

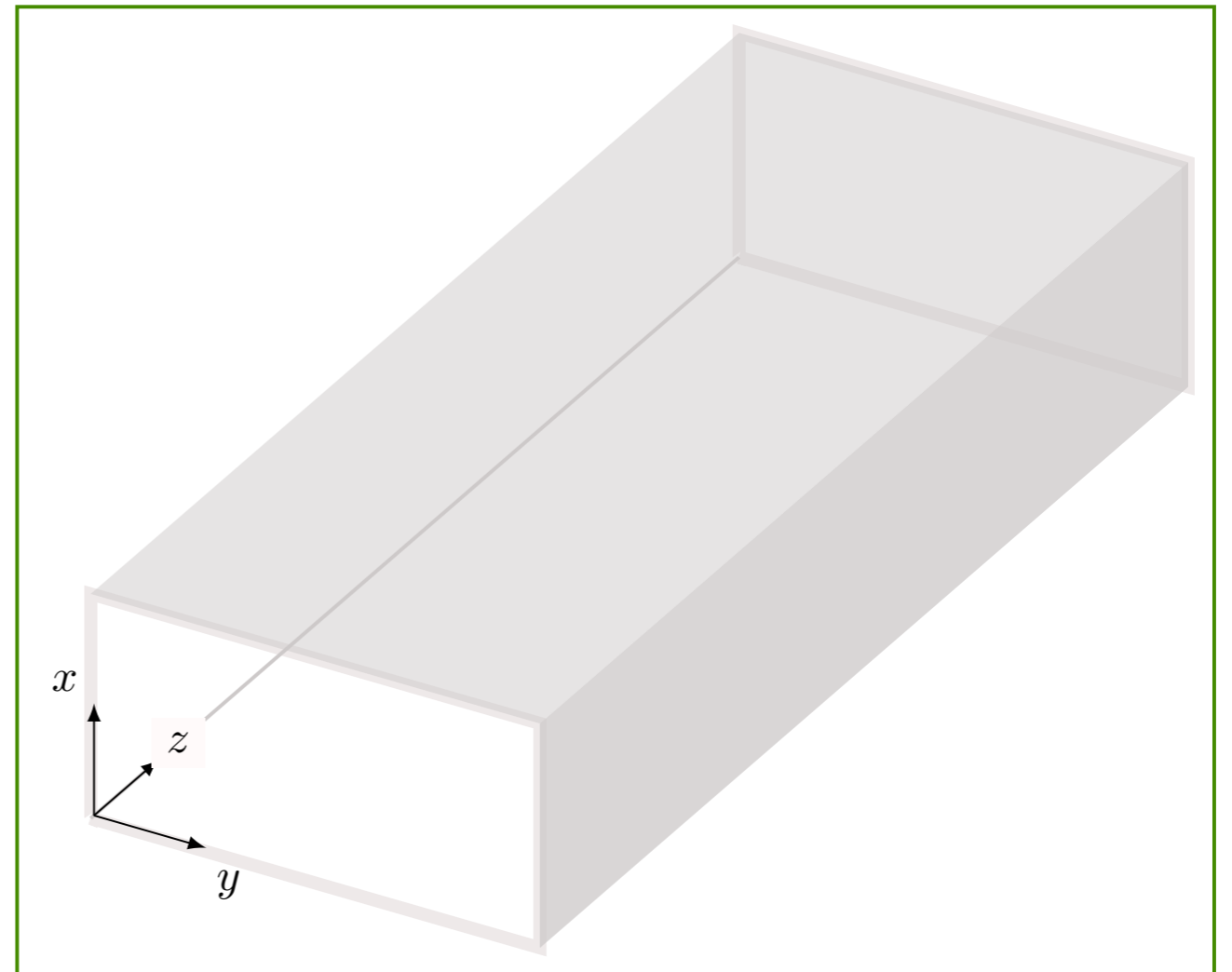
# Eletromagnetismo Avançado

2º ciclo  
Aula de 13 outubro

# Guias de ondas

$$\mathbf{E}_{\parallel} = 0$$

$$B_{\perp} = 0$$



# Guias de ondas

$$\mathbf{E}_{\parallel} = 0$$

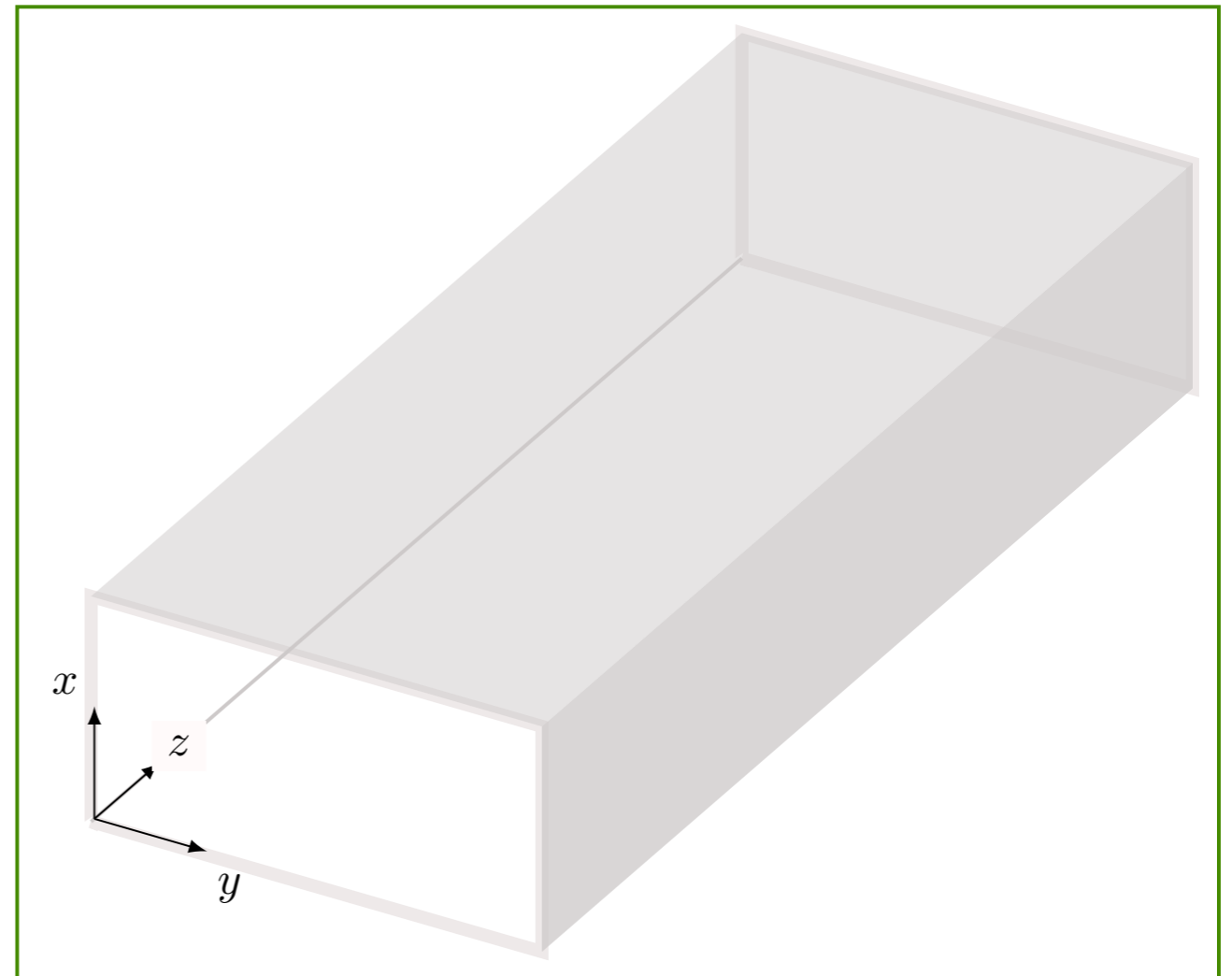
$$B_{\perp} = 0$$

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$$\tilde{\mathbf{B}} = \tilde{\mathbf{B}}_0(x, y)e^{ikz - \omega t}$$

CAMPOS  $\vec{E}_0$  e  $\vec{B}_0$  NÃO  
PODEM SER CONSTANTES,  
PORQUE TÊM DE

SATISFAZER AS CONDIÇÕES DE CONTORNO



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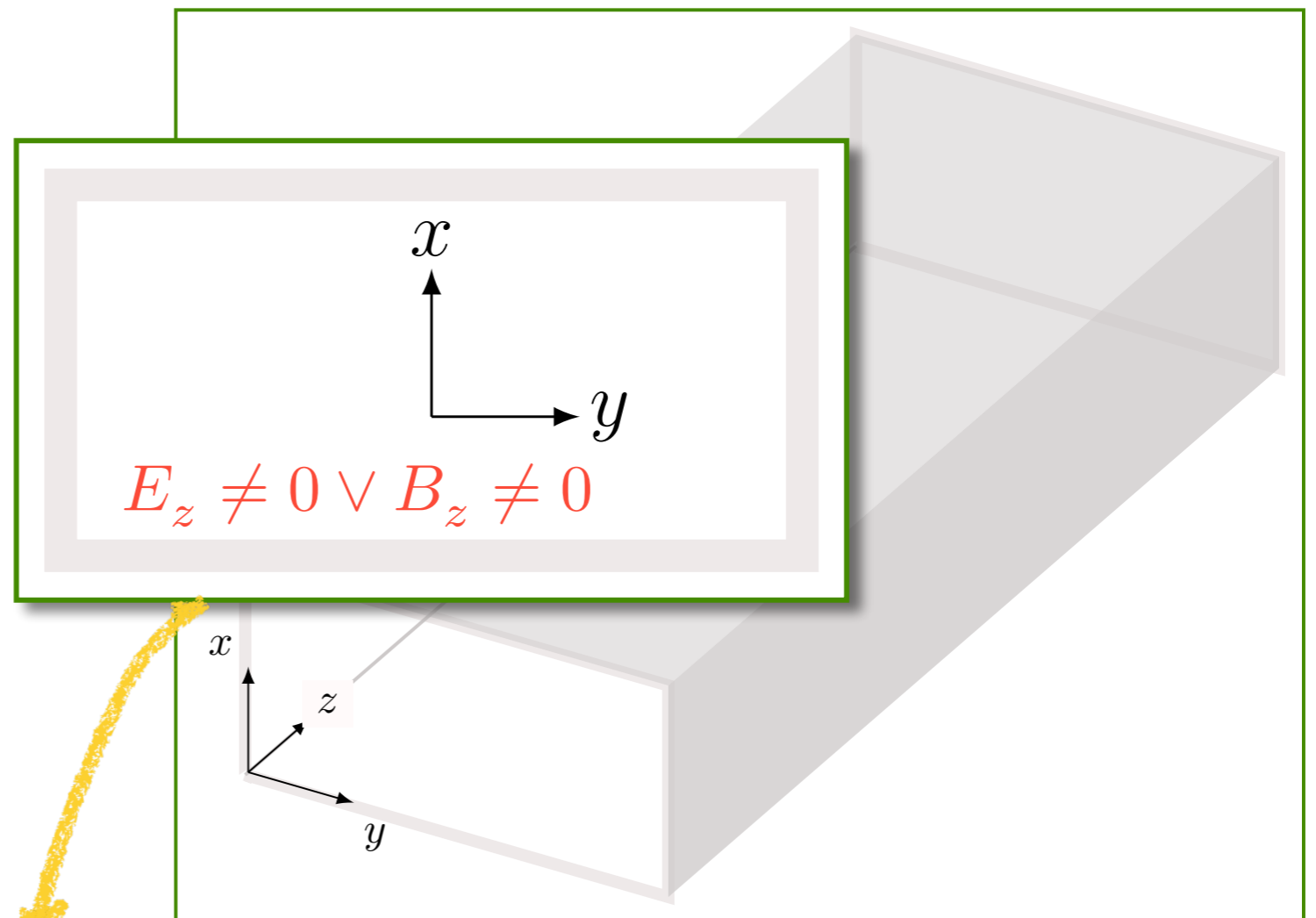
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$$\tilde{\mathbf{B}}_0 = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

Ao menos um dos campos  $\tilde{\mathbf{E}}_0$  e  $\tilde{\mathbf{B}}_0$  tem componente em  $\hat{z}$



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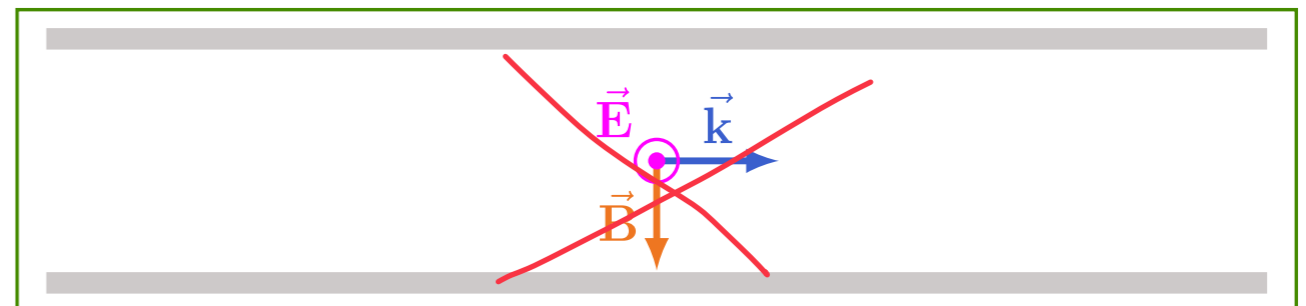
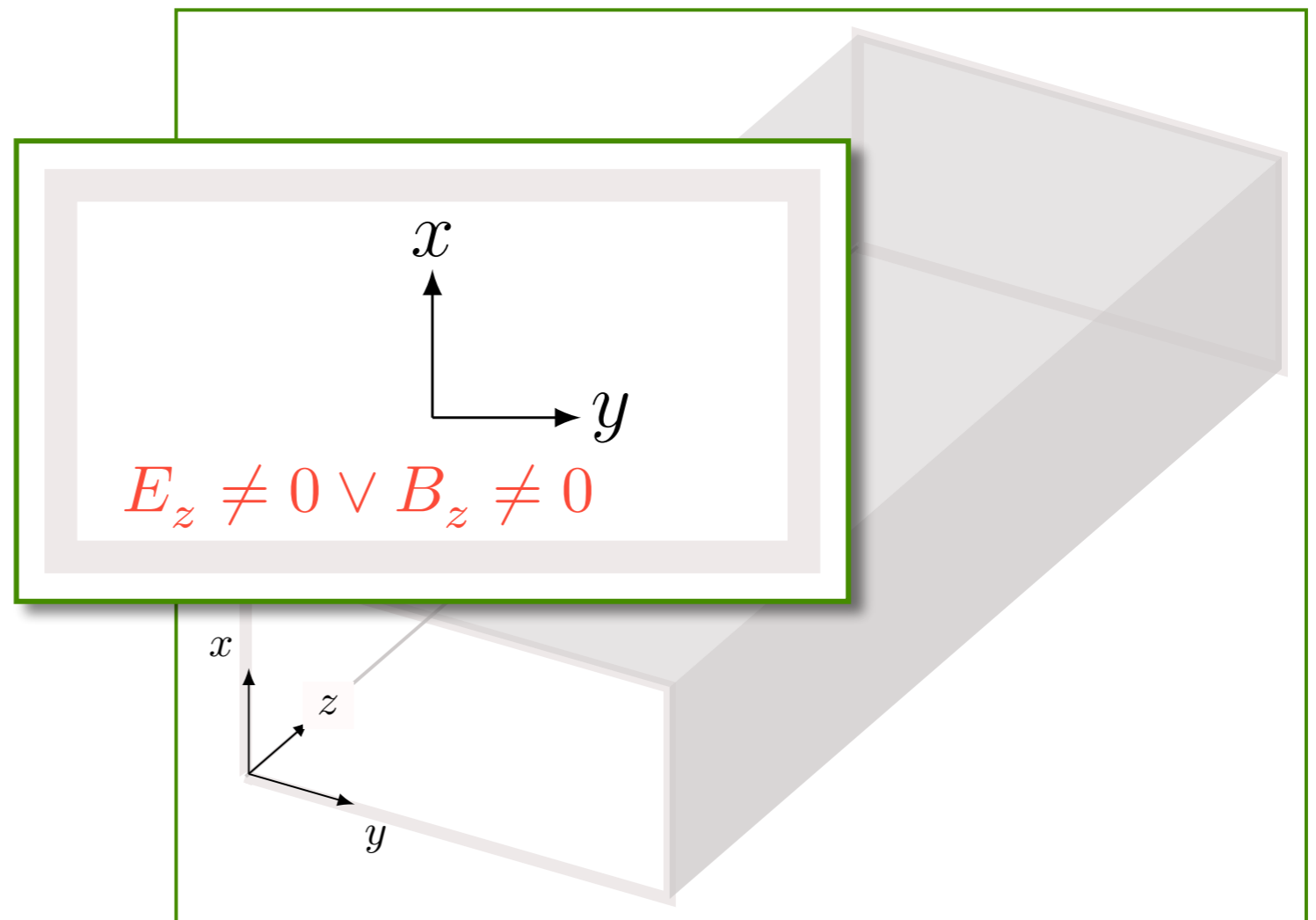
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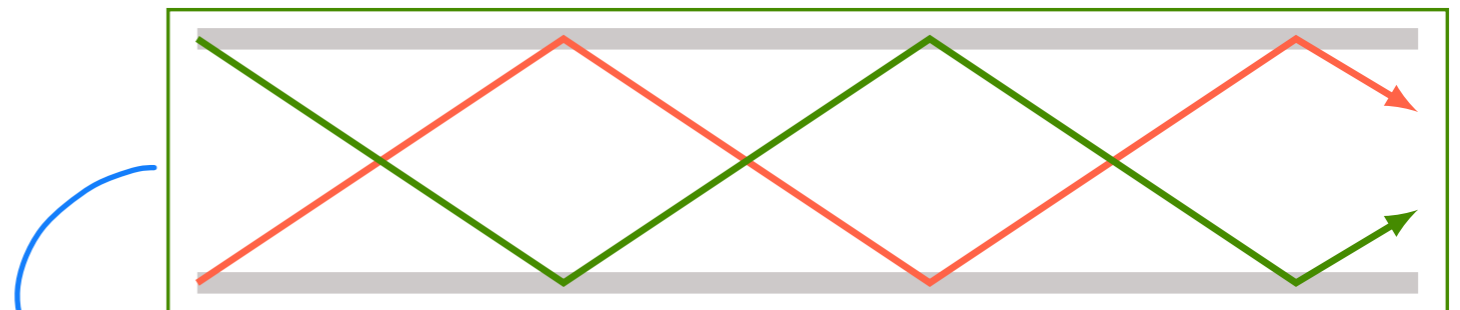
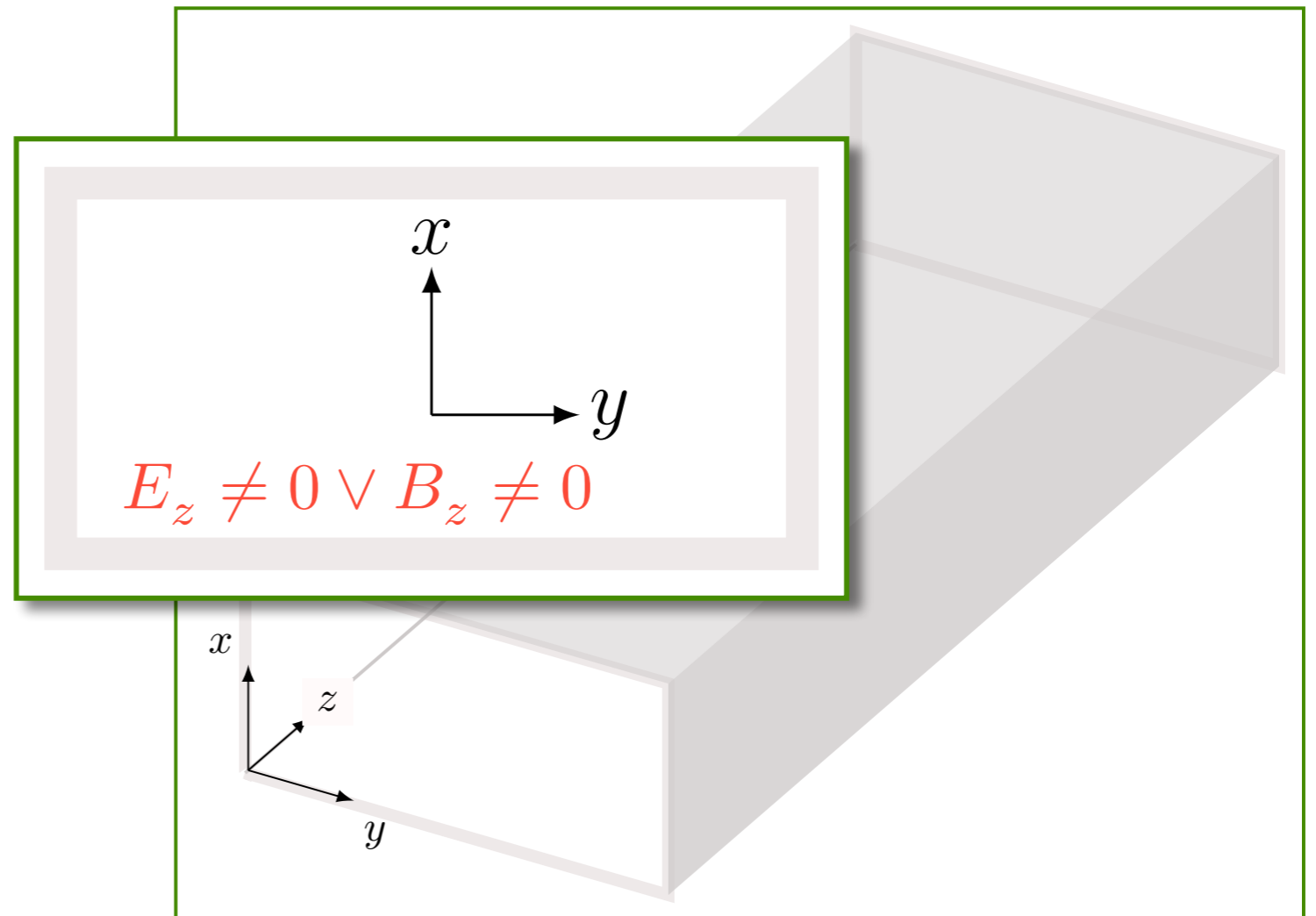
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CAMPOS COMBINAM NAS  
ONDAS OBLÍQUAS.  
SOMA SE PROPAGA EM  $\hat{z}$ .

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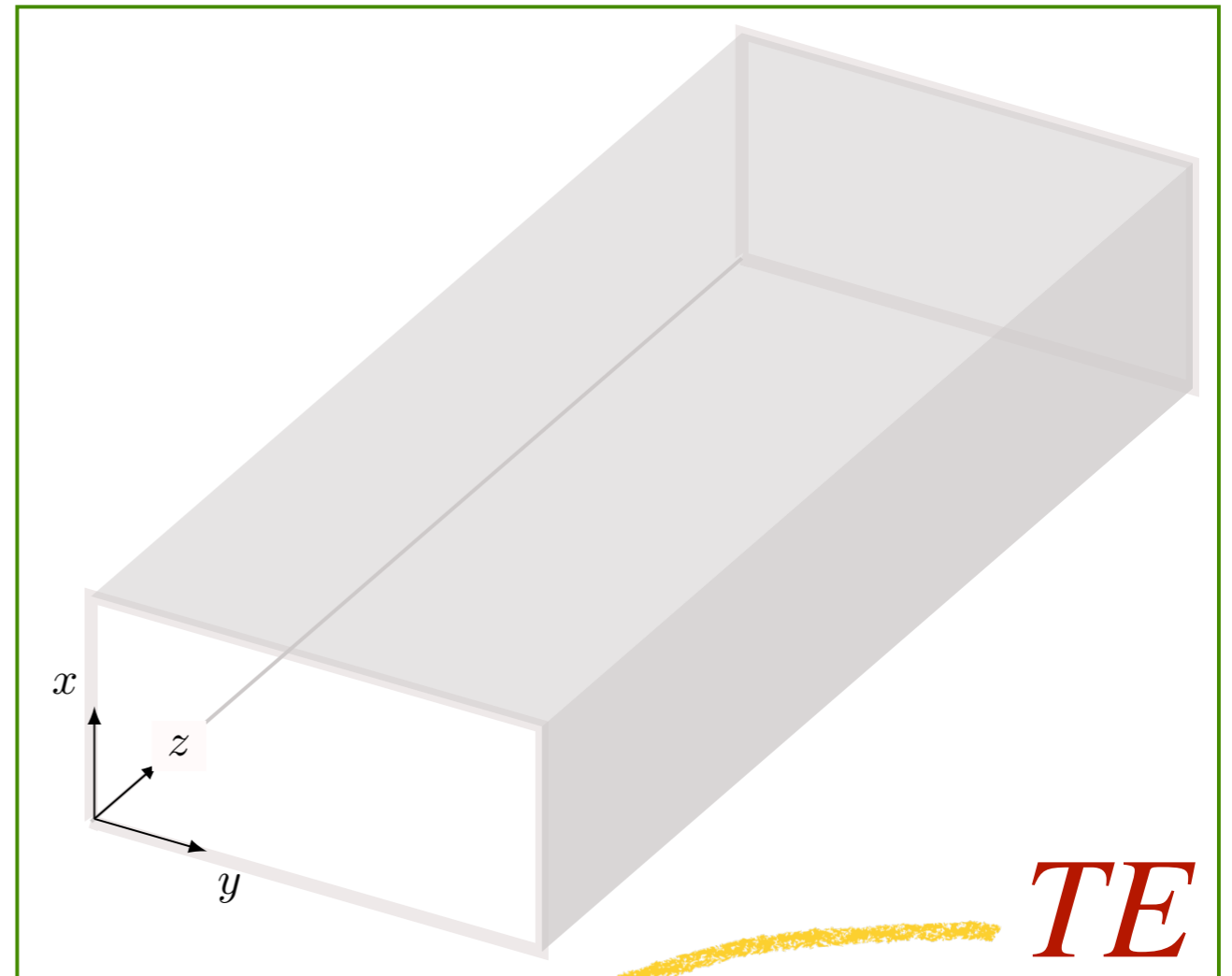
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$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \tilde{B}_z = \frac{1}{c^2} \frac{\partial^2 \tilde{B}_z}{\partial t^2}$$



$$E_z = 0$$

$$B_z \neq 0$$

# Guias de ondas

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0(x, y)e^{ikz - \omega t}$$

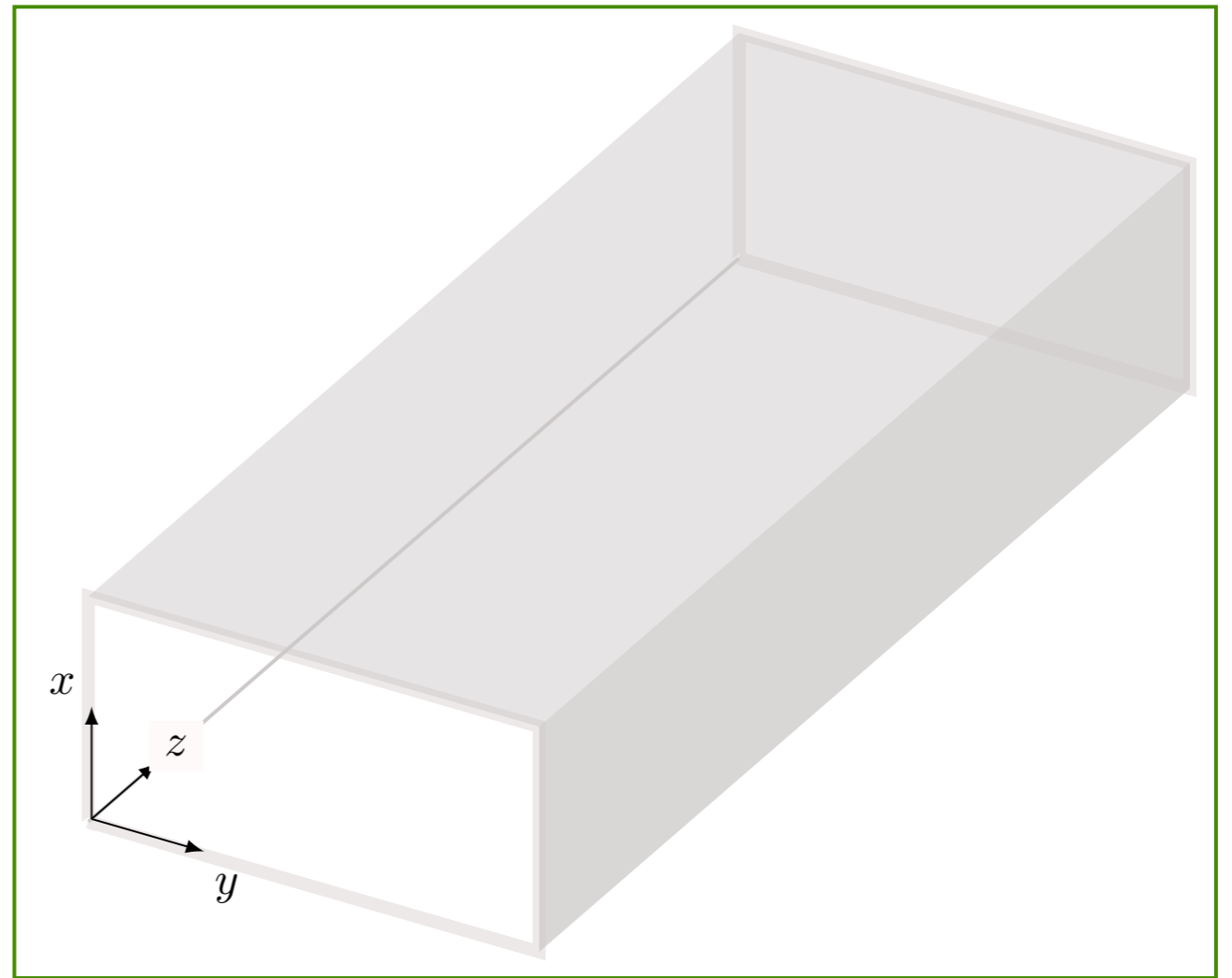
$$\tilde{\mathbf{B}} = \tilde{\mathbf{B}}_0(x, y)e^{ikz - \omega t}$$

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$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - k^2 + \left( \frac{\omega}{c} \right)^2 \right) B_z = 0$$

$$B_z = X(x)Y(y)$$

$$\hookrightarrow Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} = \left[ k^2 - \left( \frac{\omega}{c} \right)^2 \right] XY \Rightarrow \frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} = k^2 - \left( \frac{\omega}{c} \right)^2$$



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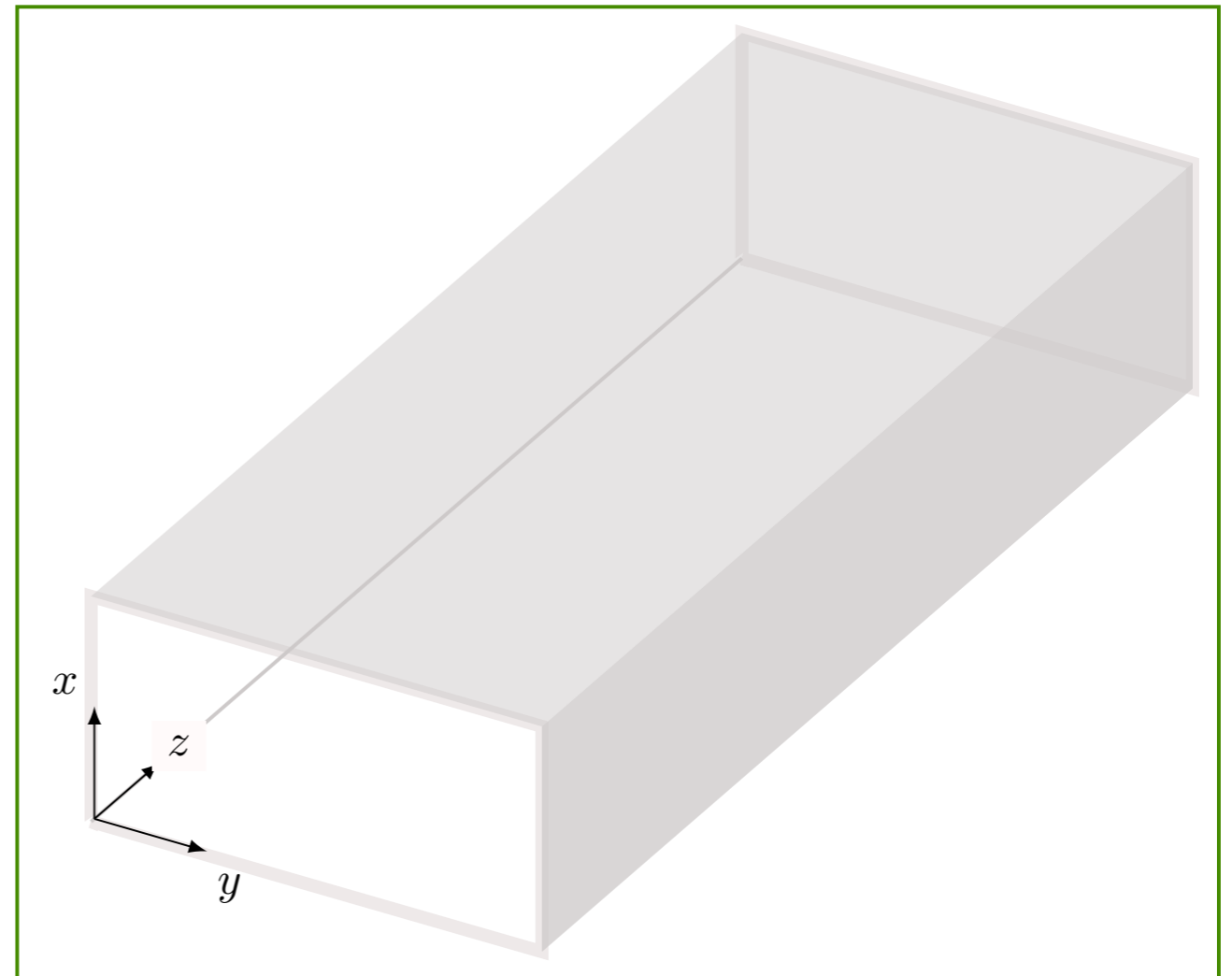
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$$B_z = X(x)Y(y)$$

$$X(x) = A \cos(k_x x) + B \sin(k_x x)$$

$$k^2 - \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_y^2$$



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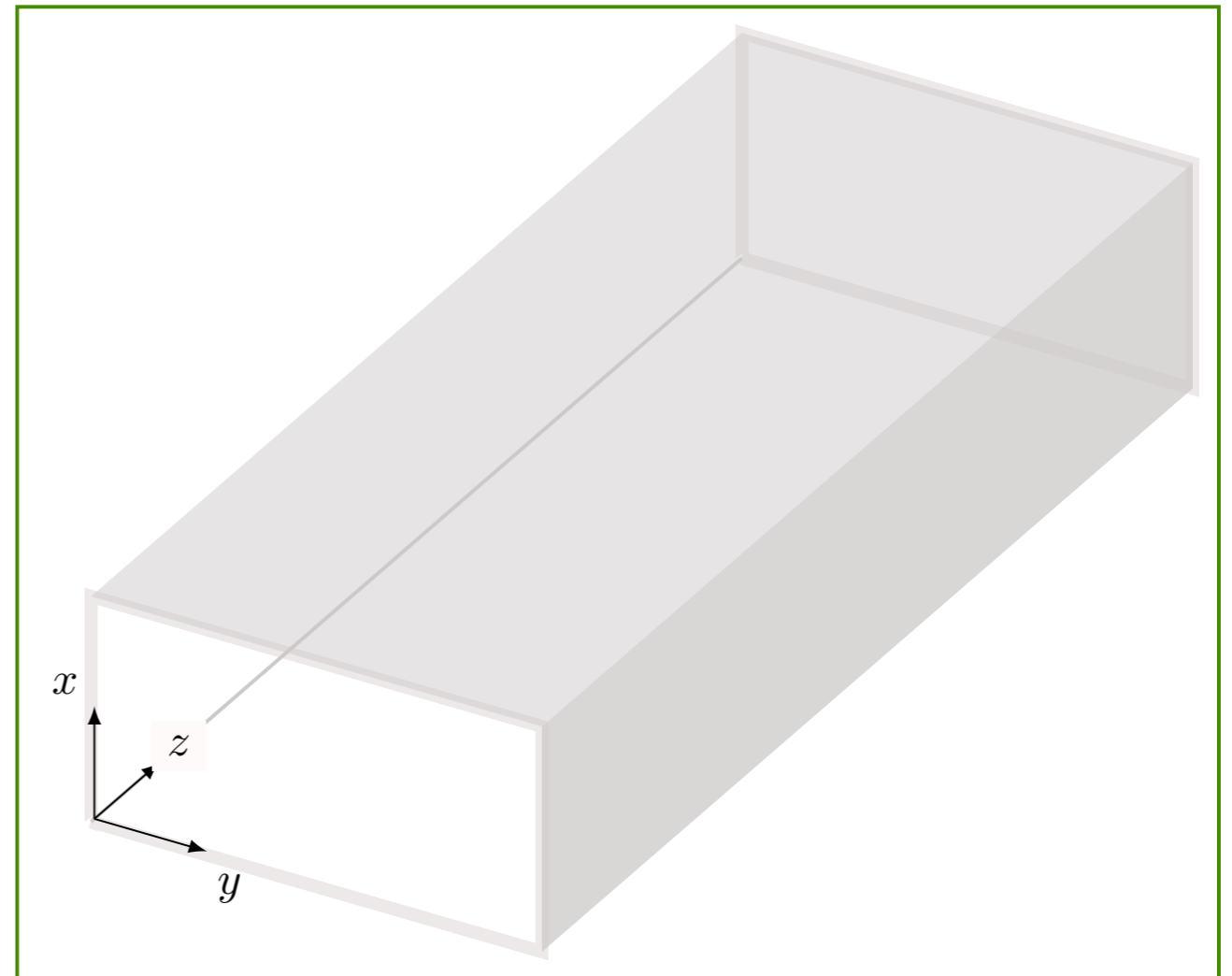
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$$B_z = X(x)Y(y)$$

$$B_z(x, y) = B_0 \cos\left(n \frac{\pi x}{a}\right) \cos\left(m \frac{\pi y}{b}\right) \quad (m, n = 0, 1, 2, \dots)$$



CONDIÇÕES DE CONTORNO

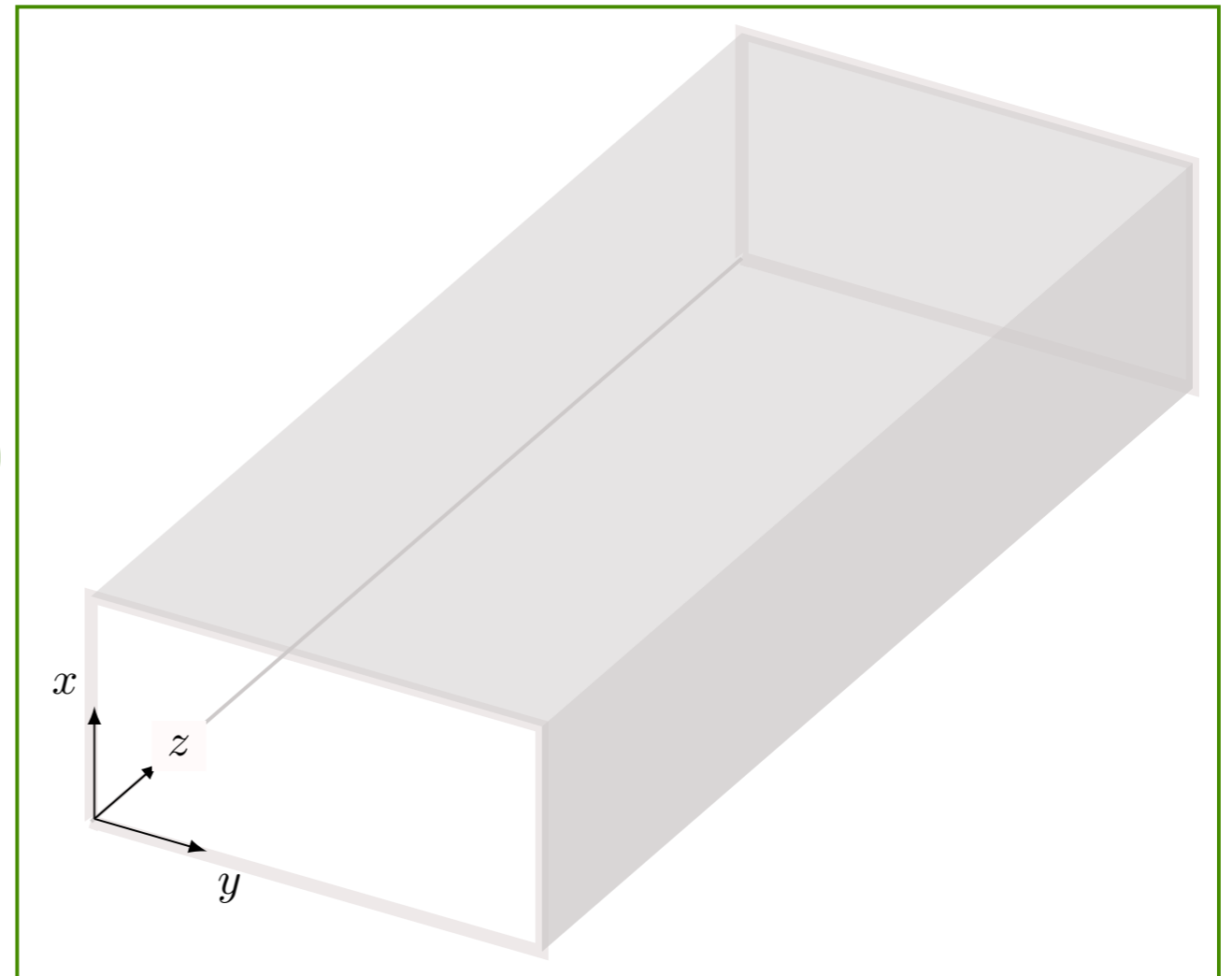
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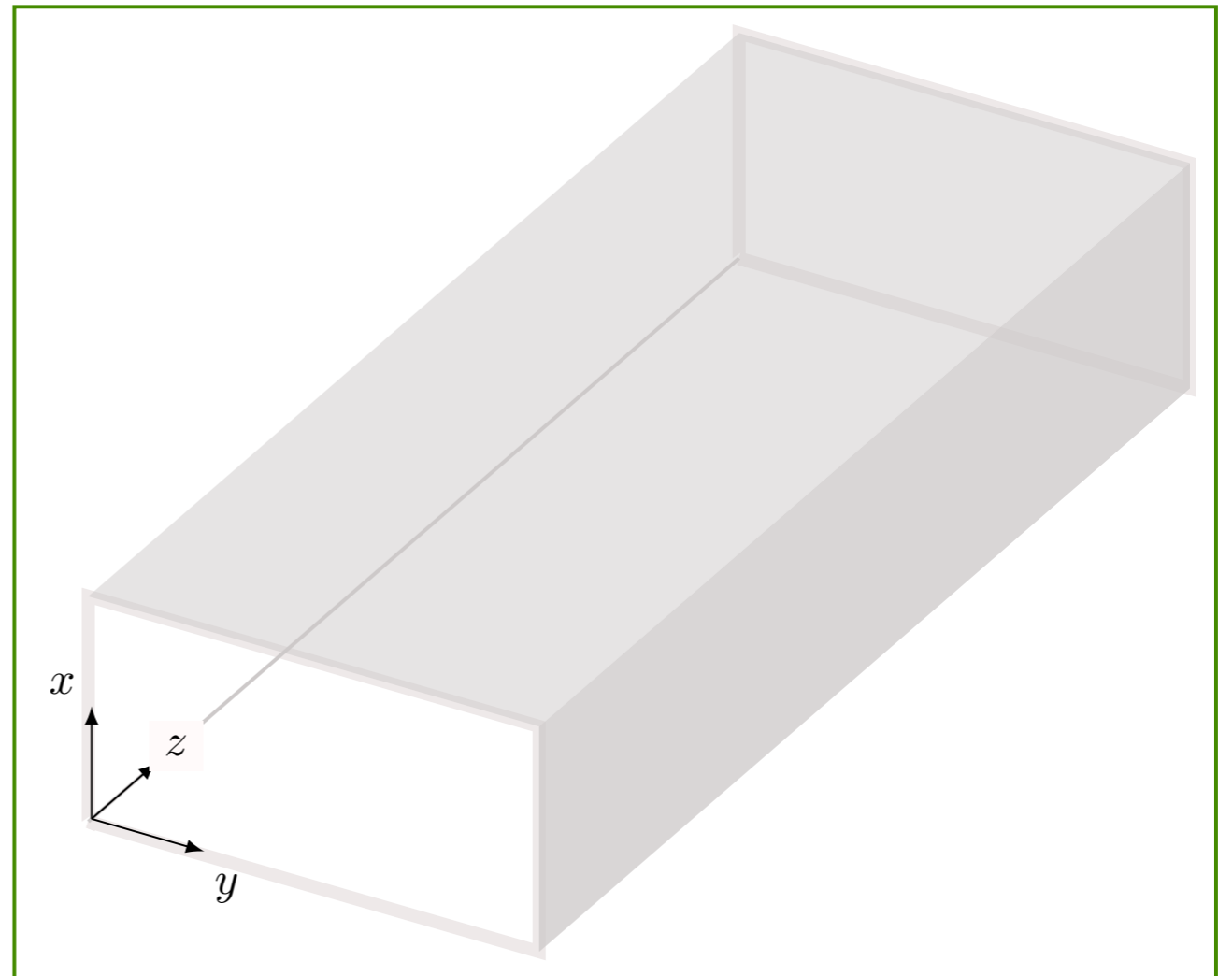
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$$\omega^2 = c^2 \left( k^2 + \left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{a}\right)^2 \right)$$

$$\omega_{mn}^2 = c^2 \left( \left(\frac{\pi m}{b}\right)^2 + \left(\frac{\pi n}{a}\right)^2 \right) \Rightarrow \omega > \omega_{mn}$$

↪ FREQUÊNCIA DE CORTE:  $\omega < \omega_{mn} \Rightarrow$  CAMPO  $\sim e^{-Kz}$  (DECAI)



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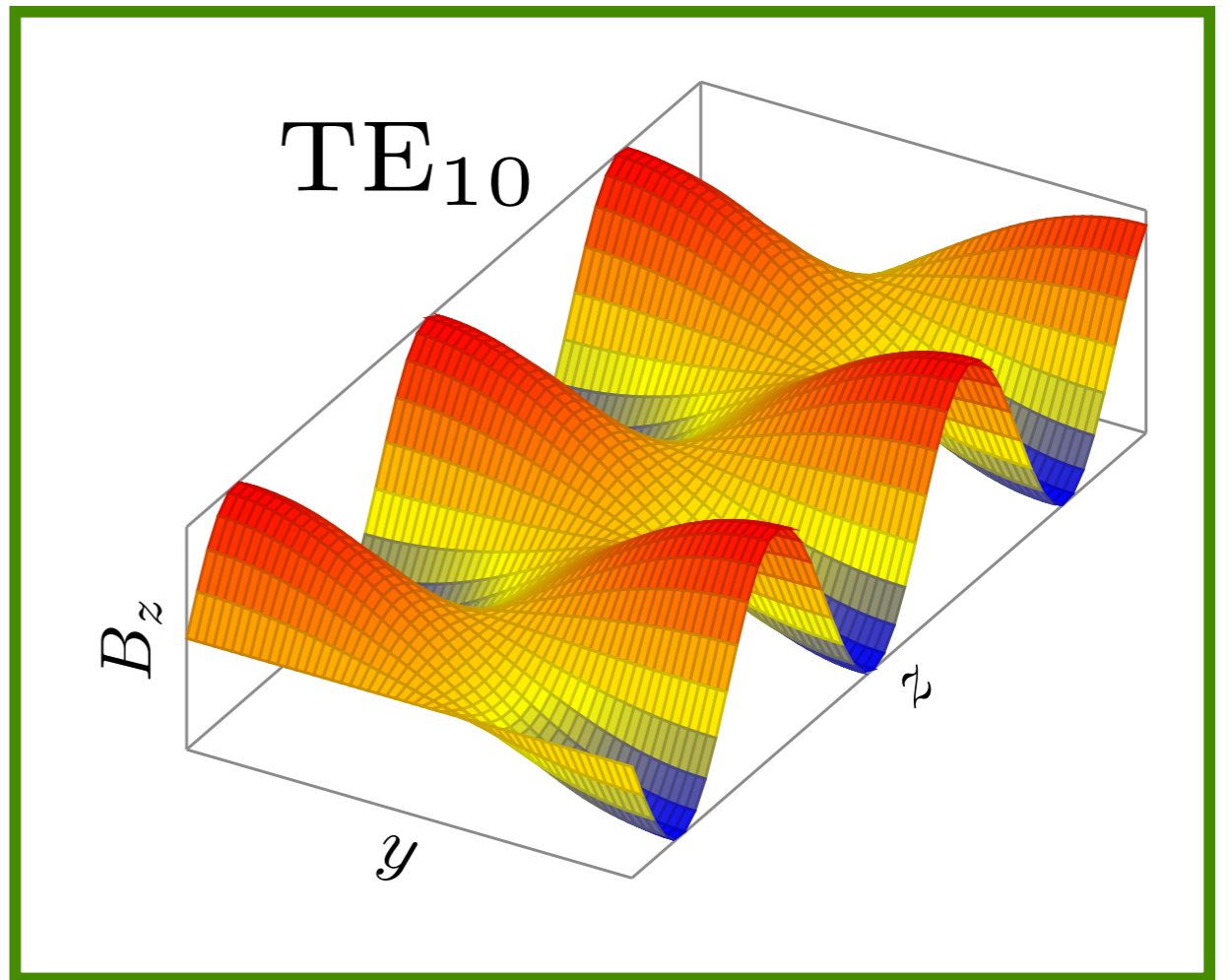
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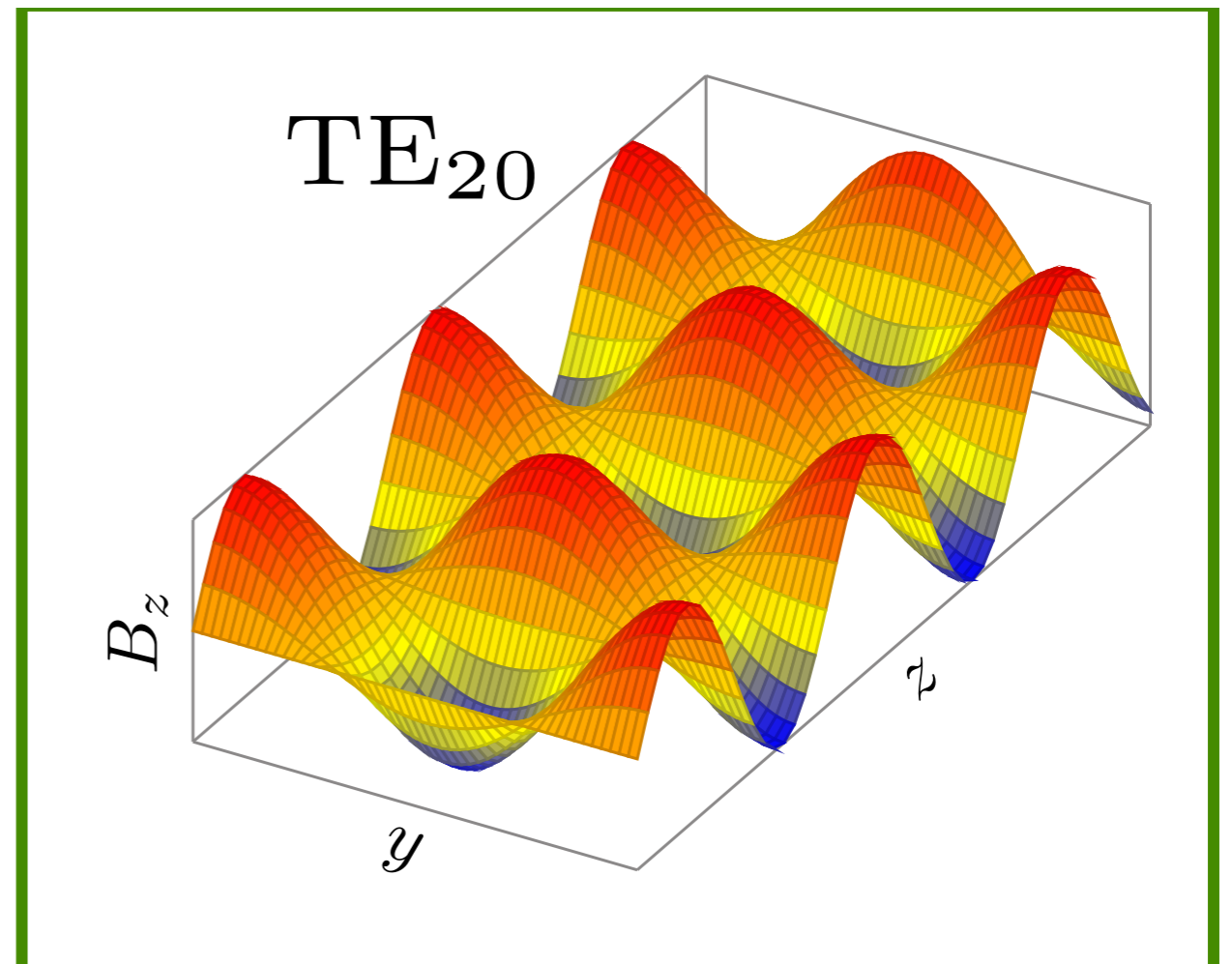
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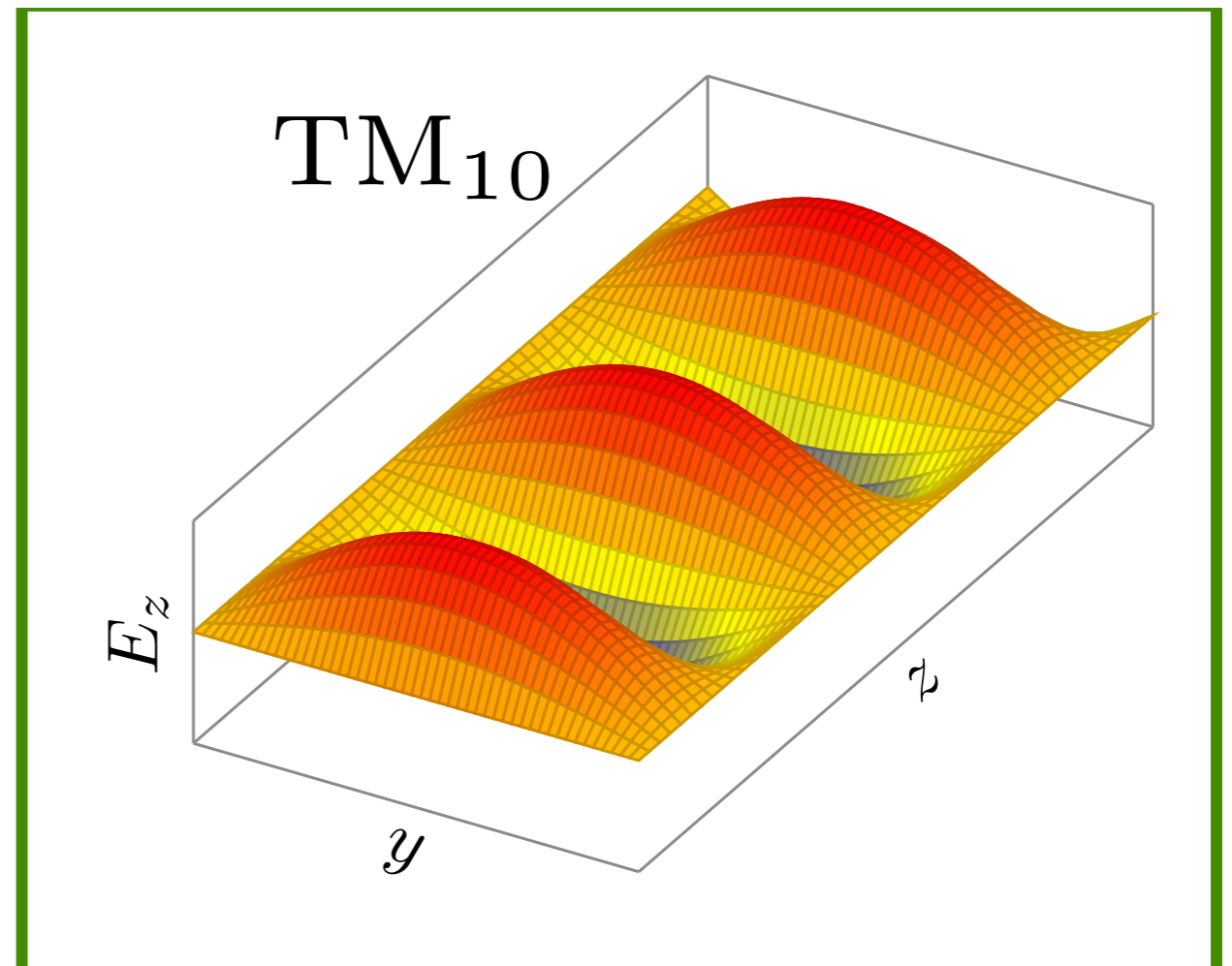
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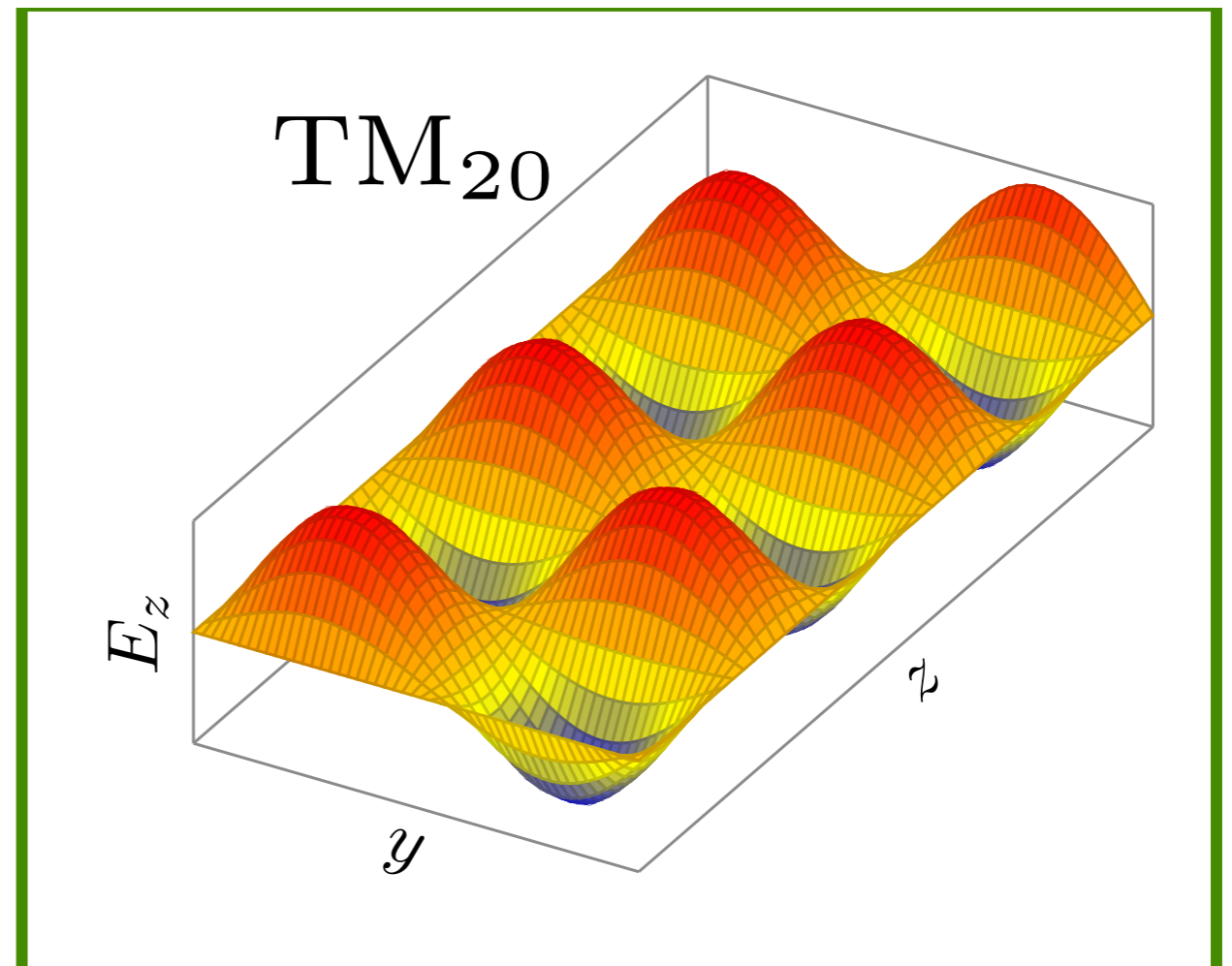
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