UNIVERSIDADE DE SÃO PAULO – PIRASSUNUNGA

ZEB0562
CÁLCULO NUMÉRICO

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DEPTO. ENGENHARIA DE BIOSSISTEMAS

USP

FZEA

ZEB0562 – TÓPICO 05: PARTE 3/5

AJUSTE DE CURVAS: LINEAR - COM INCERTEZAS

- MÍNIMOS QUADRADOS: FUNÇÃO LINEAR
  ↓
  DADOS EXPERIMENTAIS COM INCERTEZAS
- IMPLEMENTAÇÃO VIA PLANILHAS MS EXCEL
- CURVA AJUSTADA vs. LINHA DE TENDÊNCIA

Mínimos quadrados → função linear

- Ajustar função linear $f(x) = a_0 + a_1 x$ contra (x_i, y_i, σ_i)
  - Obter a_0 , a_1 minimizando: $\chi^2 = \sum_{i=1}^{n} \left[\frac{r_i}{\sigma_i} \right]^2 = \sum_{i=1}^{n} \left[\frac{y_i - f(x_i)}{\sigma_i} \right]^2$

$$\partial\chi^2/\partial a_0 = 0 \qquad \partial\chi^2/\partial a_1 = 0$$

Sistema linear:

$$\left.\begin{aligned} a_0 S_1 + a_1 S_x &= S_y \\ a_0 S_x + a_1 S_{xx} &= S_{yx} \end{aligned}\right\} \Rightarrow \begin{bmatrix} S_1 & S_x \\ S_x & S_{xx} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} S_y \\ S_{yx} \end{bmatrix}$$

Somatórios:

$$S_1 = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}, \; S_x = \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}, \; S_y = \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2}, \; S_{xx} = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2}, \; S_{yx} = \sum_{i=1}^{n} \frac{y_i x_i}{\sigma_i^2}$$

Mínimos quadrados → função linear

- Ajustar função linear $f(x) = a_0 + a_1\, x$ contra $(x_i\,, y_i\,, \sigma_i)$
  - Solução do sistema linear (forma matricial): regra de Cramer

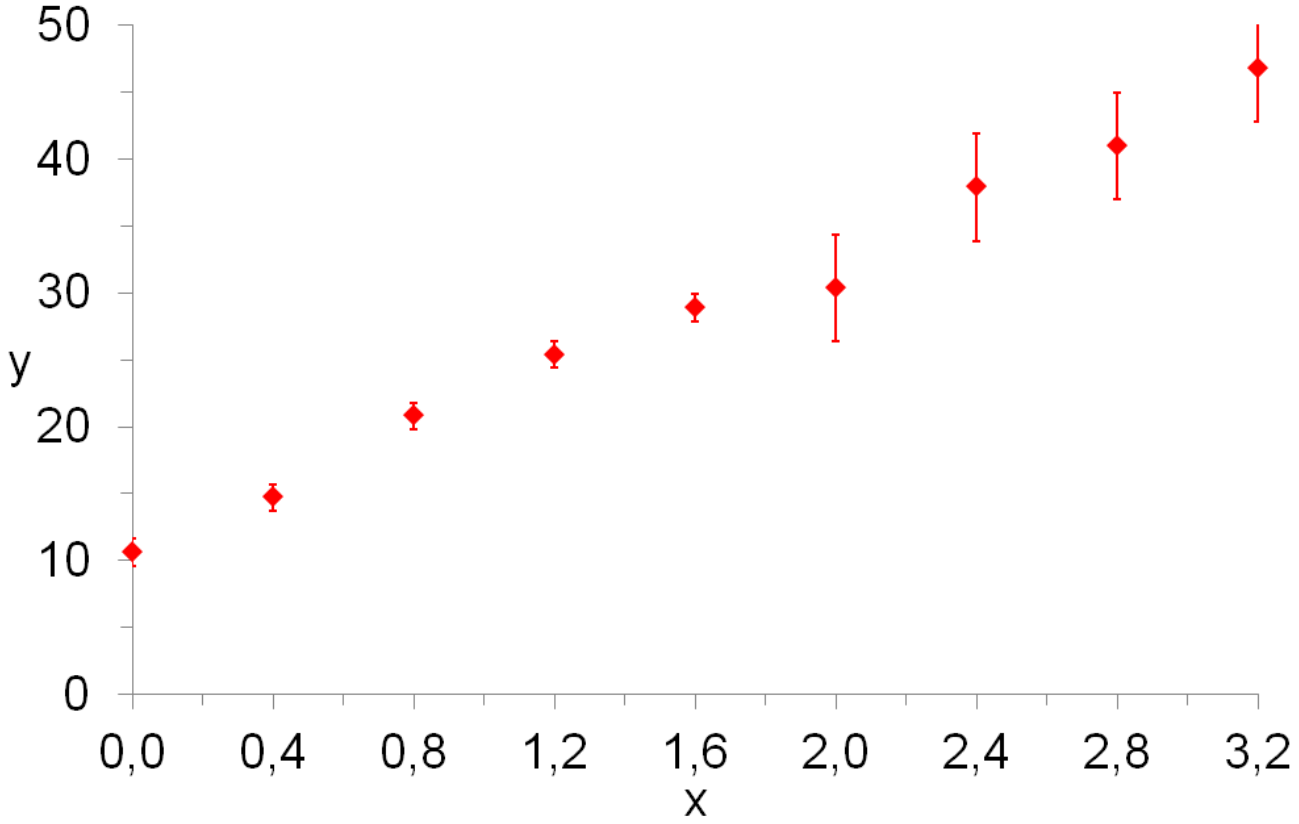
$$\begin{bmatrix} S_1 & S_x \\ S_x & S_{xx} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} S_y \\ S_{yx} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} S_1 & S_x \\ S_x & S_{xx} \end{bmatrix} \quad , \quad \mathbf{A_0} = \begin{bmatrix} S_y & S_x \\ S_{yx} & S_{xx} \end{bmatrix} \quad , \quad \mathbf{A}_1 = \begin{bmatrix} S_1 & S_y \\ S_x & S_{yx} \end{bmatrix}$$

Coef. linear: $$a_0 \pm \sigma_{a0} \quad \rightarrow \quad a_0 = \frac{\det(\mathbf{A_0})}{\det(\mathbf{A})} \quad , \quad \sigma_{a0} = \sqrt{\frac{S_{xx}}{\det(\mathbf{A})}}$$

Coef. angular: $$a_1 \pm \sigma_{a1} \quad \rightarrow \quad a_1 = \frac{\det(\mathbf{A_1})}{\det(\mathbf{A})} \quad , \quad \sigma_{a1} = \sqrt{\frac{S_1}{\det(\mathbf{A})}}$$

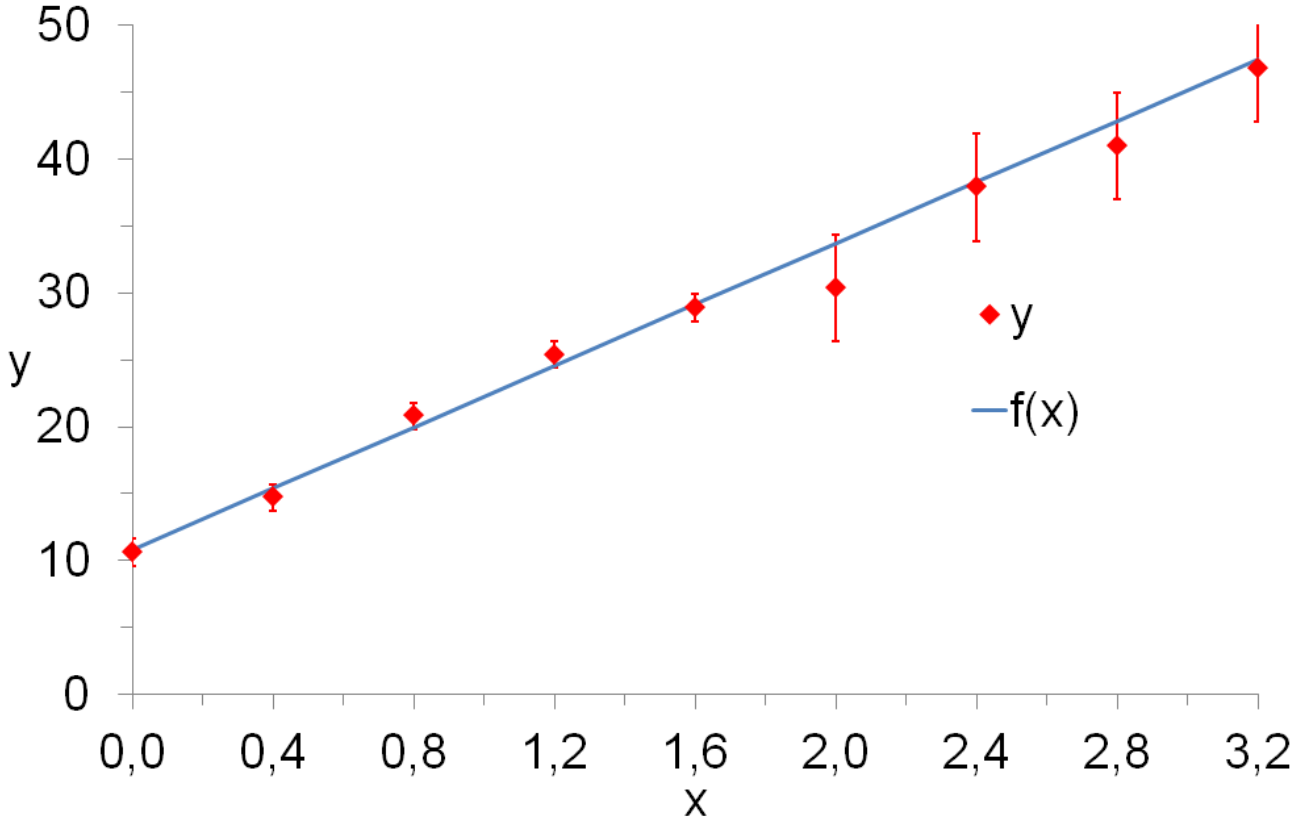
Ajuste de pontos com incertezas



x	y	σ
0,0	10,6	1,0
0,4	14,7	1,0
0,8	20,8	1,0
1,2	25,4	1,0
1,6	28,9	1,0
2,0	30,4	4,0
2,4	37,9	4,0
2,8	41,0	4,0
3,2	46,8	4,0

$$y = a_0 + a_1 x \qquad \begin{cases} a_0 \pm \sigma_{a_0} \, ? \\ a_1 \pm \sigma_{a_1} \, ? \end{cases}$$

Ajuste de pontos com incertezas



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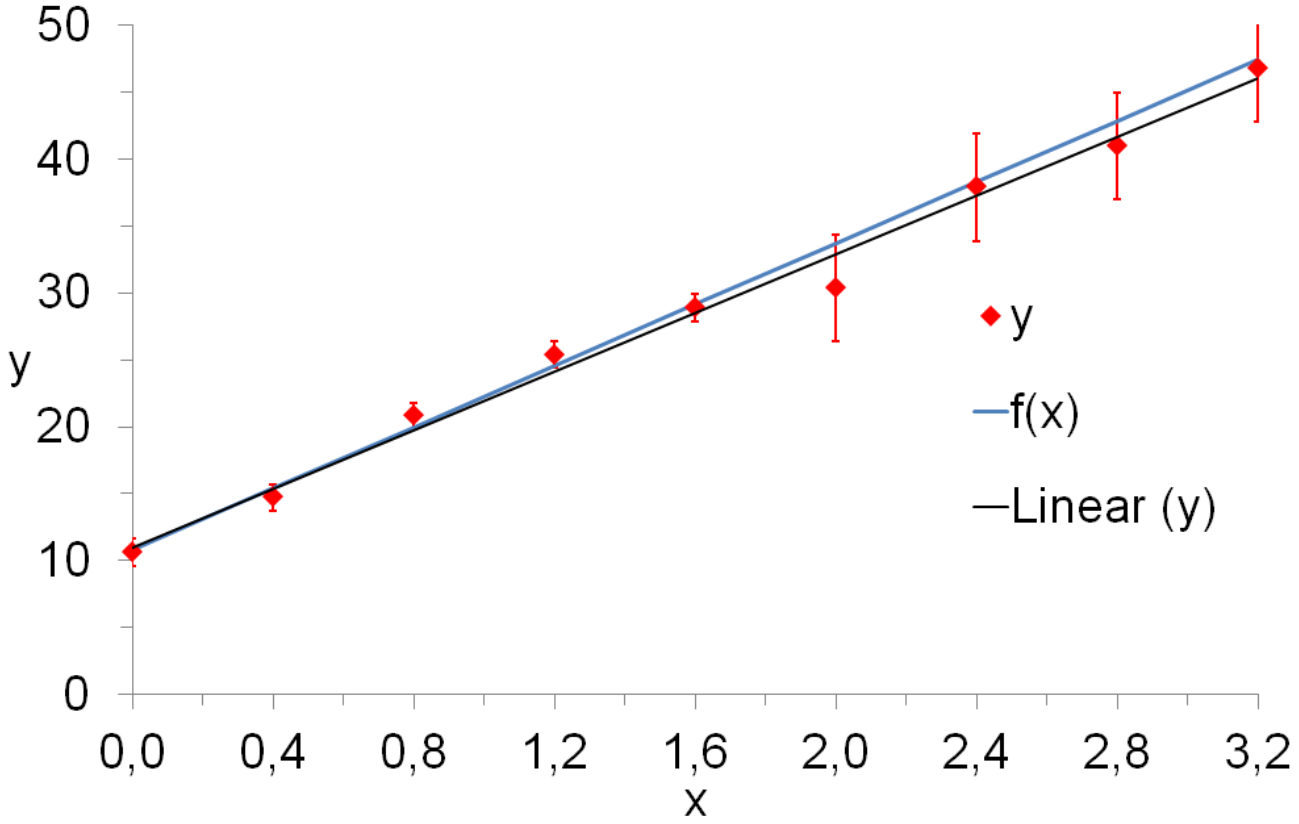
$$y = a_0 + a_1 x \qquad \begin{cases} a_0 : 10{,}75 \pm 0{,}72 \\ a_1 : 11{,}44 \pm 0{,}64 \end{cases}$$

Ajuste de pontos com incertezas



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