Eletromagnetismo

Dielétricos

- Temos novos elementos para jogar

  com o campo em meios materiais.

- Como podemos relacionar a polarização P macroscópica

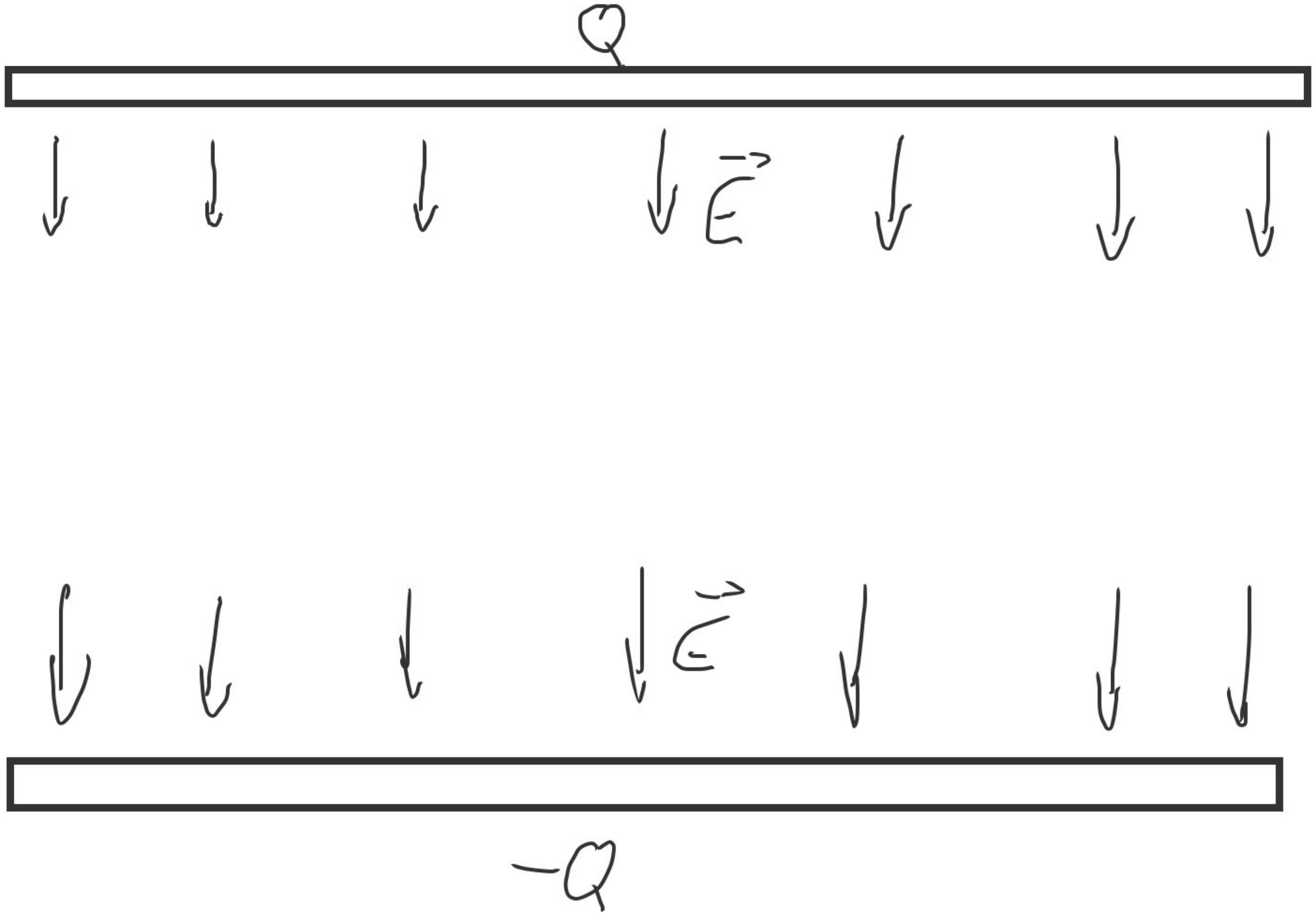
  com o dipolo microscópico p?

- O que acontece na interface de um meio dielétrico?
- Qual a diferença entre o vetor deslocamento D e o campo elétrico E?
- Como fica a energia na presença do dielétrico?

$$u = \frac{\varepsilon_0 E^2}{2}$$

$$\mathcal{E} = \int u \cdot dv$$

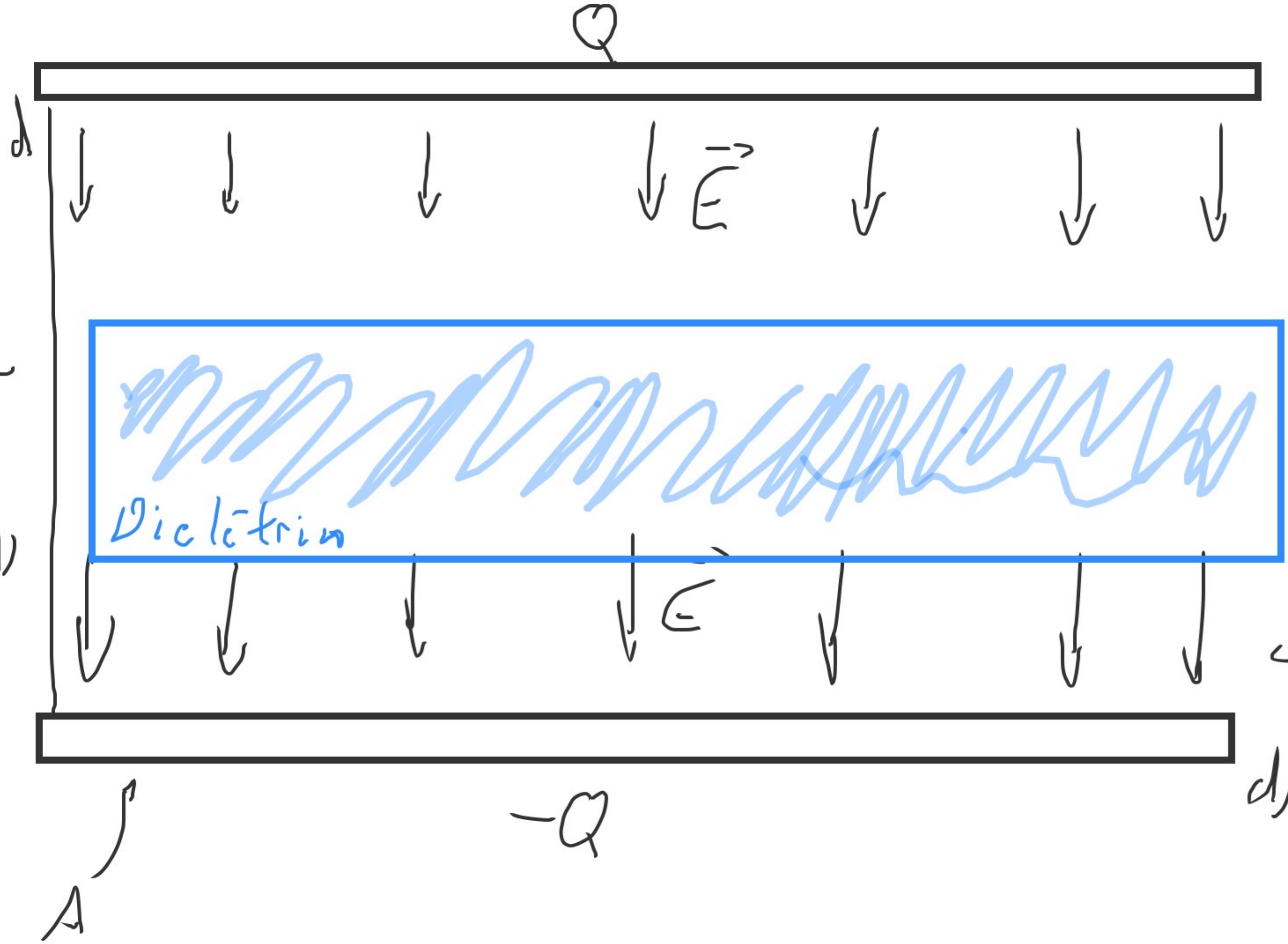
$$= \frac{\varepsilon_0 E^2}{2} \cdot (A \cdot d)$$



$$u = \frac{\varepsilon_0 E^2}{2}$$

$$\mathcal{E} = \int u \cdot dv$$

$$= \frac{\varepsilon_0 E^2}{2} \cdot \underbrace{(A \cdot d)}_{Vol.}$$

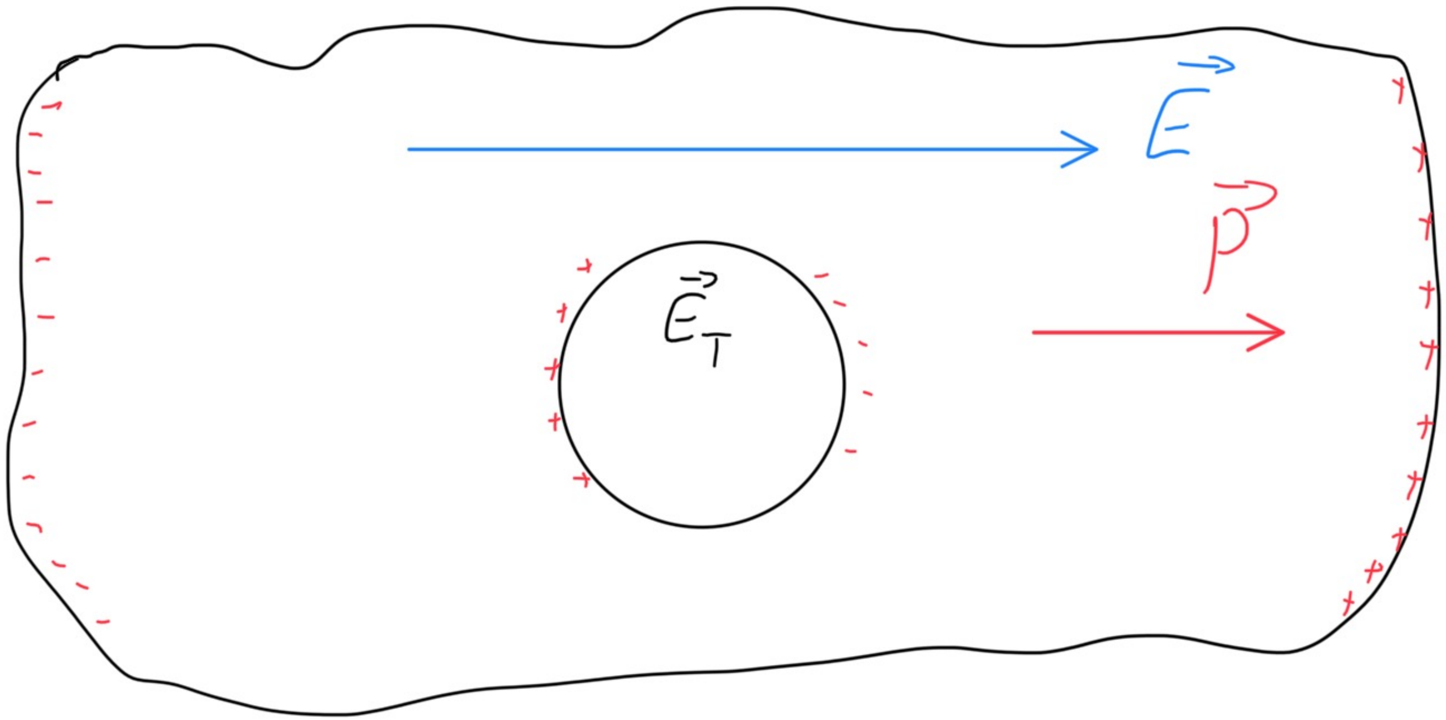


a) $\mathcal{E}_f > \mathcal{E}$

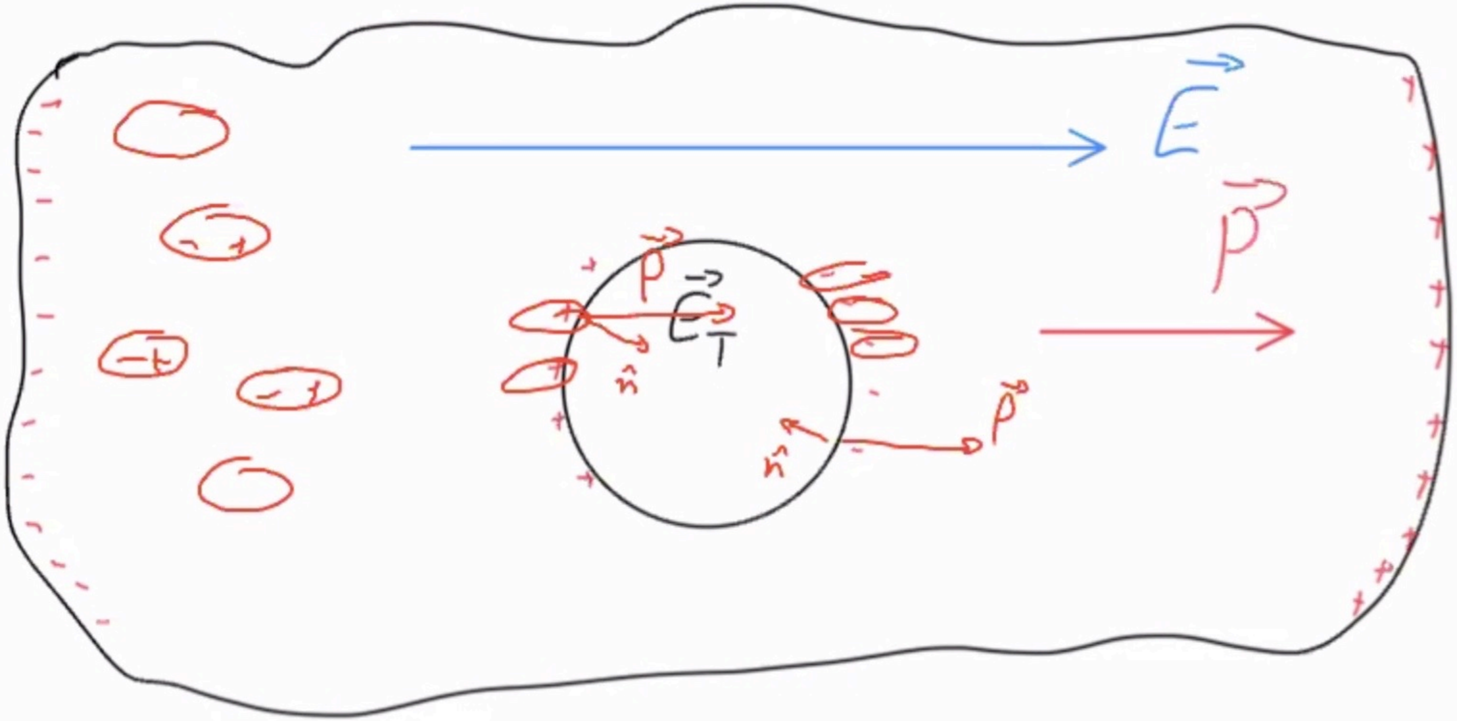
b) $\mathcal{E}_f < \mathcal{E}$

c) $\mathcal{E}_f = 0$

d) $\mathcal{E}_f = \mathcal{E}$

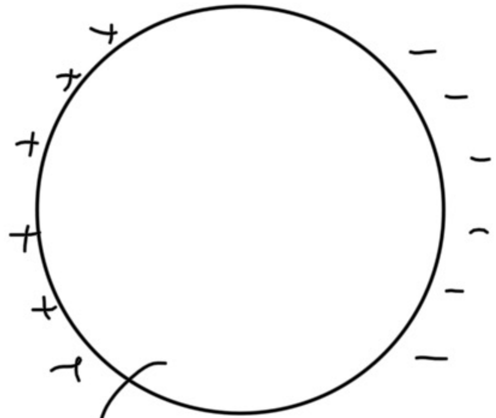


Campo total: $\vec{E}_T = \vec{E} + \vec{E}_p = \vec{E} + \dfrac{\vec{P}}{3\varepsilon_0}$



Campo total: $\vec{E}_T = \vec{E} + \vec{E}_p = \vec{E} + \dfrac{\vec{P}}{3\varepsilon_0}$

$\sigma_p = \vec{P} \cdot \hat{n}$



A molécula neste volume é polarizada (induzida ou orientada) pelo campo $\vec{E}_T$

Volume para uma molécula $\rightarrow \frac{4}{3}\pi R^3$

Densidade $n = 1/\frac{4}{3}\pi R^3$

Polarização atômica: $\vec{p} = \alpha \vec{E}_T$

Polarização do meio: $\vec{P} = \frac{\sum \vec{p}_i}{V}$ $\rightarrow$ média no volume

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$$\Rightarrow \vec{P} = n \cdot \vec{p} = \frac{3\alpha}{4\pi R^3} \vec{E}_T$$

$$\vec{P} = \frac{3\alpha}{4\pi R^3} \vec{E} + \frac{\alpha}{4\pi R^3} \frac{\vec{P}}{\varepsilon_0}$$

$$\vec{P} \left(1 - \frac{\alpha}{4\pi R^3} \frac{1}{\varepsilon_0} \right) = \frac{3\alpha}{4\pi R^3} \vec{E}$$

$$\vec{P} = \frac{3\alpha}{4\pi R^3 \varepsilon_0 - \alpha} \varepsilon_0 \vec{E} = \chi \varepsilon_0 \vec{E}$$

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$$\chi = \frac{3\alpha}{4\pi R^3 \varepsilon_0 - \alpha} \Rightarrow \varepsilon = \varepsilon_0 (\chi + 1) = \varepsilon_0 \left(\frac{3\alpha + 4\pi R^3 \varepsilon_0 - \alpha}{4\pi R^3 \varepsilon_0 - \alpha} \right)$$

$$2\alpha + 4\pi R^3 \varepsilon_0 = \frac{\varepsilon}{\varepsilon_0} \cdot 4\pi R^3 \varepsilon_0 - \frac{\varepsilon}{\varepsilon_0} \alpha$$

$$\alpha \left(\frac{\varepsilon}{\varepsilon_0} + 2 \right) = 4\pi R^3 \varepsilon_0 \cdot \left(\frac{\varepsilon}{\varepsilon_0} - 1 \right)$$

$$\Rightarrow \alpha = \frac{3\varepsilon_0}{n} \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} \right) \rightarrow \text{Eq. Clausius-Mossotti}$$

$\uparrow$ micro $\quad\quad$ $\uparrow$ macro

$$\vec{p} = \frac{3\alpha}{4\pi R^3 \varepsilon_0 - \alpha} \varepsilon_0 \vec{E} = \chi \varepsilon_0 \vec{E}$$

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$$2\alpha + 4\pi R^3 \varepsilon_0 = \frac{\varepsilon}{\varepsilon_0} \cdot 4\pi R^3 \varepsilon_0 - \frac{\varepsilon}{\varepsilon_0} \alpha$$

$$\frac{\varepsilon}{\varepsilon_0} = \chi + 1$$

$$\alpha \left(\frac{\varepsilon}{\varepsilon_0} + 2 \right) = 4\pi R^3 \varepsilon_0 \cdot \left(\frac{\varepsilon}{\varepsilon_0} - 1 \right)$$

$$\frac{\varepsilon}{\varepsilon_0} - 1 = \chi$$

$$\frac{\varepsilon}{\varepsilon_0} + 2 = \chi + 3$$

$$\Rightarrow \alpha = \frac{3\varepsilon_0}{n} \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} \right) \rightarrow \text{Eq. Clausius-Mossoti}$$

$$= \frac{3\varepsilon_0}{n} \left(\frac{\chi}{\chi + 3} \right)$$

micro

macro

Ferroeletricidade

Depois de aplicarmos o campo $\vec{E}$, o meio pode manter-se polarizado: $\vec{E}_T = \frac{\vec{P}}{3\varepsilon_0}$, onde

$$\vec{P} = \frac{\vec{p}}{V} = n\vec{p} \; ; \quad \vec{p} = \alpha \vec{E}_T$$

$$\vec{P} = n \alpha \frac{\vec{P}}{3\varepsilon_0}$$

$$\vec{P} = 0 \quad ; \; \alpha = \frac{3\varepsilon_0}{n}$$

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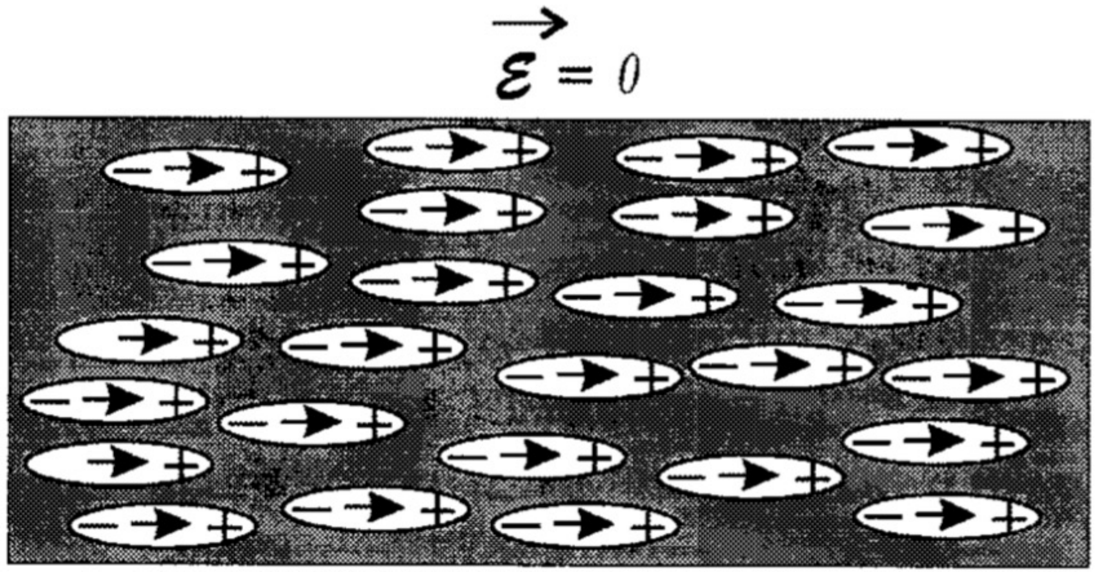

Extraído de Kleber Daum Machado, Teoria do Eletromagnetismo, vol. 1

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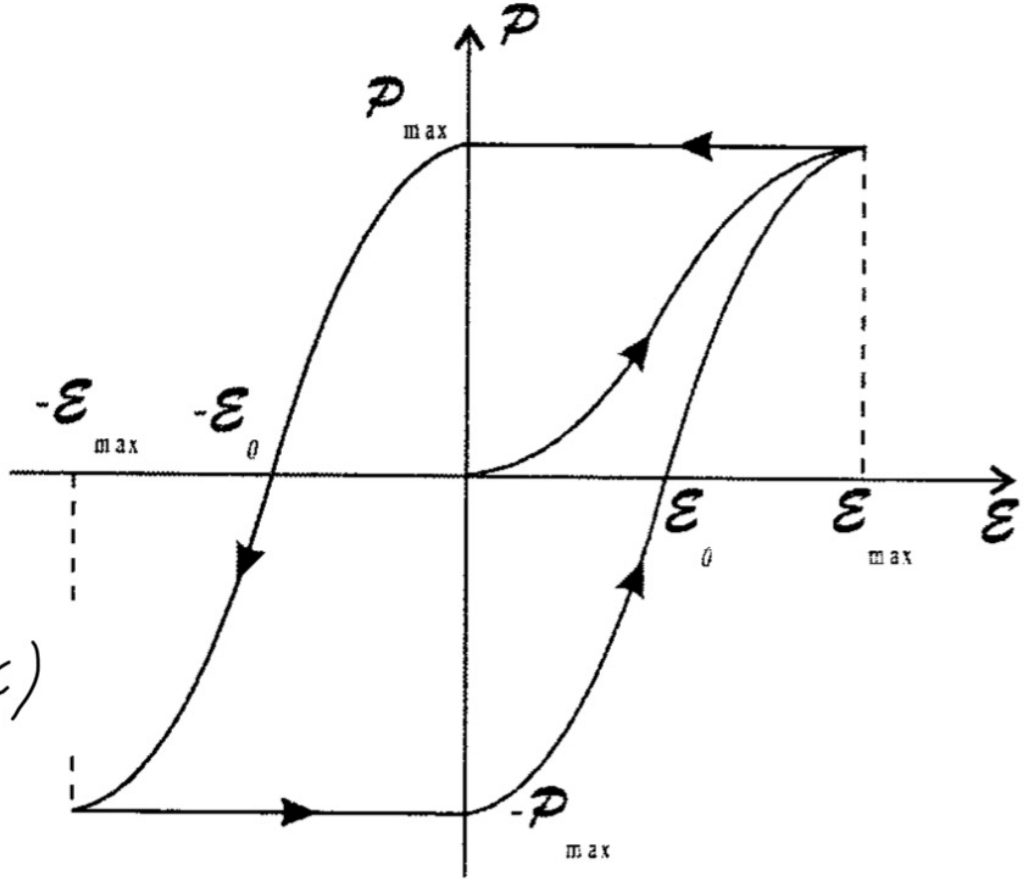
Ex: $BaTiO_3$ ($T < 393\,K$ / $120\,°C$)



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Condições de Contorno

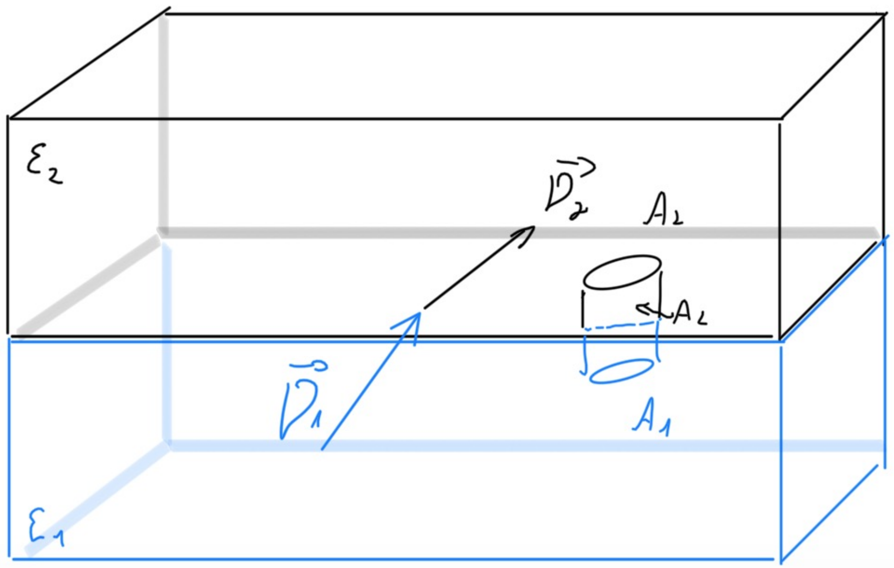
Campo do Vetor Deslocamento $\vec{D}$ => cargas livres

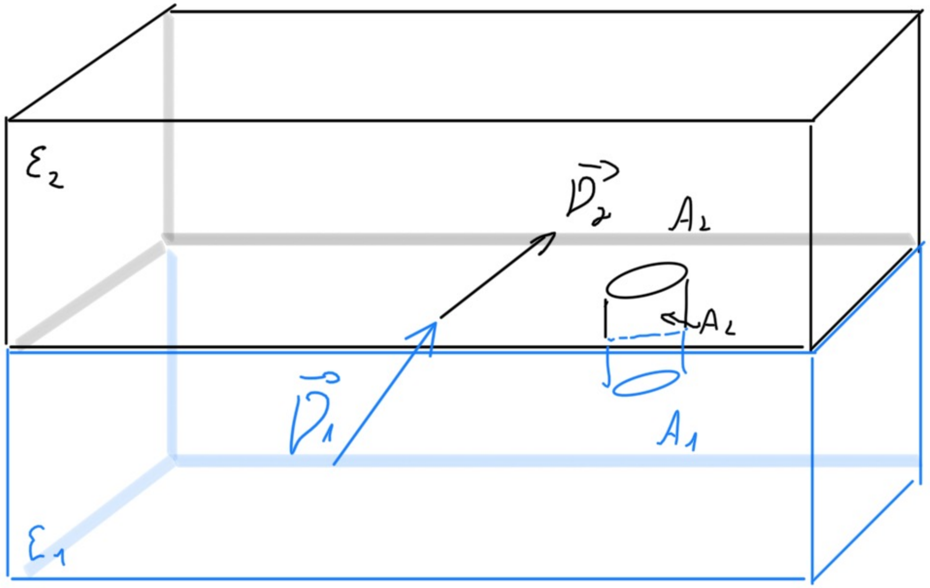
$$\nabla \cdot \vec{D} = \rho \Rightarrow \oint_S \vec{D} \cdot \hat{n} \, da = 0$$

Campo Elétrico $\vec{E} = -\nabla V \Rightarrow \nabla \times \vec{E} = 0$

$$\Rightarrow V(\vec{r}_B) - V(\vec{r}_A) = -\int_{r_A}^{r_B} \vec{E} \cdot d\vec{\ell}$$

O que ocorre entre dois dielétricos?





$$\oint \vec{D}\cdot\vec{n}\, da = \int_{A_1} \vec{D}_1\cdot\hat{n}_1\, da_1 + \int_{A_2} \vec{D}_2\cdot\hat{n}_2\, da_2 + \int_{A_\ell} \vec{D}_\ell\cdot\vec{n}_\ell\, da_\ell$$

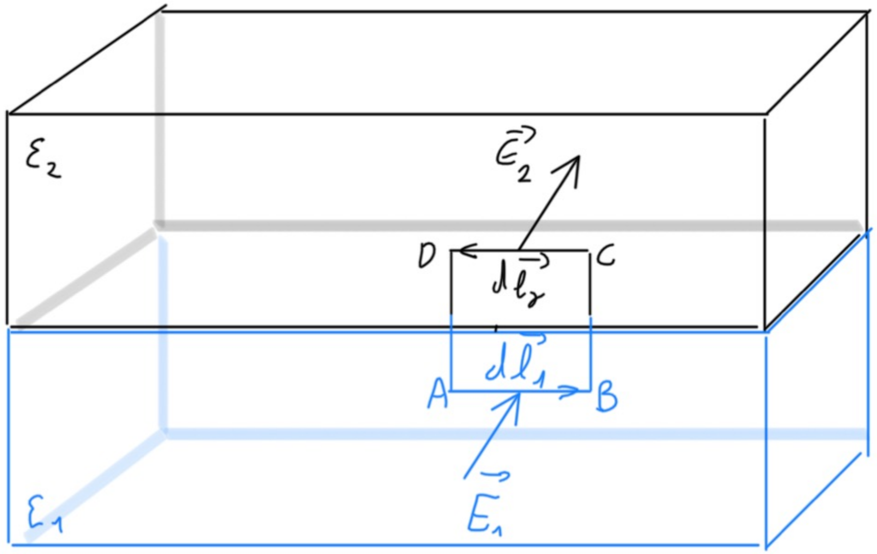
$$\rightarrow 0$$

$$\hat{n}_1 = -\hat{n}_2 \;;\; da_1 = da_2$$

$$\oint \vec{D}\cdot\hat{n}\, da = \int_{A_2} (\vec{D}_2 - \vec{D}_1)\, \hat{n}_2\, da_2$$

Carga nula (livre) $\Rightarrow$ $(\vec{D}_2 - \vec{D}_1)\cdot\hat{n}_2 = 0$

$$\Rightarrow (\varepsilon_2 \vec{E}_2 - \varepsilon_1 \vec{E}_1)\, \hat{n}_2 = 0$$



$$V_{BA} = V_B - V_A = -\int_A^B \vec{E}_1 \cdot d\vec{l}_1 \quad ; \quad V_{CB} = -\int_B^C \vec{E} \cdot d\vec{l}$$

$$V_{DC} = V_D - V_C = -\int_C^D \vec{E}_2 \, d\vec{l}_2 \quad ; \quad V_{AD} = -\int_D^A \vec{E} \, d\vec{l}$$

$$L \to 0 \Rightarrow V_{CB} = V_{AD} = 0$$

$V_{BA} + V_{CB} + V_{DC} + V_{AD} = 0 \rightarrow$ volta completa

$$\Rightarrow V_{BA} - V_{DC} = 0 \qquad d\vec{l}_1 = -d\vec{l}_2$$

$$\Rightarrow \int_A^B (\vec{E}_1 - \vec{E}_2) \, d\vec{l}_2 = 0$$

$$\begin{cases} \text{Componente transversal:} & \vec{E}_1 \, d\vec{l}_2 = \vec{E}_2 \, d\vec{l}_2 \\ \text{Componente perpendicular:} & \vec{D}_1 \cdot \hat{n} = \vec{D}_2 \cdot \hat{n} \end{cases}$$

Outra consequência: $\oint \vec{D} \cdot d\vec{l}$ não é nulo na interface

Metal: $\vec{E}_1 \, d\vec{l}_2 = \vec{E}_2 \, d\vec{l}_2$

meio 1 → condutor → $\vec{E}_1 = 0 \Rightarrow \vec{E}_2 \cdot d\vec{l}_2 = 0 \Rightarrow \vec{E}_2 = E \, \hat{n}_2$

Energia em dielétricos

Como vimos: $\mathcal{E} = \frac{1}{2}\int_{v} V(\vec{r}\,')\cdot dq(\vec{r}\,') = \frac{1}{2}\int_{v} V(\vec{r}\,')\rho(\vec{r}\,')\,dv$

$1/2 \rightarrow$ contagem dupla de cargas

$$\rho(\vec{r}) = \nabla\cdot\vec{D} \Rightarrow \mathcal{E} = \frac{1}{2}\int V\, \nabla\cdot\vec{D}\, dv$$

Como vimos $\nabla\cdot(\phi\cdot\vec{A}) = (\nabla\phi)\cdot\vec{A} + \phi\cdot(\nabla\cdot\vec{A})$

$$\Rightarrow \mathcal{E} = -\frac{1}{2}\int \underbrace{(\nabla\cdot V)}_{-\vec{E}}\cdot\vec{D}\, dv + \frac{1}{2}\int \nabla\cdot(V\cdot\vec{D})\, dv$$

$$\Rightarrow \mathcal{E} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\, dv + \frac{1}{2}\oint_{s} (V\cdot\vec{D})\cdot d\vec{a}$$

$$\Rightarrow \mathcal{E} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,dv + \frac{1}{2}\oint_S (V\cdot\vec{D})\cdot d\vec{a}$$

Integrando no espaço $\rightarrow \oint (V\cdot\vec{D})\cdot d\vec{a} = 0$

pois $V \propto \frac{1}{r}$; $\vec{D} \propto \frac{1}{r^2}$; $da \propto r^2 dr$

(se monopolo, mais rápido em multipolos)

<u>Conclusão</u>: $\mathcal{E} = \int u\,dv$

Densidade de energia $u = \frac{\vec{E}\cdot\vec{D}}{2}$

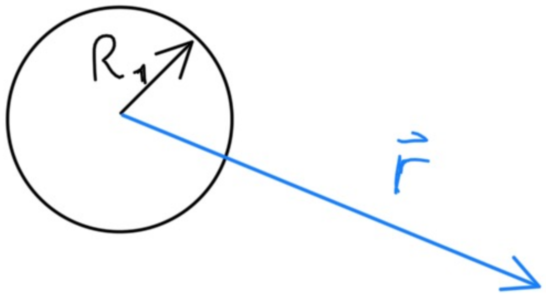
No vácuo recuperamos $u = \varepsilon_0 \frac{E^2}{2} =$

Em um dielétrico $u = \varepsilon \frac{E^2}{2}$, $\varepsilon > \varepsilon_0$

Maior ou menor? $u = \frac{D^2}{2\varepsilon}$

Esfera metálica, carregada, no vácuo (caso A)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



$$\mathcal{E}_A = \frac{1}{2}\int V\, dq =$$

$$= \frac{1}{2}\int V(R)\cdot\sigma\cdot da$$

$$= \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R}\int \sigma\cdot da = \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

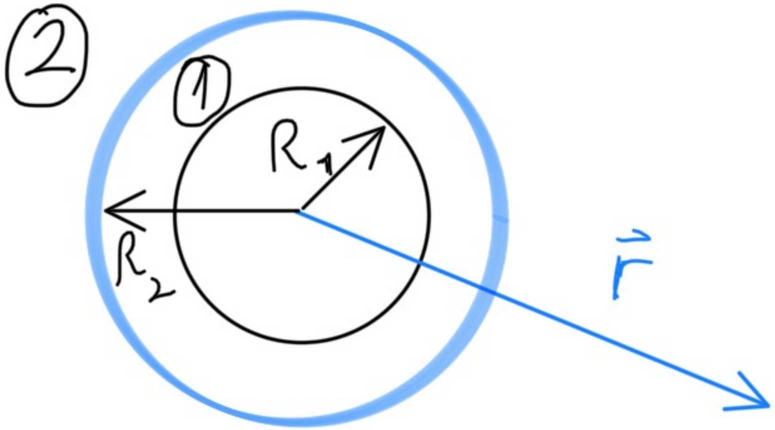
$$\mathcal{E} = \frac{1}{2}\int_R^\infty \varepsilon_0 \frac{1}{(4\pi\varepsilon_0)^2} Q^2 \frac{1}{r^4}\, r^2\, dr\, \operatorname{sen}\theta\, d\theta\, d\varphi$$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\int \operatorname{sen}\theta\, d\theta\, d\varphi = 4\pi$$

$$\mathcal{E}_A = \frac{1}{2}\frac{Q^2}{(4\pi)^2\varepsilon_0}(4\pi)\int_R^\infty \frac{1}{r^2}\, dr = \frac{Q^2}{8\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^\infty = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Esfera envolta por carga dielétrica (caso B)



$$\mathcal{E}_B = \frac{1}{2}\int_{R_1}^{R_2} \varepsilon \vec{E}\cdot\vec{E}\, dv + \frac{1}{2}\int_{R_2}^{\infty} \varepsilon_0 \vec{E}\cdot\vec{E}\, dv$$

Região 1 Região 2

$$\mathcal{E}_1 + \mathcal{E}_2$$

$$\vec{D} = \frac{1}{4\pi}\frac{Q}{r^2}\hat{r} \ ; \ r > R_1$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \ ; \ r > R_2$$

$$= \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\hat{r} \ ; \ R_2 > r > R_1$$

$$= 0 \qquad ; \ R_1 > r$$

$$\vec{D} = \frac{1}{4\pi} \frac{Q}{r^2} \hat{r} \quad ; \quad r > R_1$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \quad ; \quad r > R_2$$

$$= \frac{1}{4\pi\varepsilon} \frac{Q}{r^2} \hat{r} \quad ; \quad R_2 > r > R_1$$

$$= 0 \quad ; \quad R_1 > r$$

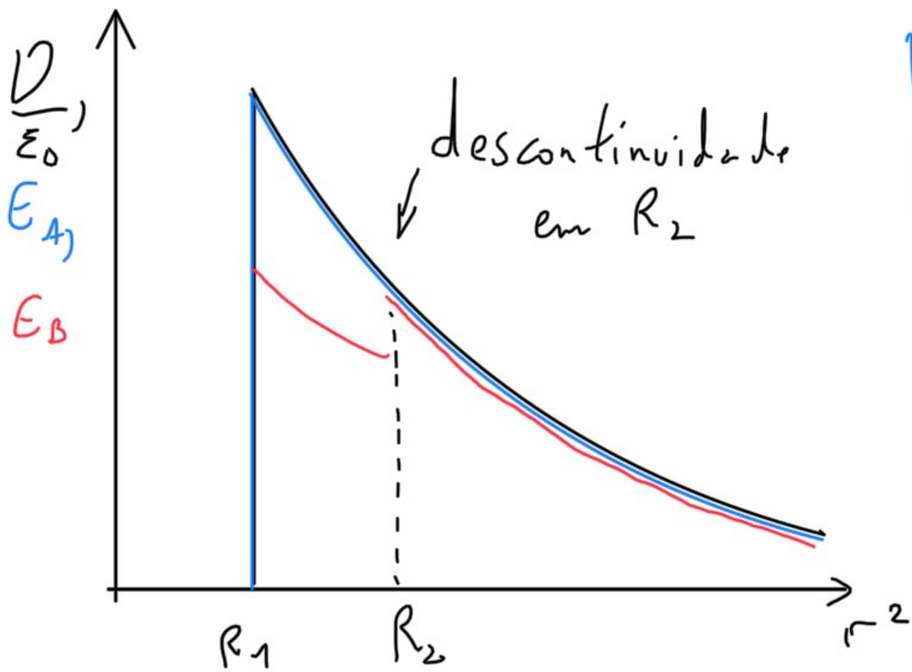
$$\mathcal{E}_2 = \frac{Q^2}{8\pi\varepsilon_0} \frac{1}{R_2} \quad ; \quad \mathcal{E}_1 = \frac{Q^2}{8\pi\varepsilon} \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

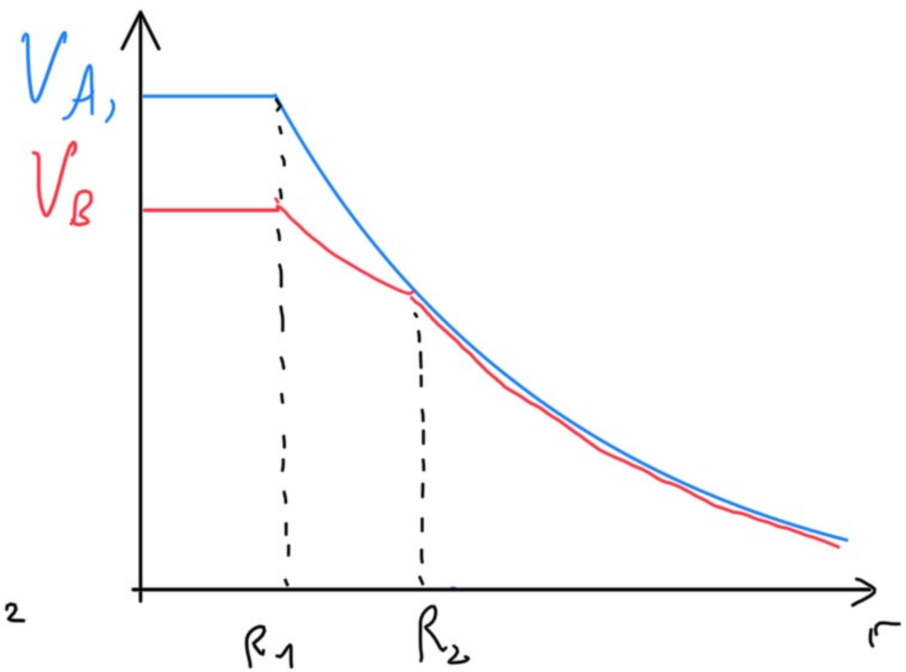
$$\mathcal{E}_B = \frac{Q^2}{8\pi\varepsilon_0} \left[\frac{1}{R_2} + \frac{\varepsilon_0}{\varepsilon} \frac{1}{R_1} - \frac{\varepsilon_0}{\varepsilon} \frac{1}{R_2} \right]$$

$$\mathcal{E}_B - \mathcal{E}_A = \frac{Q^2}{8\pi\varepsilon_0}\left[\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right)\frac{1}{R_2} + \frac{\varepsilon_0}{\varepsilon}\frac{1}{R_1} - \frac{1}{R_1}\right]$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{\varepsilon-\varepsilon_0}{\varepsilon}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\right]$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right)\left(\frac{R_1-R_2}{R_1R_2}\right) < 0$$





Polarização: $\vec{P} = \varepsilon_0 \chi \vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\hat{r} \; ; R_2 > r > R_1 \Rightarrow \vec{P} = \frac{\varepsilon_0 \chi}{\varepsilon}\frac{Q}{4\pi r^2}\hat{r}$$

Nos superfícies: $r = R_2 \Rightarrow \sigma_{p_2} = \vec{P}\cdot\hat{n} = \vec{P}\cdot\hat{r} = \frac{\varepsilon_0 \chi}{\varepsilon}\frac{Q}{4\pi R_2^2}$

$$r = R_1 \Rightarrow \sigma_{p_1} = \vec{P}\cdot\hat{n} = \vec{P}\cdot(-\hat{r}) = -\frac{\varepsilon_0 \chi}{\varepsilon}\frac{Q}{4\pi R_1^2}$$

"Carga": $\int \sigma_{p_2}\, da_2 = \frac{\varepsilon_0 \chi}{\varepsilon} Q = -\int \sigma_{p_1}\, da_1$

No volume: $\rho_p = -\nabla\cdot\vec{P}$

$$\nabla\cdot\vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 P_r = \frac{1}{r^2}\frac{\partial}{\partial r}\frac{\varepsilon_0 \chi}{\varepsilon_0}\frac{Q}{4\pi r^2}\cdot r^2 = 0$$

Eletromagnetismo

Capacitores

- Podemos armazenar a energia eletrostática!
- Gastamos energia para montar uma configuração de cargas.
- Isto inclui cargas livres e rearranjo de cargas em meios materiais.
- Esta energia está acoplada à própria distribuição do campo elétrico no espaço.
- Como podemos aumentar a eficiência?
- Como aumentar a capacidade de acumular energia?
- Vamos falar de acumuladores, ou *capacitores*.

Acumulando energia

Distribuindo cargas

$$\mathcal{E} = \frac{1}{2} \int_V V(\vec{r})\rho(\vec{r})\, dv$$

$$= \frac{1}{2} \sum V(r_i)\, q_i$$

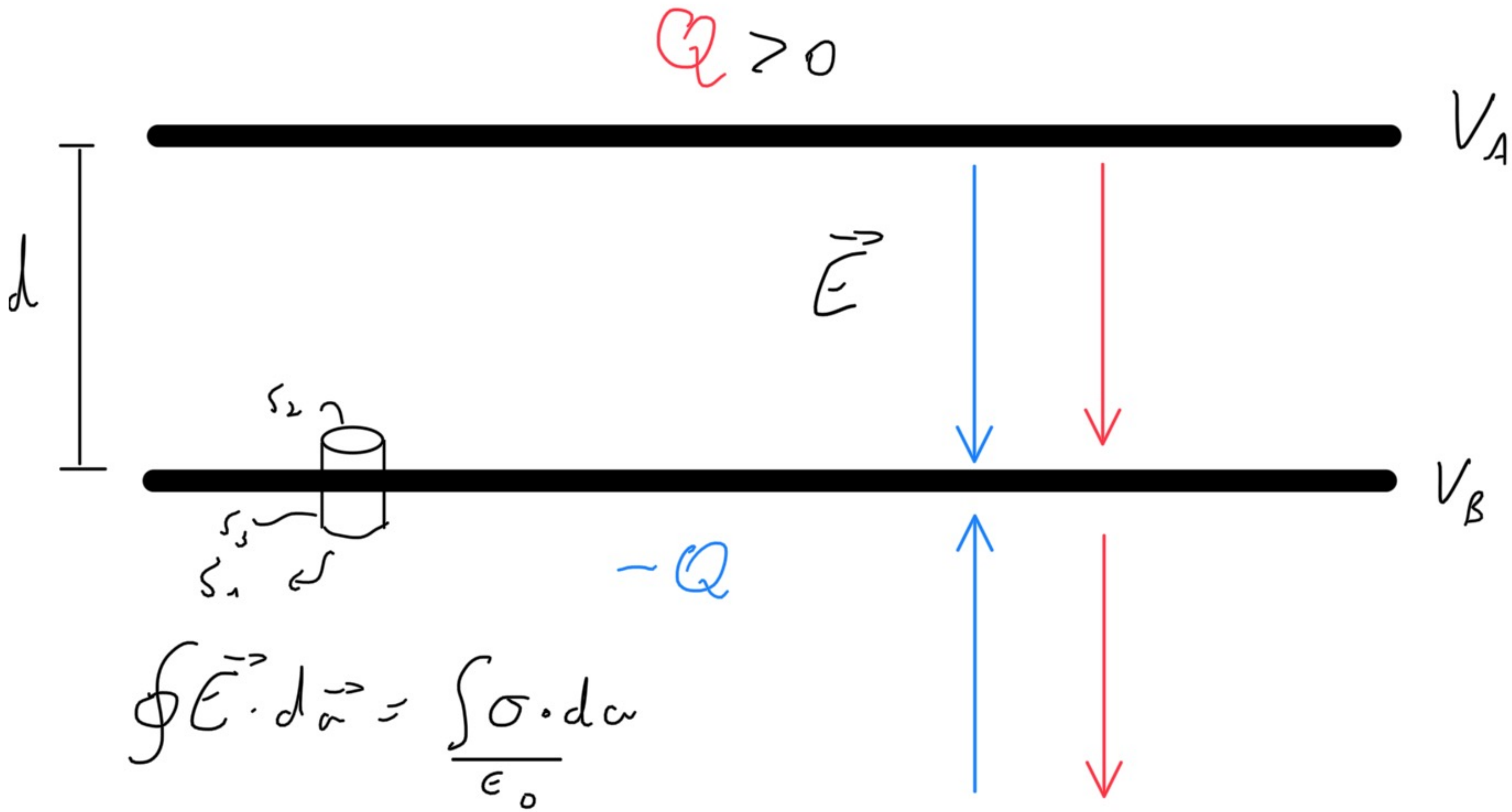
Criando campo

$$\mathcal{E} = \frac{1}{2} \int_V \vec{E} \cdot \vec{D}\, dv$$

$$\vec{E} = -\nabla V$$

$$\nabla \cdot \vec{D} = \rho$$

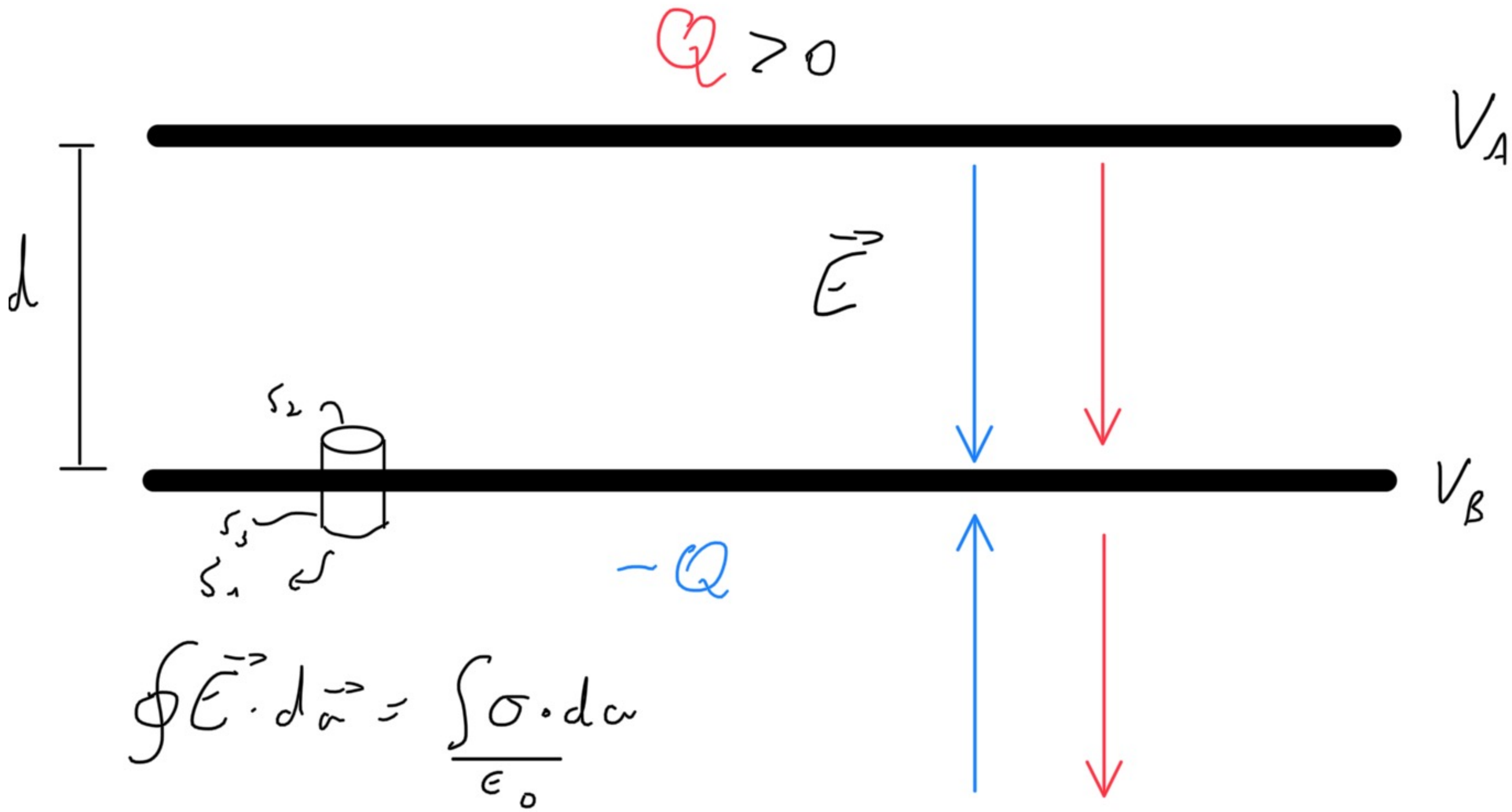
Energia em um arranjo de cargas



$$\oint \vec{E} \cdot d\vec{a} = \frac{\int \sigma \cdot da}{\epsilon_0}$$

$$\int_{S_1} \vec{E} \cdot d\vec{a} = 0 \,, \quad \int_{S_3} \vec{E} \cdot d\vec{a} = 0$$

Energia em um arranjo de cargos



$$\oint \vec{E} \cdot d\vec{a} = \frac{\int \sigma \cdot da}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{a} = \int E \cdot da = \int \frac{\sigma}{\epsilon_0} da \Rightarrow E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_A - V_B = -\int_B^A \vec{E} \cdot d\vec{l} = \frac{Q}{\epsilon_0 A} d$$

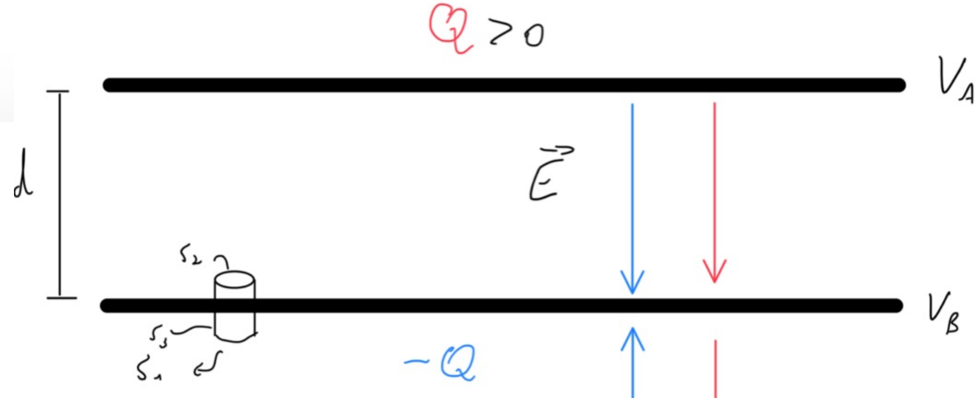
$$V_A - V_B = -\int_B^A \vec{E}\cdot d\vec{l} = \frac{Q}{\epsilon_0 A} d$$

$\Delta V \propto Q \rightarrow$ variações na proporcionalidade advém da geometria

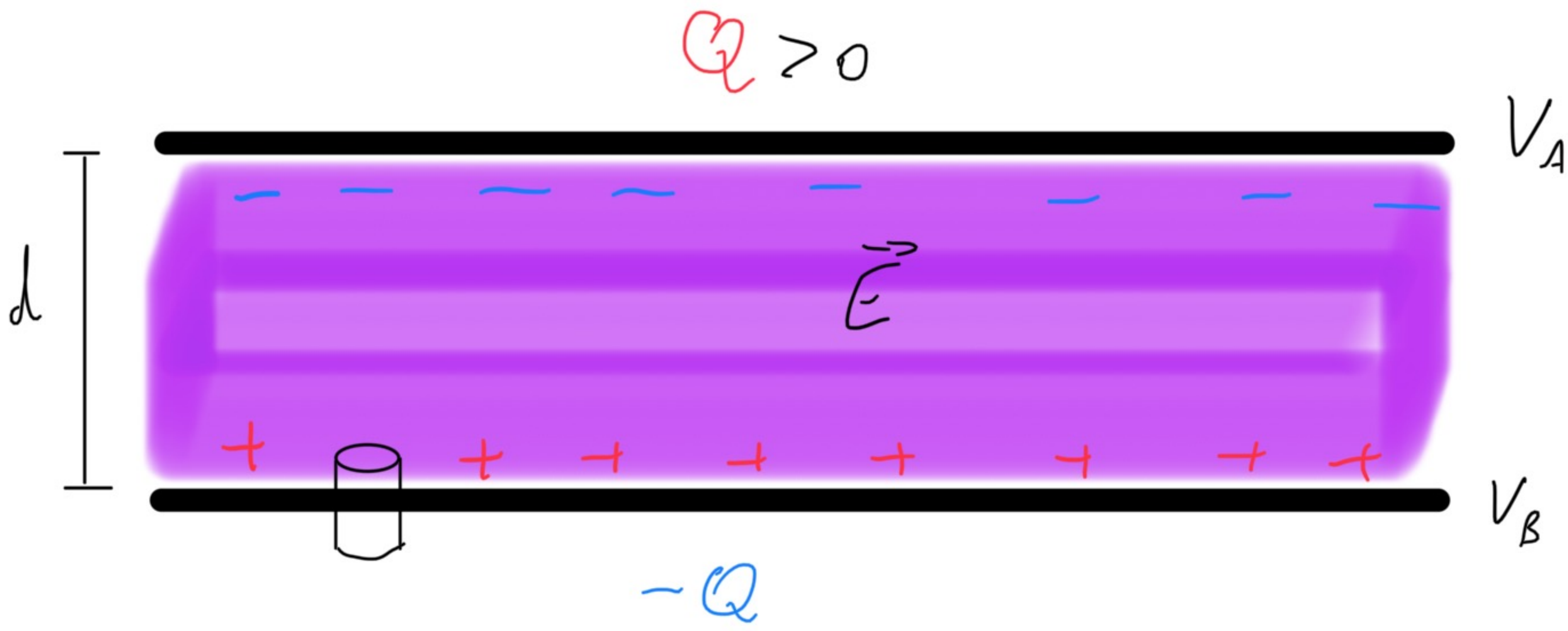
$$\Rightarrow \quad Q = C\,\Delta V \quad \Rightarrow \quad C = \frac{Q}{\Delta V} \qquad [C] = F = \frac{C}{V}$$

Temos um dispositivo de dois contatos (dipolar) que associa carga acumulada com diferença de potencial!

Duas placas paralelas: $C = \epsilon_0 \frac{A}{d}$



Com um meio dielétrico



Quanto vale $\vec{E}$? Temos a carga de polarização $\sigma_p = \vec{P} \cdot \hat{n}$ reduzindo a carga efetiva

$$\oint \vec{E} \cdot d\vec{a} = \int \frac{\sigma \cdot da}{\epsilon_0} - \int \frac{\sigma_p}{\epsilon_0} da = \int \frac{\sigma}{\epsilon_0} da - \int \frac{\vec{P}}{\epsilon_0} \hat{n} \, d\vec{a}$$

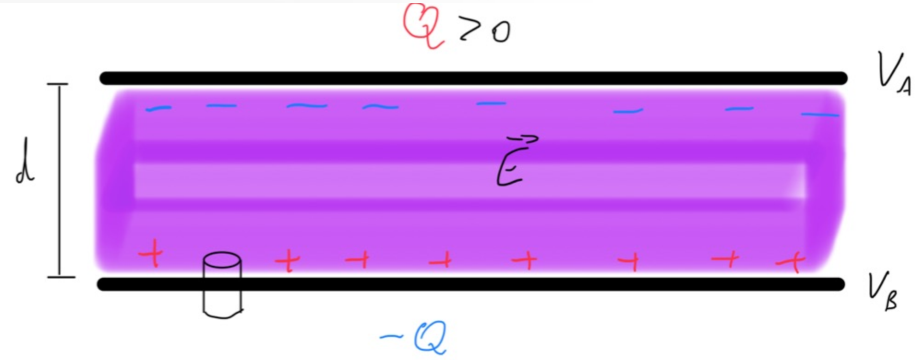
$$\oint \vec{E} \cdot d\vec{a} = \int \frac{\sigma}{\epsilon_0} \cdot da - \int \frac{\sigma_p}{\epsilon_0} \, da = \int \frac{\sigma}{\epsilon_0} \cdot da - \int \frac{\vec{P}}{\epsilon_0} \hat{n} \, d\vec{a}$$

$$\therefore \int (\epsilon_0 \vec{E} + \vec{P}) \, d\vec{a} = \int \sigma \, da \Rightarrow \int \vec{D} \, d\vec{a} = \int \sigma \, da$$

$$\Delta V = E \cdot d \quad ; \quad E = \frac{D}{\epsilon} \quad ; \quad D = \frac{Q}{A}$$

$$\Delta V = \frac{d}{\epsilon A} \cdot Q = \frac{Q}{C_\epsilon} \Rightarrow C_\epsilon = \epsilon \frac{A}{d}$$

Para a mesma carga: $\Delta V \propto 1/C$



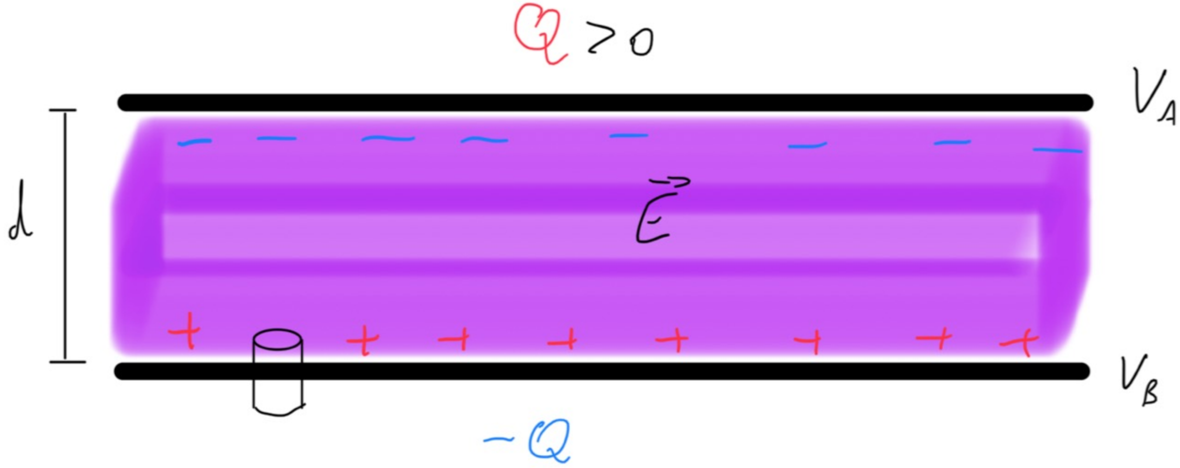
Qual a energia armazenada?

$$\mathcal{E} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,dv = \frac{\epsilon}{2} d\cdot A\cdot E^2 = \frac{\epsilon\, d\cdot A}{2}\cdot\left(\frac{\Delta V}{d}\right)^2$$

$$\mathcal{E} = \frac{\epsilon A}{d}\frac{\Delta V^2}{2} = \frac{C\Delta V^2}{2} \quad ; \quad C = \frac{Q}{\Delta V}$$

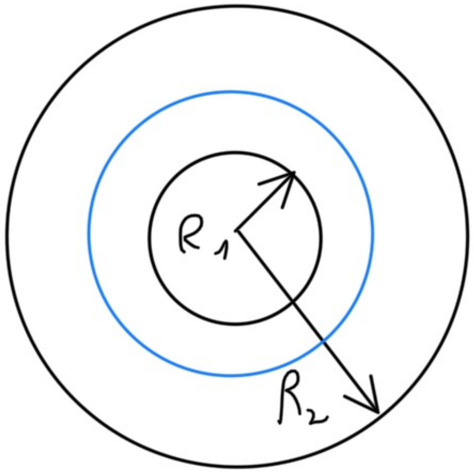
$$= \frac{Q\cdot\Delta V}{2} = \frac{1}{2}\int V\cdot\rho\,dv$$

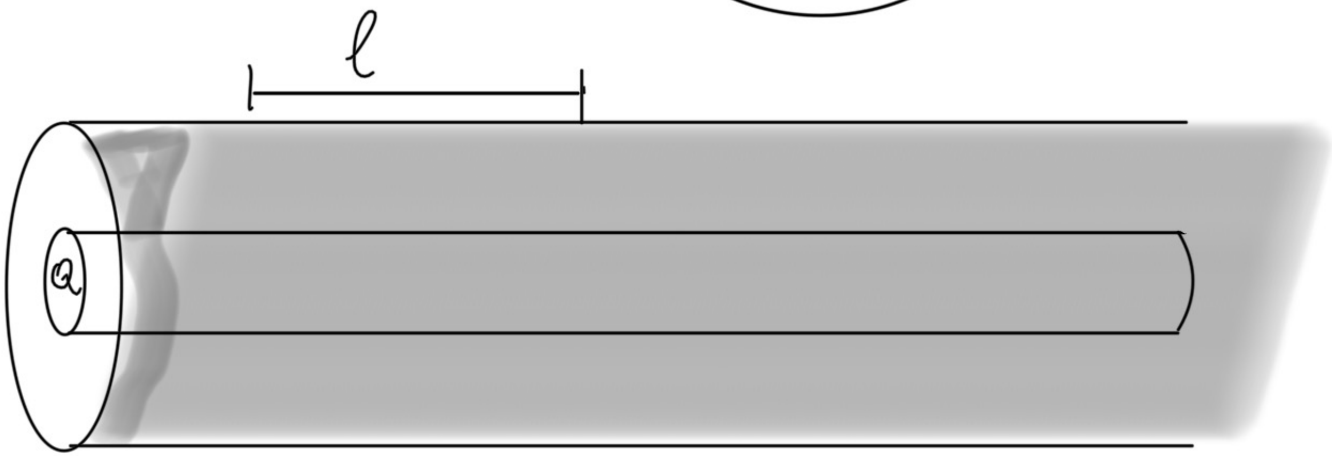
$$\mathcal{E} = \frac{Q^2}{2C}$$



Explorando geometrias

Tubular:





Superfície gaussiana: cilindro de raio r

$$\int \vec{D} \cdot d\vec{a} = D \cdot 2\pi r \cdot \ell = Q$$

$$\int \vec{D} \cdot d\vec{a} = D \cdot 2\pi r \cdot l = Q$$

$$E = \frac{D}{\epsilon} = \frac{Q}{2\pi r l \cdot \epsilon} \longrightarrow \Delta V = -\int_{R_2}^{R_1} \vec{E} \cdot d\vec{r}$$

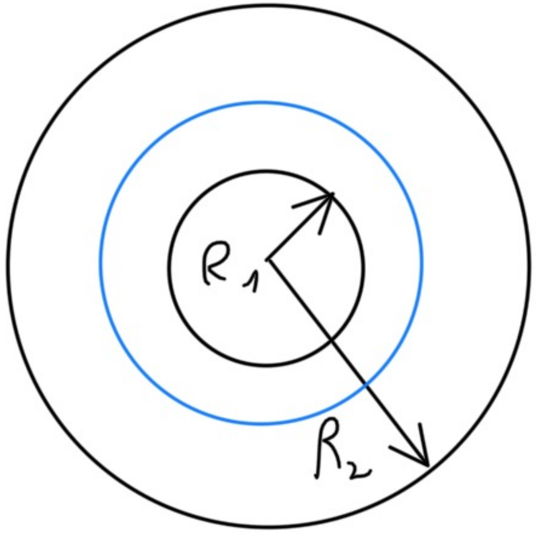
$$\Delta V = \frac{-Q}{2\pi l \epsilon} \int_{R_2}^{R_1} \frac{1}{r}\, dr = \frac{-Q}{2\pi l \epsilon} \left[\ln r \right]_{R_2}^{R_1}$$

$$= \frac{Q}{2\pi l \epsilon} \left[\ln R_2 - \ln R_1 \right]$$

$$\Delta V = V_{R_1} - V_{R_2} = \frac{Q}{2\pi l \epsilon} \ln\left(\frac{R_2}{R_1}\right) \Longrightarrow \frac{C}{l} = \frac{2\pi \epsilon}{\ln\left(\frac{R_2}{R_1}\right)}$$

Capacitor esférico

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{r} \; ; \; R_1 < r < R_2$$



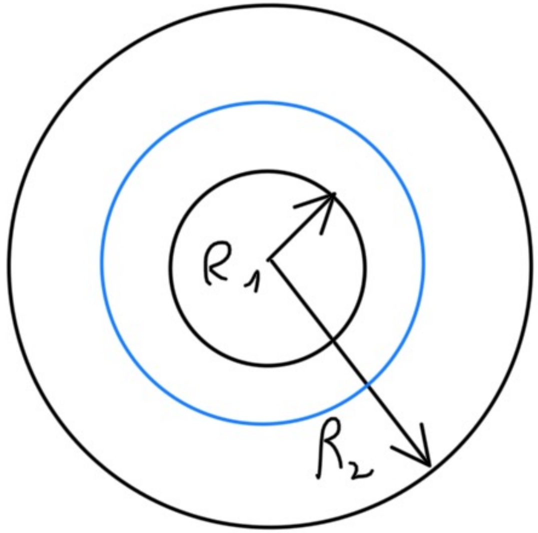
$$\Delta V = V_{R_1} - V_{R_2} = -\int_{R_2}^{R_1} \vec{E}\cdot d\vec{l} = \int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon}\cdot\frac{1}{r^2}\cdot\hat{r}\cdot\hat{r}\,dr$$

$$= \frac{Q}{4\pi\varepsilon}\int_{R_1}^{R_2}\frac{1}{r^2}\,dr = \frac{-Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{R_1}^{R_2}$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Capacitor esférico

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{r} \; ; \; R_1 < r < R_2$$

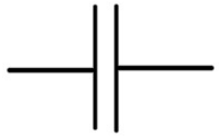


$$C = 4\pi\epsilon \frac{R_1 R_2}{R_2 - R_1}$$

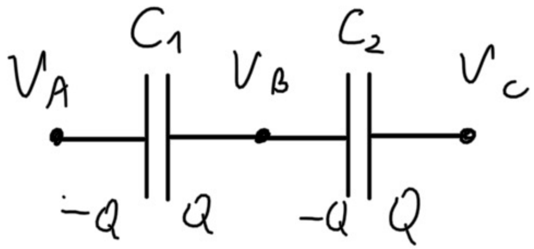
Note: $\mathcal{E} = \dfrac{Q^2}{2C} = \dfrac{Q^2}{8\pi\epsilon}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

No vácuo, se $\lim\limits_{R_2 \to \infty} \mathcal{E} = \dfrac{Q^2}{8\pi\epsilon_0 R_1}$ → Energia de uma casca esférica carregada!

Associações: Capacitor



Série:



$$V_C - V_B = \frac{Q}{C_2} = V_2 \qquad V_B - V_A = \frac{Q}{C_1} = V_1$$

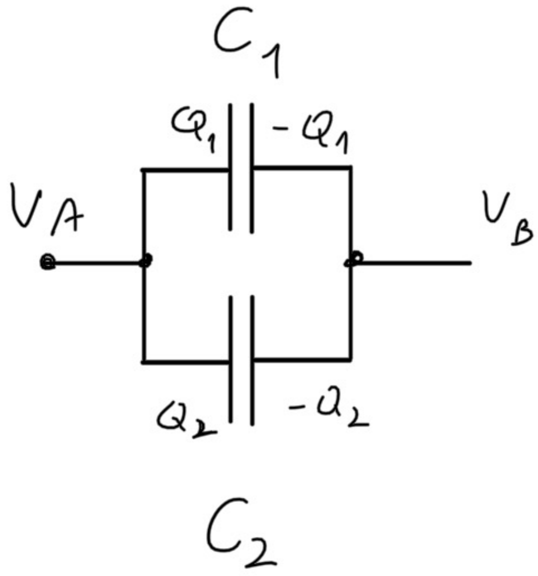
$$\Delta V = V_C - V_A = V_C - V_B + V_B - V_A = Q\left(\frac{1}{C_2} + \frac{1}{C_1}\right) = \frac{Q}{C_{eq}}$$

$$C_{eq} = \frac{C_2 C_1}{C_1 + C_2}$$

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2} \implies V_1 \cdot C_1 = V_2 \cdot C_2$$

Paralelo:

$$V_A - V_B = \frac{Q_1}{C_1} = \Delta V$$



$$V_A - V_B = \frac{Q_2}{C_2} = \Delta V$$

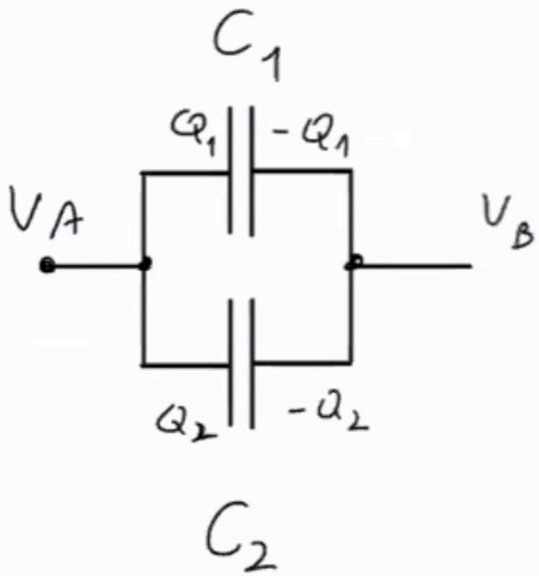
$$Q = Q_1 + Q_2 = C_1 \cdot \Delta V + C_2 \Delta V = \Delta V \cdot (C_1 + C_2)$$

$$C_{eq} = C_1 + C_2$$

$$V_1 = V_2 \qquad \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

Paralelo:

$$V_A - V_B = \frac{Q_1}{C_1} = \Delta V$$



$$V_A - V_B = \frac{Q_2}{C_2} = \Delta V$$

$$Q = Q_1 + Q_2 = C_1 \cdot \Delta V + C_2 \Delta V = \Delta V \cdot (C_1 + C_2)$$

$$C_{eq} = C_1 + C_2$$

$$V_1 = V_2 \qquad \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$V_1 = V_A - V_B = V_2$$

$\hookrightarrow C_1$ $\qquad$ $\hookrightarrow C_2$

Qual o trabalho da inserção de um dielétrico?