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16/05/2022 - Gabarito lista 2 MAP0151

1. $P(x) = x^5 + 2x^3 - 1$, $x_0 = 1{,}85$

p	1	0	2	0	0	-1
x_0	1,85	1,85	1,85	1,85	1,85	1,85
b	1	1,85	5,4225	10,0316	18,5585	33,3332

$\Rightarrow b_0 = P(x_0) \approx 33{,}332$ e $q(x) = x^4 + 1{,}85x^3 + 5{,}4225x^2 + 10{,}0316 \cdot x + 18{,}5585$
é tal que $P(x) = q(x)(x - x_0) + P(x_0)$

2. MEG sem pivotação:

$$\begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -1 \\ -8 & 2 & 2 \end{bmatrix} \cdot x = \begin{bmatrix} 6 \\ 6 \\ -8 \end{bmatrix} \equiv \left[\begin{array}{ccc:c} 2 & 1 & -1 & 6 \\ 4 & 0 & -1 & 6 \\ -8 & 2 & 2 & -8 \end{array}\right] = [A|b] \Rightarrow$$

$$[A|b] \xrightarrow{L_2' = L_2 - (4/2)L_1} \left[\begin{array}{ccc:c} 2 & 1 & -1 & 6 \\ 0 & -2 & 1 & -6 \\ -8 & 2 & 2 & -8 \end{array}\right] \xrightarrow{L_3' = L_3 - (-8/2)L_1} \left[\begin{array}{ccc:c} 2 & 1 & -1 & 6 \\ 0 & -2 & 1 & -6 \\ 0 & 6 & -2 & 16 \end{array}\right] \Rightarrow$$

$$\xrightarrow{L_3' = L_3 - (6/-2)L_2} \left[\begin{array}{ccc:c} 2 & 1 & -1 & 6 \\ 0 & -2 & 1 & -6 \\ 0 & 0 & 1 & -2 \end{array}\right] \Rightarrow \begin{cases} x_3 = -2/1 = -2 \\ x_2 = (-6 - 1(-2))/-2 = 2 \\ x_1 = (6 + 1(-2) - 1 \cdot (2))/2 = 1 \end{cases}$$

Validação: $\begin{cases} 2 \cdot (1) + 1 \cdot (2) - 1(-2) = 6 \Leftrightarrow 6 = 6 \ (V) \\ 4 \cdot (1) + 0 \cdot (2) - 1(-2) = 6 \Leftrightarrow 6 = 6 \ (V) \\ -8 \cdot (1) + 2 \cdot (2) + 2(-2) = -8 \Leftrightarrow -8 = -8 \ (V) \end{cases} \therefore x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

spiral

3. $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ {·Por MEG sem pivotação (1º) ·Por MEG com pivotação (2º)

·Por MEG sem pivotação:

$$\underbrace{\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}}_{A} \cdot X = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{b} \Rightarrow [A|b] = \left[\begin{array}{ccc:c} 1 & -2 & 0 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -2 & 1 & 1 \end{array}\right] \xrightarrow{L_2' = L_2 - (-2/1)L_1}$$

$$\left[\begin{array}{ccc:c} 1 & -2 & 0 & 1 \\ 0 & -3 & -2 & 3 \\ 0 & -2 & 1 & 1 \end{array}\right] \xrightarrow{L_3' = L_3 - (-2/-3)L_2} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 1 \\ 0 & -3 & -2 & 3 \\ 0 & 0 & 7/3 & -1 \end{array}\right] \Rightarrow$$

$x_3 = -1 \cdot (3/7) = \underline{-3/7},$
$x_2 = (3 + 2(-3/7))/(-3) = \underline{-5/7},$
$x_1 = (1 - 0(-3/7) + 2(-5/7))/1$
$= \underline{-3/7},$

Validação:
$1 \cdot (-3/7) - 2(-5/7) + 0 \cdot (-3/7) = 1 \Leftrightarrow 1 = 1$ (✓)
$-2(-3/7) + 1(-5/7) - 2 \cdot (-3/7) = 1 \Leftrightarrow \frac{7}{7} = 1$ (✓)
$0 \cdot (-3/7) - 2(-5/7) + 1 \cdot (-3/7) = 1 \Leftrightarrow \frac{7}{7} = 1$ (✓)

$$\Rightarrow X = \begin{bmatrix} -3/7 \\ -5/7 \\ -3/7 \end{bmatrix}$$

·Por MEG com pivotação:

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow [A|b] = \left[\begin{array}{ccc:c} 1 & -2 & 0 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -2 & 1 & 1 \end{array}\right] \xrightarrow{L_2 \leftrightarrow L_1}$$

$$\left[\begin{array}{ccc|c} -2 & 1 & -2 & 1 \\ 1 & -2 & 0 & 1 \\ 0 & -2 & 1 & 1 \end{array}\right] \xrightarrow{L_2' = L_2 - (1/-2)L_1} \xrightarrow{L_3 <-> L_2} \left[\begin{array}{ccc|c} -2 & 1 & -2 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & -3/2 & -1 & 3/2 \end{array}\right] \xrightarrow{L_3' = L_3 - (-3/2/-2)L_2}$$

$$\left[\begin{array}{ccc|c} -2 & 1 & -2 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -7/4 & 3/4 \end{array}\right] \Rightarrow \begin{cases} x_3 = 3/4 / -7/4 = -3/7 \\ x_2 = (1 - (-3/7))/(-2) = -5/7 \\ x_1 = (1 + 2(-3/7) - 1\cdot(-5/7))/(-2) = -3/7 \end{cases} \rightarrow X = \begin{bmatrix} -3/7 \\ -5/7 \\ -3/7 \end{bmatrix}$$

(Como matriz X igual a sem pivotação, validação é a mesma)

4. $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \quad P(2,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow$ $P(2,3) \cdot A \cdot P(2,3)$ é triangular superior sob quais condições em A?

$$P(2,3)\cdot A \cdot P(2,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1\cdot a_{11} + 0\cdot 0 + 0\cdot 0 & 1\cdot a_{12} + 0\cdot a_{22} + 0\cdot 0 & 1\cdot a_{13} + 0\cdot a_{23} + 0\cdot a_{33} \\ 0\cdot a_{11} + 0\cdot 0 + 1\cdot 0 & 0\cdot a_{12} + 0\cdot a_{22} + 1\cdot 0 & 0\cdot a_{13} + 0\cdot a_{23} + 1\cdot a_{33} \\ 0\cdot a_{11} + 1\cdot 0 + 0\cdot 0 & 0\cdot a_{12} + 1\cdot a_{22} + 0\cdot 0 & 0\cdot a_{13} + 1\cdot a_{23} + 0\cdot a_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & a_{33} \\ 0 & a_{22} & a_{23} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{13} & a_{12} \\ 0 & a_{33} & 0 \\ 0 & a_{23} & a_{23} \end{bmatrix} \div B \Rightarrow$$

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$$\Rightarrow B \text{ é do tipo: } \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix}, \text{ com } b_{ij} \in \mathbb{R},\ i,j \in \{1,2,3\}: j \geq i, \Leftrightarrow \{ a_{23} = 0$$

5. Ache a decomposição em LU da matriz $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} = A$$

$$\begin{array}{cc} U & L \\ \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ -2 & 1 & -2 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \Rightarrow \left\{ \begin{array}{l} \cdot\, m_{ij} = \dfrac{a_{ij}}{a_{jj}} \Rightarrow m_{21} = \dfrac{-2}{1} \text{ e } m_{31} = \dfrac{0}{1} \\ \cdot\, U: L_2' = L_2 - (m_{21})L_1 \end{array} \right\} \Rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left\{ \begin{array}{l} \cdot\, m_{32} = \dfrac{-2}{-3} \\ \cdot\, U: L_3' = L_3 - (m_{32})L_2 \end{array} \right\} \Rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & 0 & 7/3 & 0 & 2/3 & 1 \end{array}\right] \Rightarrow U = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{bmatrix} \text{ e } L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}$$

Validação:

$$A = L \cdot U \Leftrightarrow \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -1 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 0 \\ 0 & -3 & -2 \\ 0 & 0 & 7/3 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -1 \\ 0 & -2 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} //$$