

Eletromagnetismo

Campo Elétrico na Matéria

- As equações descritas são válidas sempre.
- Em meios materiais, precisamos lidar com a distribuição local de cargas, inclusive com aquelas *induzidas* ou *rearranjadas* pelo próprio campo!
- Neste caso, podemos combinar a resposta do meio ao campo aplicado.
- Veremos o caso de cargas livres (condutores) e meios dielétricos.
- Começando pelos condutores: cargas livres para se deslocarem!

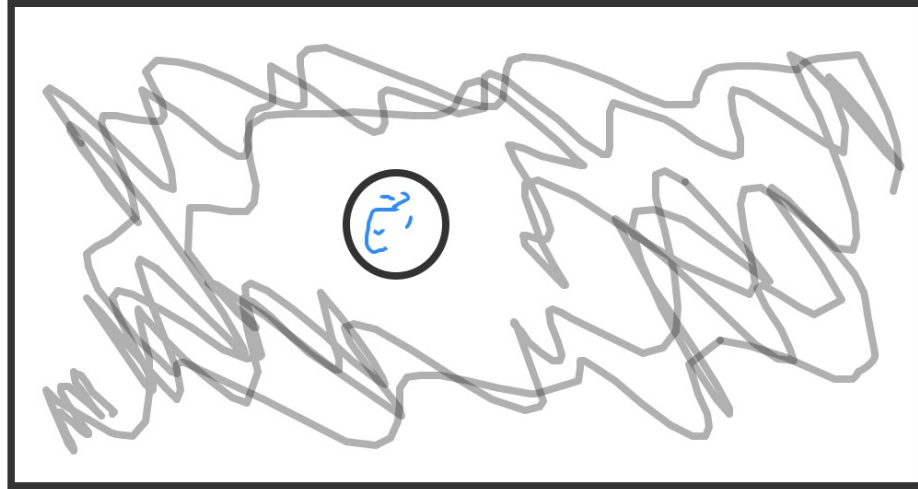
\vec{E}



$$|E'| < E \quad (a)$$

$$|E'| = 0 \quad (b)$$

$$|E'| > E \quad (c)$$



\vec{E}



Qual o campo no caso de um material condutor?
E no caso de um meio isolante?

Condutor \rightarrow condição estática \Rightarrow equilíbrio

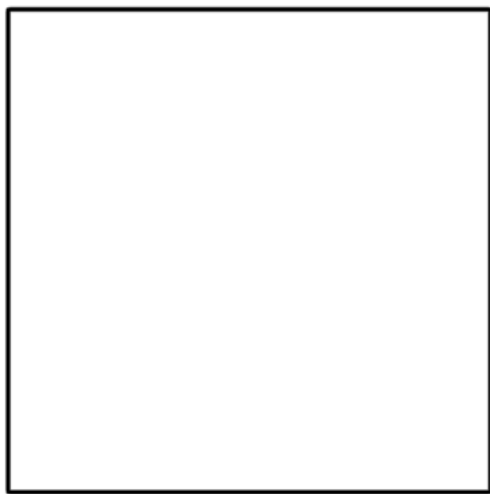
Cargas livres: elétrons em metais

Em líquidos: íons negativos (ânions) e positivos (cátions)

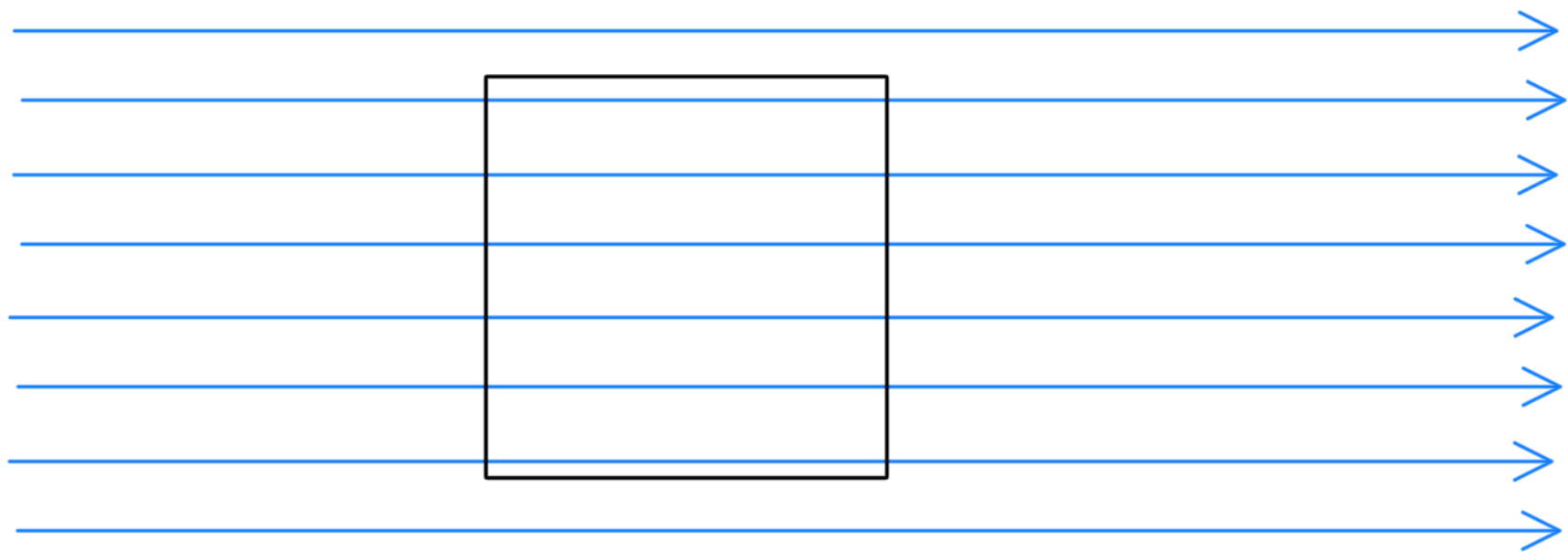
Em quantidade ilimitada para todos propósitos práticos

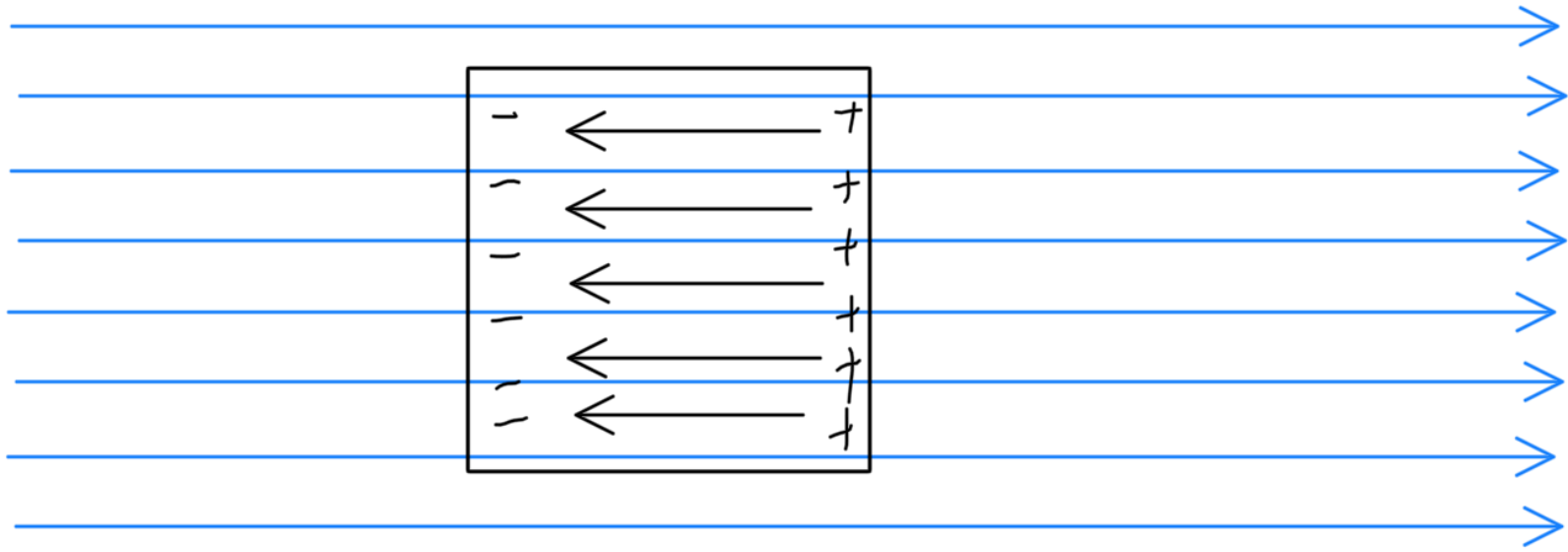
Implicações: $E = 0 \rightarrow$

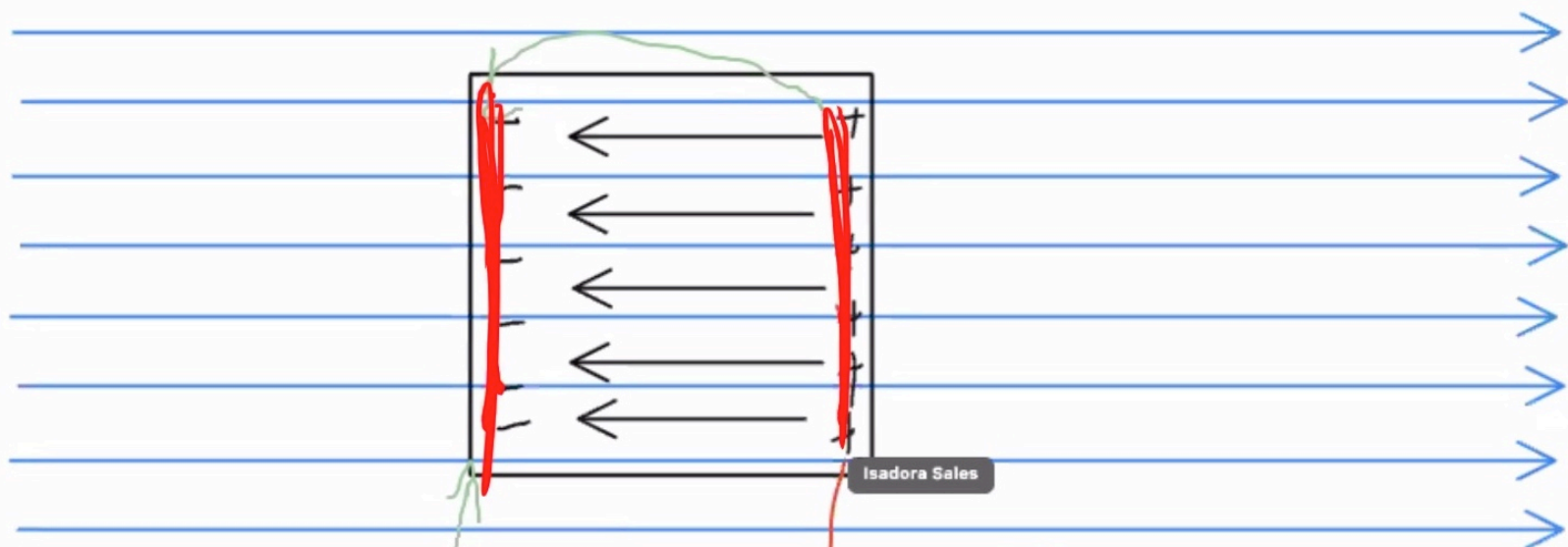
se houvesse E , haveria rearranjo de cargas









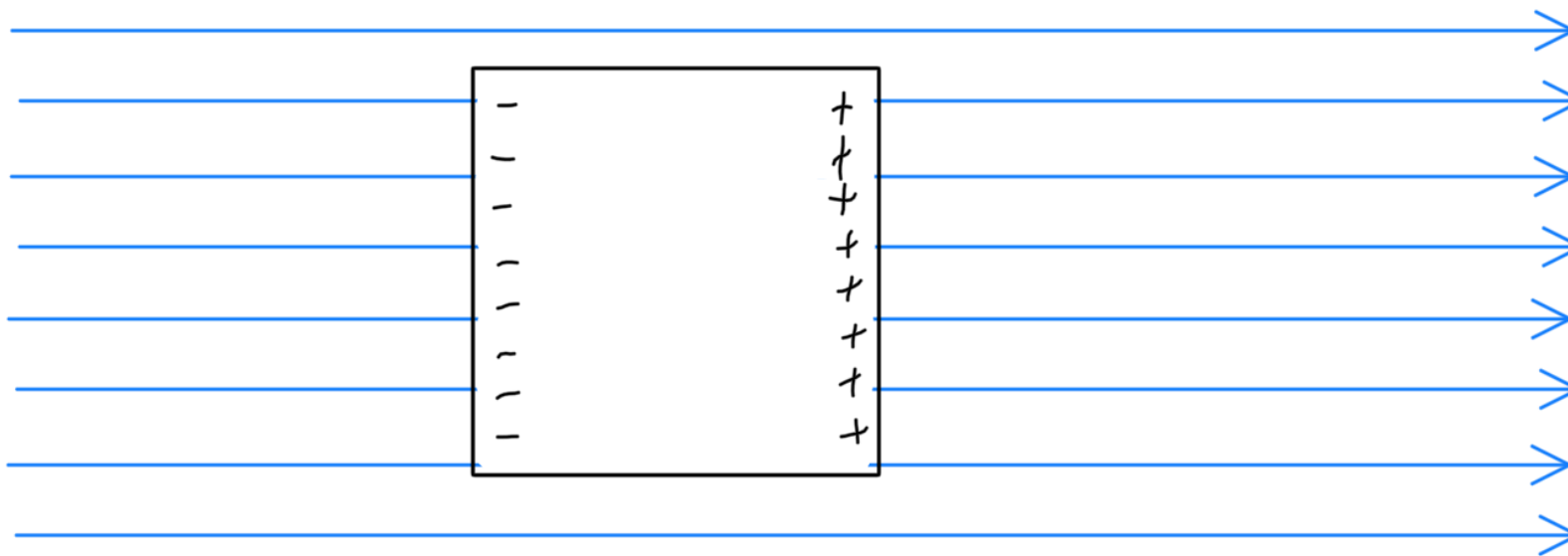


Isadora Sales

→ íons positivos



$$\vec{F} = -e \vec{E}$$

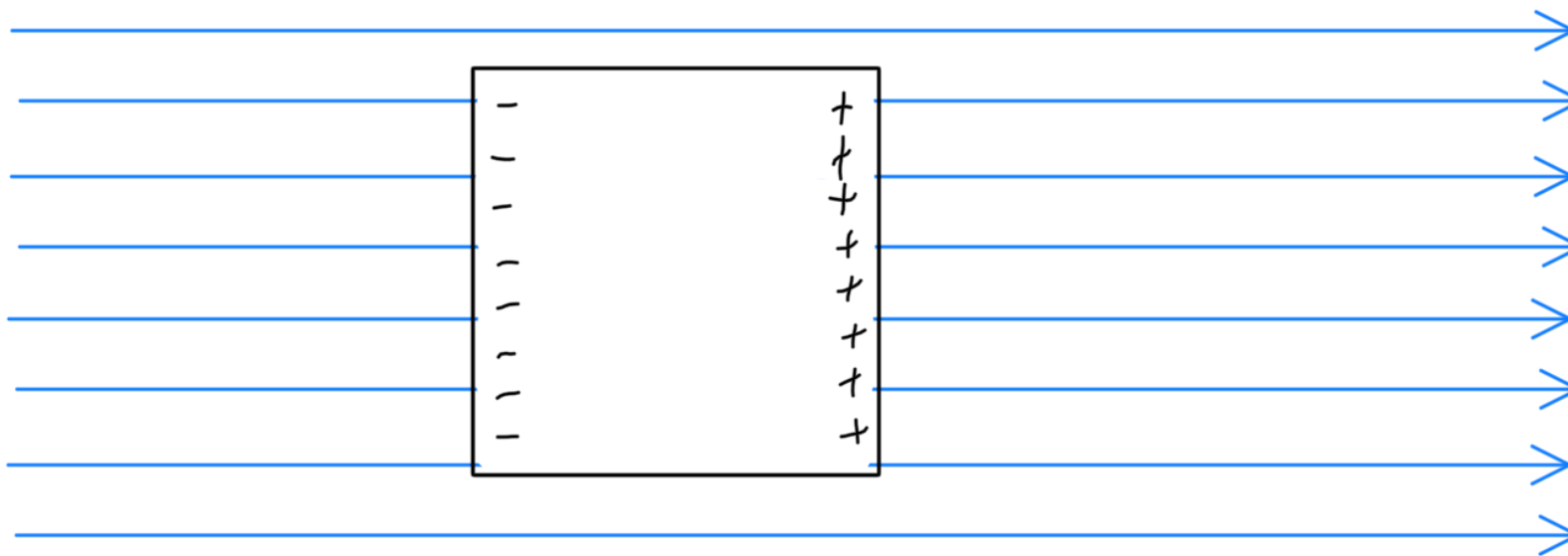


$$\Rightarrow E = 0 \rightarrow \nabla \cdot E = 0 \Rightarrow \rho = 0$$

→ "sem carga líquida" no volume

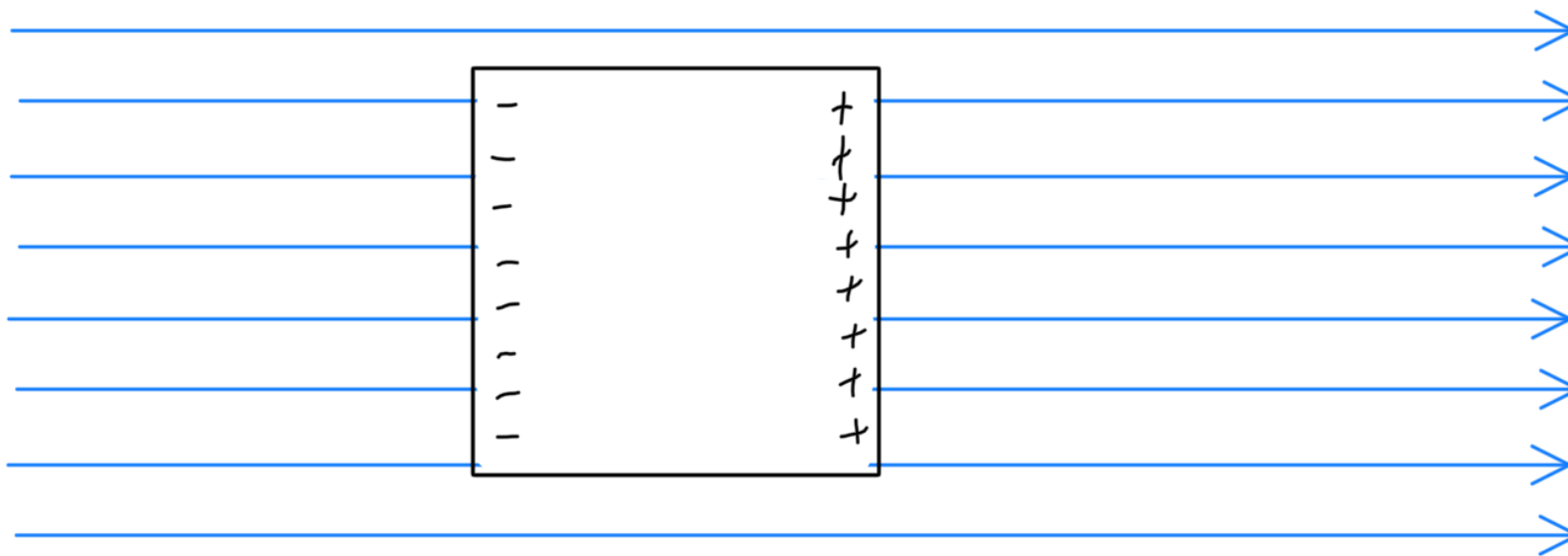
existem cargas (elétrons e cátions), mas em balanço

→ carga total nula



$$\Rightarrow \text{Se } E=0 \quad V(b)-V(a)=-\int_a^b \vec{E} d\vec{l} = 0$$

Mesmo potencial



⇒ Toda carga está na superfície

(ok, em uma camada bem fina)

⇒ Isto minimiza a energia eletrostática

\Rightarrow Isto minimiza a energia eletrostática

$$\rho = \text{cte}$$

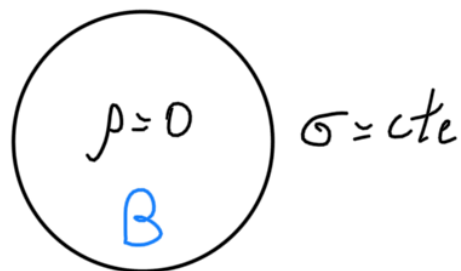
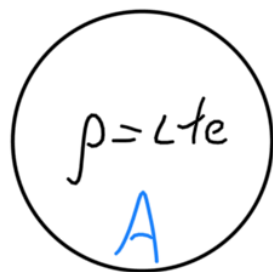
A

$$\rho = 0$$

B

$$\sigma = \text{cte}$$

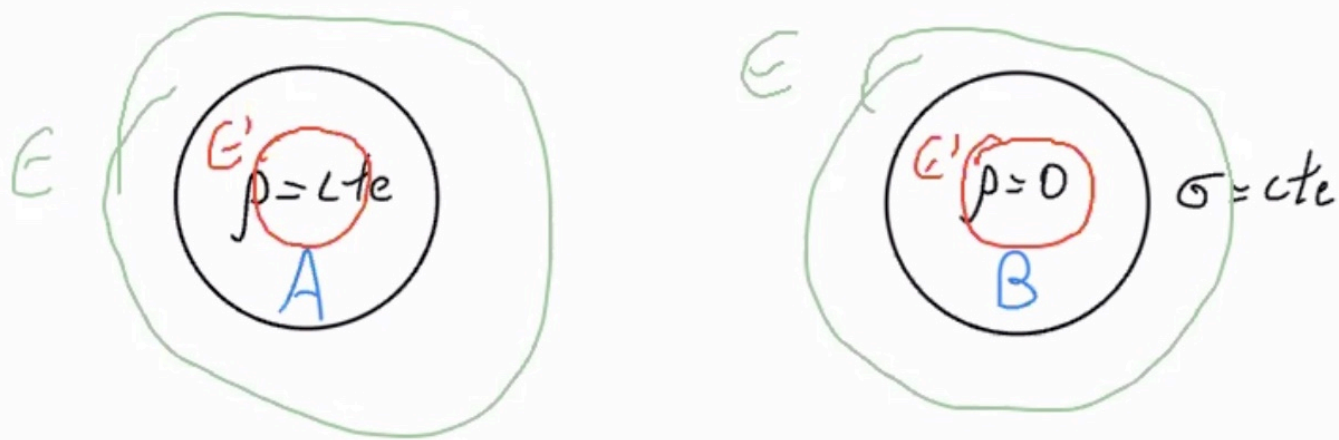
\Rightarrow Isto minimiza a energia eletrostática



Mesma carga, energia $E = \frac{1}{2} \epsilon_0 \int E^2 dV$ distinto

$$E_A > E_B$$

⇒ Isto minimiza a energia eletrostática



Mesma carga, energia $E = \frac{1}{2} \epsilon_0 \int E^2 dv$ distinto

$$E_A > E_B$$

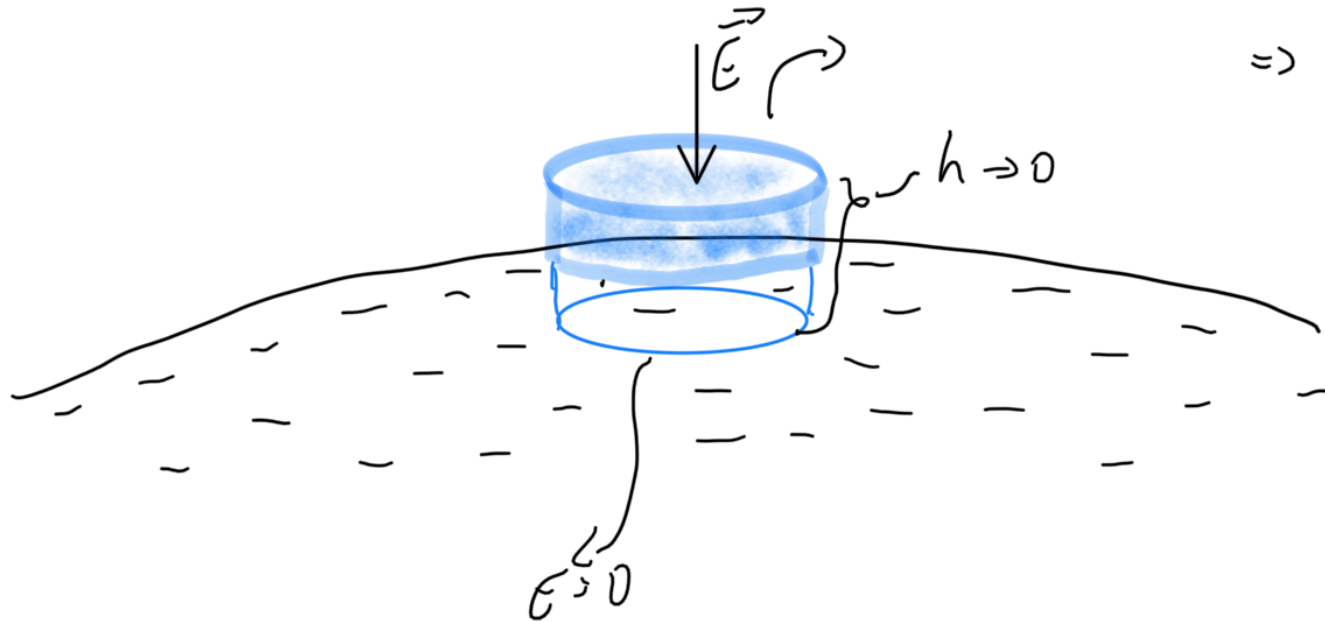
$$\int \vec{E} \cdot d\vec{\sigma} = 0$$

↳ E' em B

\Rightarrow Campo na superfície

$$\int_S \vec{E} \cdot d\vec{\alpha} = \int_S \sigma \cdot d\alpha / \epsilon_0$$

$$\Rightarrow E_n = \frac{\sigma}{\epsilon_0}$$



$$\vec{E} = E_n \hat{n} + E_p \hat{p} \rightarrow$$

se $E_p \neq 0 \rightarrow$ força sobre as cargas

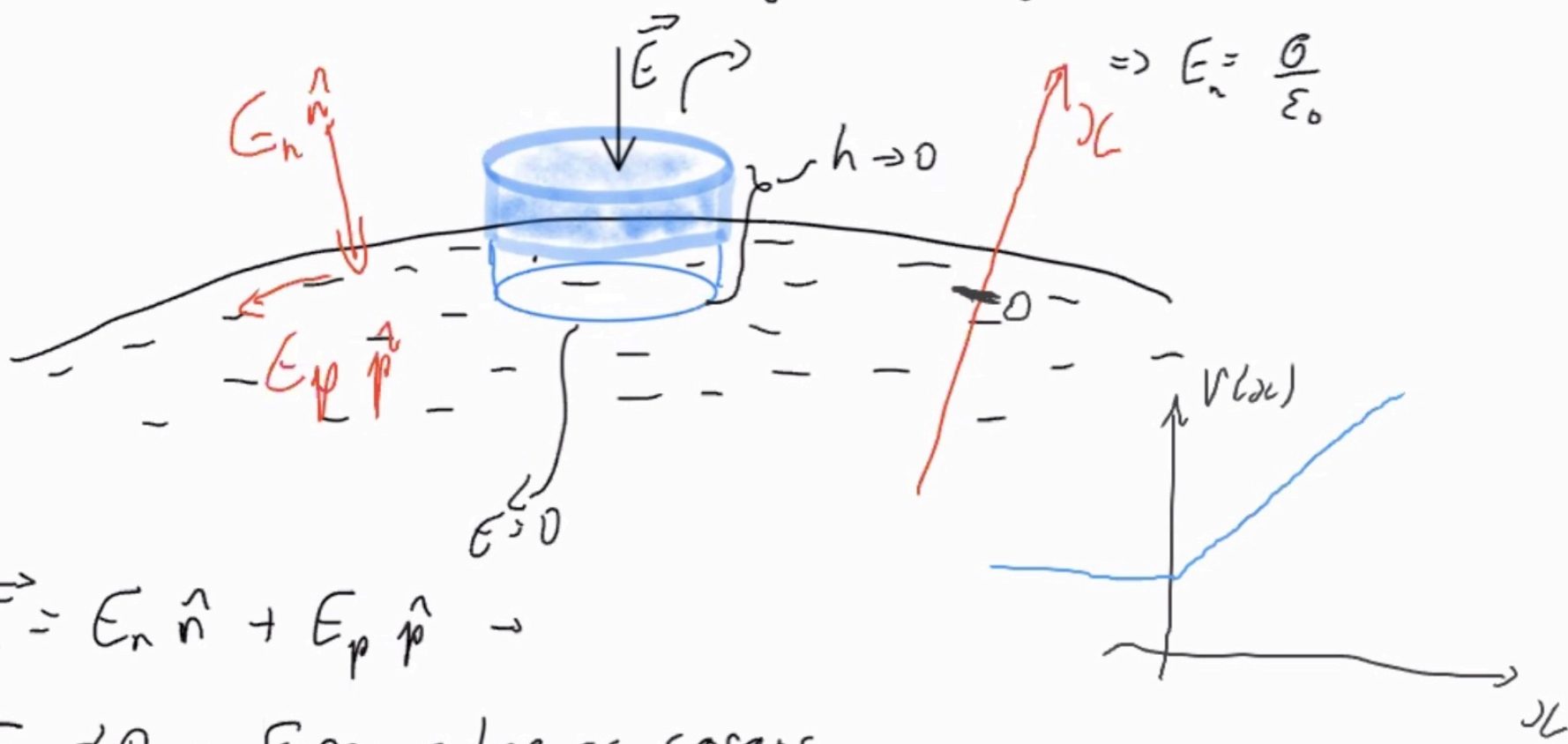
\rightarrow fora do equilíbrio! $\Rightarrow E_p = 0$

$$\text{Como } \vec{E} = -\nabla \cdot V \Rightarrow \sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

\Rightarrow Campo na superfície

$$\int_S \vec{E} \cdot d\vec{\alpha} = \int_S \sigma \cdot d\alpha / \epsilon_0$$

$$\Rightarrow E_n = \frac{\sigma}{\epsilon_0}$$



$$\vec{E} = E_n \hat{n} + E_p \hat{p} \rightarrow$$

se $E_p \neq 0 \rightarrow$ força sobre as cargas

\rightarrow fora do equilíbrio! $\Rightarrow E_p = 0$

Como $\vec{E} = -\nabla V \Rightarrow \sigma = -\epsilon_0 \frac{\partial V}{\partial n}$

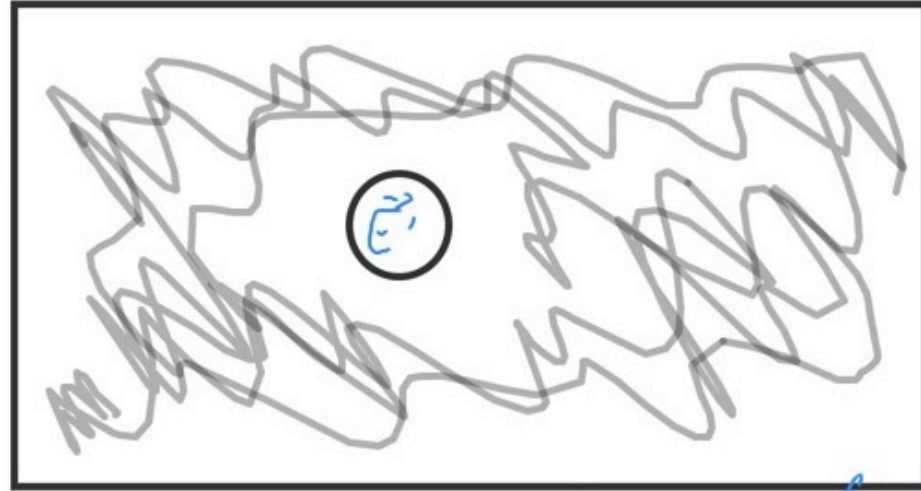
\vec{E}



$$|E'| < E \quad (a)$$

$$\underline{|E'| = 0 \quad (b)}$$

$$|E'| > E \quad (c)$$



metálica
(condutora)

Blindagem / Gaiola de Faraday

\vec{E}

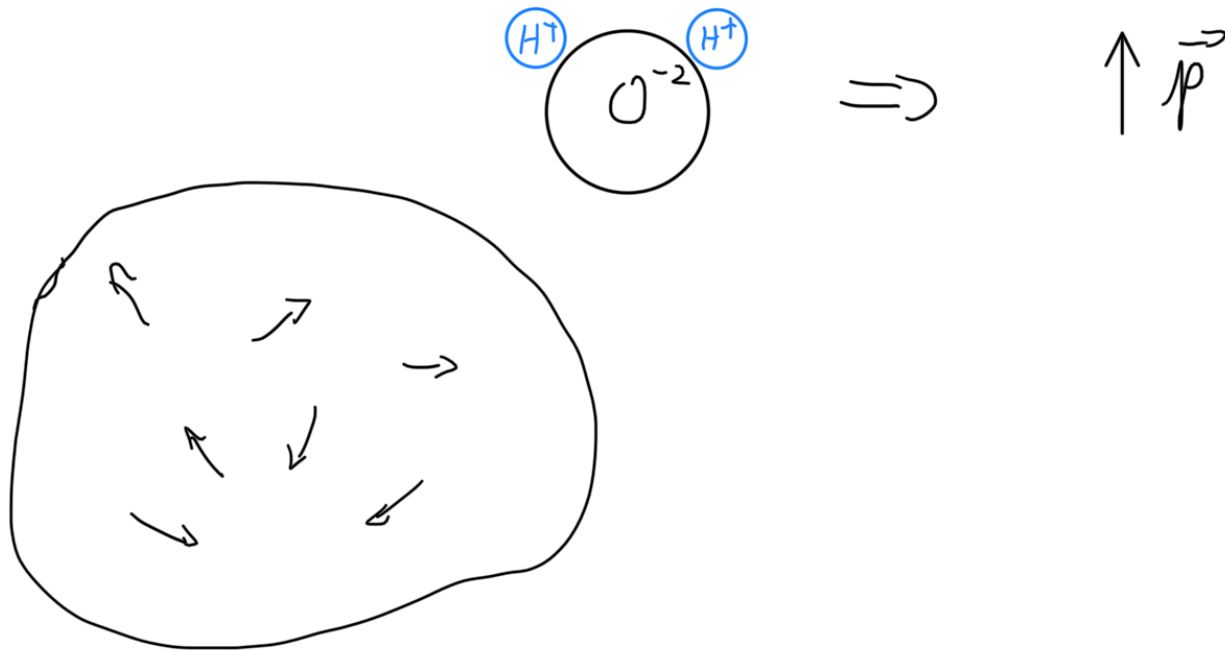


Aula 7: Meios Dielétricos

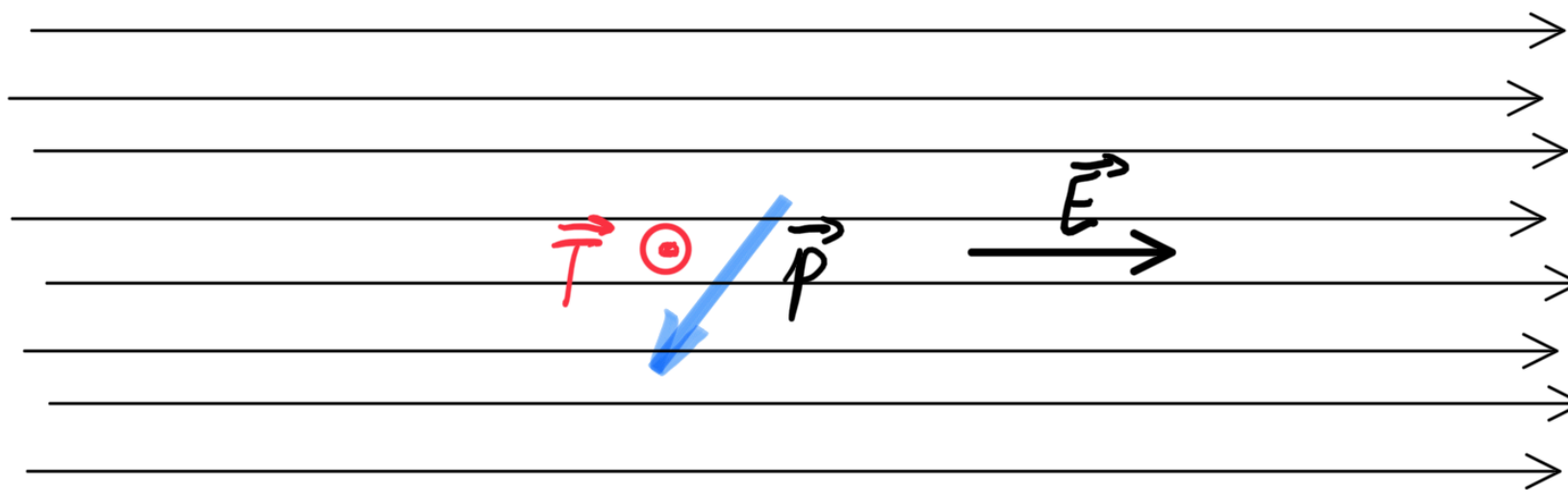
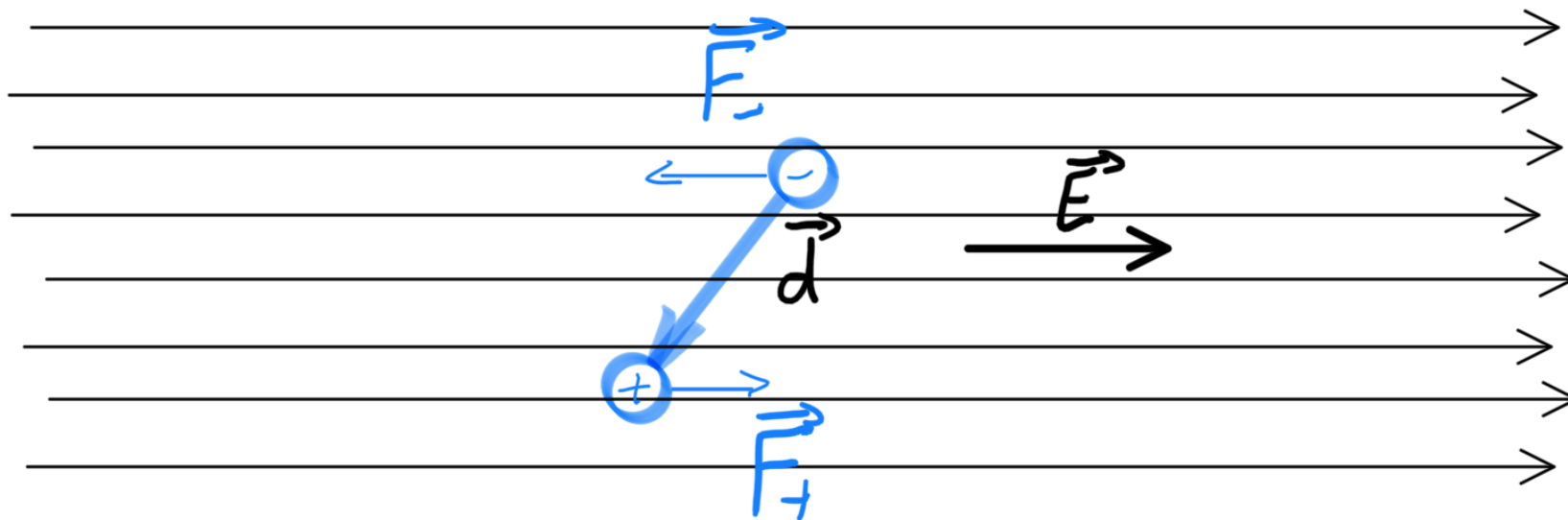
Sem cargas livres: Dipolos

Intrínsecos Induzidos

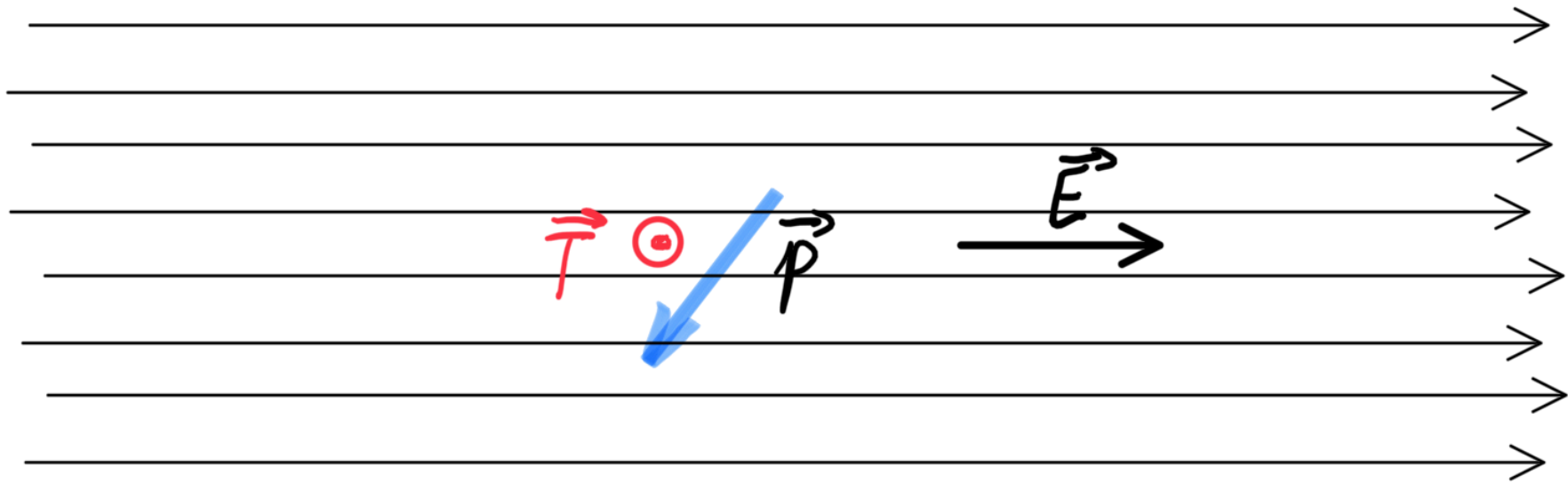
Dipolos Intrínsecos: p.ex. fluido \rightarrow molécula polar



Como vimos: Campo externo leva a um torque



$$\vec{T} = \vec{p} \times \vec{E}$$



$$\vec{T} = \vec{p} \times \vec{E}$$

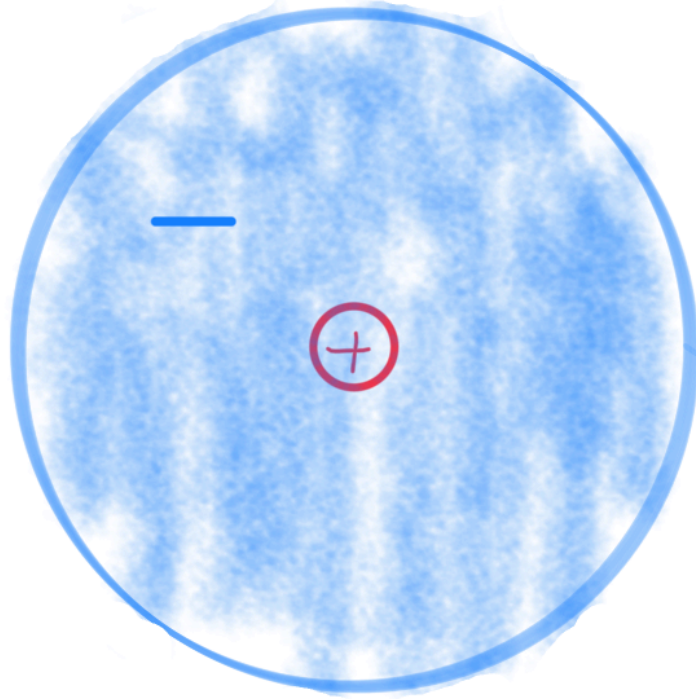
Condição de equilíbrio estável $\vec{p} \propto \vec{E}$ ($\vec{T} = 0$)

→ O dipolo se alinha ao campo

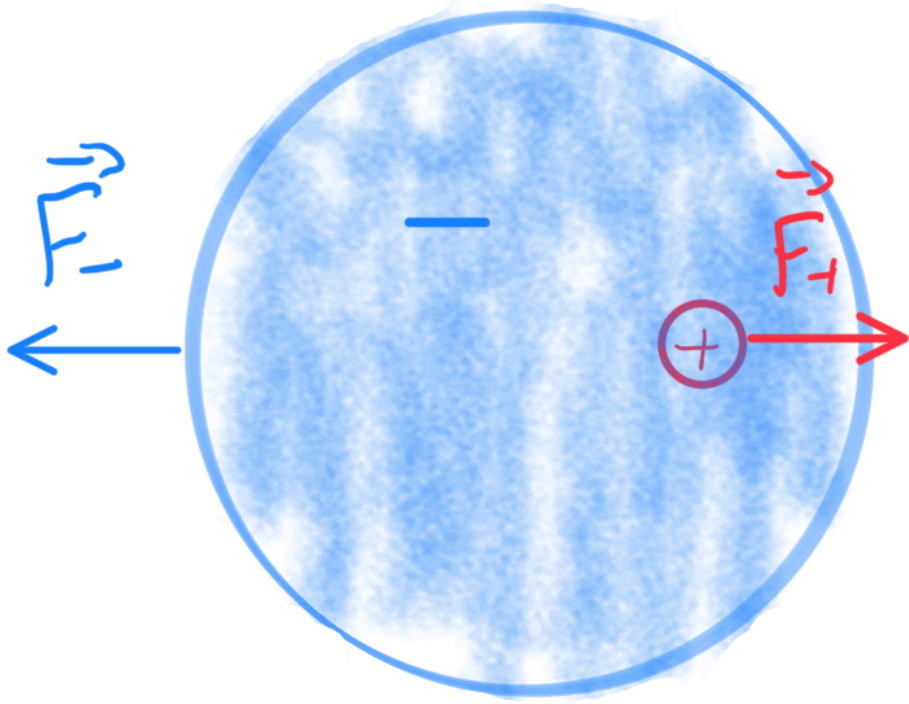
→ Alinhamento limitado pela agitação térmica

Maior agitação → menor alinhamento

Dipolos induzidos

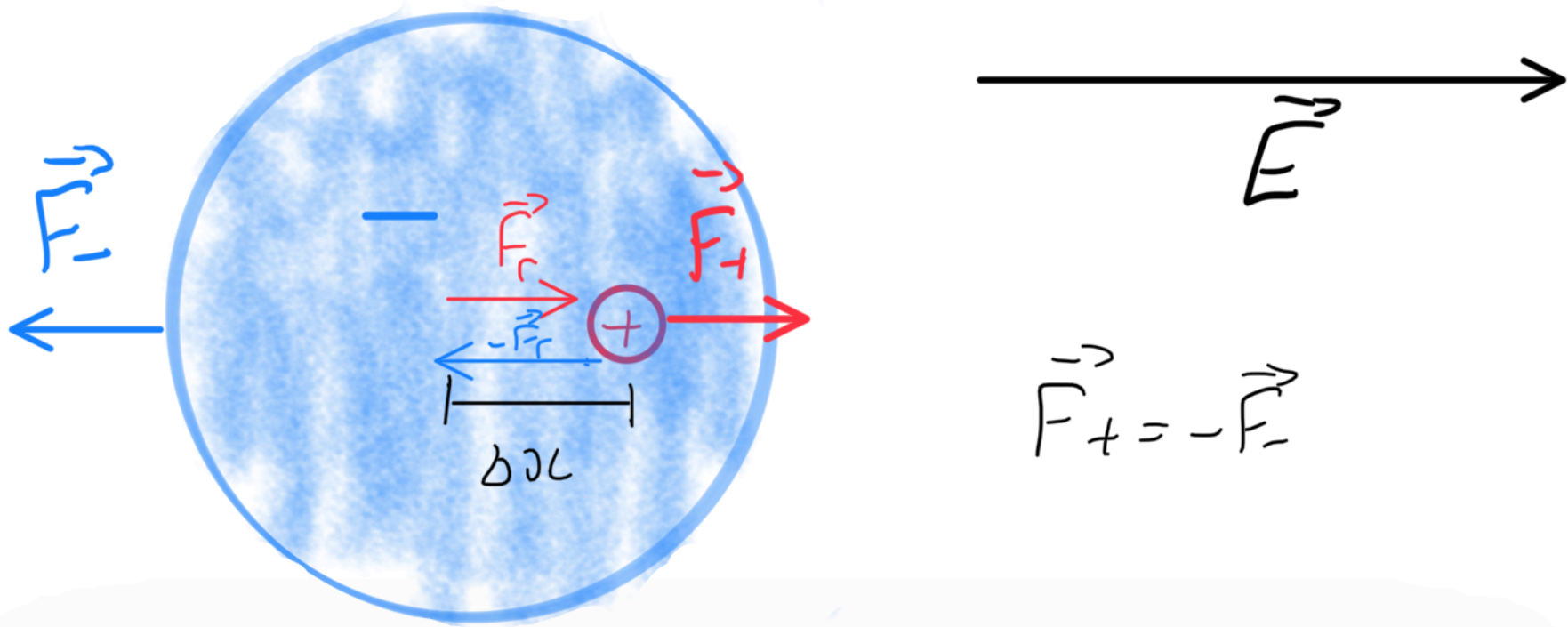


Dipolos induzidos



$$\vec{F}_+ = -\vec{F}_-$$

Dipolos induzidos



Limite linear: novo equilíbrio $\vec{F}_+ = -\vec{F}_r$

$$\vec{F}_- = \vec{F}_r$$

onde $\vec{F}_r \rightarrow$ força de restauração entre cargas + e -

Limite linear: novo equilíbrio $\vec{F}_+ = -\vec{F}_r$
 $\vec{F}_- = \vec{F}_r$

onde $\vec{F}_r \rightarrow$ força de restauração entre cargas $+$ e $-$

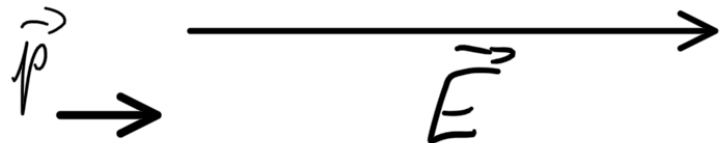
Expandindo em série de Taylor

$$F_r = -k \Delta x + \alpha \Delta x^2 + \beta \Delta x^3 + \dots$$

e mantendo somente o termo linear

$$\Delta x = \frac{q \cdot E}{k} \rightarrow p \propto \Delta x$$

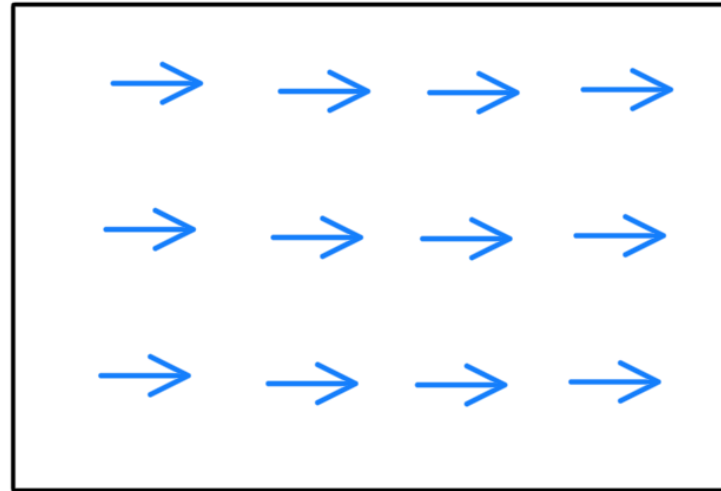
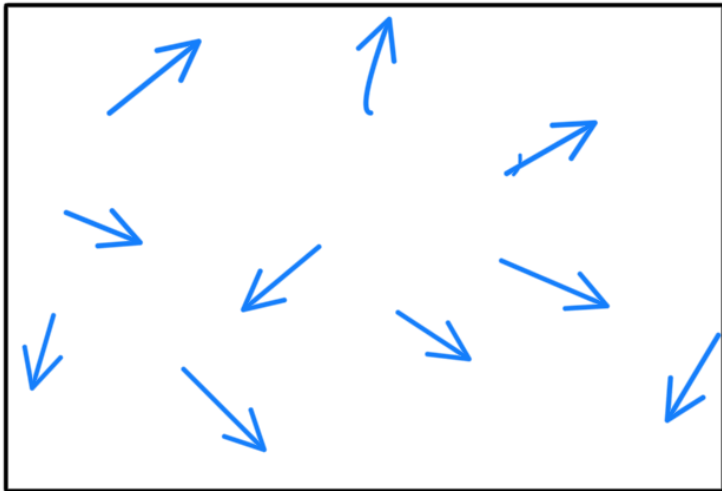
Dipolo induzido $\Rightarrow \vec{p} \propto \vec{E}$ (isotrópico)

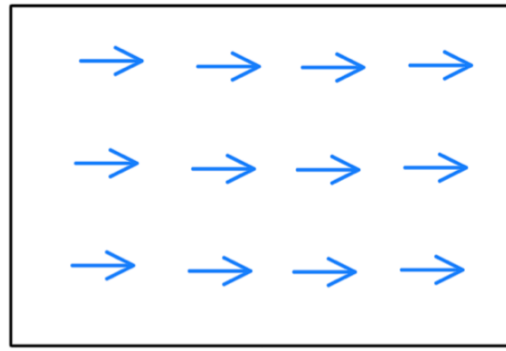
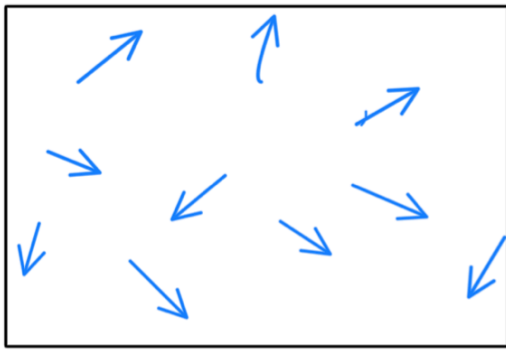


Polarização: Densidade de dipolos

$$\vec{P} = \frac{\sum \vec{p}_i}{V}$$

$$[P] = C/m^2$$





Potencial produzido: pelo dipolo médio

$$d\vec{p} = \vec{p} \cdot dV$$

↳ elementos
de volume

pelos cargas: $dq = \rho(\vec{r}') dV$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{|\vec{r} - \vec{r}'|} + \frac{1}{4\pi\epsilon_0} \frac{d\vec{p} \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad \left/ \begin{array}{l} dV = dV(\vec{r}, \vec{r}') \\ \vec{r} \text{ posição no espaço} \\ \vec{r}' \text{ pos. da Fonte de potencial} \end{array} \right.$$

$$V(\vec{r}) = \int dV = \frac{1}{4\pi\epsilon_0} \int \left[\frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} + \frac{\vec{P}(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right] dV$$

$$V(\vec{r}) = \int dV = \frac{1}{4\pi\epsilon_0} \int \left[\frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} + \frac{\vec{P}(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right] dV'$$

Como vimos: $\nabla \left[\frac{1}{|\vec{r} - \vec{r}'|} \right] = - \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$ derivando em \vec{r}

$\nabla' \left[\frac{1}{|\vec{r} - \vec{r}'|} \right] = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$ derivando em \vec{r}'

$$\Rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} + \vec{P}(\vec{r}') \cdot \nabla' \left[\frac{1}{|\vec{r} - \vec{r}'|} \right] dV'$$

$$\Rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} + \vec{P}(\vec{r}') \cdot \nabla' \left[\frac{1}{|\vec{r}-\vec{r}'|} \right] d\tau$$

Vamos nos concentrar aqui!

$$\nabla' \left[\frac{\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right] = \vec{P}(\vec{r}') \cdot \nabla' \left[\frac{1}{|\vec{r}-\vec{r}'|} \right] + \frac{1}{|\vec{r}-\vec{r}'|} \cdot \nabla' \vec{P}(\vec{r}')$$

$$\int_V \nabla' \left[\frac{\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right] \cdot d\tau = \int_V \vec{P}(\vec{r}') \cdot \nabla' \left[\frac{1}{|\vec{r}-\vec{r}'|} \right] d\tau + \int_V \frac{\nabla' \vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\tau$$

$$\Downarrow$$

$$\oint_S \frac{\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{a}'$$

Três termos contribuem para o potencial

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

→ dens. de cargas isoladas

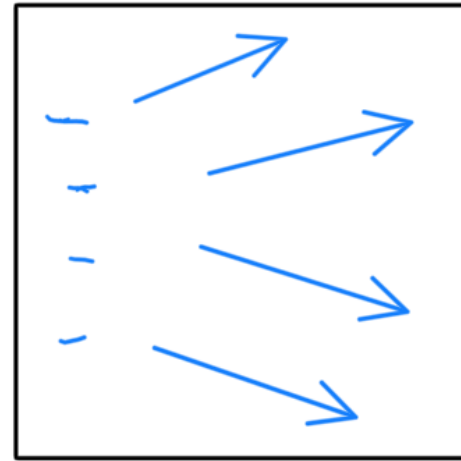
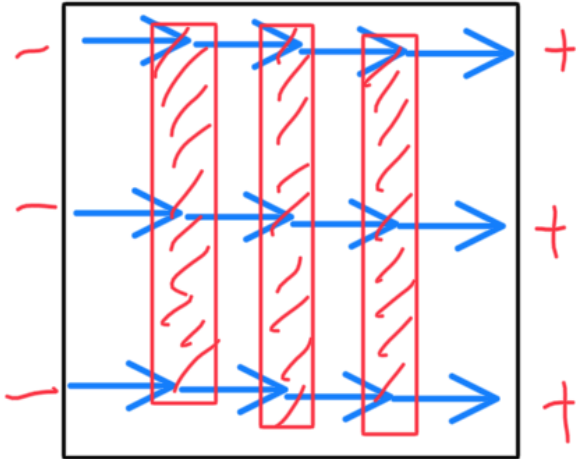
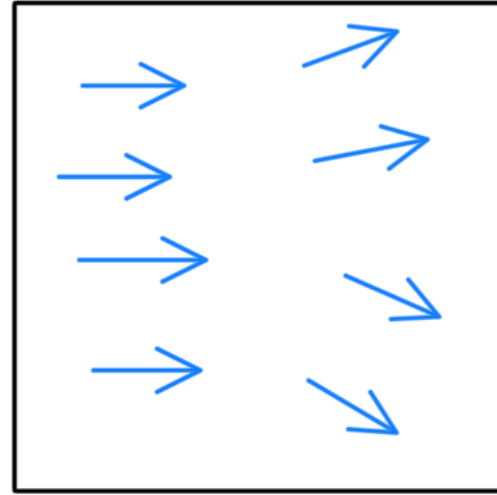
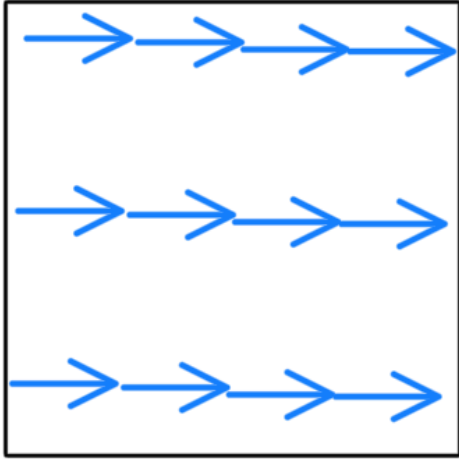
$$+ \frac{1}{4\pi\epsilon_0} \oint_S \frac{\vec{P}(\vec{r}') \cdot \hat{n}}{|\vec{r} - \vec{r}'|} da$$

→ carga de superfície do dipolo σ_p

$$- \frac{1}{4\pi\epsilon_0} \int_V \frac{\nabla' \cdot \vec{P}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

→ carga volumétrica de gradiente de dipolo ρ_p

$$\sigma_p = \vec{p} \cdot \vec{n} \quad ; \quad \rho_p = -\nabla \cdot \vec{p}$$



Três termos contribuem para o potencial

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV \quad \rightarrow \text{dens. de cargas isoladas}$$

$$+ \frac{1}{4\pi\epsilon_0} \oint_S \frac{\vec{P}(\vec{r}') \cdot \hat{n}}{|\vec{r} - \vec{r}'|} da \quad \rightarrow \text{carga de superfície do dipolo } \sigma_p$$

$$- \frac{1}{4\pi\epsilon_0} \int_V \frac{\nabla' \cdot \vec{P}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV \quad \rightarrow \text{carga volumétrica de gradiente de dipolo } \rho_p$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV + \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_p(\vec{r}')}{|\vec{r} - \vec{r}'|} dV + \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_p(\vec{r}')}{|\vec{r} - \vec{r}'|} da$$

Campo dentro do dielétrico:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') - \nabla \cdot \vec{P}}{|\vec{r} - \vec{r}'|} dV$$

Lei de Gauss: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow$ agora modificada pelo dipolo

$$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho - \nabla \cdot \vec{P}}{\epsilon_0} \Rightarrow \rho = \nabla \cdot [\epsilon_0 \vec{E} + \vec{P}]$$

A carga livre não é mais associada ao campo elétrico
mas a um novo campo: deslocamento elétrico \vec{D}

Campo dentro do dielétrico:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') - \nabla \cdot \vec{P}}{|\vec{r} - \vec{r}'|} dV$$

Lei de Gauss: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow$ agora modificada pelo dipolo

$$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho - \nabla \cdot \vec{P}}{\epsilon_0} \Rightarrow \rho = \nabla \cdot [\epsilon_0 \vec{E} + \vec{P}]$$

afetado
por \vec{P}

A carga livre não é mais associada ao campo elétrico
mas a um novo campo: deslocamento elétrico \vec{D}



$$\nabla \cdot \vec{D} = \rho \quad ; \quad \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

↳ forma geral da lei de Gauss, associada a
cargas livres

Se $\vec{P} = 0 \Rightarrow \vec{D} = \epsilon_0 \vec{E}$ (permissividade do vácuo)

Se $\vec{P} \propto \vec{E}$, $\vec{P} = \epsilon_0 \chi \vec{E}$ ($\chi \rightarrow$ susceptibilidade elétrica)

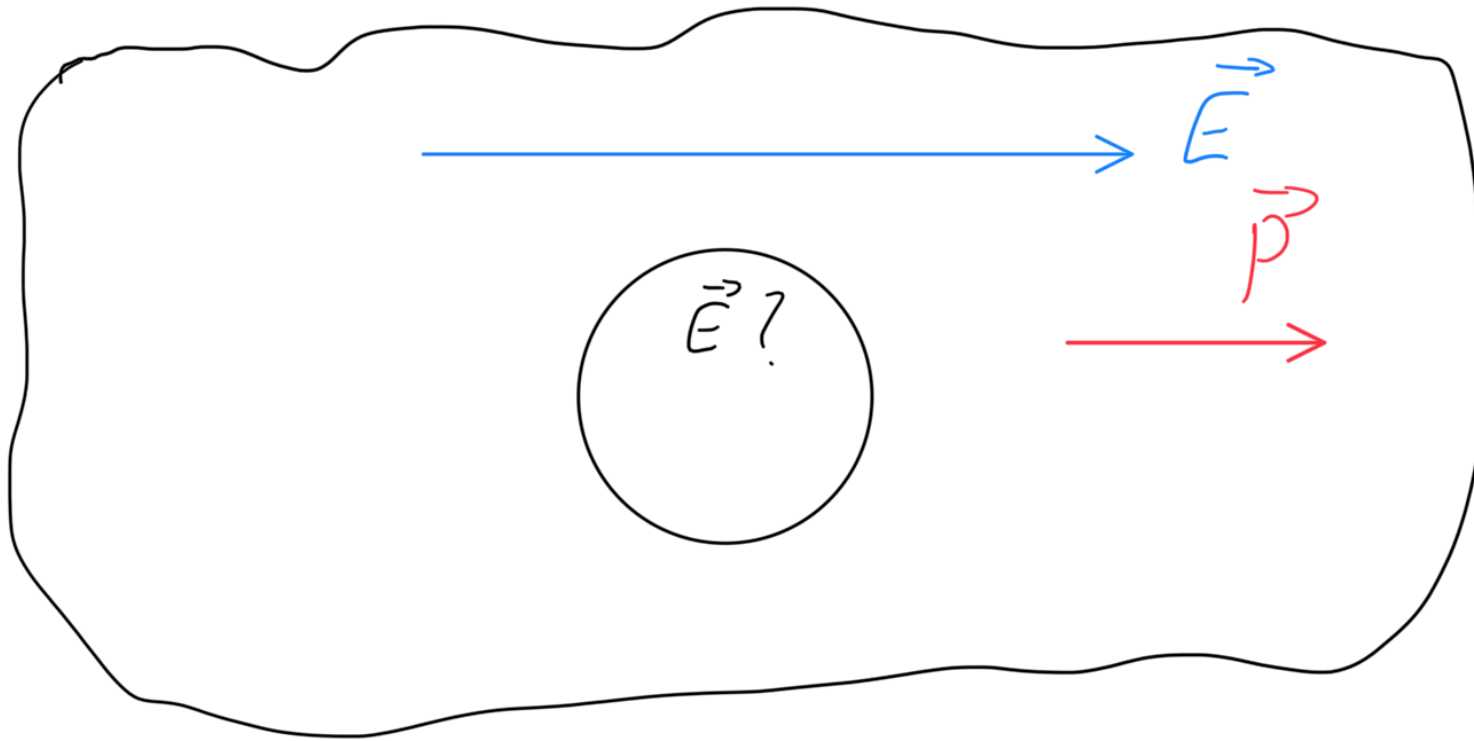
$$\vec{D} = \epsilon_0 (\chi + 1) \vec{E} = \epsilon \vec{E}$$

↳ permissividade elétrica

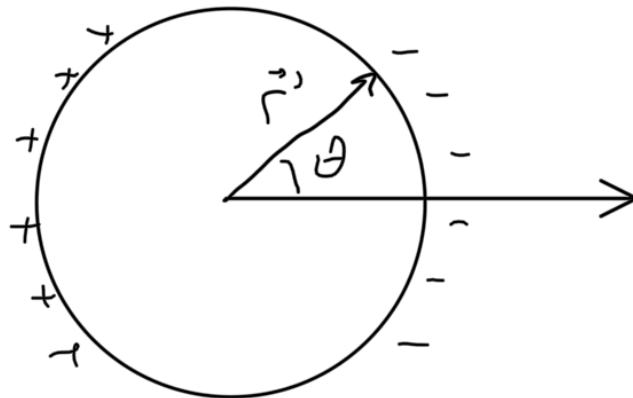
$$\nabla \cdot \vec{E} = \frac{\nabla \cdot \vec{D}}{\epsilon} = \frac{\rho}{\epsilon} \quad , \quad \nabla^2 V = -\frac{\rho}{\epsilon}$$

As relações permanecem válidas localmente

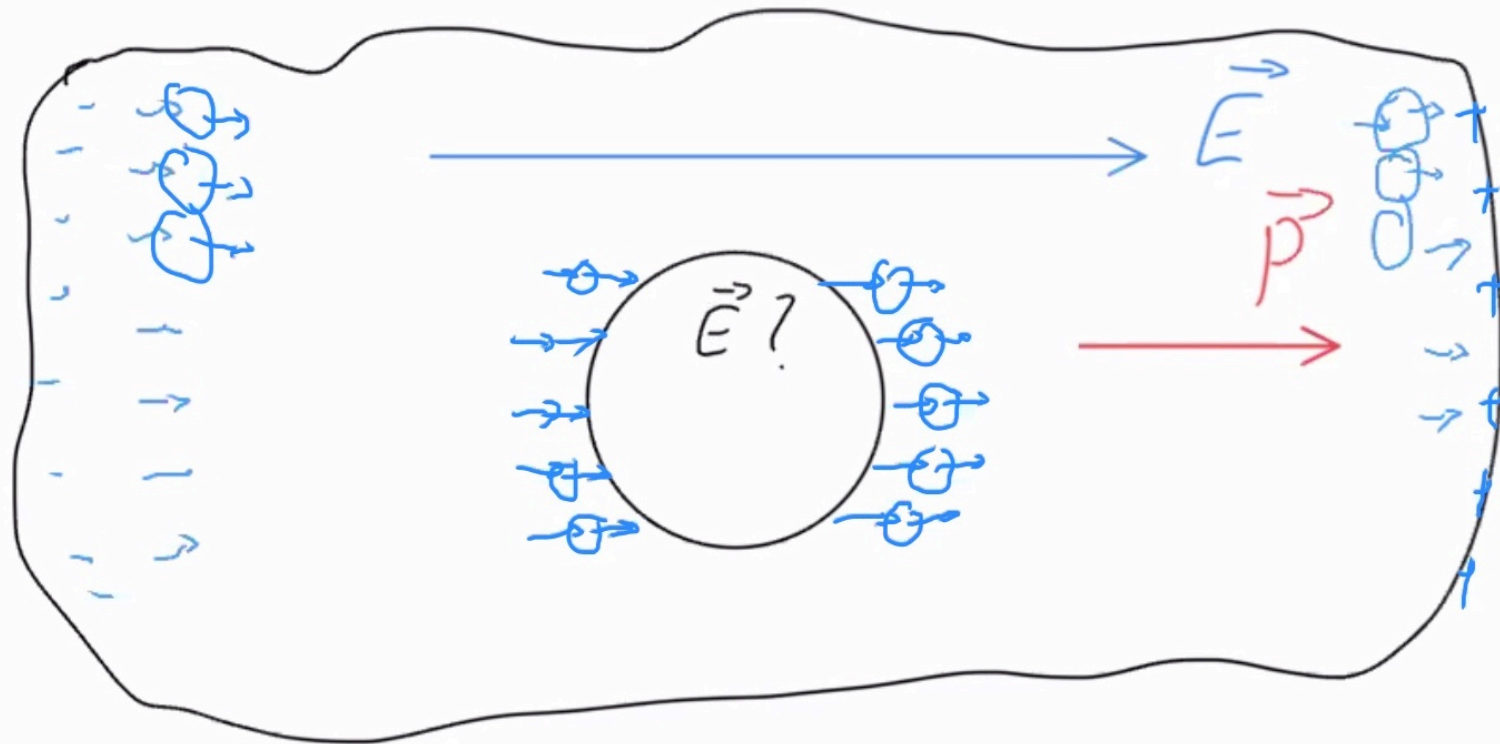
Cavidade dentro do dielétrico



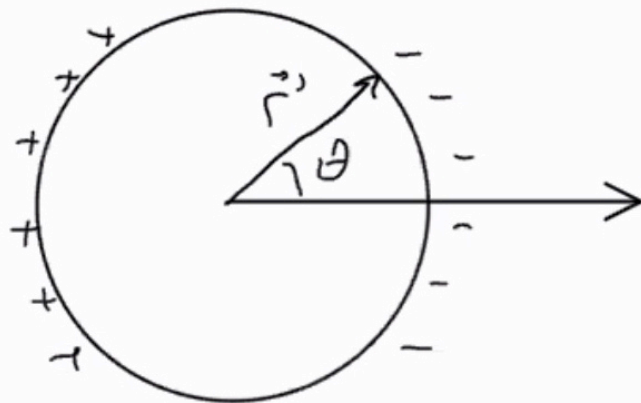
Densidade superficial de cargas



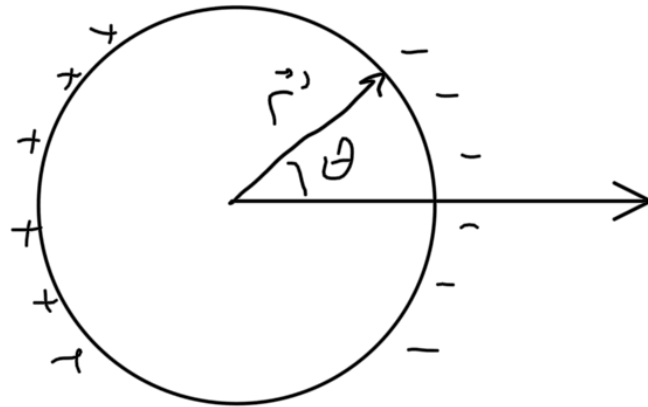
Cavidade dentro do dielétrico



Densidade superficial de cargas



Densidade superficial
de cargas



$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \oint_S \sigma_p \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} da$$

$$\vec{r} = 0 \Rightarrow \vec{E}_p = -\frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_p R}{R^3} \vec{r}' da$$
$$\vec{r}' = R \cdot \hat{r}$$

$$\sigma_p = \vec{P} \cdot \hat{n} = \vec{P} \cdot (-\hat{r}) = -P \cos\theta$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \oint_S \sigma_p \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} da$$

$$\vec{r} = 0 \Rightarrow \vec{E}_p = -\frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_p R}{R^3} \vec{r}' da$$

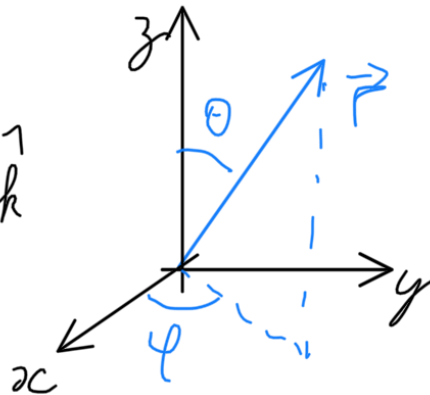
$$\vec{r}' = R \cdot \hat{r}$$

$$\sigma_p = \vec{P} \cdot \hat{n} = \vec{P} \cdot (-\hat{r}) = -P \cos\theta$$

$$da = R d\theta \cdot R \sin\theta d\varphi$$

$$\vec{E}_p = \frac{P}{4\pi\epsilon_0 R^2} \int_0^{2\pi} \int_0^{\pi} \hat{r} \cdot R^2 \cos\theta \sin\theta d\theta d\varphi$$

$$\hat{r} = \cos\varphi \sin\theta \hat{i} + \sin\varphi \sin\theta \hat{j} + \cos\theta \hat{k}$$



$$E_x = \frac{\rho}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^{\pi} \cos\varphi \cos\theta \sin^2\theta \, d\theta \, d\varphi = 0$$

$\left(\int_0^{2\pi} \cos\varphi \, d\varphi = 0 \right)$

$$E_y = \frac{\rho}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^{\pi} \sin\varphi \cos\theta \sin^2\theta \, d\theta \, d\varphi = 0$$

$\left(\int_0^{2\pi} \sin\varphi \, d\varphi = 0 \right)$

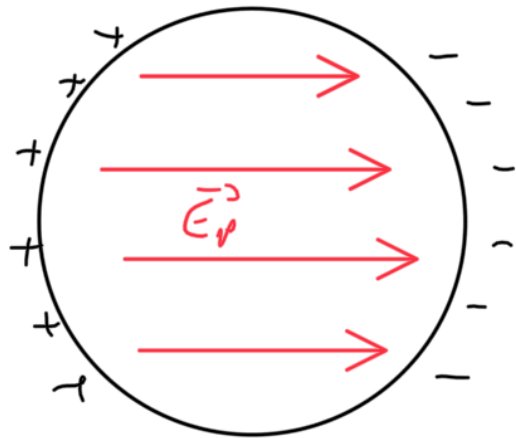
$$E_z = \frac{\rho}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta$$

$$\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = -\int_1^{-1} x^2 \, dx = \frac{x^3}{3} \Big|_{-1}^1 = \frac{2}{3}$$

$$x = \cos\theta \quad \frac{dx}{d\theta} = -\sin\theta$$

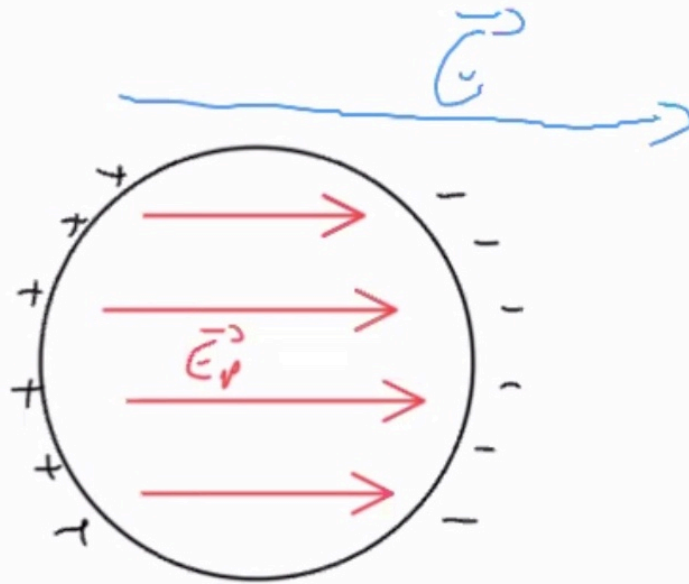
$$\vec{E}_p = E_z \hat{k} = \frac{\rho}{4\pi\epsilon_0} \cdot 2\pi \cdot \frac{2}{3} k \hat{k} = \frac{\rho}{3\epsilon_0} \hat{k}$$

Campo total $\vec{E}_T = \vec{E} + \vec{E}_p = \vec{E} + \frac{\rho}{3\epsilon_0}$



$$\vec{E}_p = \epsilon_3 \vec{h} = \frac{\rho}{4\pi\epsilon_0} \cdot 2\pi \cdot \frac{2}{3} h = \frac{\rho}{3\epsilon_0} \vec{h}$$

Campo total $\vec{E}_T = \vec{E} + \vec{E}_p = \vec{E} + \frac{\rho}{3\epsilon_0}$



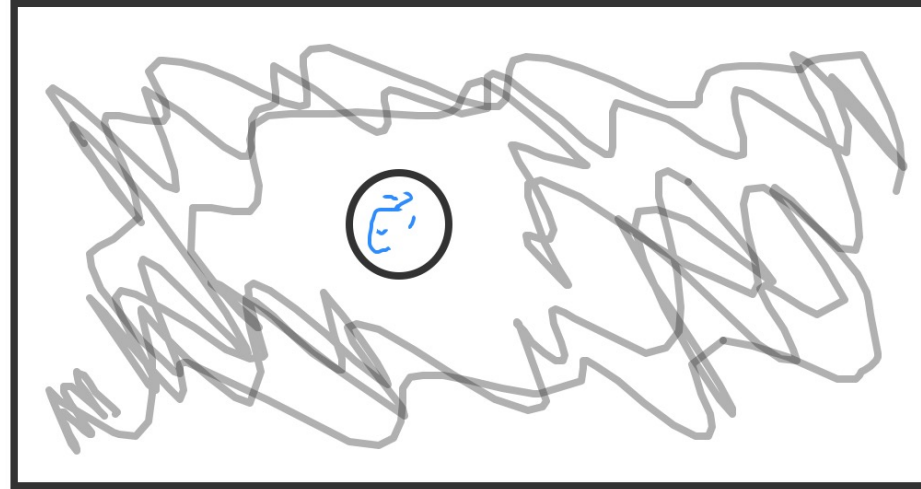
\vec{E}



$$|E'| < E \quad (a)$$

$$|E'| = 0 \quad (b)$$

$$|E'| > E \quad (c)$$



Qual o campo no caso de um material condutor?
E no caso de um meio isolante?

\vec{E}

