“Buracos Negros”

Esmerindo Bernardes

IFSC, USP

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Conteúdo I

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- Minkowski: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (cdt)^2 - d\mathbf{r}\cdot d\mathbf{r}$
  - ✓ tipo tempo (*timelike*): $ds^2 > 0$
  - ✓ tipo espaço (*spacelike*): $ds^2 < 0$
  - ✓ tipo luz (*lightlike*): $ds^2 = 0$
- Métrica estacionária: $\dfrac{\partial g_{\mu\nu}}{\partial x^0} = \partial_0 g_{\mu\nu} = g_{\mu\nu,0} = 0, \quad g_{00} > 0$
- Métrica estática: ds^2 invariante a reversão temporal (TR)
  - ♦ $ds^2(-x^0) = ds^2(x^0) \implies g_{0i} = 0$
- Métrica estacionária e estática (subespaço 2D esférico):

$$ds^2 = g_{tt}(r)(cdt)^2 + g_{rr}(r)dr^2 + f(r)\left(d\theta^2 + \operatorname{sen}^2\theta\, d\varphi^2\right)$$

  - ♦ $f < 0$
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Conteúdo I

Schwarzschild–de Sitter

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Conteúdo I

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Conteúdo I

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- Métrica: $\boxed{ds^2 = Q(cdt)^2 - Q^{-1}dr^2 - r^2\, d\varphi^2}$, $Q = 1 - \frac{r_s}{r} + \frac{\Lambda}{3}r^2$
- Energia: $\boxed{Q\frac{dt}{d\tau} = \frac{E}{mc^2}}$

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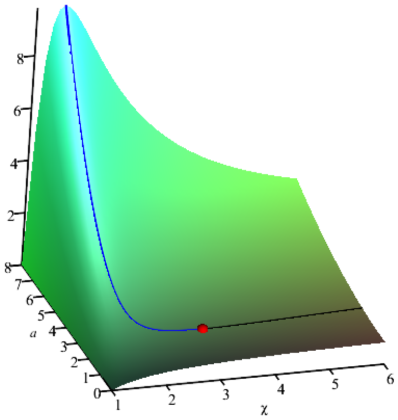
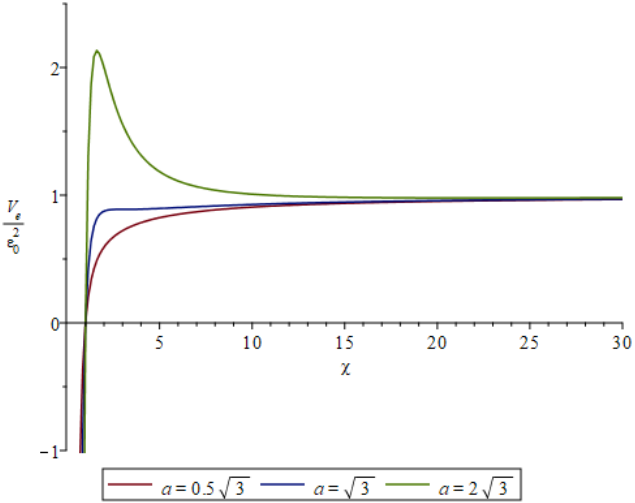
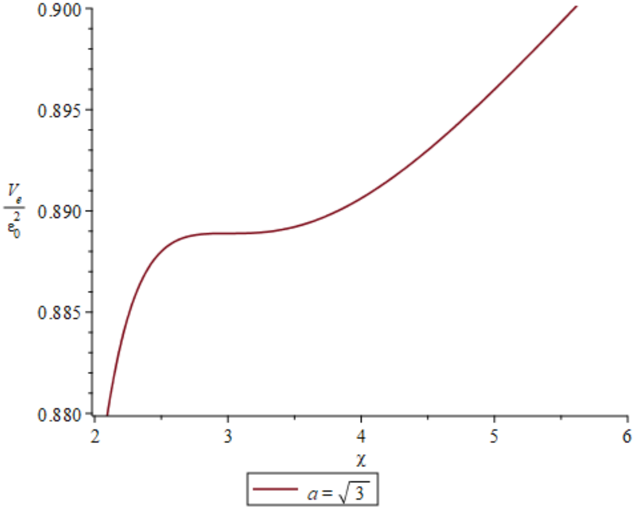


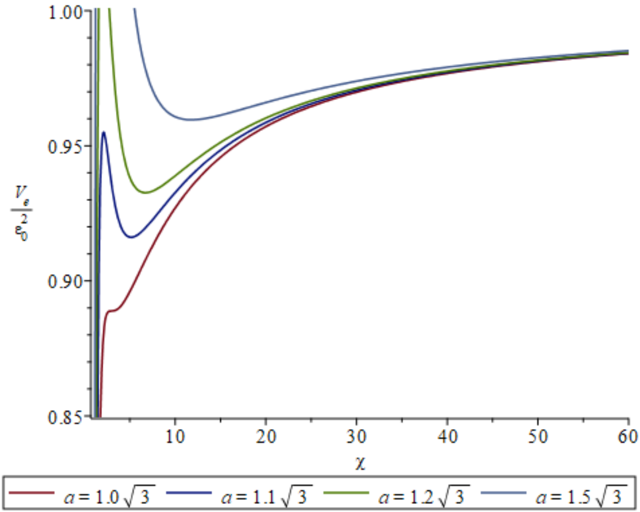
Potencial Efetivo–Geral



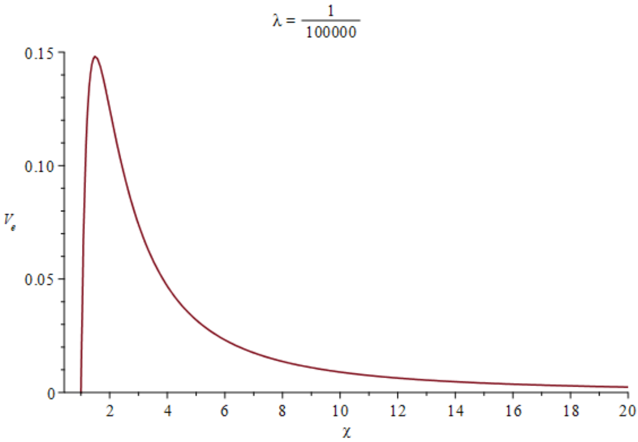
Potencial Efetivo–Inflexão



Potencial Efetivo–Mínimos



Potencial Efetivo–Luz



Conteúdo I

EDOs: Partículas

- $\varepsilon = \dfrac{E}{mc^2}$, $r_s = \dfrac{2MG}{c^2}$, $a = \dfrac{L}{r_s mc}$, $\tau_s = \dfrac{r_s}{c}$, $\bar{\tau} = \dfrac{\tau}{\tau_s}$, $\bar{t} = \dfrac{t}{\tau_s}$

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$$\left(\frac{d\chi}{d\bar{\tau}}\right)^2 = \varepsilon^2 - Q\left(1 + \frac{a^2}{\chi^2}\right) = \varepsilon^2 - \frac{(\lambda\chi^3 + \chi - 1)(\chi^2 + a^2)}{\chi^3}$$

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- Trajetória (posição):

$$\frac{d\chi}{d\varphi} = \frac{d\chi}{d\bar{\tau}}\frac{d\bar{\tau}}{d\varphi} = \frac{\chi^2}{a}\frac{d\chi}{d\bar{\tau}} = \pm\frac{1}{a}\sqrt{\varepsilon^2\chi^4 - (\lambda\chi^3 + \chi - 1)(\chi^2 + a^2)\chi}$$

EDOs: Partículas

- $\varepsilon = \frac{E}{mc^2}$, $r_s = \frac{2MG}{c^2}$, $a = \frac{L}{r_s mc}$, $\tau_s = \frac{r_s}{c}$, $\bar{\tau} = \frac{\tau}{\tau_s}$, $\bar{t} = \frac{t}{\tau_s}$
- $\boxed{Q = 1 - \frac{1}{\chi} + \lambda\chi^2}$, $\chi = \frac{r}{r_s}$, $\lambda = \frac{\Lambda r_s^2}{3}$
- Energia: $\boxed{\frac{dt}{d\tau} = \frac{\varepsilon}{Q} = \frac{\varepsilon\chi}{\lambda\chi^3 + \chi - 1}}$ M. Angular: $\boxed{\frac{d\varphi}{d\bar{\tau}} = \frac{a}{\chi^2}}$
- Dispersão:

$$\boxed{\left(\frac{d\chi}{d\bar{\tau}}\right)^2 = \varepsilon^2 - Q\left(1 + \frac{a^2}{\chi^2}\right) = \varepsilon^2 - \frac{(\lambda\chi^3 + \chi - 1)(\chi^2 + a^2)}{\chi^3}}$$

- Trajetória (posição):

$$\boxed{\frac{d\chi}{d\varphi} = \frac{d\chi}{d\bar{\tau}}\frac{d\bar{\tau}}{d\varphi} = \frac{\chi^2}{a}\frac{d\chi}{d\bar{\tau}} = \pm\frac{1}{a}\sqrt{\varepsilon^2\chi^4 - (\lambda\chi^3 + \chi - 1)(\chi^2 + a^2)\chi}}$$

- Trajetória (tempo): $\boxed{\frac{d\bar{t}}{d\varphi} = \frac{d\bar{t}}{d\bar{\tau}}\frac{d\bar{\tau}}{d\varphi} = \frac{\chi^2}{a}\frac{d\bar{t}}{d\bar{\tau}} = \frac{\varepsilon}{a}\frac{\chi^3}{\lambda\chi^3 + \chi - 1}}$

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EDOs: Luz

- $\frac{\tau}{m} = \xi$, $r_s = \frac{2MG}{c^2}$, $\tau_s = \frac{r_s}{c}$, $\bar{\chi} = \frac{\chi}{\tau_s}$, $\bar{t} = \frac{t}{\tau_s}$

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- M. Angular: $mr^2\frac{d\varphi}{d\tau} = L$

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Conteúdo I

Geodésicas

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Conteúdo I

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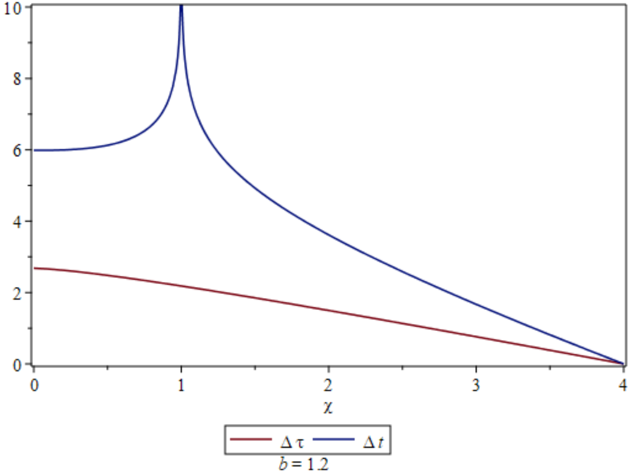
- Tempo distante: $\dfrac{d\bar{t}}{d\chi}\dfrac{d\chi}{d\bar{\tau}} = \dfrac{\varepsilon\chi}{\chi - 1} \Longrightarrow \left(\dfrac{d\bar{t}}{d\chi}\right)^2 = \dfrac{(1+b)\chi^3}{(\chi-1)^2(1+b\chi)}$

$$\pm 2b^{3/2}\bar{\tau}(\chi) = 2b^{3/2}\tanh^{-1}\left[\frac{1 + (1+2b)\chi}{2\sqrt{1+b}\sqrt{\chi(1+b\chi)}}\right]$$
$$-\sqrt{1+b}\left[2\sqrt{b\chi(1+b\chi)} + (2b-1)\ln\left[1 + 2b\chi + 2\sqrt{b\chi(1+b\chi)}\right]\right]$$

- $\lim\limits_{b\to 0} \bar{\tau} = \pm\dfrac{2}{3}\chi^{3/2},\ \lim\limits_{b\to 0} \bar{t} = \pm\dfrac{2}{3}\sqrt{\chi}(3+\chi) \mp \tanh^{-1}\left(\dfrac{1+\chi}{2\sqrt{\chi}}\right)$

Movimento Radial



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