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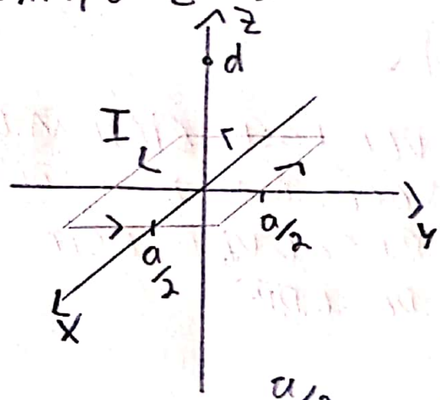
BIOT-SAVART: $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times (\vec{r}-\vec{r}')}{\|\vec{r}-\vec{r}'\|^3}$

$\vec{r}$ -> OBSERVADOR; $\vec{r}'$ -> FONTE

PELA SIMETRIA TEMOS NO PONTO DE INTERESSE QUE O CAMPO É SO EM $\vec{z}$



$\vec{r} = (0,0,d)$

$\vec{r}' = (x, a/2, 0)$

$d\vec{l} = dx\,\hat{x}$

↳ PARA 1 SO LADO, DEPOIS MULTIPLICAMOS POR 4.

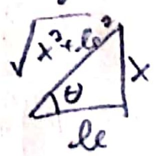
$$\frac{1}{4} B_{TOT} = \left(\frac{\mu_0 I}{4\pi}\int_{-a/2}^{a/2} \frac{dx\,\hat{x}\times\left(-x\hat{x} - \frac{a}{2}\hat{y} + \hat{z}d\right)}{\left(x^2+\frac{a^2}{4}+d^2\right)^{3/2}}\right)\cdot\vec{z}$$

$$= \frac{\mu_0 I}{4\pi}\int_{-a/2}^{a/2} \frac{-\frac{a}{2}\,dx}{\left(x^2+\underbrace{\frac{a^2}{4}+d^2}_{\ell^2}\right)^{3/2}} = \frac{-\mu_0 I a}{8\pi}\int_{-a/2}^{a/2} \frac{dx}{(\ell^2+x^2)^{3/2}}$$

$$= -\frac{\mu_0 I a}{8\pi \ell^3}\int_{-a/2}^{a/2} \frac{dx}{\left(1+\frac{x^2}{\ell^2}\right)^{3/2}} \quad ; \quad \frac{x}{\ell} = Tg\,\theta \Rightarrow dx = \ell(1+Tg^2\theta)\,d\theta$$

$$= -\frac{\mu_0 I a}{8\pi\ell^3}\int_{-a/2}^{a/2} \frac{\ell(1+Tg^2\theta)\,d\theta}{(1+Tg^2\theta)^{3/2}} = \frac{-\mu_0 I a}{8\pi\ell^2}\int_{-a/2}^{a/2}\frac{d\theta}{(1+Tg^2\theta)^{1/2}} \quad ; \quad 1+Tg^2\theta = \sec^2\theta \Rightarrow$$

$$= -\frac{\mu_0 I a}{8\pi\ell^2}\int_{-a/2}^{a/2} \frac{d\theta}{\sec\theta} = -\frac{\mu_0 I a}{8\pi \ell^2}\, \sin\theta \Big|_{-a/2}^{a/2} \quad ;$$



$\Rightarrow \sin\theta = \frac{x}{\sqrt{x^2+\ell^2}}$

$$= -\frac{\mu_0 I a}{8\pi\ell^2}\left(\frac{x}{\sqrt{x^2+\ell^2}}\right)\Big|_{-a/2}^{a/2} \Rightarrow B_{TOT} = \frac{\mu_0 I a^2}{2\pi}\left(\frac{1}{d^2+\frac{a^2}{4}}\right)\frac{1}{\sqrt{d^2+\frac{a^2}{2}}}$$