Álgebra do Momento Angular

$[L_i, L_j] = i\hbar\, \epsilon_{ijk} L_k \Rightarrow \epsilon_{ijk} = \begin{cases} +1: \text{cíclico} \\ -1: \text{n cíclico} \\ 0: \text{repetições} \end{cases}$

$i,j,k = \{1, 2, 3\}$

$[L_x, L_y] = i\hbar L_z;\quad [L_y, L_z] = i\hbar L_x;\quad [L_z, L_x] = i\hbar L_y.$ $\qquad [S_x, S_y] = i\hbar S_z,\quad [S_y, S_z] = i\hbar S_x,\quad [S_z, S_x] = i\hbar S_y.$

$\boxed{\vec{J} = \vec{L} + \vec{S}}$

$[J_x, J_y] = i\hbar J_z, \quad [J_y, J_z] = i\hbar J_x, \quad [J_z, J_x] = i\hbar J_y$

CSCO: $\{\hat{H}, \hat{J}^2, J_z\}$

$$\boxed{\begin{aligned} J_+ &= J_x + iJ_y \\ J_- &= J_x - iJ_y \end{aligned}}$$

$\hat{a}^\dagger a|\alpha\rangle = \alpha|\alpha\rangle$

$$\begin{aligned} J^2|j,m\rangle &= \hbar^2 j(j+1)|j,m\rangle \\ J_z|j,m\rangle &= m\hbar|j,m\rangle \end{aligned}$$

$$\frac{1}{2}(J_+ + J_-) = \frac{1}{2}(J_x + iJ_y + J_x - iJ_y) = J_x$$

$$\frac{1}{2i}(J_+ - J_-) = \frac{1}{2i}(J_x + iJ_y - J_x + iJ_y) = J_y$$

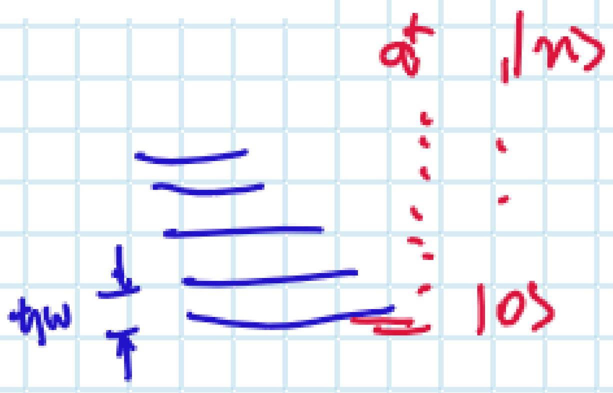
$$[J^2, J_+] = [J^2, J_x + iJ_y] = [J^2, J_x] + i[J^2, J_y] = 0 \Rightarrow J^2 J_+|j,m\rangle = J_+ J^2|j,m\rangle$$

$$[J_z, J_\pm] = [J_z, J_x \pm iJ_y] = [J_z, J_x] \pm i[J_z, J_y] = i\hbar J_y \pm i(-i\hbar J_x) = \hbar(J_x \pm iJ_y) = \pm\hbar J_\pm$$

$$\boxed{\begin{aligned} J^2 &= J_-J_+ + J_z^2 + \hbar J_z \\ J^2 &= J_z^2 + \tfrac{1}{2}(J_+J_- + J_-J_+) \end{aligned}}$$

$$\boxed{\begin{aligned} [J_z, J_+] &= \hbar J_+ \\ [J_z, J_-] &= -\hbar J_- \end{aligned}}$$

$[a, a^\dagger] = 1$



$\hookrightarrow [\hat{A}, \hat{B}] = \gamma\hat{C} = \pm\hbar B$

$\rightarrow \hat{A}|\phi\rangle = \alpha|\phi\rangle$

$$\hat{A}\hat{B}|\phi\rangle = ([\hat{A},\hat{B}] + \hat{B}\hat{A})|\phi\rangle$$

$\hat{A}\hat{B} - \hat{B}\hat{A}$ $\qquad \alpha|\phi\rangle$

$$= (\gamma\hat{C} + \alpha\hat{B})|\phi\rangle$$

$$J_z\, \boxed{J_+|j,m\rangle} = ([J_z, J_+] + J_+J_z)|j,m\rangle$$

$$(\hbar J_+ + J_+ m\hbar)|j,m\rangle$$

$$\hbar(m+1)\,\boxed{J_+|j,m\rangle}$$

Autovetor

Autovalor

$(m \rightarrow m+1)$

$\boxed{J_+|j,m\rangle} \propto |j', m+1\rangle$

$J_-|j,m\rangle \propto |j', m-1\rangle$

determinar coeficientes...

$$J_+|j,m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}\,|j, m+1\rangle$$
$$J_-|j,m\rangle = \hbar\sqrt{j(j+1) - m(m-1)}\,|j, m-1\rangle$$

$$J_+|j, m=j\rangle = 0$$
$$J_-|j, m=-j\rangle = 0$$

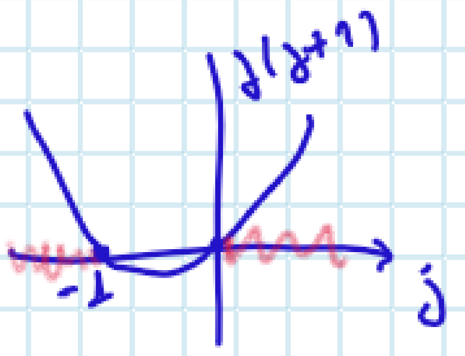
Análogo ao osc. harmônico

$a|0\rangle = 0$

$$\langle j,m|J^2|j,m\rangle = \hbar^2 j(j+1)\,|j,m\rangle$$

$$\langle j,m|J^\dagger J|j,m\rangle = \| J|j,m\rangle \|^2$$

$j(j+1) \geq 0$



$j \geq 0 \rightsquigarrow 2j = \mathbb{Z}$

$\{0, 1, 2, \ldots\}$

$m = \{j, j-1, \ldots, -j\}$

$$\boxed{j = \{0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots\}}$$

$l = \{0, 1, 2, \ldots\}$

$s = \{0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots\}$

(Spinor) $\leftarrow$ Autovetor

$$\vec{J} = \begin{cases} \vec{L} \hookrightarrow Y_l^m \quad \text{harmônico esférico} \\ \vec{S} \rightsquigarrow (\chi_+) \quad \text{Spinor} \end{cases}$$

$$\left(J^2(J_\pm|j,m\rangle)\right) = J_\pm J^2|j,m\rangle$$
$$= \hbar^2 j(j+1)\, J_\pm|j,m\rangle$$

$J_- = (J_+)^\dagger$

$\langle j,m|J_\mp J_\pm|j,m\rangle \quad = \ldots$

c_{jm}

$$J_\mp J_\pm = (J^2 - J_z^2 \mp \hbar J_z) \rightsquigarrow |c_{jm}|^2 = \hbar^2(j(j+1) - m(m\pm1))$$

$$c_{jm} = \hbar\sqrt{j(j+1) - (m\pm1)}$$

Conjunto completo de op. que comutam (CSCO)

$[A, B] = 0$ ⇝ Consigo encontrar uma base comum $\{|\phi_1\rangle, |\phi_2\rangle \dots |\phi_n\rangle\}$

$A|\phi_i\rangle = a_i|\phi_i\rangle$ $\quad i = \{1, 2, \dots, n\}$

$B|\phi_i\rangle = b_i|\phi_i\rangle$

$[H, A] = 0$

$U(t) \propto e^{\frac{iHt}{\hbar}}$ ← $H|\psi\rangle = i\hbar \frac{\partial|\psi\rangle}{\partial t}$

Mecânica Clássica

$i = \{1, \dots N\}$

$\{q_j^i, p_j^i\}$

q_1^i, q_2^i, q_3^i
$\downarrow \downarrow \downarrow$
$x \quad y \quad z$

$\Rightarrow [x, P_x] = i\hbar$

Grandezas físicas

- $E_c = \frac{p^2}{2m}$
- $V(r) = f(r)$
- $L(r, p)$

MQ ⇒ Observáveis
↳ op. Hermitiano

$\hat{H}$ → Dinâmica: $H = \frac{p^2}{2m} + V(r)$
$\hat{H}$ → Energia

$\hat{r} = (\cancel{\hat{x}}, \cancel{\hat{y}}, \cancel{\hat{z}})$

$\hat{p} = (\cancel{\hat{P}_x}, \cancel{\hat{P}_y}, \cancel{\hat{P}_z})$

↳ $\hat{L} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$

L^2

$[\hat{x}, \hat{y}] = [\hat{i}, \hat{j}] = 0$

$[\hat{P}_x, \hat{P}_y] = 0$

$[\hat{x}, \hat{P}_y] = 0$

$\{H, L^2, L_z\}$