The Weinberg-Salam Model

The gauge group is $SU(2)_L \otimes U(1)_Y$

The left handed fermions couple to $SU(2)_L$ gauge fields but right handed fermions are scalars

Lepton numbers are separately conserved $(e\,,\ \mu\,,\ \tau)$

Each family transforms under separate representation

$$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \qquad R = l_R \qquad \nu_L = \frac{(1-\gamma_5)}{2}\,\nu \qquad l_{L/R} = \frac{(1\pm\gamma_5)}{2}\,l$$

$$Y\,(L) = -1 \qquad Y\,(R) = -2 \qquad Q = T_3 + \frac{Y}{2}$$

Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\,\text{Tr}\left(F_{\mu\nu}\,F^{\mu\nu}\right) - \frac{1}{4}\,G_{\mu\nu}\,G^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[\,A_\mu\,,\,A_\nu\,]$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$A_\mu = A_\mu^a\,T_a \qquad [\,T_a\,,\,T_b\,] = i\,\varepsilon_{abc}\,T_c \qquad T_a = \frac{\sigma_a}{2}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Lepton sector

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \bar{R}\, i\, \gamma^\mu\, D_\mu R + \bar{L}\, i\, \gamma^\mu\, D_\mu L \\ &= \bar{R}\, i\, \gamma^\mu\, \partial_\mu R + \bar{L}\, i\, \gamma^\mu\, \partial_\mu L - \frac{g'}{2}\, J^Y_\mu\, B^\mu - g\, J^a_\mu\, A^\mu_a\end{aligned}$$

$$D_\mu L = \partial_\mu L + i \frac{g}{2}\, \sigma_a\, A^a_\mu\, L + i\, \frac{g'}{2}\, Y\,(L)\; B_\mu\, L$$

$$D_\mu R = \partial_\mu R + i\, \frac{g'}{2}\, Y\,(R)\; B_\mu\, R$$

$$J^Y_\mu = \bar{L}\, \gamma_\mu Y\,(L)\; L + \bar{R}\, \gamma_\mu Y\,(R)\; R = -\left[\bar{\nu}_L\, \gamma_\mu\, \nu_L + \bar{l}_L\, \gamma_\mu\, l_L + 2\, \bar{l}_R\, \gamma_\mu\, l_R\right]$$

$$J^a_\mu = \bar{L}\, \gamma_\mu \frac{\sigma_a}{2}\; L$$

$$\begin{aligned}J^1_\mu &= \frac{1}{2}\left[\bar{l}_L\, \gamma_\mu\, \nu_L + \bar{\nu}_L\, \gamma_\mu\, l_L\right] \\ J^2_\mu &= \frac{i}{2}\left[\bar{l}_L\, \gamma_\mu\, \nu_L - \bar{\nu}_L\, \gamma_\mu\, l_L\right] \\ J^3_\mu &= \frac{1}{2}\left[\bar{\nu}_L\, \gamma_\mu\, \nu_L - \bar{l}_L\, \gamma_\mu\, l_L\right]\end{aligned}$$

$$\begin{aligned}-\frac{g'}{2}\, J^Y_\mu\, B^\mu - g\, J^3_\mu\, A^\mu_3 &= \bar{\nu}_L\, \gamma_\mu \nu_L\, \left(\frac{g'}{2}\, B_\mu - \frac{g}{2}\, A^3_\mu\right) \\ &+ \bar{l}_L\, \gamma_\mu\, l_L \left(\frac{g'}{2}\, B_\mu + \frac{g}{2}\, A^3_\mu\right) + \bar{l}_R\, \gamma_\mu\, l_R\, g'\, B_\mu\end{aligned}$$

$$
\begin{aligned}
-\frac{g'}{2}\, J^Y_\mu\, B^\mu - g\, J^3_\mu\, A^\mu_3 &= \bar{\nu}_L\, \gamma_\mu \nu_L \left(\frac{g'}{2}\, B_\mu - \frac{g}{2}\, A^3_\mu \right) \\
&+ \bar{l}_L\, \gamma_\mu\, l_L \left(\frac{g'}{2}\, B_\mu + \frac{g}{2}\, A^3_\mu \right) + \bar{l}_R\, \gamma_\mu\, l_R\, g'\, B_\mu
\end{aligned}
$$

$$
\begin{pmatrix} B_\mu \\ A^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} \mathcal{A}_\mu \\ Z_\mu \end{pmatrix}
$$

$$
\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}
$$

$$
\begin{aligned}
B_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}\, [g\, \mathcal{A}_\mu - g'\, Z_\mu] \\
A^3_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}\, [g'\, \mathcal{A}_\mu + g\, Z_\mu]
\end{aligned}
$$

$$
\frac{g'}{2}\, B_\mu - \frac{g}{2}\, A^3_\mu = -\frac{1}{2}\, \sqrt{g^2 + g'^2}\, Z_\mu
$$

$$
\frac{g'}{2}\, B_\mu + \frac{g}{2}\, A^3_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left[g\, g'\, \mathcal{A}_\mu + \frac{1}{2} \left(g^2 - g'^2 \right) Z_\mu \right]
$$

$$
\begin{aligned}
-\frac{g'}{2}\, J^Y_\mu\, B^\mu - g\, J^3_\mu\, A^\mu_3 &= -\frac{1}{2}\, \sqrt{g^2 + g'^2}\, \bar{\nu}_L\, \gamma_\mu \nu_L\, Z^\mu \\
&+ \frac{1}{\sqrt{g^2 + g'^2}} \left[\frac{1}{2} \left(g^2 - g'^2 \right) \bar{l}_L\, \gamma_\mu\, l_L - g'^2\, \bar{l}_R\, \gamma_\mu\, l_R \right] Z^\mu \\
&+ e\, \bar{l}\, \gamma_\mu\, l\, \mathcal{A}^\mu
\end{aligned}
$$

$$
e \equiv \frac{g\, g'}{\sqrt{g^2 + g'^2}} = g\, \sin\theta_W = g'\, \cos\theta_W
$$

The Higgs Sector

The Higgs is a doublet of $SU(2)_L$ and has hypercharge 1

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad\qquad Y(\phi) = 1$$

$$Q = T_3 + \frac{Y}{2} \qquad\qquad Q(\phi^+) = 1 \qquad\qquad Q(\phi^0) = 0$$

Covariant derivative

$$D_\mu \phi = \partial_\mu \phi + i\frac{g}{2}\,\sigma_i\,A_\mu\,\phi + i\,\frac{g'}{2}\,Y(\phi)\;B_\mu\,\phi$$

Potential

$$V = \mu^2\,\phi^\dagger\,\phi + \lambda\,\left(\phi^\dagger\,\phi\right)^2 = \lambda\left(\phi^\dagger\,\phi - \frac{v^2}{2}\right)^2 - \frac{v^4}{4}\,\lambda \qquad\qquad v^2 = -\frac{\mu^2}{\lambda} > 0$$

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu\phi\right)^\dagger\;D^\mu\phi - V$$

Parameterize the Higgs as

$$\phi = e^{i\,\sigma_a\,\chi^a/2\,v}\left(\begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array}\right)$$

$$D_\mu\phi = e^{i\,\sigma_a\,\chi^a/2\,v}\left[\partial_\mu + i\frac{g}{2}\,\sigma_a\,A'^{\,a}_\mu + i\,\frac{g'}{2}\,Y\,(\phi)\;B_\mu\right]\left(\begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array}\right)$$

$$A'_\mu = h^{-1}\,A_\mu\,h - \frac{i}{g}\,h^{-1}\partial_\mu h \qquad\qquad h = e^{i\,\sigma_a\,\chi^a/2\,v}$$

$\chi^a \equiv$ these are the 3 Goldstone bosons eaten by the gauge fields

Denote

$$g\,A'^{\,a}_\mu\,\sigma_a + g'\,B_\mu = \left(\begin{array}{cc} g\,A'^{\,3}_\mu + g'\,B_\mu & g\left(A'^{\,1}_\mu - i\,A'^{\,2}_\mu\right) \\ g\left(A'^{\,1}_\mu + i\,A'^{\,2}_\mu\right) & -g\,A'^{\,3}_\mu + g'\,B_\mu \end{array}\right)$$

$$\left(g\,A'^{\,a}_\mu\,\sigma_a + g'\,B_\mu\right)\left(\begin{array}{c} 0 \\ 1 \end{array}\right) = g\left(\begin{array}{c} \sqrt{2}\,A'^{\,+}_\mu \\ -\frac{Z_\mu}{\cos\theta_W} \end{array}\right)$$

$$A'^{\,\pm}_\mu = \frac{A'^{\,1}_\mu \mp i\,A'^{\,2}_\mu}{\sqrt{2}} \qquad\qquad Z_\mu = \frac{g\,A'^{\,3}_\mu - g'\,B_\mu}{\sqrt{g^2 + g'^2}}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$| \, i \, \frac{v}{\sqrt{2}} \, \frac{1}{2} \, (g \, {A'_\mu}^a \, \sigma_a + g' \, B_\mu) \begin{pmatrix} 0 \\ 1 \end{pmatrix} |^2 \;=\; \frac{g^2 \, v^2}{4} \, {A'_\mu}^+ \, {A'_\mu}^- + \frac{g^2 \, v^2}{8 \, \cos^2 \theta_W} \, Z_\mu \, Z^\mu$$

$$= \frac{1}{2} \, M_W^2 \left[\left({A'_\mu}^1 \right)^2 + \left({A'_\mu}^2 \right)^2 \right] + \frac{1}{2} \, M_Z^2 \, Z_\mu \, Z^\mu$$

$$M_W = \frac{g \, v}{2} \qquad\qquad M_Z = \frac{g \, v}{2 \, \cos \theta_W} = \frac{M_W}{\cos \theta_W}$$

$$\frac{g}{2\sqrt{2}} = \sqrt{\frac{M_W^2 \, G_F}{\sqrt{2}}} \qquad \rightarrow \qquad M_W^2 = \frac{\sqrt{2}}{8} \, \frac{g^2}{G_F}$$

$$v^2 = \frac{1}{\sqrt{2} \, G_F} \qquad \rightarrow \qquad v = 247 \, GeV$$

$$M_W = (80.379 \pm 0.012) \; GeV$$

$$M_Z = (91.1876 \pm 0.0021) \; GeV$$

$$\sin^2 \theta_W = 0.22343 \pm 0.00007$$

(PDG 2018)

The Higgs Mass

$$\phi = e^{i\,\sigma_a\,\chi^a/2\,v}\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} V = \mu^2\,\phi^\dagger\,\phi + \lambda\,\left(\phi^\dagger\,\phi\right)^2 &= \frac{\mu^2}{2}\,(v+H)^2 + \frac{\lambda}{4}\,(v+H)^4 \\ &= \left(\frac{\mu^2}{2} + \frac{\lambda}{2}\,v^2 + \lambda\,v^2\right)\,H^2 + \ldots \\ &= \frac{1}{2}\,M_H^2\,H^2 + \ldots \end{aligned}$$

$$v^2 = -\frac{\mu^2}{\lambda} \qquad\qquad M_H = \sqrt{-2\,\mu^2}$$

$$M_H = (125.18 \pm 0.16)\ GeV$$

(PDG 2018)

Lepton masses: the Yukawa coupling

A term like $M_l\,(\bar{l}_R\,l_L + \bar{l}_L\,l_R)$ would break $SU(2)_L$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -G_l\left(\bar{R}\left(\phi^\dagger\,L\right) + \left(\bar{L}\phi\right)R\right) \\
&= -G_l\frac{(v+H)}{\sqrt{2}}\left[\bar{l}_R\,(0\;1)\begin{pmatrix}\nu_l\\ l_L\end{pmatrix} + (\bar{\nu}_L\;\bar{l}_L)\begin{pmatrix}0\\ 1\end{pmatrix}l_R\right] \\
&= -G_l\frac{v}{\sqrt{2}}\,\bar{l}\,l - G_l\frac{H}{\sqrt{2}}\,\bar{l}\,l
\end{aligned}$$

$$M_l = \frac{G_l\,v}{\sqrt{2}}$$

Higgs coupling $\quad \dfrac{M_l}{v}\,\bar{l}\,l\,H$