} \Rightarrow \underline{\sin \theta_I \cdot n_1 = n_2 \sin \theta_T}$$

Como ficam as amplitudes?

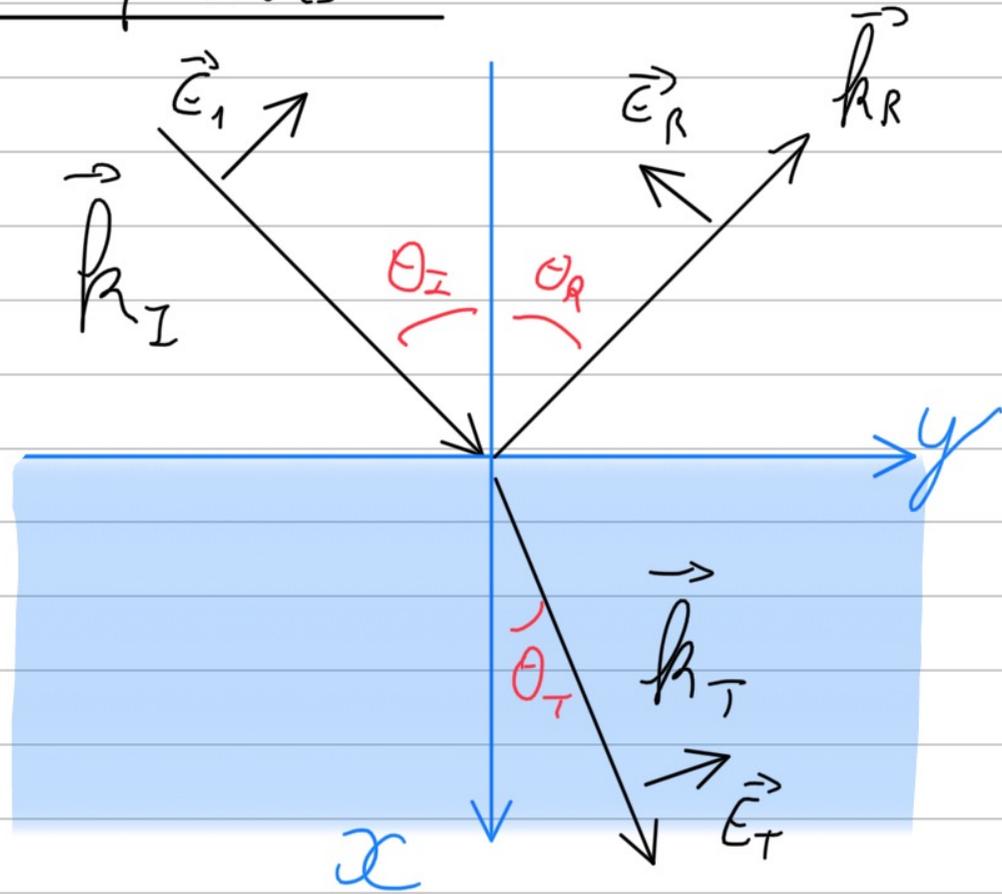
$$\underline{E_{1\parallel} = E_{2\parallel}}$$

$$\Rightarrow \epsilon_{Iy} + \epsilon_{Ry} = \epsilon_{Ty}$$

$$\epsilon_{Iz} + \epsilon_{Rz} = \epsilon_{Tz}$$

$$\underline{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$$\epsilon_1 (\epsilon_{Ix} + \epsilon_{Rx}) = \epsilon_{Tx} \cdot \epsilon_2$$



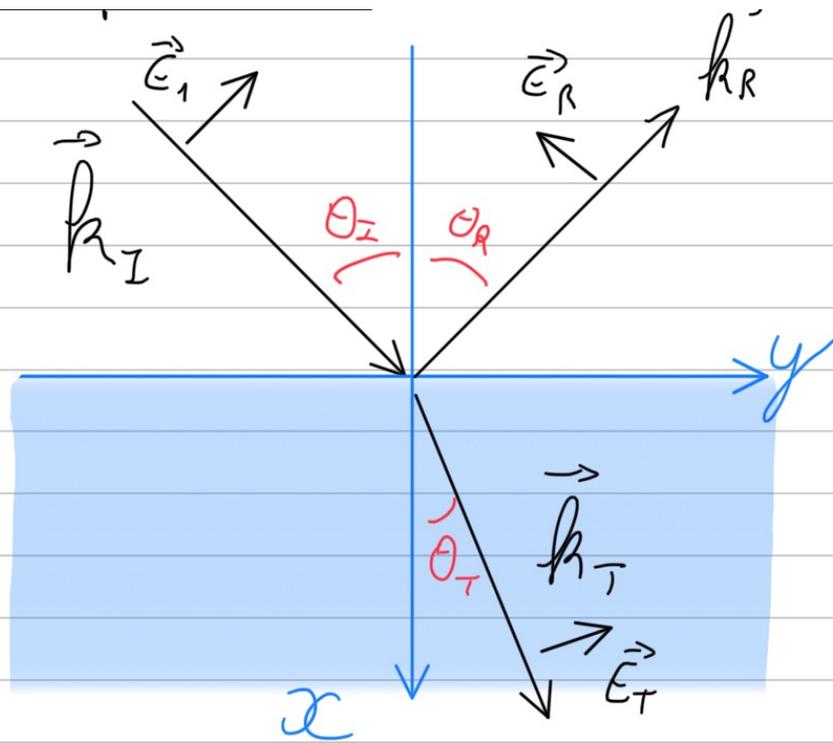
$$\underline{E_{1\parallel} = E_{2\parallel}}$$

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$$\underline{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$$\epsilon_1 (\epsilon_{Ix} + \epsilon_{Rx}) = \epsilon_{Tx} \cdot \epsilon_2$$



Para o campo magnético:

$$\underline{B_{1n} = B_{2n}} \Rightarrow B_{Ix} + B_{Rx} = B_{Tx}$$

$$\underline{\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}} \Rightarrow \frac{1}{\mu_1} [B_{Iy} + B_{Ry}] = \frac{1}{\mu_2} [B_{Ty}]$$

$$\frac{1}{\mu_1} [B_{Iz} + B_{Rz}] = \frac{1}{\mu_2} [B_{Tz}]$$

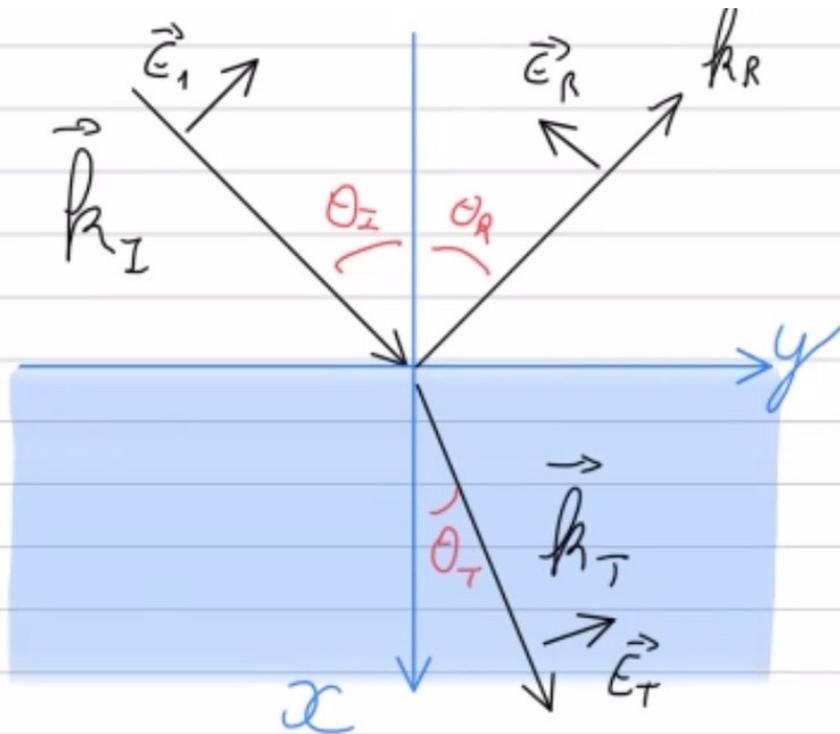
$$\underline{E_{1||} = E_{2||}}$$

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Para o campo magnético:

$$\underline{B_{1n} = B_{2n}} \Rightarrow \underline{B_{Ix} + B_{Rx} = B_{Tx}}$$

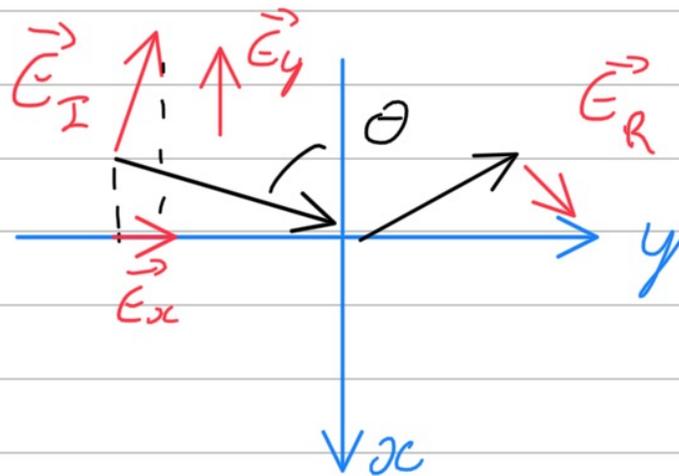
$$\underline{\frac{B_{1||}}{\mu_1} = \frac{B_{2||}}{\mu_2}} \Rightarrow \underline{\frac{1}{\mu_1} [B_{Iy} + B_{Ry}] = \frac{1}{\mu_2} [B_{Ty}]}$$

$$\underline{\frac{1}{\mu_1} [B_{Iz} + B_{Rz}] = \frac{1}{\mu_2} [B_{Tz}]}$$

Compo \vec{E} no plano $xy \rightarrow$

$$\vec{E} = E_x \hat{x} + E_y \hat{y}$$

$$= E \cdot \text{sen } \theta \hat{x} + E \text{ cos } \theta \hat{y}$$



$$x \Rightarrow \epsilon_1 [E_I \text{ sen } \theta_I - E_R \text{ sen } \theta_R] = \epsilon_2 [E_T \text{ sen } \theta_T]$$

$$E_I - E_R = \left(\frac{\epsilon_2 \cdot \text{sen } \theta_T}{\epsilon_1 \text{ sen } \theta_I} \right) E_T$$

$$y \Rightarrow [E_I \text{ cos } \theta_I + E_R \text{ cos } \theta_R] = E_T \text{ cos } \theta_T \Rightarrow (E_I + E_R) = \frac{\text{cos } \theta_T}{\text{cos } \theta_I} E_T$$

$$z \Rightarrow \text{Como } B_z = \frac{h}{\omega} E = \frac{1}{v} E$$

$$\Rightarrow \frac{1}{\mu_1 v_1} (E_I - E_R) = \frac{1}{\mu_2 v_2} (E_T)$$

$$(\varepsilon_I - \varepsilon_R) = \frac{\mu_1 v_1}{\mu_2 v_2} \varepsilon_T = \frac{\mu_1}{\mu_2} \frac{n_2}{n_1} \varepsilon_T$$

$$\varepsilon_I + \varepsilon_R = \alpha \varepsilon_T \quad / \quad \varepsilon_I - \varepsilon_R = \beta \varepsilon_T$$

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} \cdot \frac{\sin \theta_T}{\sin \theta_I} = \frac{\varepsilon_2}{\varepsilon_1} \cdot \frac{n_1}{n_2} = \frac{\mu_1 v_1^2}{\mu_2 v_2^2} \cdot \frac{c}{v_1} \cdot \frac{v_2}{c} = \frac{\mu_1 \cdot v_1}{\mu_2 v_2} = \frac{\mu_1}{\mu_2} \frac{n_2}{n_1}$$

$$\varepsilon_i = \frac{1}{\mu_i v_i^2}$$

Redundância

$$2 \varepsilon_I = (\alpha + \beta) \varepsilon_T \Rightarrow t = \frac{\varepsilon_T}{\varepsilon_I} = \frac{2}{\alpha + \beta}$$

$$\varepsilon_R = \alpha \varepsilon_T - \varepsilon_I = \left(\frac{2\alpha}{\alpha + \beta} - 1 \right) \varepsilon_I \Rightarrow r = \frac{\varepsilon_R}{\varepsilon_I} = \frac{\alpha - \beta}{\alpha + \beta}$$

Equações de Fresnel

Note que tipicamente $\mu_1 = \mu_2$, $\beta = \frac{n_2}{n_1}$

$$\alpha = \frac{\omega n_T}{\omega n_I} = \frac{\sqrt{1 - \sin^2 \theta_T}}{\omega n_I} = \sqrt{\frac{1 - \left(\frac{n_1}{n_2} \sin \theta_I\right)^2}{1 - \sin^2 \theta_I}}$$

$$n_2 \sin \theta_I = n_1 \sin \theta_T$$

$$\Rightarrow \alpha = \sqrt{\frac{1 - \frac{\sin^2 \theta_T}{\beta^2}}{1 - \sin^2 \theta_I}} \quad \beta = \frac{n_T}{n_I}$$

$$t_{||} = \frac{2}{\frac{\omega n_T}{\omega n_I} + \frac{n_T}{n_I}} = \frac{2 n_I \omega n_I}{n_I \omega n_T + n_T \omega n_I}$$

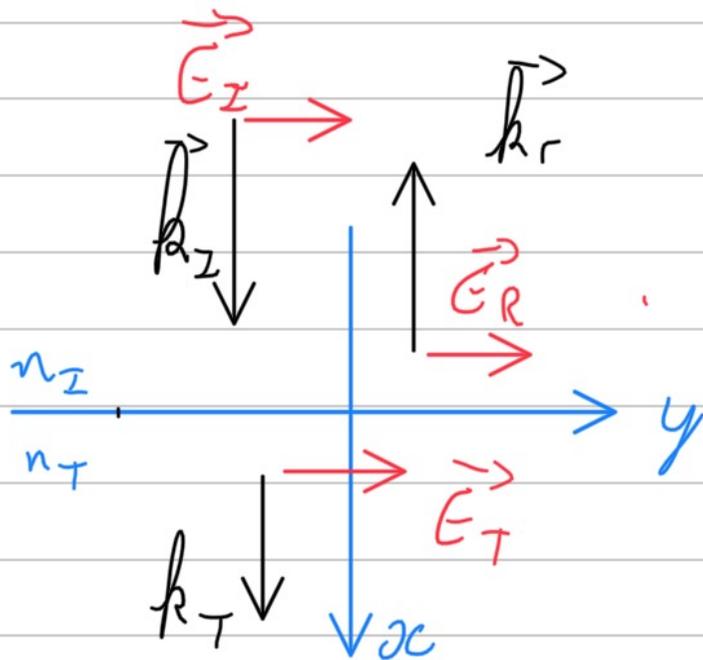
$$r_{||} = \frac{n_I \omega n_T - n_T \omega n_I}{n_I \omega n_T + n_T \omega n_I}$$

Incidência normal: $\theta_I = 0 \Rightarrow \alpha = 1$

$$\Gamma = \frac{1-\beta}{1+\beta} = \frac{n_I - n_T}{n_I + n_T}, \quad t = \frac{2n_I}{n_I + n_T}$$

$$n_I > n_T \rightarrow \Gamma > 1; \quad n_I < n_T \rightarrow \Gamma < 1$$

Inversão!



$$n_I = \frac{c}{v_I} \quad n_T = \frac{c}{v_T}$$

$$\Gamma = \frac{v_T - v_I}{v_T + v_I}$$

Onda em corda: $v = \sqrt{T/\mu}$ (tensão / dens. linear)

$\lambda \rightarrow$

$\lambda \rightarrow$

$v_1 > v_2$

$\leftarrow v$

Limite: Parede

$\lambda \rightarrow$

$\lambda \rightarrow$

$v_1 < v_2$

$\leftarrow \lambda$

Limite: ponta livre

Reflexão nula: $\alpha = \beta \Rightarrow r = 0!$

$$r = \frac{\alpha - \beta}{\alpha + \beta}$$

$$\alpha = \sqrt{\frac{1 - \frac{\sin^2 \theta_I}{\beta^2}}{1 - \sin^2 \theta_I}} \quad / \quad \beta = \frac{n_I}{n_T}$$

$$\Rightarrow \frac{1 - \sin^2 \theta_I}{\beta^2} = \beta^2 (1 - \sin^2 \theta_I)$$

$$\beta^2 - \sin^2 \theta_I = \beta^4 - \beta^4 \sin^2 \theta_I$$

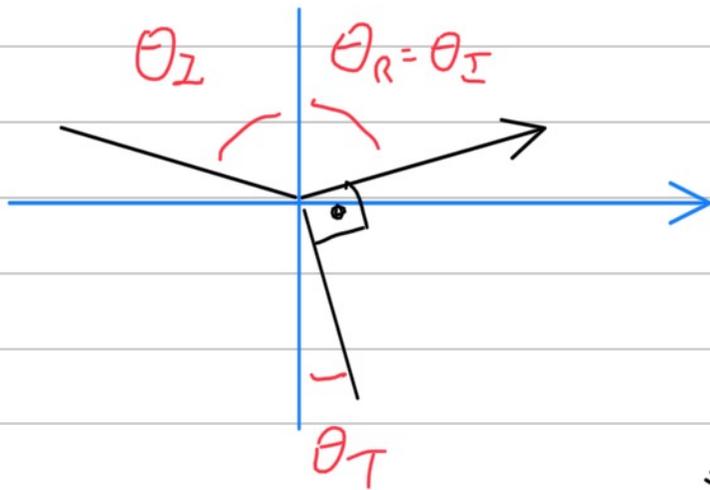
$$\sin^2 \theta_I (1 - \beta^4) = \beta^2 - \beta^4 \Rightarrow \sin^2 \theta_I = \frac{\beta^2 - \beta^4}{1 - \beta^4}$$

Note porém que $\gamma_{\theta}^2 = \frac{\sin^2 \theta}{\omega^2 \theta} = \frac{\sin^2 \theta}{1 - \sin^2 \theta}$

$$\cos^2 \theta = 1 - \sin^2 \theta = \frac{1 - \beta^4 - \beta^2 + \beta^4}{1 - \beta^4} = \frac{1 - \beta^2}{1 - \beta^4}$$

$$\tan^2 \theta = \beta^2 \frac{(1 - \beta^2)}{1 - \beta^4} \cdot \frac{1 - \beta^4}{1 - \beta^2} = \beta^2 \frac{\cancel{1 - \beta^4}}{\cancel{1 - \beta^2}} \therefore \boxed{\tan \theta_B = \frac{n_T}{n_I}}$$

Ângulo de Brewster (1781-1868)



$$\sin \theta_T = \frac{n_I}{n_T} \sin \theta_I$$

$$= \frac{1}{\tan \theta_B} \sin \theta_B = \cos \theta_B \quad \checkmark$$

$$\Rightarrow \sin \theta_T = \cos \theta_B \rightarrow \text{ângulo reto} \quad \checkmark$$

Por quê? Qual a origem física?

Reflexão rasante: $\theta_1 \rightarrow \pi/2 \Rightarrow \sin \theta_2 = 1$, $\cos \theta_2 \rightarrow 0$

$$r = \frac{\alpha - \beta}{\alpha + \beta}$$

$$\alpha = \frac{\sqrt{1 - \frac{\sin^2 \theta_2}{\beta^2}}}{\cos \theta_2} \quad / \quad \beta = \frac{n_T}{n_I}$$

$$\lim_{\theta \rightarrow \pi/2} r = \lim_{\theta \rightarrow \pi/2} \frac{\alpha - \beta}{\alpha + \beta} = \lim_{\theta \rightarrow \pi/2} \left(\frac{\sqrt{\beta^2 - \sin^2 \theta} - \beta \cos \theta}{\sqrt{\beta^2 - \sin^2 \theta} + \beta \cos \theta} \right)$$

$$\underline{r \approx 1} /$$

Reflexão total

$$r = \left(\frac{\alpha - \beta}{\alpha + \beta} \right)$$

$$\alpha = \sqrt{\frac{1 - \frac{\sin^2 \theta_2}{\beta^2}}{1 - \sin^2 \theta_1}} \quad / \quad \beta = \frac{n_T}{n_I}$$

$$\alpha \in \mathbb{R} \Rightarrow 1 - \frac{\sin^2 \theta_2}{\beta^2} > 0 \rightarrow \sin^2 \theta_2 < \beta^2$$

$$\text{se } n_T > n_I \rightarrow \theta_2 \in [0, \pi/2]$$

$$\text{se } n_T < n_I \rightarrow \sin \theta_2 = \beta \text{ e } \cdot \text{ o limite para haver transmissão!}$$

$$\text{Acima do ângulo crítico } \theta_c = \arcsen\left(\frac{n_T}{n_2}\right)$$

a condição de propagação deve ser revista,

$$|r| = 1$$

Além do ângulo crítico!

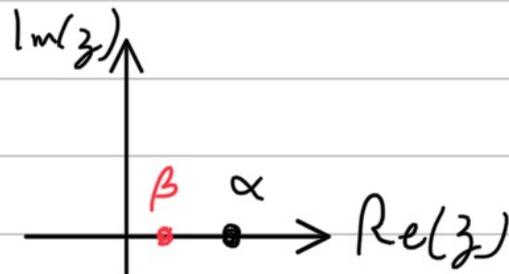
$$r = \left(\frac{\alpha - \beta}{\alpha + \beta} \right)$$

$$\alpha = \sqrt{\frac{1 - \frac{\sin^2 \theta_I}{\beta^2}}{1 - \sin^2 \theta_I}} \quad / \quad \beta = \frac{n_I}{n_T}$$

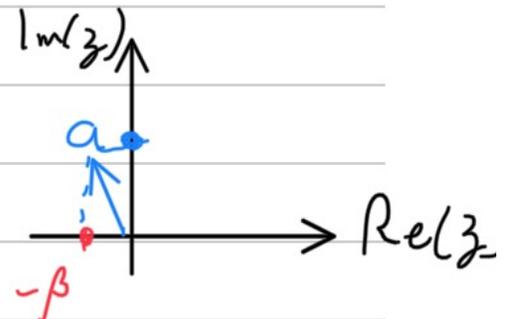
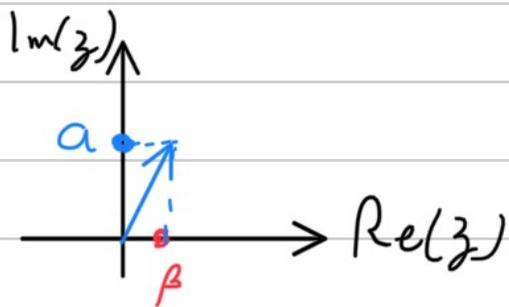
$$1 - \frac{\sin^2 \theta_I}{\beta^2} < 0 \Rightarrow \alpha \in \mathbb{C} \rightarrow \alpha = ia$$

$$a \in \mathbb{R}$$

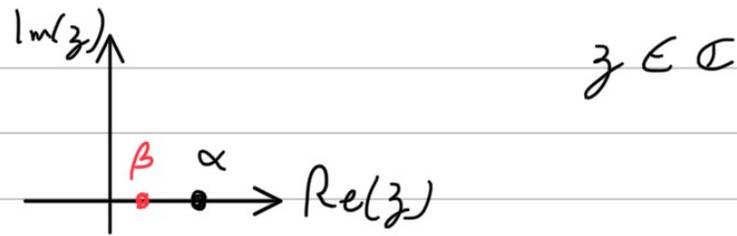
$$\Gamma = \frac{ia - \beta}{ia + \beta}$$



$$\begin{aligned} |\Gamma|^2 &= \frac{ia - \beta}{ia + \beta} \cdot \frac{-ia - \beta}{-ia - \beta} \\ &= \frac{a^2 + \beta^2}{a^2 + \beta^2} = \underline{1} \end{aligned}$$

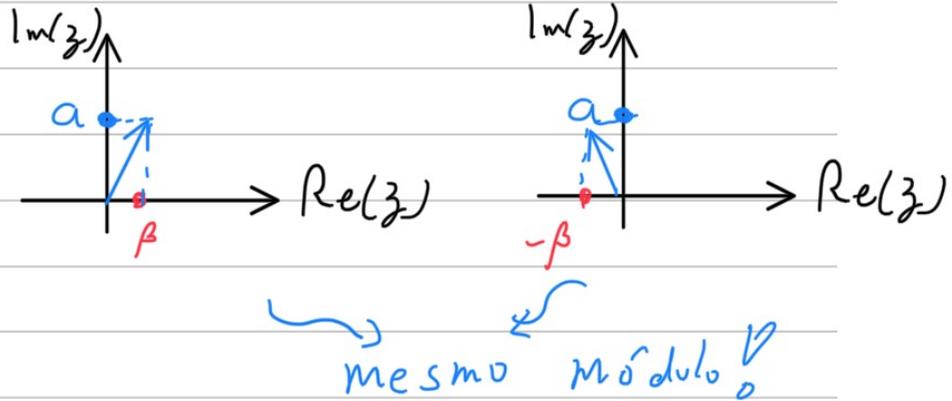


$$\Gamma = \frac{ia - \beta}{ia + \beta}$$



$$|\Gamma|^2 = \frac{ia - \beta}{ia + \beta} \cdot \frac{-ia - \beta}{-ia + \beta}$$

$$= \frac{a^2 + \beta^2}{a^2 + \beta^2} = 1$$



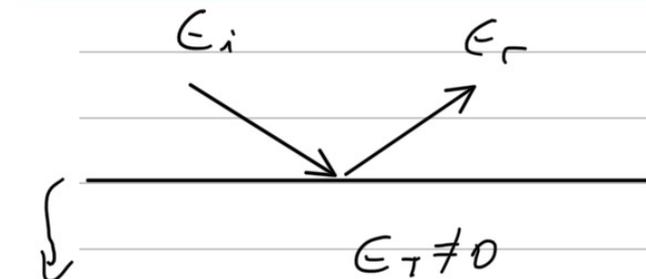
$$R = |\Gamma|^2 = 1$$

$$T = 1 - R = 0$$

$$\Gamma = |\Gamma| e^{i\theta} = \frac{ia - \beta}{ia + \beta} \cdot \frac{-ia + \beta}{-ia + \beta} =$$

$$= \frac{-(ia - \beta)^2}{\beta^2 + a^2} = \frac{-\beta^2 + a^2 + 2ia}{\beta^2 + a^2}$$

$$\tan \theta = \left(\frac{2a}{a^2 - \beta^2} \right)$$



na interface:
descontínuo: $E_{1n} = E_{2n} = E_2$

$$E_{1||} = E_{2||}$$

On da evanescente!

Campo elétrico transversal?

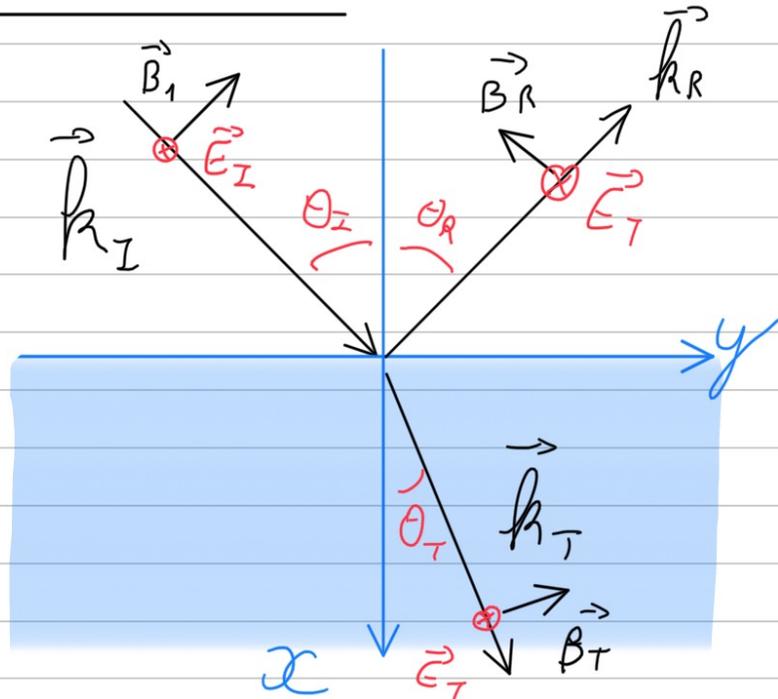
$$\underline{E_{1\parallel} = E_{2\parallel}}$$

$$\Rightarrow \cancel{E_{Iy} + E_{Ry} = E_{Ty}}$$

$$E_{Iz} + E_{Rz} = E_{Tz}$$

$$\cancel{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$$\cancel{\epsilon_1 (E_{Ix} + E_{Rx}) = \epsilon_2 E_{Tx}}$$



Para o campo magnético:

$$\underline{B_{1n} = B_{2n}} \Rightarrow B_{Ix} + B_{Rx} = B_{Tx}$$

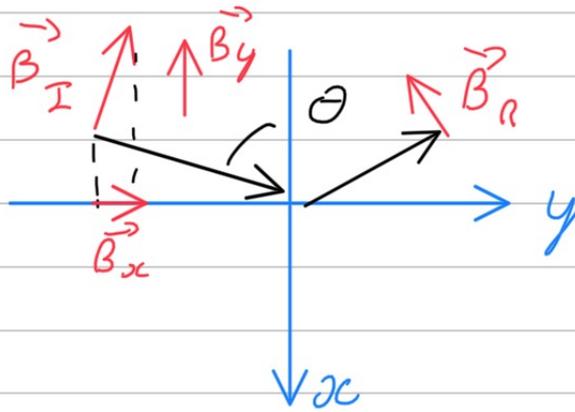
$$\underline{\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}} \Rightarrow \frac{1}{\mu_1} [B_{Iy} + B_{Ry}] = \frac{1}{\mu_2} [B_{Ty}]$$

$$\cancel{\frac{1}{\mu_1} [B_{Iz} + B_{Rz}] = \frac{1}{\mu_2} [B_{Tz}]}$$

Compõe \vec{B} no plano $xy \rightarrow$

$$\vec{B} = B_x \hat{x} + B_y \hat{y}$$

$$= B \cdot \sin \theta \hat{x} + B \cos \theta \hat{y}$$



$$\Rightarrow \text{xc} \Rightarrow B_I \sin \theta_I + B_R \sin \theta_R = B_T \sin \theta_T$$

$$B_I + B_R = \frac{\sin \theta_T}{\sin \theta_I} B_T = \frac{n_I}{n_T} B_T \quad \text{(I)}$$

$$y \Rightarrow (B_I + B_R) \cos \theta_I = \frac{\mu_I}{\mu_T} B_T \cos \theta_T \quad \text{(II)}$$

$$E_I + E_R = E_T$$

$$B = \frac{1}{v} E = \frac{n}{c} E \Rightarrow \frac{E_I + E_R}{c} \cdot n_I = \frac{E_T}{c} \frac{n_T \cdot n_I}{n_T} \quad \text{(I)}$$

(redundante)

$$\text{(II)} \Rightarrow (E_I - E_R) \cos \theta_I \cdot \frac{n_I}{c} = \frac{\mu_I}{\mu_T} \cos \theta_T \frac{n_T}{c} E_T$$

$$\therefore \varepsilon_I + \varepsilon_R = \varepsilon_T$$

$$\varepsilon_I - \varepsilon_R = \kappa \varepsilon_T$$

$$\kappa = \frac{\mu_I \cdot n_T \cdot \cos \theta_T}{\mu_T \cdot n_I \cdot \cos \theta_I}$$

$$\varepsilon_T = \frac{2}{1 + \kappa} \varepsilon_I \quad \Rightarrow \quad t_{\perp} = \frac{2 n_I \cos \theta_I}{n_T \cos \theta_T + n_I \cos \theta_I}$$

$$\varepsilon_R = \frac{1 - \kappa}{2} \varepsilon_T = \frac{1 - \kappa}{1 + \kappa} \varepsilon_I \quad \Rightarrow \quad r_{\perp} = \frac{n_I \cos \theta_I - n_T \cos \theta_T}{n_T \cos \theta_T + n_I \cos \theta_I}$$

Campo Paralelo ao plano de incidência

$$t_{\parallel} = \frac{2 n_I \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I}$$

$$r_{\parallel} = \frac{n_I \cos \theta_T - n_T \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I}$$

$$\varepsilon_T = \frac{2}{1+\kappa} \varepsilon_I \quad \Rightarrow \quad t_{\perp} = \frac{2 n_I \cos \theta_I}{n_T \cos \theta_T + n_I \cos \theta_I}$$

$$\varepsilon_R = \frac{1-\kappa}{2} \varepsilon_I = \frac{1-\kappa}{1+\kappa} \varepsilon_I \quad \Rightarrow \quad \Gamma_{\perp} = \frac{n_I \cos \theta_I - n_T \cos \theta_T}{n_T \cos \theta_T + n_I \cos \theta_I}$$

Campo Paralelo ao plano de incidência

$$t_{\parallel} = \frac{2 n_I \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I}$$

$$\Gamma_{\parallel} = \frac{n_I \cos \theta_T - n_T \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I}$$

Paralelo? Nem tanto

$$\Gamma_{\parallel} = 0 \Rightarrow n_I \cos \theta_I = n_T \cos \theta_T$$

$$\text{Mas } n_I \sin \theta_I = n_T \sin \theta_T$$

$$\Rightarrow \cos \theta_I \cdot n_T \frac{\sin \theta_T}{\sin \theta_I} = n_T \cos \theta_T$$

$$\sin \theta_I = \sin \theta_T \rightarrow \theta_I = \theta_T \quad \cancel{A}$$

Fluxo de Energia: $\vec{S} = \vec{E} \times \vec{H}$ $B = E/v$

$$I = \langle |\vec{S}| \rangle = \frac{\langle |\vec{E} \times \vec{B}| \rangle}{\mu_0} = \frac{1}{v} \frac{1}{\mu_0} \langle E^2 \rangle = \frac{c^2 \epsilon_0}{v} \frac{\langle E \rangle^2}{2}$$

$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

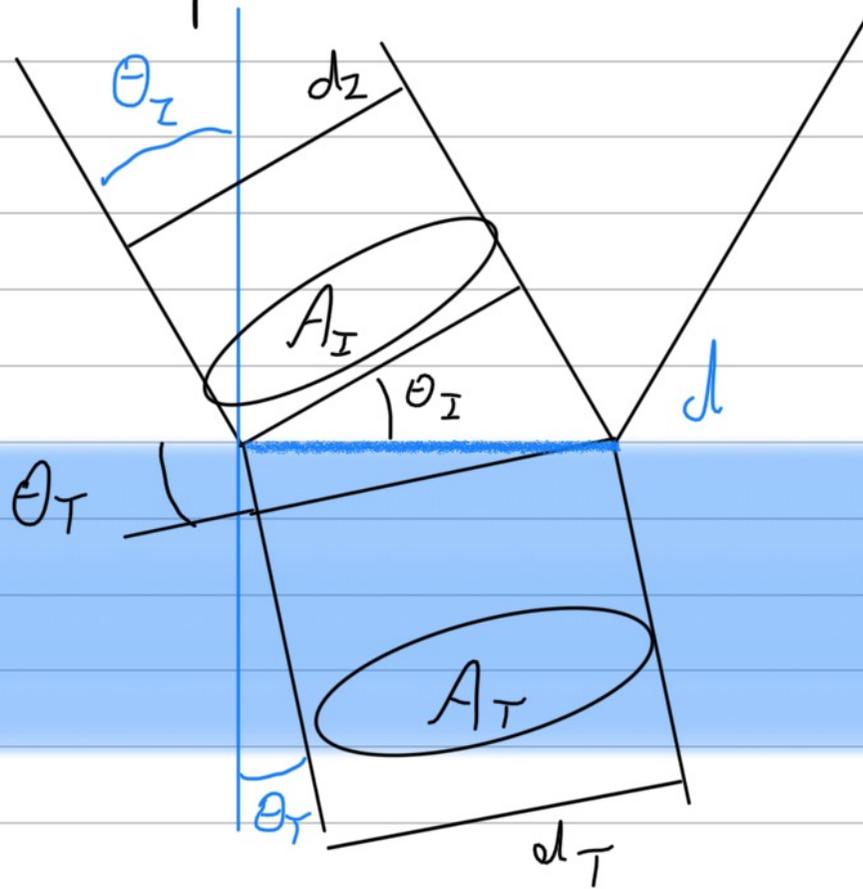
$$I = n \cdot \frac{c \epsilon_0}{2} \langle E \rangle^2$$

$$\frac{I_T}{I_I} = \frac{n_T}{n_I} \cdot |t|^2$$

$$\frac{I_R}{I_I} = |r|^2$$

Qual a potência total transmitida?

Qual a potência total transmitida?



Precisamos integrar sobre a área ∇

Potência Incidente: $P_I = \int_{A_I} I_I \cdot da_i = I_I \cdot A_I$

Potência Transmitida: $P_T = \int_{A_T} I_T da_T = I_T \cdot A_T$

$$\text{Potência Incidente: } P_I = \int_{A_I} I_I \cdot da_i = I_I \cdot A_I$$

$$\text{Potência Transmitida: } P_T = \int_{A_T} I_T \cdot da_T = I_T \cdot A_T$$

Usando uma área pequena o suficiente para ter $I = \text{cte}$

só para facilitar a dedução

$$T: \frac{P_T}{P_I} = \frac{I_T}{I_I} \cdot \frac{A_T}{A_I} = \frac{n_T}{n_I} \left(\frac{t}{l}\right)^2 \frac{A_T}{A_I}$$

Note que no processo de refração temos uma contração ou extensão na dimensão paralela ao plano. Pela figura, vemos que o segmento d é comum aos dois triângulos, o que leva a

$$\cos \theta_I = \frac{d_I}{d} \quad ; \quad \cos \theta_T = \frac{d_T}{d} \quad ; \quad A_I \propto d_I \quad , \quad A_T \propto d_T$$

Note que no processo de refração temos uma contração ou extensão na dimensão paralela ao plano. Pela figura, vemos que o segmento d é comum aos dois triângulos, o que leva a

$$\cos \theta_I = \frac{d_I}{d} \quad ; \quad \cos \theta_T = \frac{d_T}{d} \quad ; \quad A_I \propto d_I \quad , \quad A_T \propto d_T$$

$$\Rightarrow \frac{A_T}{A_I} = \frac{\cos \theta_T}{\cos \theta_I}$$

$$\therefore T = \frac{n_T \cos \theta_T}{n_I \cos \theta_I} \cdot (t)^2$$

$$\text{Como } R = \frac{P_R}{P_I} = |\Gamma|^2 \Rightarrow T + R = 1$$

Para \perp & \parallel Verifique!

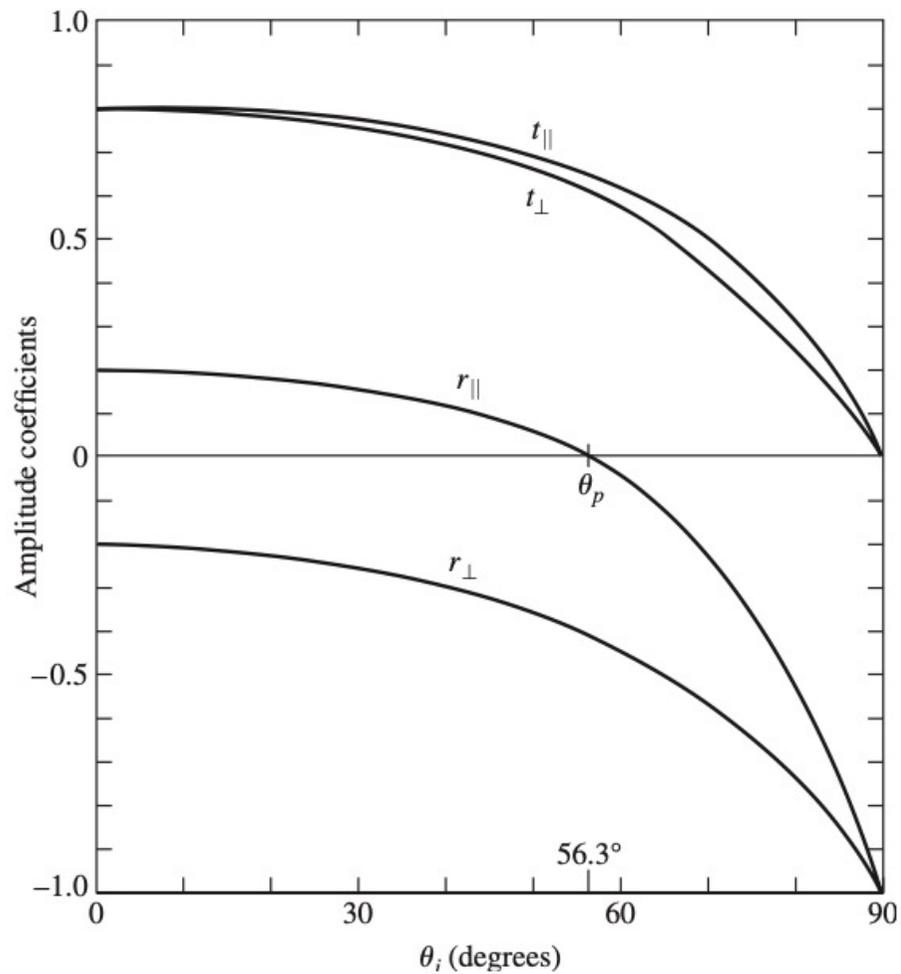


Figure 4.49 The amplitude coefficients of reflection and transmission as a function of incident angle. These correspond to external reflection $n_t > n_i$ at an air-glass interface ($n_{ti} = 1.5$).

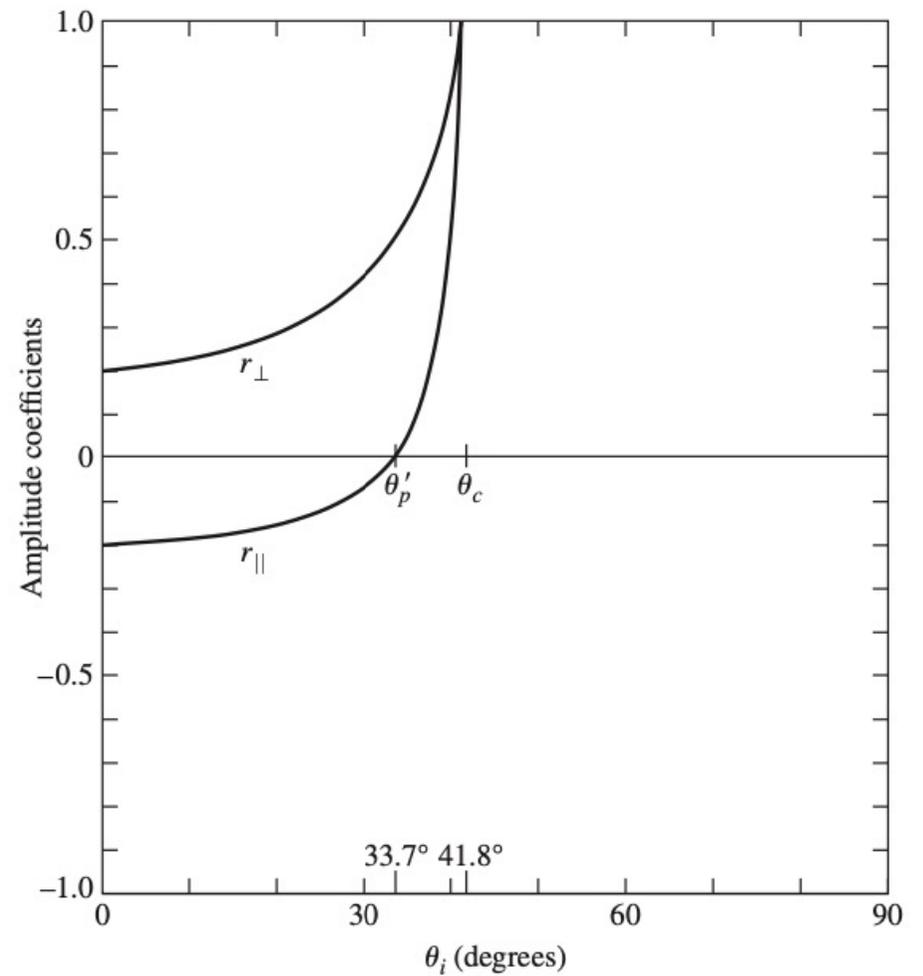


Figure 4.50 The amplitude coefficients of reflection as a function of incident angle. These correspond to internal reflection $n_t < n_i$ at an air-glass interface ($n_{ti} = 1/1.5$).