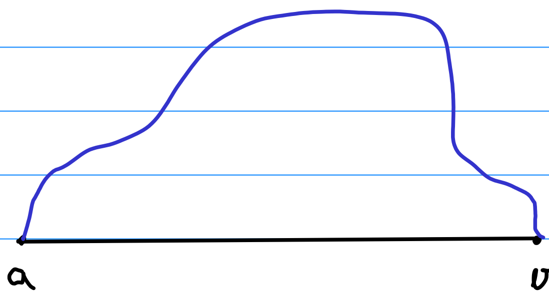
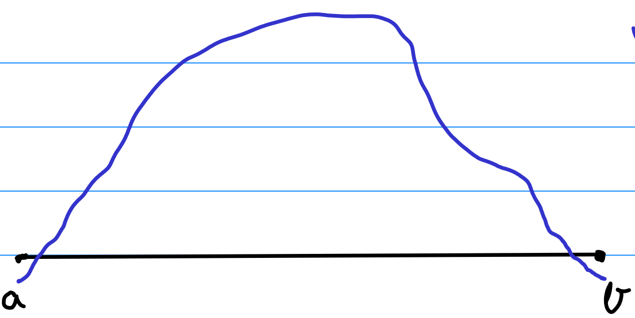


$y = f(x)$

$$I = \int_a^b f(x)\,dx > 0$$

$$T = \frac{b-a}{2}\left(\overset{\nearrow d}{\cancel{f}}(a) + \overset{\nearrow c}{\cancel{f}}(b)\right) = 0$$

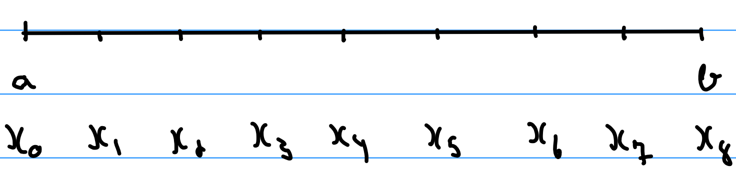


$y = f(x)$

$f(a) < 0$; $f(b) < 0$

$T < 0$; $I > 0$

$n = 8$; $h = \frac{b-a}{8}$



$$x_k = a + k \cdot h = a + k \cdot \frac{b-a}{n}, \quad 0 \le k \le n$$

$$x_0 = a; \; x_n = b; \qquad x_k - x_{k-1} = h = \frac{b-a}{n}, \quad 1 \le k \le n$$

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f(x)\,dx \simeq \sum_{i=1}^{n} \frac{h}{2}\left[f(x_{i-1}) + f(x_i)\right]$$

$$= \frac{h}{2}\left[f(x_0) + f(x_1) + f(x_1) + f(x_2) + \cdots + f(x_{n-2}) + f(x_{n-1}) + f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{h}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$= h\cdot\left[\frac{1}{2}\underbrace{f(x_0)}_{a} + \sum_{i=1}^{n} f(x_i) + \frac{1}{2}\underbrace{f(x_n)}_{b}\right] = T_n \text{ ou } T(h)$$

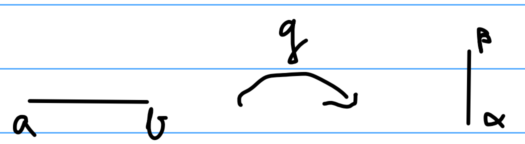
$$-\left[f(x) - P_1(x)\right] = \frac{f''(t_x)}{2}\underbrace{(x-a)(b-x)}_{\geq 0 \text{ p/ } x \in [a,b]}$$

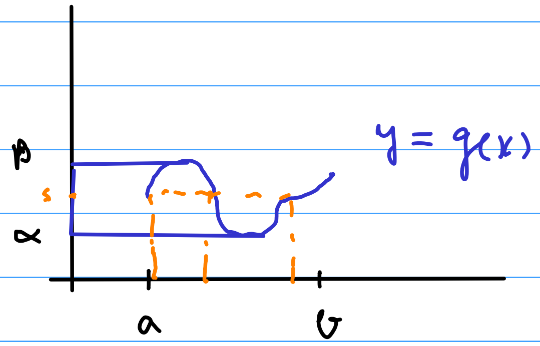
$$\int_a^b (x-a)(b-x) = \frac{(b-a)^3}{6}$$

<u>TEOR</u> $g : [a,b] \longrightarrow \mathbb{R}$ <u>CONTÍNUA</u>

ENTÃO, A IMAGEM DO INTERVALO $[a,b]$ POR g É O INTERVALO $[\alpha, \beta]$, ONDE

$$\alpha = \min_{x\in[a,b]} g(x); \quad \beta = \max_{x\in[a,b]} g(x)$$





PARA TODO s TAL QUE $\alpha \leq s \leq \beta$, EXISTE $t \in [a,b]$ TAL QUE $s = g(t)$, POIS s PERTENCE À IMAGEM DE g

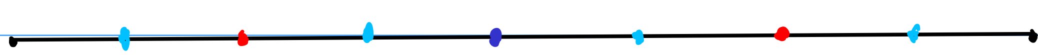
<u>OBS</u> $g: [a,b] \to \mathbb{R}$ CONTÍNUA ; $g([a,b]) = [\underset{\text{MIN } g}{\alpha}, \underset{\text{MAX } g}{\beta}]$

CONSIDERE $t_1, \dots, t_n \in [a,b]$. ENTÃO

$$\alpha \leq \underbrace{\frac{1}{n} \sum_{i=1}^{n} g(t_i)}_{\text{PERTENCE À IMAGEM DE } g} \leq \beta$$

$\exists\, t \in [a,b]$ TAL QUE

$$\frac{1}{n} \sum_{i=1}^{n} g(t_i) = g(t)$$



$b-a, \frac{b-a}{2}, \frac{b-a}{3}, \frac{b-a}{4}, \frac{b-a}{6}, \frac{b-a}{8}, \frac{b-a}{12}, \ldots$