PLANETARY TIDES AND SUNSPOT CYCLES

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Abstract. There is an empirical function of the heights of tides on the Sun produced by Venus, Earth, and Jupiter whose period is nearly equal to that of the 11-yr sunspot cycle (Wood, 1972). This period match has been used in suggestions that planetary tides cause sunspots and, indirectly, terrestrial climate changes and earthquakes. We derive the period of the tidal function in terms of the planetary orbital periods and show that it is artificially lengthened by aliasing. Furthermore, there exists a class of functions whose measure in frequency space is so great that, in the absence of a physical justification for preferring one member, no statistically significant period match can possibly be made with current sunspot data.

A close correlation between the periods of the 11-yr sunspot cycle and a function of planetary tidal height on the Sun has recently been found by Wood (1972). Aside from being intrinsically interesting, it merits further study because it has been used in models which predict long-term global climate changes (Gribbin, 1973) and major earthquakes (Gribbin and Plagemann, 1974). No physical motivation for the choice of Wood's rather complicated function – the smoothed magnitude of the change in tidal height raised by Venus and Earth in the direction of Jupiter between successive alignments (either oppositions or conjunctions) of Venus and Earth – has been given, nor has the basis of its 11-yr periodicity been explained. The purpose of this article is to derive the period of Wood's function in terms of planetary orbital periods and to discuss the physical and statistical significance of its match to the sunspot cycle.

A simple expression for the total tidal height in the direction of Jupiter can be obtained if the orbits of Venus, Earth, and Jupiter are taken to be circular. The small errors resulting from this simplification have zero mean value, so they do not affect the calculated mean period of Wood's tidal function. The orbital angle θ travelled in time t by a planet of orbital frequency ν is $\theta = 2\pi\nu t$. Thus, the interval between successive alignments of Venus and Earth is $\tau_a = [2(\nu_V - \nu_E)]^{-1} \approx 0.8$ yr. Starting with $\theta = 0$ and $t = 0$ at a conjunction of all three planets, the angle θ_{VJ} between Venus and Jupiter develops as $\theta_{VJ} = 2\pi(\nu_V - \nu_J)\,t$; for the Earth, $\theta_{EJ} = 2\pi\,(\nu_E - \nu_J)\,t$. The tidal height due to a planet is given by $H = 1.5\,H_M\,(\cos^2\phi - \frac{1}{3})$, where H_M is the maximum tidal height and ϕ is the azimuthal angle between the line joining the planet to the Sun and the line going from the center of the Sun to the point where the tide is measured. The above relations are sufficient to yield the total sub-Jupiter tide height H_T on the Sun due to Venus, Earth, and Jupiter (Figure 1A)

$$H_T = 1.5\,H_{MV}(\cos^2\theta_{VJ} - \tfrac{1}{3}) + 1.5\,H_{ME}(\cos^2\theta_{EJ} - \tfrac{1}{3}) + H_{MJ}\,. \qquad (1)$$

Alignments of Venus and Earth occur at times $t = N\tau_a$, where N is any integer. At these times $\cos^2\theta_{VJ} = \cos^2\theta_{EJ}$, so the change ΔH_T in tidal height in the direction of

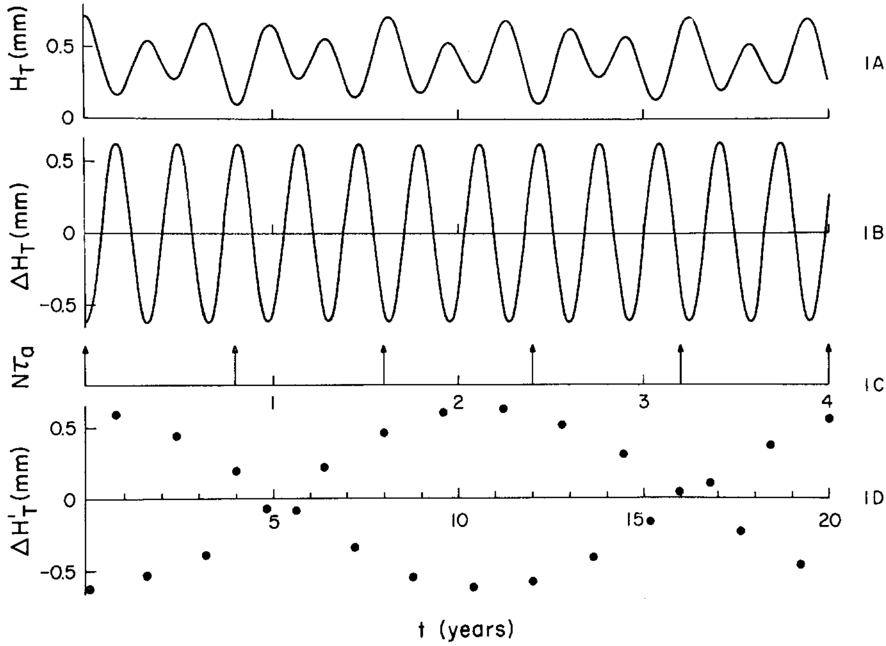

Fig. 1. The sub-Jupiter tidal height H_T (1A) due to Venus, Earth, and Jupiter changes by $\Delta H'_T$ (1B) between successive alignments of Venus and Earth. Sampling ΔH_T at intervals τ_a (1C) yields $\Delta H'_T$ (1D) whose absolute value is a rectified sine wave with an 11.07 yr period. Note that the time scale has been compressed by a factor of 5 in 1D.

Jupiter between successive Earth-Venus alignments is

$$\Delta H_T = 1.5\,(H_{MV} + H_{ME})\,\{\cos^2[\theta_{VJ}(t + \tau_a)] - \cos^2[\theta_{VJ}(t)]\} \tag{2}$$

when this continuous function is sampled at $t = N\tau_a$. Because θ_{VJ} is linear in time, this equation can be reduced to the purely sinusoidal form

$$\Delta H_T = -\,1.5\,(H_{MV} + H_{ME})\sin[\theta_{VJ}(\tau_a)]\,\sin[2\theta_{VJ}(t) + \theta_{VJ}(\tau_a)]\,, \tag{3}$$

whose frequency ν_T is $2(\nu_V - \nu_J)$. The continuous function ΔH_T (Figure 1B) is sampled at intervals τ_a (Figure 1C) to yield the discontinuous function $\Delta H'_T$ (Figure 1D) whose smoothed magnitude was found to have the same period as the sunspot number cycle.

The low frequency ν'_T of the smoothed $|\Delta H'_T|$ results from a beating between the frequency $\nu_T = 2(\nu_V - \nu_J)$ and an odd harmonic of the critical sampling frequency $\nu_c = (2\tau_a)^{-1}$. This beating is manifestation of the phenomenon called 'aliasing' which results from undersampling (Bracewell, 1965). Smoothing selects the lowest-frequency beat note which is a solution of

$$\nu'_T = |M(\nu_V - \nu_E) - 2(\nu_V - \nu_J)|\,, \qquad M = \pm 1, \pm 3, \pm 5, \ldots . \tag{4}$$

This solution is $M = 5$ and

$$\nu'_T = 3\,\nu_V - 5\,\nu_E + 2\,\nu_J\,. \tag{5}$$

Planetary orbital periods $(\nu_V)^{-1}=0.61521$, $(\nu_E)^{-1}=1.00004$, and $(\nu_J)^{-1}=11.86223$ tropical years (Allen, 1955) yield $(\nu'_T)^{-1}=22.13$ yr, nearly the period of a full sunspot cycle including magnetic polarity reversal. The rectified sine wave $|\Delta H'_T|$ has a period of 11.07 yr, which is within the 0.1% uncertainty of the measured sunspot number period of 11.06 yr (Cole, 1973).

The derivation of Equation (5) demonstrates that the period of Wood's tidal function need not arise from comparably long planetary orbital periods. The significance of this point will become clearer if some of the physical characteristics of planetary tides on the Sun are presented.

The tidal acceleration a_T produced a distance r from the center of the Sun by a planet of mass m and orbital radius d is $a_T=2Grm/d^3$. The hydrostatic equilibrium equation, which must be obeyed whenever accelerations are applied for times longer than the 10^3 s mechanical relaxation time scale of the Sun (Schwarzschild, 1958), requires that the tidal height $H_M=R_\odot a_T/(3\ g)$, g being the acceleration due to gravity on the solar surface. Thus the maximum tide height produced by the Earth is only $H_{ME}=0.1$ mm. The tide due to Mercury is comparable, and those caused by Venus and Jupiter are about twice as large. The tidal effects of all the remaining planets are much smaller.

Since it once appeared that the 11.9-yr orbital period of Jupiter was a necessary part of any tidal explanation of the 11-yr sunspot cycle, suggestions were made (Wood, 1972; Gribbin, 1973; Gribbin and Plagemann, 1974) that the approximately 180-yr periodicity observed in sunspot numbers (Cole, 1973) is related to tides raised by the long-period planets Uranus, Neptune, Pluto. Now that we have shown that long orbital periods are not required to produce long tidal cycles, the physical fact that these outer planets can cause a tide height of less than 1 micron on the Sun makes it very unlikely that they have any tidal effect on the long-term sunspot cycle. We note also that it is not valid to ignore Mercury on the grounds that its orbital period is much less than 11 yr (Wood, 1972). For example, we can generate a tidal function very similar to Wood's using only Mercury, Venus, and Earth – the smoothed magnitude in the change in tidal height raised by Venus and Earth in the direction of Mercury between successive conjunctions of Venus and Earth – for which the analog of Equation (5) is $\nu'_T=10\,\nu_V-8\,\nu_E-2\,\nu_M$, yielding a predicted sunspot number cycle 10.2 yr long.

Thus Wood's tidal function is one member of a class of functions of comparable physical plausibility which yield various predicted sunspot cycles. By itself, the fact that it matches the observed sunspot period to within 0.1% implies a 0.2% *a priori* probability that the match is just a coincidence. But the probability that at least one member of the class has the right period is at least an order of magnitude larger, just on the basis of the example given above, which is within 10% of the correct period. In conclusion, it does not appear possible to use such a period match statistically to imply that planetary tides affect sunspots without either a physical justification or a much closer period match than can be obtained with current sunspot data.

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